Symmetries, Physical Modes and Inflationary Ward Identities

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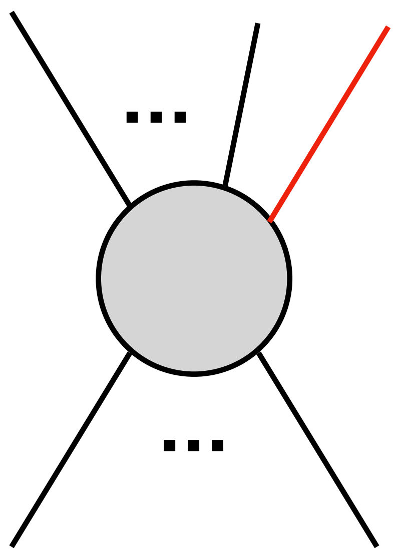
Outline

- Introduction
- Ultra slow roll puzzle
- Toy model illustration
    - Test for known identities
- New identity on initial-late time correlation
    - Test using the toy model
- Physical symmetry and its relation to late time identities
- Comment on inflation

[Lam Hui, Austin Joyce and SW, to appear]

S-Matrix Ward Identities, Soft Theorems



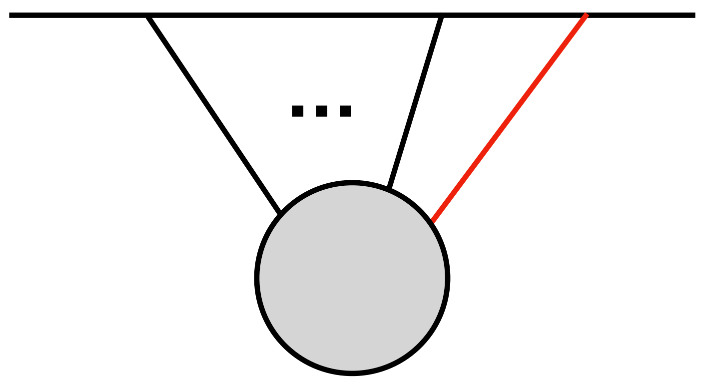
pion, photon, graviton

Weinberg's soft theorem

Soft pion theorem (Adler's zero)

Ward identity, Slavnov Taylor identity

Consistency Relations



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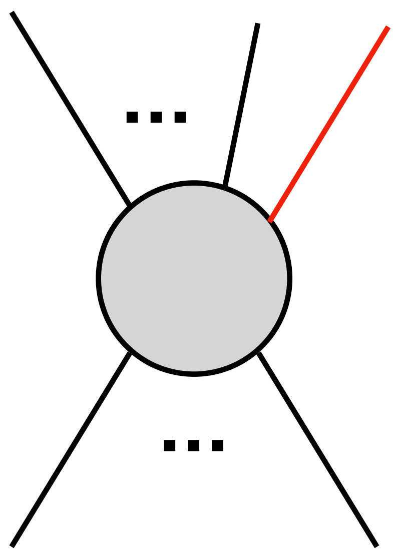

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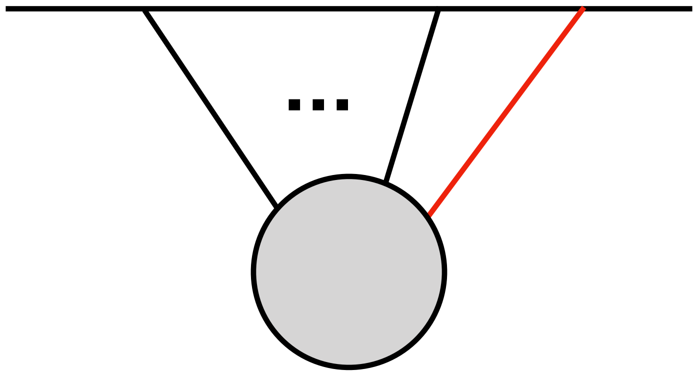
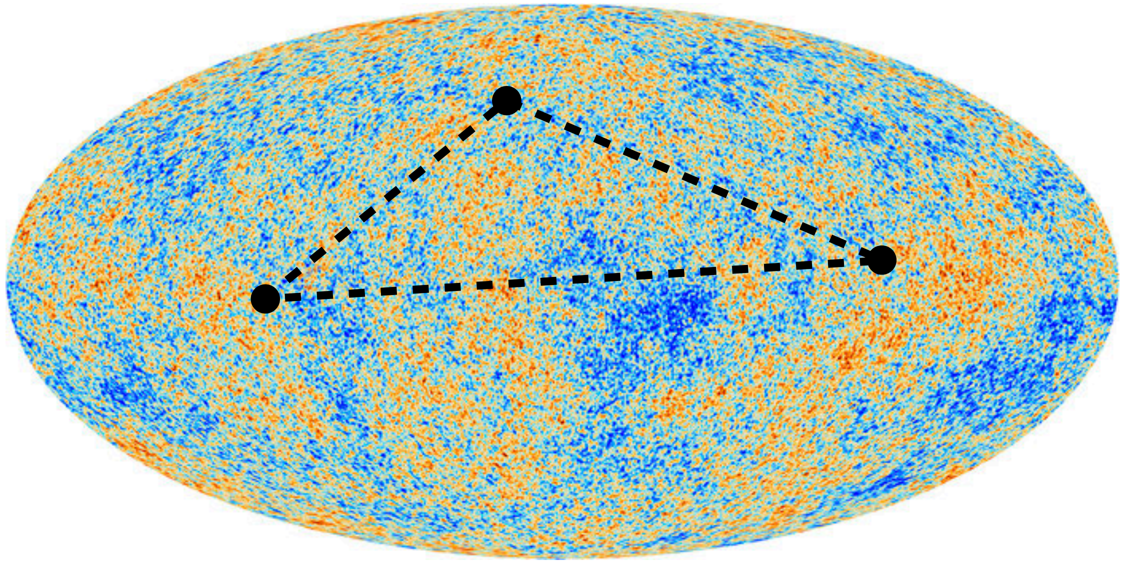


Non-gaussianities in the squeezed limit

$$\lim_{\vec{q}\to 0} \frac{1}{P_\zeta(q)} \langle \zeta_{\vec{q}} \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle' = (1 - n_{\rm S}) P_\zeta(k_1)$$

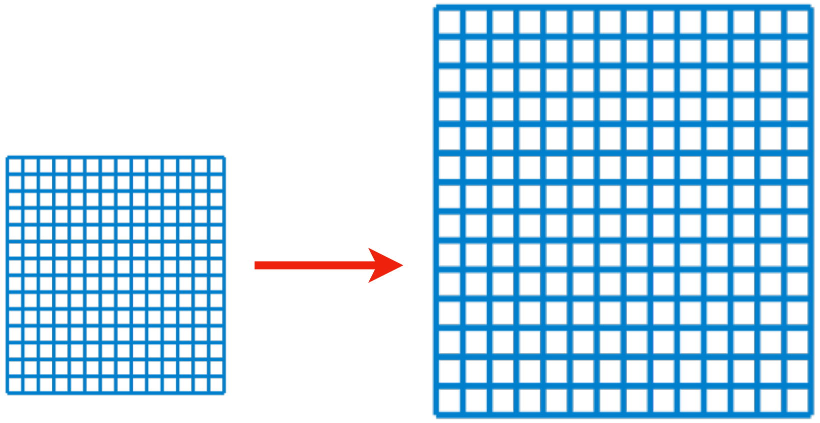


Well Known Consistency Relations

The leading analytic structure of the soft momentum:

$$\frac{1}{P_\zeta(q)}\langle\zeta(\vec{q})\zeta(\vec{k}_1)\zeta(\vec{k}_2)\rangle_t' \stackrel{q\to 0}{=} c_0(k_1,k_2)q^0 + c_1(k_1,k_2)q^1 + \mathcal{O}(q^2)$$

Symmetry: $\delta_D\zeta = 1 + \vec{x}\cdot\vec{\nabla}\zeta$



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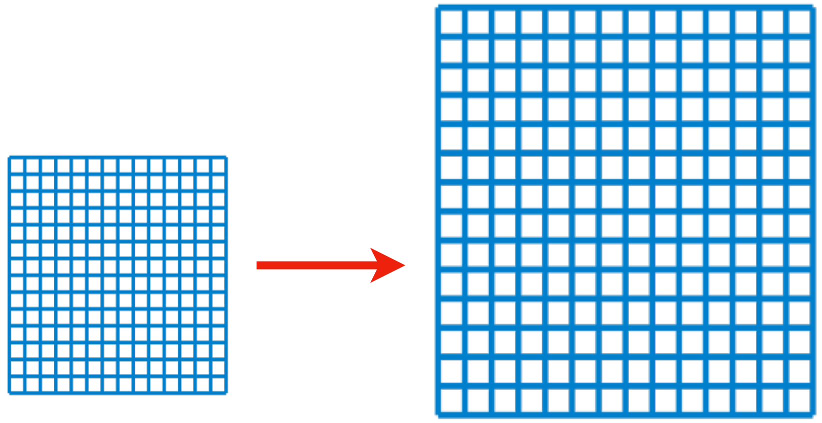

Maldacena's relation for inflation

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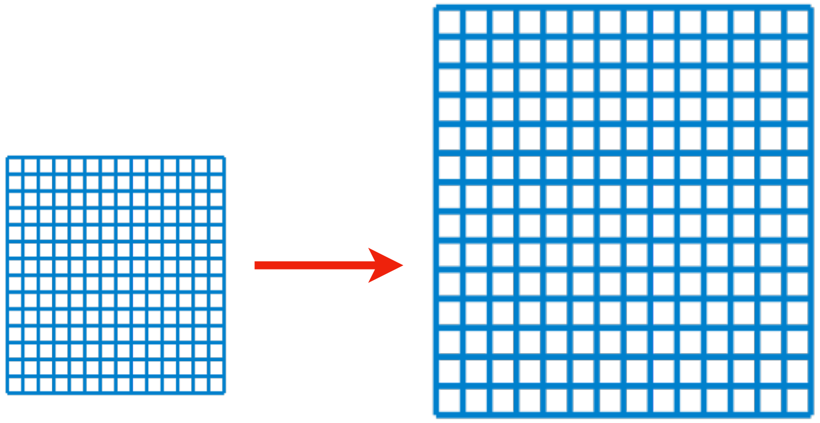

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General form:

$$\lim_{\vec{q}\to 0}\frac{1}{P(q)}\langle\zeta_{\vec{q}}\zeta_{\vec{k}_1}\cdots\zeta_{\vec{k}_N}\rangle' = -\left(3(N-1)+\sum_{a=1}^{N}\vec{k}_a\cdot\vec{\nabla}_{k_a}\right)\langle\zeta_{\vec{k}_1}\cdots\zeta_{\vec{k}_N}\rangle'$$

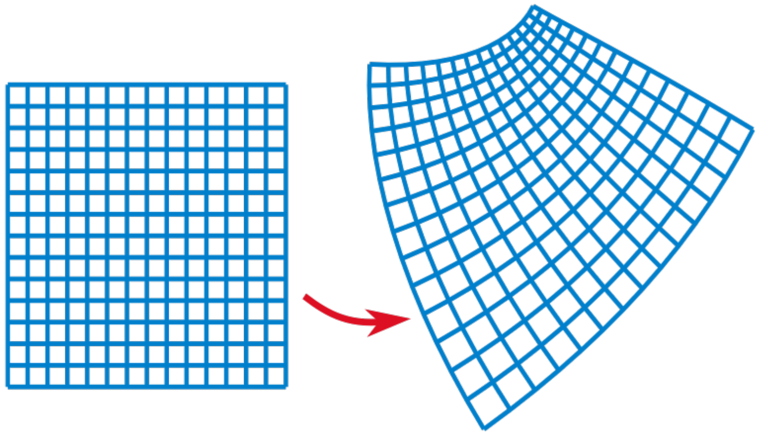
[Assassi, Baumann & Green '12; Hinterbichler, Hui & Khoury, '13]

Well Known Consistency Relations

The leading analytic structure of the soft momentum:

$$\frac{1}{P_\zeta(q)}\langle\zeta(\vec{q})\zeta(\vec{k}_1)\zeta(\vec{k}_2)\rangle'_t \overset{q\to 0}{=} c_0(k_1,k_2)q^0 + c_1(k_1,k_2)q^1 + \mathcal{O}(q^2)$$

Symmetry: $\delta_{K^i}\zeta = 2x^i + \left(2x^i\vec{x}\cdot\vec{\nabla} - x^2\nabla^i\right)\zeta$



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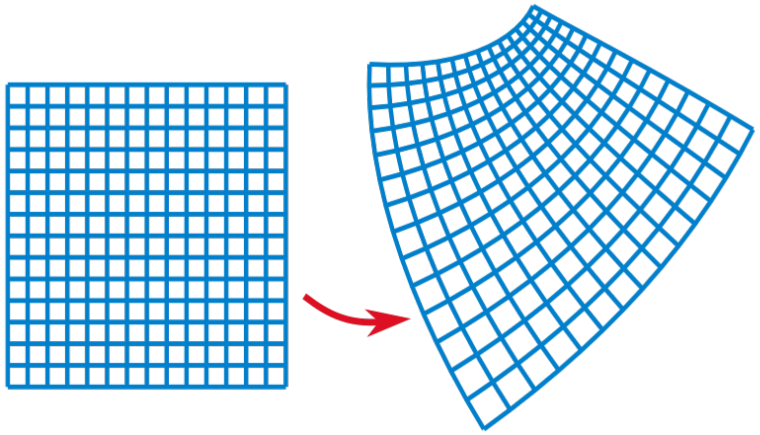

General form:

$$\lim_{\vec{q}\to 0}\frac{\partial}{\partial q^i}\left(\frac{1}{P(q)}\langle\zeta_{\vec{q}}\zeta_{\vec{k}_1}\cdots\zeta_{\vec{k}_N}\rangle'\right) = \frac{1}{2}\sum_{a=1}^{N}\left(-6\nabla^i_{k_a} + k^i_a\nabla^2_{k_a} - 2\vec{k}_a\cdot\vec{\nabla}_{k_a}\nabla^i_{k_a}\right)\langle\zeta_{\vec{k}_1}\cdots\zeta_{\vec{k}_N}\rangle'$$

[Creminelli, Norena & Simonovic '12; Hinterbichler, Hui & Khoury, '13]

A Challenger Comes in...

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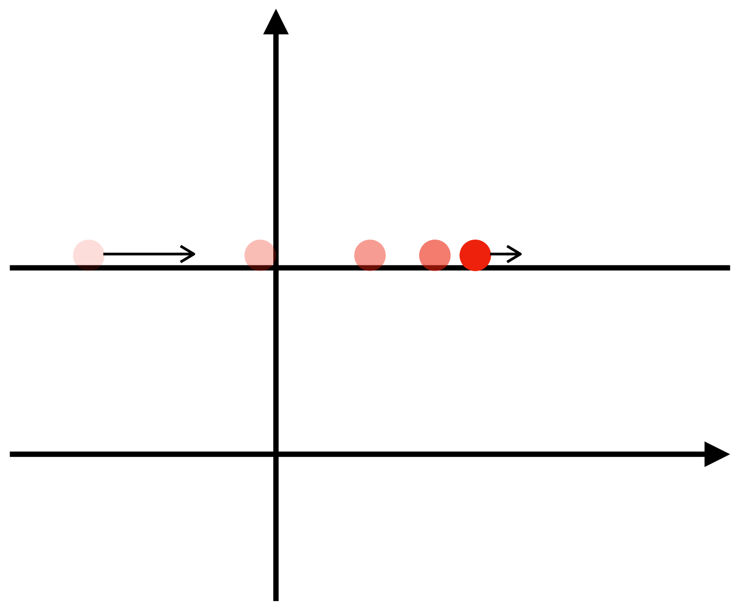

Ultra slow roll: same action as in slow roll

$$\epsilon \approx \frac{\epsilon_0}{a^6}, \quad \eta = -6 + 2\epsilon.$$

Same symmetries

$$\delta_D \zeta = 1 + \vec{x} \cdot \vec{\nabla} \zeta,$$
$$\delta_{K^i} \zeta = 2x^i + \left(2x^i \vec{x} \cdot \vec{\nabla} - x^2 \nabla^i \right) \zeta.$$

$$\lim_{\vec{q} \to 0} \frac{1}{P_\zeta(q)} \langle \zeta_{\vec{q}} \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle' \approx 6 P_\zeta(k_1) \neq (-3 - k_1 \partial_{k_1}) \langle \zeta_{\vec{k}_1} \zeta_{-\vec{k}_1} \rangle'$$

[Namjoo, Firouzjahi & Sasaki '12]

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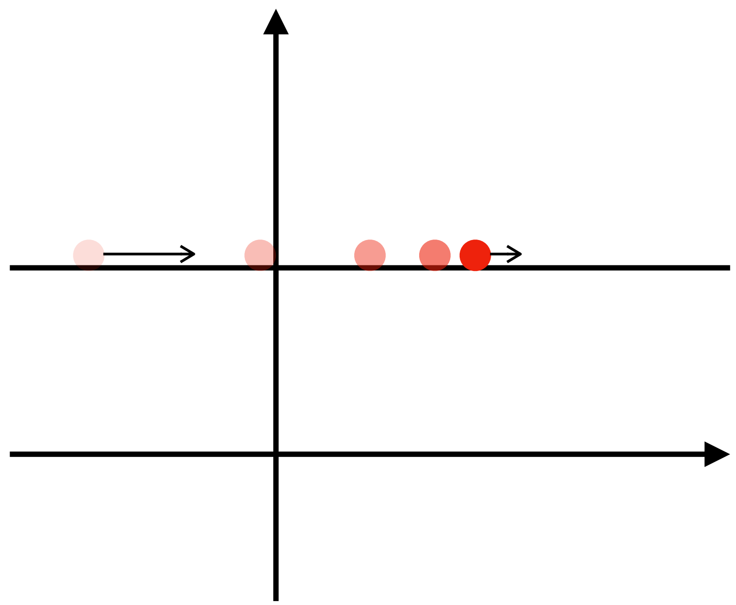

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[Namjoo, Firouzjahi & Sasaki ’12]

Non conservation of ζ outside the horizon $\longrightarrow$ non-local action.

An Illustrative Toy Model

$$
\begin{aligned}
S &= M^2 \int dt d^3x f(t) \left(e^{3\zeta}\dot{\zeta}^2 - e^{\zeta}(\nabla\zeta)^2 \right) \\
&= M^2 \int dt d^3x f(t) \left(\dot{\zeta}^2 - (\nabla\zeta)^2 + 3\zeta\dot{\zeta}^2 - \zeta(\nabla\zeta)^2 + \dots \right)
\end{aligned}
$$

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Flat space: $f(t) = \text{constant}$ $M^2 f(t) \to M_p^2 a^2 \epsilon$

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"Slow roll": $f(t) = \dfrac{1}{(-Ht)^2}, \quad t \in (-\infty, 0)$

$$\delta_T \zeta = 1 - t\partial_t \zeta$$

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$$\delta_T \zeta = \frac{1}{t^3} + \frac{1}{2t^2} \partial_t \zeta$$

Identities Satisfied in The Toy Model

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$$\delta\zeta = \delta_{\rm NL} + \delta_{\rm L}\zeta$$

$$\lim_{\vec{q}\to 0} \hat{\delta}_{\rm NL}(\vec{q}) \left(\frac{1}{P_\zeta(q)} \langle \zeta_{\vec{q}} \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle' \right) = \hat{\delta}_{\rm L} \langle \zeta_{\vec{k}_1} \zeta_{-\vec{k}_1} \rangle'$$

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"Ultra-slow roll": $f(t) = (Ht)^4$	✗	✗	✔
$f(t) = (Ht)^{2n},\ n \leq -1$	✔	✔	NA

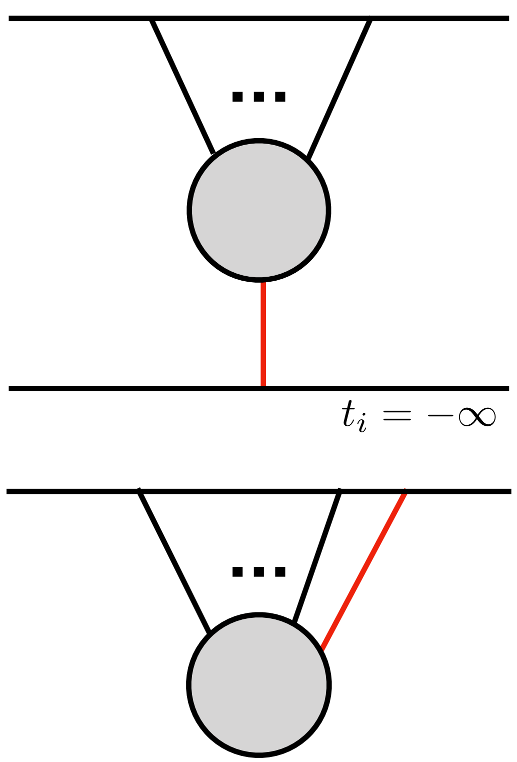
Ward Identity From Path Integral

Initial-late time identity:

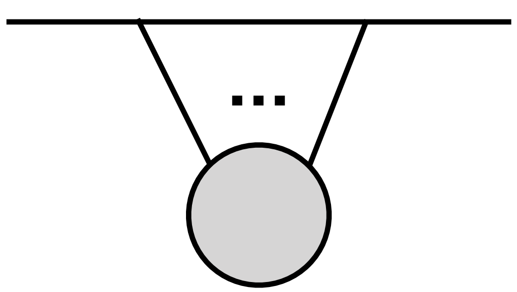
$$\hat{\delta}_{\rm NL}(\vec{q},t_i)\left(G_q\langle\mathcal{O}_{\varphi_f}(\vec{k}_1,\cdots,\vec{k}_N)\varphi_i(\vec{q})\rangle_c'\right)\Big|_{\vec{q}=0}+c.c.=\hat{\delta}_{\rm L}\langle\mathcal{O}_{\varphi_f}(\vec{k}_1,\cdots,\vec{k}_N)\rangle_c'$$

$$\delta\zeta=\delta_{\rm NL}+\delta_{\rm L}\zeta \qquad \Psi_0[\varphi_i]\propto\exp\left(-\frac{1}{2}\int\frac{d^3k}{(2\pi)^3}G_k\varphi_i(\vec{k})\varphi_i(-\vec{k})\right)$$

Spontaneously broken symmetry, change of vacuum under symmetry



Previous identities



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Example: flat space theory with shift symmetry

$$\phi\to\phi+c$$

$$\left(G_q\langle\phi_f(\vec{k}_1)\cdots\phi_f(\vec{k}_N)\phi_i(\vec{q})\rangle_c'\right)\Big|_{\vec{q}=0}+c.c.=0$$

Ward Identity From Path Integral

Initial-late time identity:

$$\hat{\delta}_{\mathrm{NL}}(\vec{q}, t_i) \left(G_q \langle \zeta_f(\vec{k}_1) \zeta_f(\vec{k}_2) \zeta_i(\vec{q}) \rangle' \right) \Big|_{\vec{q}=0} + c.c. = \hat{\delta}_{\mathrm{L}} \langle \zeta_f(\vec{k}_1) \zeta_f(-\vec{k}_1) \rangle'$$

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Observable: equal time correlation

When does it imply the equal time identity?

Physical Modes

Physical Symmetry: The non-linear part of the symmetry $\delta_{\rm NL}$ *must match the time dependence of a physical mode in the* $|\vec{k}| \to 0$ *limit.*

$$\lim_{\vec{k}\to 0} \zeta_k(t) \propto \delta_{\rm NL}(t)$$

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- $\sim$ Weinberg's adiabatic mode
- Not residual gauge symmetry
- Not associated to adiabatic quantities

Equal Time Identities

For any symmetry with non-linear transformation and linear transformation,

$$\delta\zeta = \delta_{\rm NL} + \delta_{\rm L}\zeta$$

the identity

$$\lim_{\vec{q}\to 0} \hat{\delta}_{\rm NL}(\vec{q}) \frac{1}{P_\zeta(q)} \langle \zeta_f(\vec{q}) \zeta_f(\vec{k}_1) \zeta_f(\vec{k}_2) \rangle' = \hat{\delta}_{\rm L} \langle \zeta_f(\vec{k}_1) \zeta_f(-\vec{k}_1) \rangle'$$

holds when it is a physical symmetry.

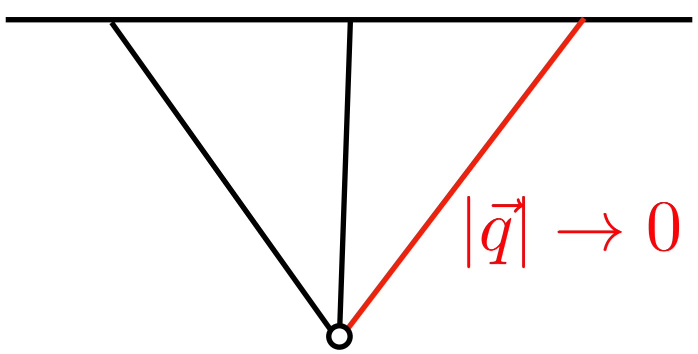
Physical Symmetry: The non-linear part of the symmetry $\delta_{\rm NL}$ must match the time dependence of a physical mode in the $|\vec{k}| \to 0$ limit.

$$\lim_{\vec{k}\to 0} \zeta_k(t) \propto \delta_{\rm NL}(t)$$

Perturbative Proof

$$\delta\zeta = \delta_{\rm NL} + \delta_{\rm L}\zeta$$

$$\delta_{NL}\mathcal{L}_{(n+1)}[\zeta] + \delta_L\mathcal{L}_{(n)}[\zeta] = \partial_\mu K^\mu_{(n)}[\zeta]$$



$\mathcal{L}_{(3)}[u_{k_1}, u_{k_2}, u_q]$

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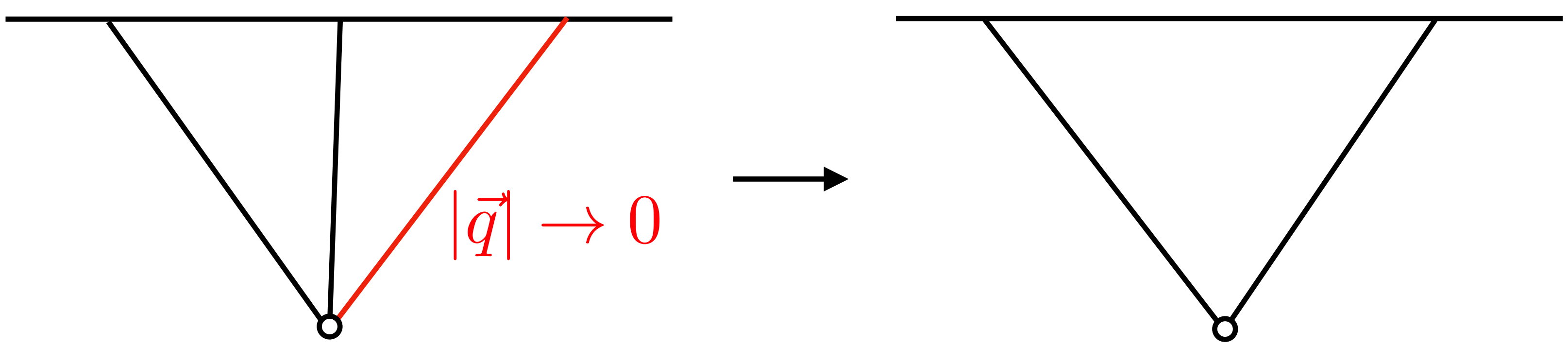

$\mathcal{L}_{(3)}[u_{k_1}, u_{k_2}, u_q]$ $\qquad$ $-\mathcal{L}_{(2)}[\delta_{\mathrm{L}} u_{k_1}, u_{k_2}] + \partial_\mu B^\mu_{(2)}[u_{k_1}, u_{k_2}]$

Physical Symmetry: The non-linear part of the symmetry δ_{NL} must match the time dependence of a physical mode in the $|\vec{k}| \to 0$ limit.

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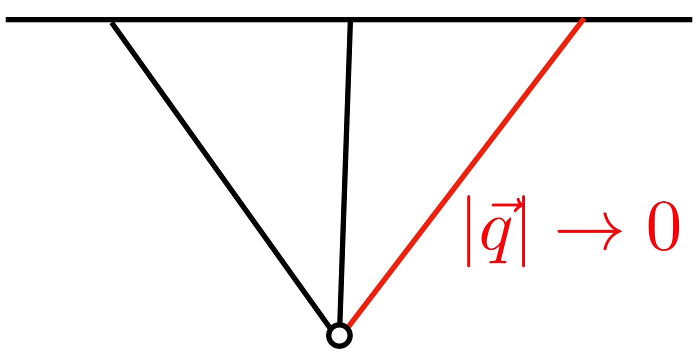
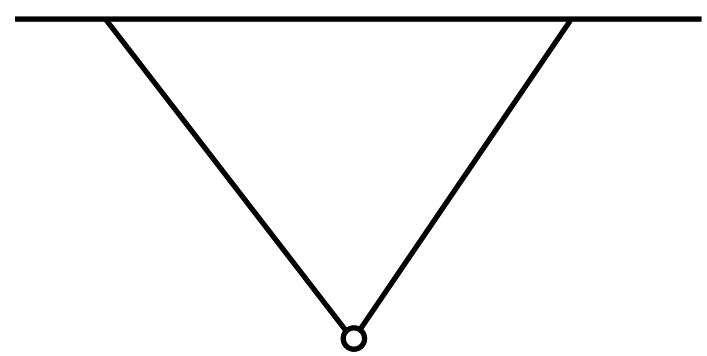


$\longrightarrow$



$\longrightarrow$ $P_\zeta(q)\hat{\delta}_{\rm L}\langle\zeta_f(\vec{k}_1)\zeta_f(-\vec{k}_1)\rangle'$

$\mathcal{L}_{(3)}[u_{k_1}, u_{k_2}, u_q]$

$-\mathcal{L}_{(2)}[\delta_{\rm L}u_{k_1}, u_{k_2}] + \partial_\mu B^\mu_{(2)}[u_{k_1}, u_{k_2}]$

Physical Symmetry: The non-linear part of the symmetry $\delta_{\rm NL}$ must match the time dependence of a physical mode in the $|\vec{k}| \to 0$ limit.

$$\lim_{\vec{k}\to 0}\zeta_k(t) \propto \delta_{\rm NL}(t)$$

Identities Satisfied In the Toy Model

Flat space: $u_k(t) = \frac{1}{\sqrt{4kM}} e^{ikt}$ $\quad\quad$ $\delta_D \zeta = 1 + \vec{x} \cdot \vec{\nabla} \zeta$

"Slow roll": $u_k(t) = \frac{H}{\sqrt{4k^3 M}} (1 - ikt) e^{ikt}$ $\quad\quad$ $\delta_D \zeta = 1 + \vec{x} \cdot \vec{\nabla} \zeta$ $\quad\quad$ $\delta_T \zeta = 1 - t\partial_t \zeta$

"Ultra-slow roll": $u_k(t) = \frac{i}{\sqrt{4k^3 M H^2 t^3}} (1 - ikt) e^{ikt}$ $\quad\quad$ $\delta_T \zeta = \frac{1}{t^3} + \frac{1}{2t^2} \partial_t \zeta$

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"Ultra-slow roll": $f(t) = (Ht)^4$	✗	✗	✔
$f(t) = (Ht)^{2n}, \ n \leq -1$	✔	✔	NA

Comment on Inflation

- Slow roll:

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should work even at finite time if ζ approaches a constant in the zero momentum limit

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should work even at finite time if ζ approaches a constant in the zero momentum limit

- Ultra slow roll: $\delta\Phi = c$

Physical symmetry: $\delta\zeta_k = (2\pi)^3 \delta(\vec{k}) \left(\frac{H}{\dot{\Phi}} + \frac{1}{2} \int^t dt' \dot{\Phi} \right) + \frac{1}{a\dot{\Phi}} \zeta_k' - \frac{1}{2} \left(\int^t dt' \dot{\Phi} \right) (3 + \vec{k} \cdot \partial_{\vec{k}}) \zeta_k$

Identity:

$$\lim_{\vec{q}\to 0} \left(\frac{H}{\dot{\Phi}} + \frac{1}{2} \int^t dt' \dot{\Phi} \right) \frac{1}{P_\zeta(q)} \langle \zeta_f(\vec{q}) \zeta_f(\vec{k}_1) \zeta_f(\vec{k}_2) \rangle'$$
$$= \frac{1}{a\dot{\Phi}} \frac{\partial}{\partial \tau} \langle \zeta_f(\vec{k}_1) \zeta_f(-\vec{k}_1) \rangle' - \frac{1}{2} \left(\int^t dt' \dot{\Phi} \right) (3 + k_1 \partial_{k_1}) \langle \zeta_f(\vec{k}_1) \zeta_f(-\vec{k}_1) \rangle'$$

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- Physical mode —> late time consistency relation!
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- Adiabatic mode (physical mode) in gauge theory, especially gravity?
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- Non-perturbative proof for the use of physical symmetry???

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Thank You!!