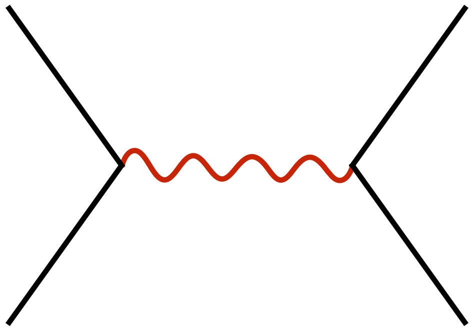
Inflationary Correlators from Symmetries and Singularities



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This talk is based on work in progress with

Nima Arkani-Hamed, Daniel Baumann, and Gui Pimentel

as well as earlier works

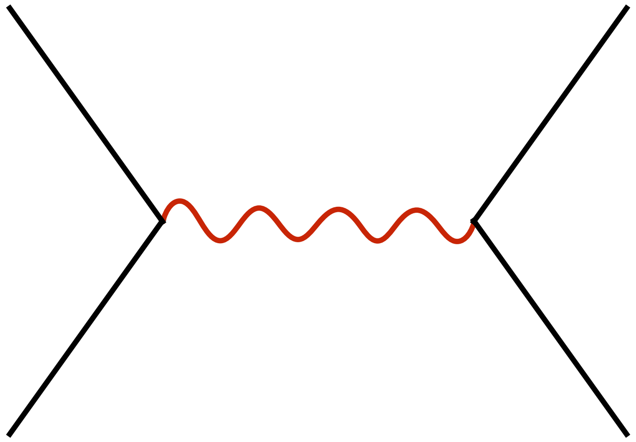
HL, Baumann, Pimentel (JHEP 1607.03735)

Baumann, Goon, HL, Pimentel (JHEP 1712.06624)

Moradinezhad, HL, Muñoz, Dvorkin (JCAP 1801.07265)

See also talks by Daniel Baumann and Yi Wang.

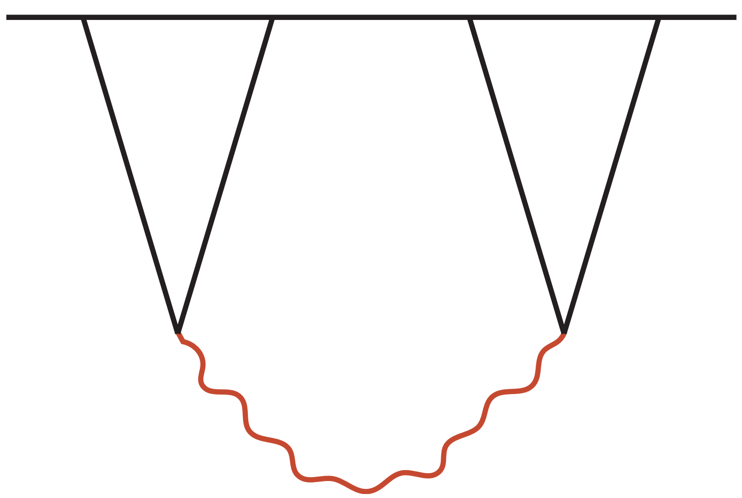
In flat space, *symmetries* and *singularities* completely fix the structure of scattering amplitudes at tree level.



$$\longrightarrow \quad g^2\,\frac{P_S\left(1+\frac{2t}{M^2-4m^2}\right)}{s-M^2}$$

No Lagrangian description is needed to get the answer!

In slow-roll inflation, boundary correlators are controlled by *(weakly broken) conformal symmetry*.



$$\longrightarrow \qquad ?$$

What is the analytic structure of these correlators?

de Sitter 4-point function

Arkani-Hamed, Baumann, HL, Pimentel (in progress)

Kinematics of the de Sitter four-point function:

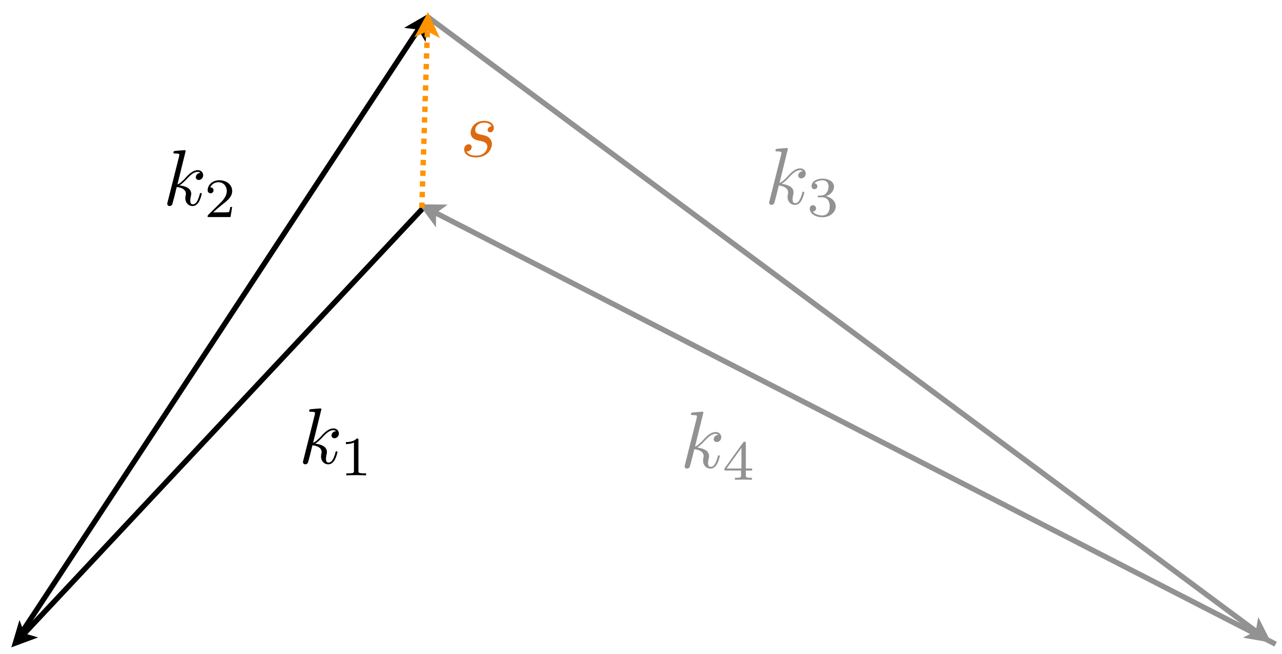
$$\langle \phi_{\vec{k}_1} \cdots \phi_{\vec{k}_4} \rangle = F(\underbrace{k_1, \cdots, k_4, s, t}_{\text{isotropy}})\, \delta^{(3)}(\underbrace{\vec{k}_1 + \cdots + \vec{k}_4}_{\text{translation}})$$

Define dimensionless variables to make *dilatation* manifest.

$$u = \frac{s}{k_1 + k_2}$$

$$v = \frac{s}{k_3 + k_4}$$

$$\hat{F} = s^{\#} F(u, v, \cdots)$$

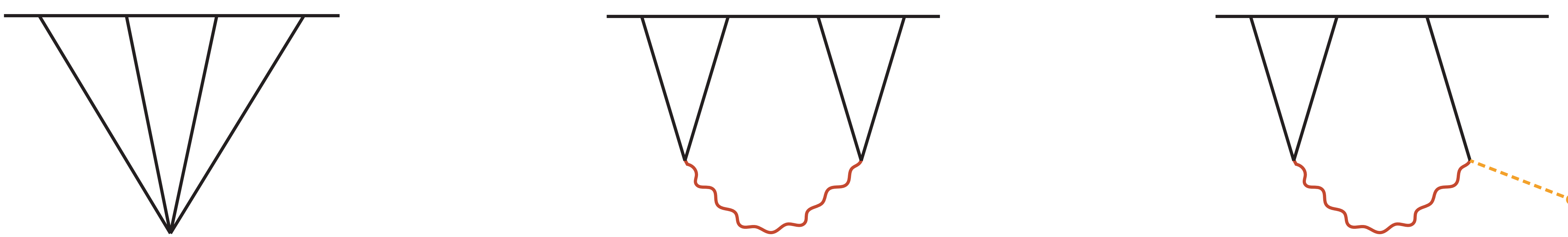


Invariance under *special conformal transformations* implies

$$(\Delta_u - \Delta_v)\hat{F}(u,v) = 0\,, \qquad \Delta_u = u^2(1-u^2)\partial_u^2 - 2u^3\partial_u$$

We have classified the solutions to this equation:

Contact $\longrightarrow$ Exchange $\longrightarrow$ Inflationary 3-pt



Contact diagrams carry the *simplest possible* analytic structure:

$$\hat{C} = \Delta_u^n \left(\frac{uv}{u+v} \right) = \Delta_u^n \left(\frac{1}{\sum k_i} \right)$$

These act as the source for the exchange diagrams:

$$\left[\Delta_u + \left(\tfrac{1}{4} + \mu^2\right)\right] \hat{F} = \hat{C} \quad \xrightarrow{\mu \to \infty} \quad \hat{F} = \mu^{-2} \hat{C}$$

Removing *unphysical singularities* uniquely fixes the solution.

For small u, the solution is

$$F = \sum_n \frac{(-1)^n}{(n+\frac{1}{2})^2 + \mu^2} \left(\frac{u}{v}\right)^n + \frac{\pi}{\cosh \pi\mu} \frac{\sin(\mu \log u/v)}{\mu}$$

↑ EFT expansion

↑ non-perturbative correction $\sim e^{-\pi\mu}$

The general solution has an analogous structure.

Exchange of spinning particles

Arkani-Hamed, Baumann, HL, Pimentel (in progress)

For spin exchange, we expand in the *polarization basis*.

$$\hat{F}_S = \sum_{\lambda=0}^{S} \hat{A}_\lambda(u,v)\,\Pi_\lambda(\text{angles})$$

The coefficient functions are related by a simple *ladder operator*.

$$\hat{A}_S = \underbrace{(uv)^2\partial_u\partial_v}_{=D_{uv}}\,\hat{A}_{S-1}\,, \qquad \hat{A}_0 = \hat{F}_{S=0}$$

i.e. all solutions follow from the *scalar-exchange* diagram!

Spin 1:

$$\hat{F}_1 = \left[\Pi_1 D_{uv} + \Pi_0 \Delta_u\right] \hat{F}_0$$

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Spin 2:

$$\hat{F}_2 = \left[\Pi_2 D_{uv}^2 + \Pi_1 D_{uv}(\Delta_u - 2) + \Pi_0 \Delta_u (\Delta_u - 2)\right] \hat{F}_0$$

Spin 1:

$$\hat{F}_1 = \left[\Pi_1 D_{uv} + \Pi_0 \Delta_u\right] \hat{F}_0$$

Spin 2:

$$\hat{F}_2 = \left[\Pi_2 D_{uv}^2 + \Pi_1 D_{uv}(\Delta_u - 2) + \Pi_0 \Delta_u(\Delta_u - 2)\right] \hat{F}_0$$

For general spin, the coefficients are determined by

$$\hat{A}_\lambda = D_{uv}^\lambda \prod_{j=1}^{S-\lambda} \big(\Delta_u - (S-j)(S-j+1)\big) \hat{F}_0$$

Inflationary 3-point function

Arkani-Hamed, Baumann, HL, Pimentel (in progress)

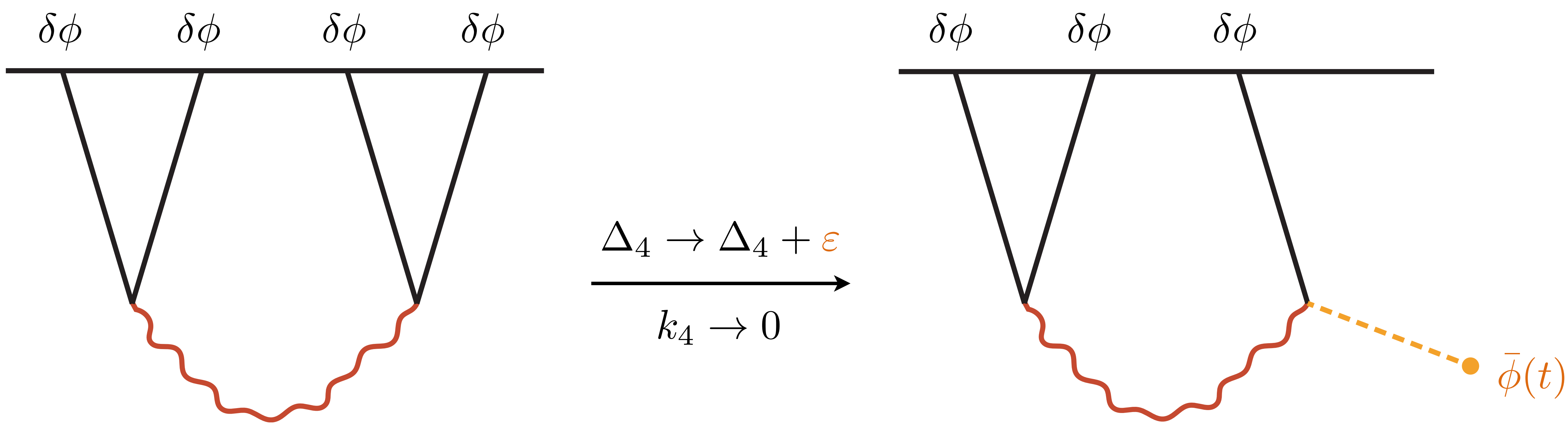
A bulk field has a characteristic conformal weight:

$$\Delta = \frac{3}{2} + \underbrace{\sqrt{\frac{9}{4} - \frac{m^2}{H^2}}}_{\equiv i\mu} = \begin{cases} 2 & \textit{conformal} \text{ scalar} \\ 3 & \textit{massless} \text{ scalar} \end{cases}$$

The 4-point function with external *inflatons* can be obtained by acting with simple *weight-shifting operators.*

$$F_{\Delta=3} = \left(1 - \frac{k_1 k_2}{k_{12}} \partial_{k_{12}}\right)\left(1 - \frac{k_3 k_4}{k_{34}} \partial_{k_{34}}\right) F_{\Delta=2}$$

To obtain the inflationary 3-point function, we evaluate one of the external legs on the *time-dependent inflaton background*.



$$= F_{\Delta=3}$$

$$= \varepsilon \lim_{v\to 1} \partial_v F_{\Delta=3}$$

How big is the signal?

$$f_{\mathrm{NL}} = \frac{\langle 3\mathrm{pt}\rangle}{\langle 2\mathrm{pt}\rangle^2} \sim \varepsilon\, e^{-\pi\mu} g^2 M_{\mathrm{pl}}^2$$

Particle production leads to an extra suppression.

Need non-gravitational interactions for observability.

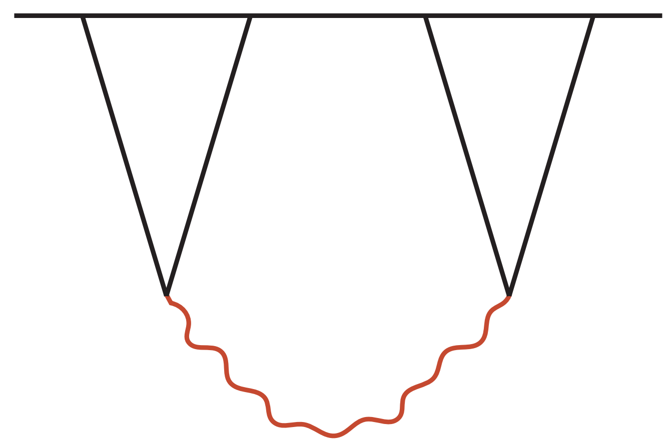
E.g. $$g^2 M_{\mathrm{pl}}^2 \sim \frac{M_{\mathrm{pl}}^2}{M_{\mathrm{string}}^2} \gg 1$$

Arkani-Hamed & Maldacena (2015)
HL, Baumann, Pimentel (2016)

Conclusions

As in flat space, *symmetries* and *singularities* can be used to fully fix the structure of the inflationary 3- and 4-point functions.

$$\left[\Delta_u + \left(\tfrac{1}{4} + \mu^2\right)\right]$$



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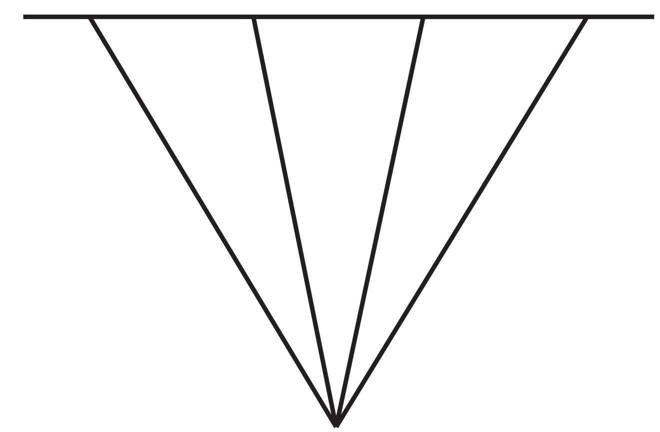

With luck, these signatures may be observable with the next-generation cosmological experiments.

Thank you!