Angular Inflation in α-attractors

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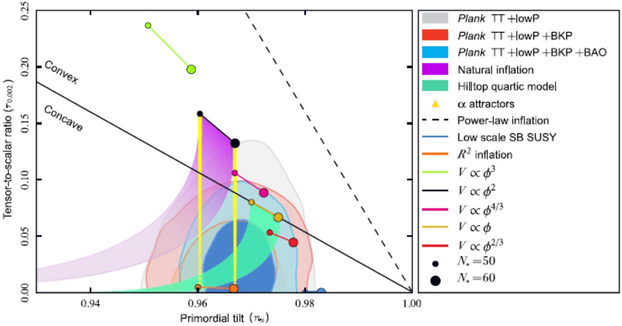
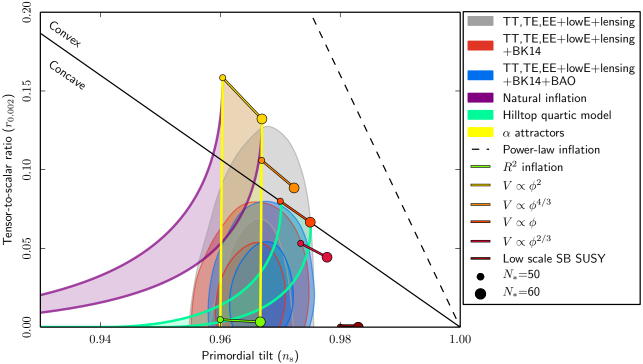
with Perseas Christodoulidis & Diederik Roest
arXiv:1803.09841 [hep-th]

Hints from the sky



Plateau models of inflation are consistent with *Planck* data.

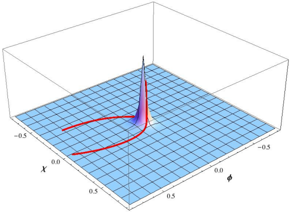
Hints from the sky



Plateau models of inflation are STILL consistent with *Planck* data,

$\Rightarrow$ the time of horizon-exit is being constrained.

Field-space effects to the rescue



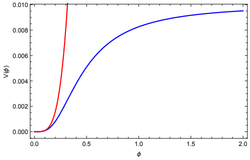
Non-minimal coupling to gravity:

$$\mathcal{L} \subset \xi \phi^2 R$$

Example: Higgs inflation

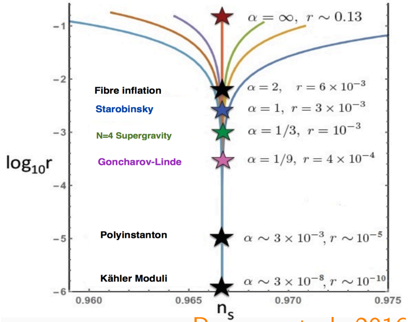
The conformal transformation from the Jordan to the Einstein frame leads to

$$\tilde{V} \sim \lambda \phi^4 \rightarrow V \sim \frac{\lambda}{\xi^2}\left(1 - \frac{2}{\xi \phi^2}\right)$$



The predictions are those of the Starobinsky model $\mathcal{L} = R + \xi R^2$

$$n_s \simeq 1 - \frac{2}{N}, \quad r \simeq \frac{12}{N^2}$$



Burgess et al. 2016

- String theory compactification: Fibre inflation
- Supergravity: T-model & E-model

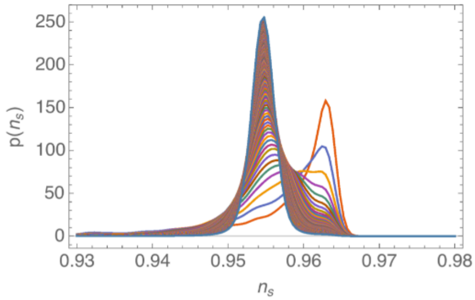
all lead to a **hyperbolic field-space**. **Canonically normalizing** the inflaton leads to a potential

$$V \sim V_0 \left(1 - e^{-\#\phi}\right)$$

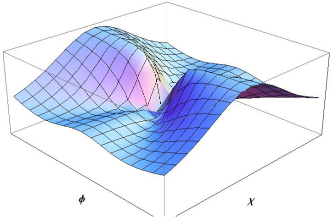
and the "universal" predictions $\boxed{n_s \simeq 1 - \frac{2}{N}, \quad r \simeq \frac{12\alpha}{N^2}}$

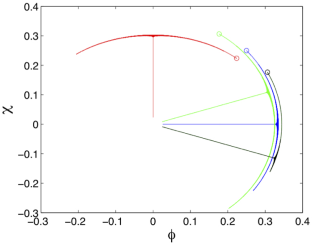
Inflation and multiple fields

Only one scalar field at high energies is rather unlikely.
How does the forest of models cope with many scalar fields?



Easther, Frazer, Peiris, Price





- Simple models shift n_s predictions ↑↑
- Non-minimal couplings lead to strong single-field attractors $\longrightarrow \longrightarrow \longrightarrow$

Kaiser & EIS 2014

Multi-field α-attractors

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2} \mathcal{G}_{IJ} g^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J - V(\phi^I) \right] .$$

with background quantities

$$H^2 = \frac{1}{3M_{\rm Pl}^2} \left[\frac{1}{2} \mathcal{G}_{IJ} \dot{\phi}^I \dot{\phi}^J + V(\phi^I) \right] , \qquad \dot{H} = -\frac{1}{2M_{\rm Pl}^2} \mathcal{G}_{IJ} \dot{\phi}^I \dot{\phi}^J .$$

and equations of motion for the fields

$$\mathcal{D}_t \dot{\varphi}^I + 3H \dot{\varphi}^I + \mathcal{G}^{IJ} V_{,J} = 0 ,$$

The field-space has a constant curvature

$$\boxed{\mathcal{R} = -\frac{4}{3\alpha}}$$

$\mathcal{G}_{IJ}$ is defined in equivalent ways. We use **Poincare disk** variables:

$$\mathcal{G}_{IJ} = \frac{6\alpha}{(1 - \phi^2 - \chi^2)^2} \delta_{IJ}$$

Model and conventions

- On the Poincare disk $-1 < \{\phi, \chi\} < 1$, the two Cartesian fields are equivalent.
- We use the transformation

$$\psi = \sqrt{6\alpha}\,\tanh^{-1}(r)$$

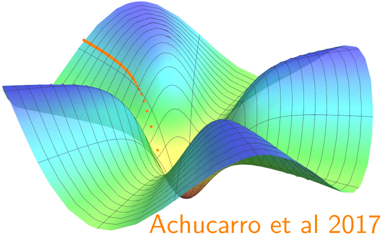
where $0 < \psi < \infty$ for $r = \phi^2 + \chi^2$ for visualization only.

- We use the simplest quadratic potential

$$V(\phi, \chi) = \frac{\alpha}{2}\left(m_\phi^2 \phi^2 + m_\chi^2 \chi^2\right)$$

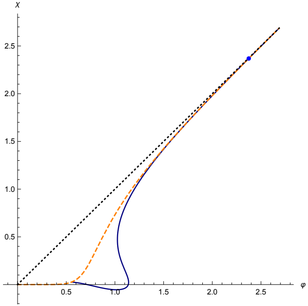
with the mass hierarchy parameter $R \equiv \frac{m_\chi^2}{m_\phi^2} > 1$.

Multi-field α-attractors



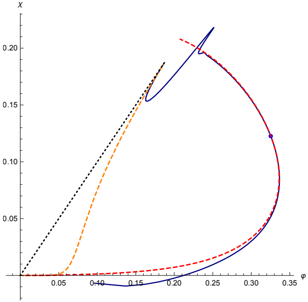
Achucarro et al 2017

For mild fild-space curvature $\alpha = \mathcal{O}(1)$, the fields perform a slow-turn motion along the potential ridge
$\Rightarrow$ **the same CMB predictions**.



$$\leftarrow \left\{ \begin{array}{c} R = 10 \\ \alpha = 1/6 \end{array} \right\}$$

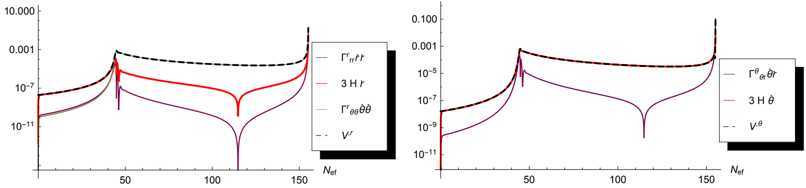
$$\left\{ \begin{array}{c} R = 10 \\ \alpha = 1/600 \end{array} \right\} \rightarrow$$



A new curvature-supported regime

$$\ddot{r} + \frac{2r\dot{r}^2}{1-r^2} - \frac{r(r^2+1)}{1-r^2}\dot{\theta}^2 + 3H\dot{r} + m_\phi^2(1-r^2)^2 r \frac{f(R,\theta)}{2} = 0\,,$$

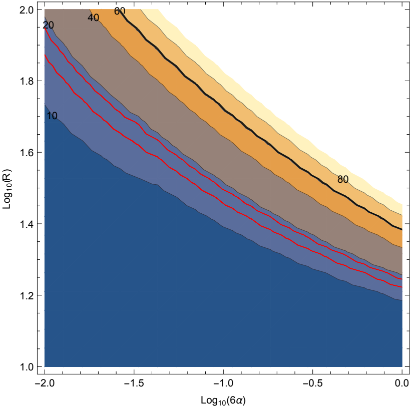
where $f(R,\theta) = 1 + R + (1-R)\cos(2\theta)$

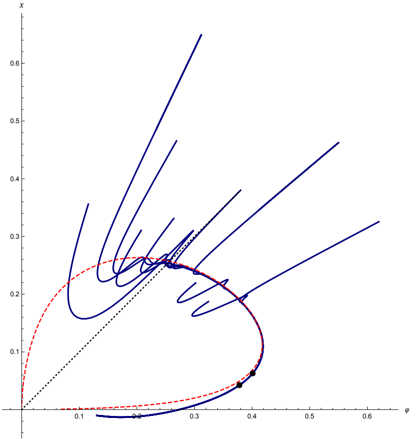


$$\ddot{\theta} + \frac{2(1+r^2)}{r(1-r^2)}\dot{r}\dot{\theta} + 3H\dot{\theta} + \frac{m_\phi^2}{2}(R-1)\left(1-r^2\right)^2\sin(2\theta) = 0\,,$$

Analytical progress can be made for the trajectory $r(\theta)$

$$1 - r^2 \simeq \frac{9\alpha(\cot^2\theta + R\tan^2\theta)^2}{2(R-1)^2}$$





and the number of e-folds N

$$N \simeq \frac{R}{27\alpha} + \mathcal{O}\left(\frac{\log R}{\alpha}\right) + \ldots$$

Fluctuations

$$\mathcal{D}_t^2 Q^I + 3H\mathcal{D}_t Q^I + \left[\frac{k^2}{a^2}\delta^I{}_J + \mathcal{M}^I{}_J - \frac{1}{M_{\rm Pl}^2 a^3}\mathcal{D}_t\left(\frac{a^3}{H}\dot\phi^I\dot\phi_J\right)\right] Q^J = 0$$

we can define

$$\dot\sigma \equiv |\dot\varphi^I| = \sqrt{\mathcal{G}_{IJ}\dot\varphi^I\dot\varphi^J}\,, \quad \hat\sigma^I \equiv \frac{\dot\varphi^I}{\dot\sigma}\,, \quad \omega^I = \mathcal{D}_t\hat\sigma^I\,, \quad \hat s^I \equiv \frac{\omega^I}{\omega}\,.$$

Decompose perturbations:

$$\boxed{Q_\sigma \equiv \hat\sigma_I Q^I} \quad , \quad \boxed{Q_s \equiv \hat s_I Q^I}$$

Define the entropic mass:

$$\mu_s^2 = \mathcal{M}_{ss} + 3\omega^2$$

where $\mathcal{M}_{ss} = \hat s_I \hat s^J \mathcal{M}^I{}_J$

and $\eta_{\sigma\sigma} \equiv \frac{\mathcal{M}_{\sigma\sigma}}{V}\,, \quad \eta_{ss} \equiv \frac{\mathcal{M}_{ss}}{V}$

Adiabatic & entropy perturbations

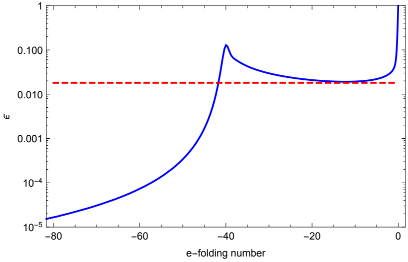
$$\boxed{R_c = \frac{H}{\dot\sigma}Q_\sigma} \quad , \quad \boxed{S = \frac{H}{\dot\sigma}Q_s}$$

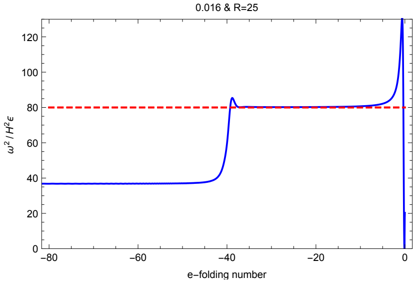
in the super-horizon limit

$$\dot R_c \simeq \alpha H S \quad , \quad \dot S \simeq \beta H S$$

$$\alpha = \frac{2\omega}{H} \quad , \quad \beta = -2\epsilon - \eta_{ss} + \eta_{\sigma\sigma} - \frac{4\omega^2}{3H^2}$$

Slow-roll quantities





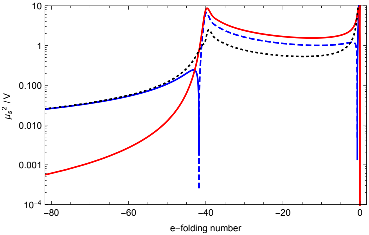
$$\epsilon \gtrsim \frac{27\alpha}{R} \sim \frac{1}{N_{\text{ang.}}}$$

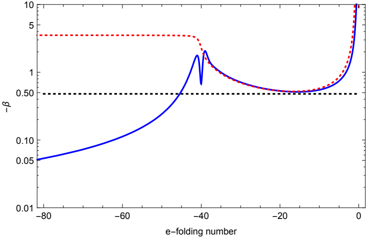
compare to result for radial α-attractor inflation $\epsilon \sim \frac{1}{N^2}$

$$\frac{\omega^2}{H^2} \simeq \frac{4}{3\alpha}\epsilon = \mathcal{R} \times \epsilon$$

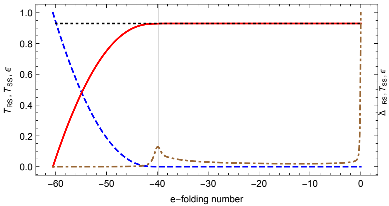
Angular inflation is generically **not** slow-turn

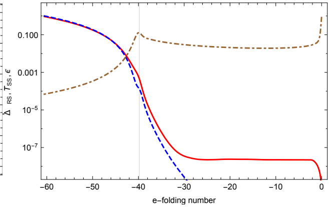
Isocurvature mode is **heavy and decays** with $-\beta \geq 12/R$



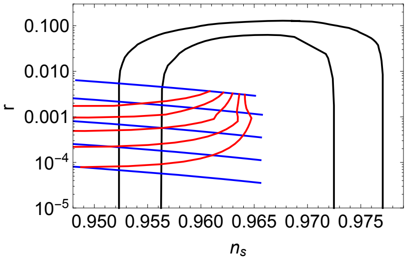


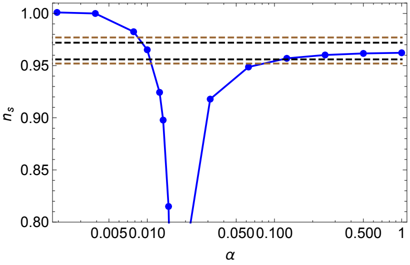
$T_{RS} \to 0\,,\; T_{RS} \to \text{const.} \Rightarrow$ **no power transfer** to adiabatic modes

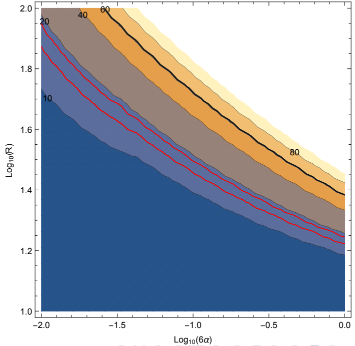




Observables







$$n_s = 1 - \frac{2}{N - N(\theta_0)}$$

$$r = \frac{12\alpha}{(N - N(\theta_0))^2}$$

Summary

The good:

- α-attractors and related models are excellent candidates for explaining CMB data
- Several constructions of α-attractors follow from String Theory or Supersymmetry
- So far α-attractors have lead to very robust predictions

The bad:

- A new attractor was found, leading to shifting predictions for CMB observables

The useful:

- Shifted predictions can **bound field-space curvature** and **constrain high energy theories**

A systematic study of α-attractors in view of **angular inflation** is essential, anticipating new data

Thank you . . .

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Equations of motion

Backrgound fields:

$$\mathcal{D}_t\dot{\phi}^I + 3H\dot{\phi}^I + \mathcal{G}^{IK}V_{,K} = 0$$

Perturbations $Q^I = \delta\phi^I + \frac{\dot{\phi}^I}{H}\psi$:

$$\mathcal{D}_t^2 Q_k^I + 3H\mathcal{D}_t Q_k^I + \left[\frac{k^2}{a^2}\delta^I_J + \mathcal{M}^I_J - \frac{1}{M_{Pl}^2 a^3}\mathcal{D}_t\left(\frac{a^3}{H}\dot{\phi}^I\dot{\phi}_J\right)\right] Q_k^I = 0$$

where

$$\mathcal{M}^I_J = \mathcal{G}^{IK}\mathcal{D}_J\mathcal{D}_K V - \mathcal{R}^I_{LMJ}\dot{\phi}^L\dot{\phi}^M$$

Fluctuation decomposition

Fluctuations along the background trajectory (adiabatic)

$$\ddot{Q}_\sigma + 3H\dot{Q}_\sigma + \left[\frac{k^2}{a^2} + \mathcal{M}_{\sigma\sigma} - \omega^2 - \frac{1}{M_{\rm pl}^2 a^3}\frac{d}{dt}\left(\frac{a^3\dot{\sigma}^2}{H}\right)\right] Q_\sigma$$
$$= 2\,\frac{d}{dt}\left(\omega\, Q_s\right) - 2\left(\frac{V_{,\sigma}}{\dot{\sigma}} + \frac{\dot{H}}{H}\right)\omega\, Q_s$$

and perpendicular to it (isocurvature)

$$\ddot{Q}_s + 3H\dot{Q}_s + \left[\frac{k^2}{a^2} + \mathcal{M}_{ss} + 3\omega^2\right] Q_s = 4M_{\rm pl}^2\,\frac{\omega}{\dot{\sigma}}\frac{k^2}{a^2}\Psi$$

Numerics

We checked the simple relation

$$n_s = 1 - \frac{2}{N - N(\theta_0)}$$

using **mTransport**, by Dias, Frazer & Seery, finding excellent agreement.