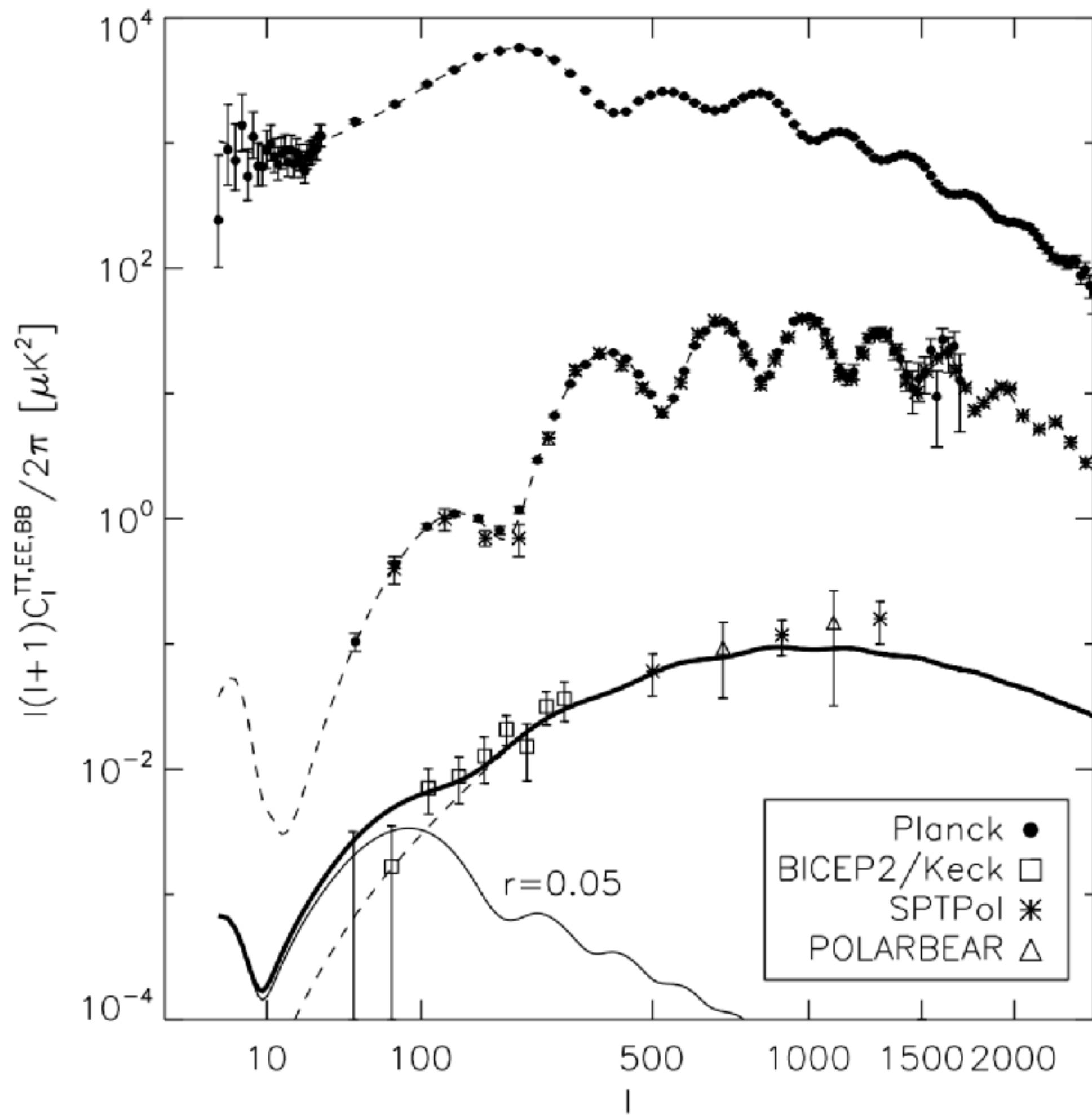


Non-Gaussian gravitational waves from inflation

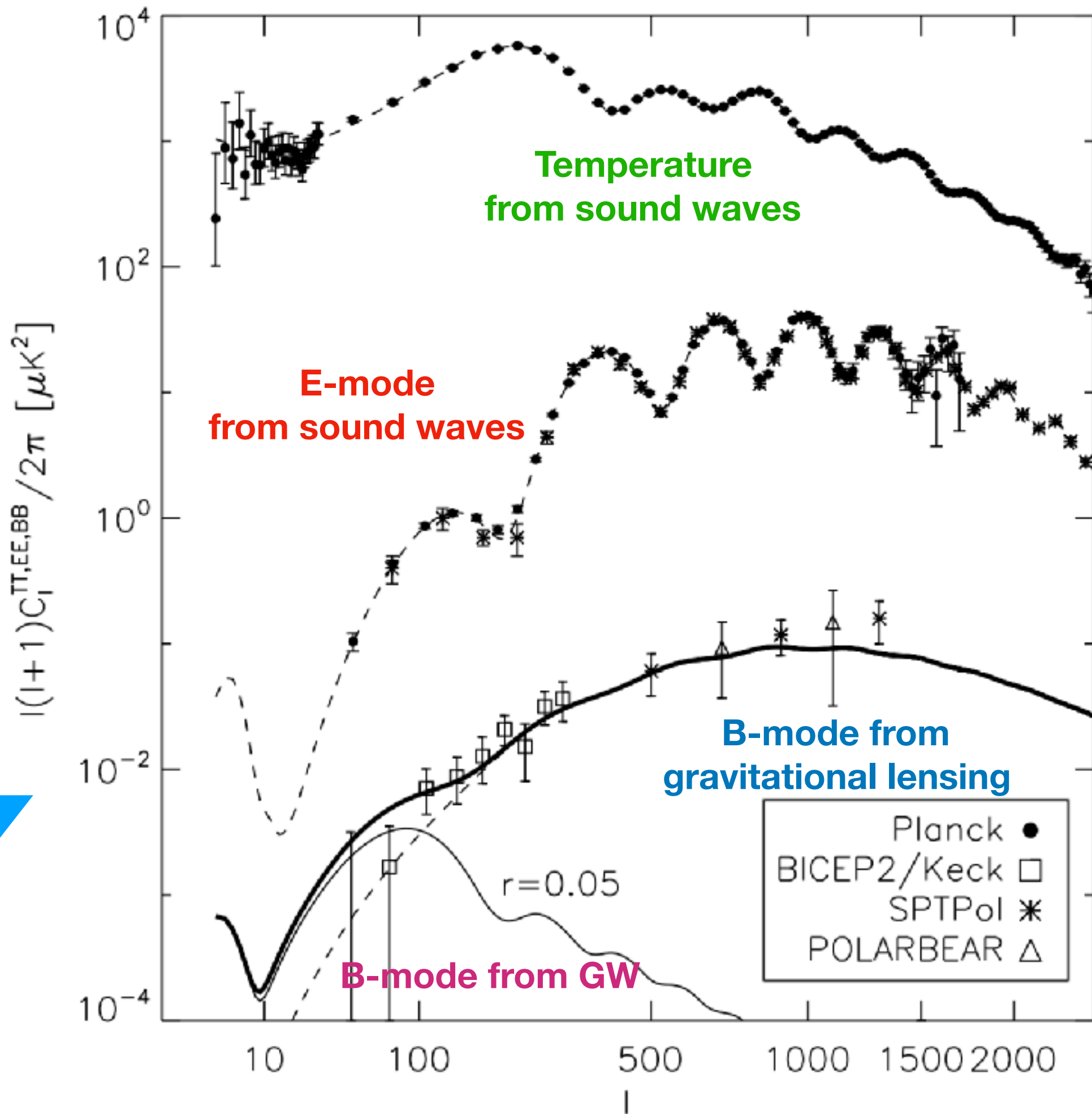
Eiichiro Komatsu

[Max Planck Institute for Astrophysics]

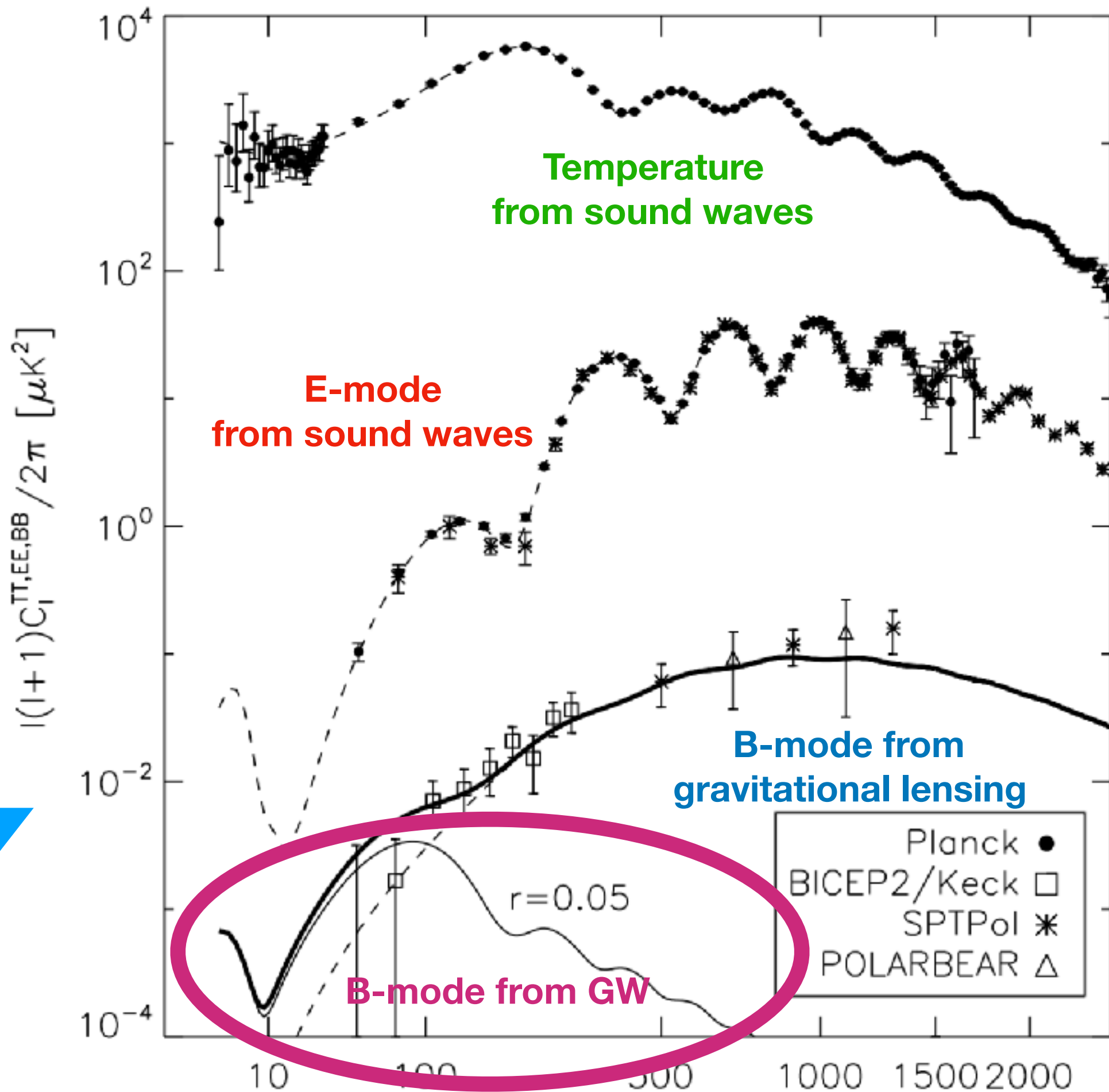
COSMO-18, IBS, Daejeon, August 28, 2018



Seven orders of magnitude in power
in “just” 25 years

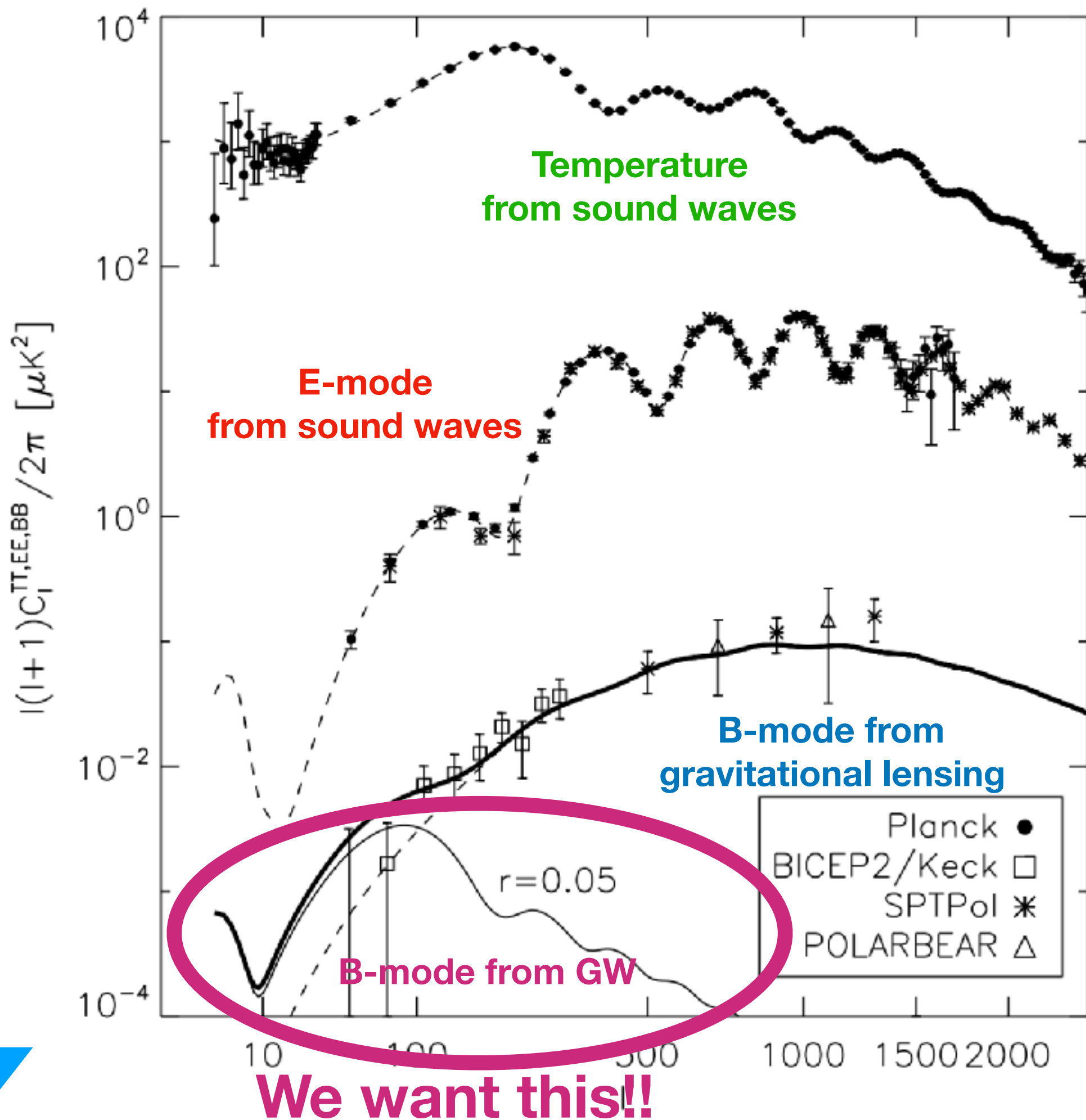
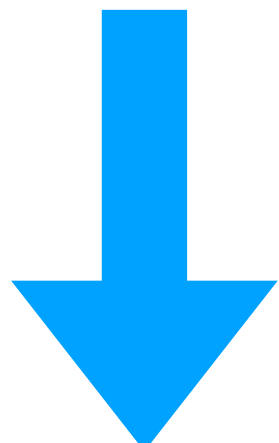


Seven orders of magnitude in power
in “just” 25 years



We want this!!

Another two orders of magnitude
in the next 10–15 years

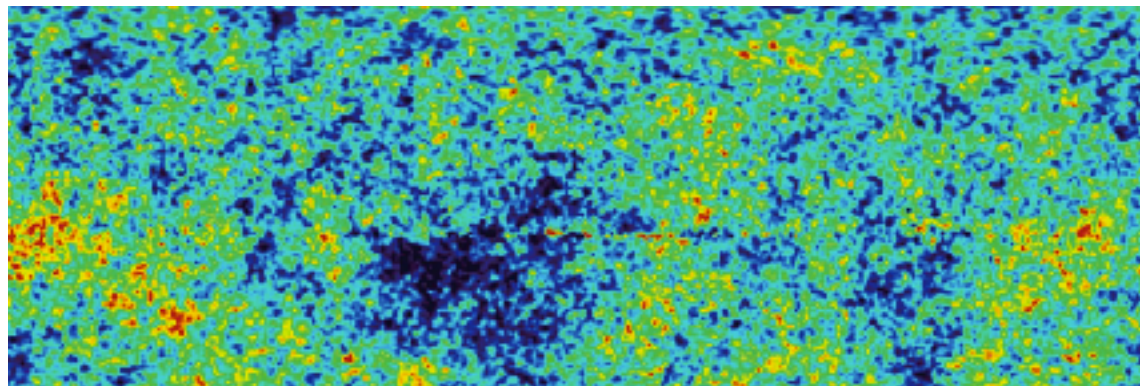
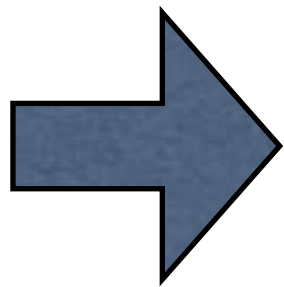


Key Predictions

 ζ

scalar
mode

- Fluctuations we observe today in CMB and the matter distribution originate from quantum fluctuations during inflation

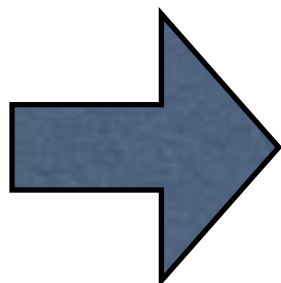


Mukhanov&Chibisov (1981)
Guth & Pi (1982)
Hawking (1982)
Starobinsky (1982)
Bardeen, Steinhardt&Turner (1983)

 h_{ij}

tensor
mode

- There should also be *ultra long-wavelength* gravitational waves generated during inflation



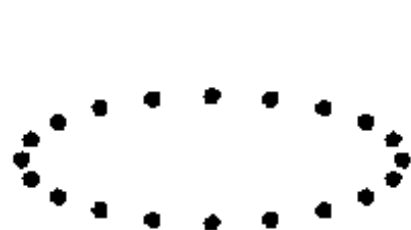
Grishchuk (1974)
Starobinsky (1979)

We measure distortions in space

- A distance between two points in space

$$d\ell^2 = a^2(t)[1 + 2\zeta(\mathbf{x}, t)][\delta_{ij} + h_{ij}(\mathbf{x}, t)]dx^i dx^j$$

- ζ : “curvature perturbation” (scalar mode)
 - Perturbation to the determinant of the spatial metric
- h_{ij} : “gravitational waves” (tensor mode)
 - Perturbation that does not alter the determinant

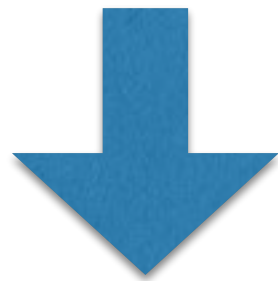


$$\sum_i h_{ii} = 0$$

Measuring GW

- GW changes distances between two points

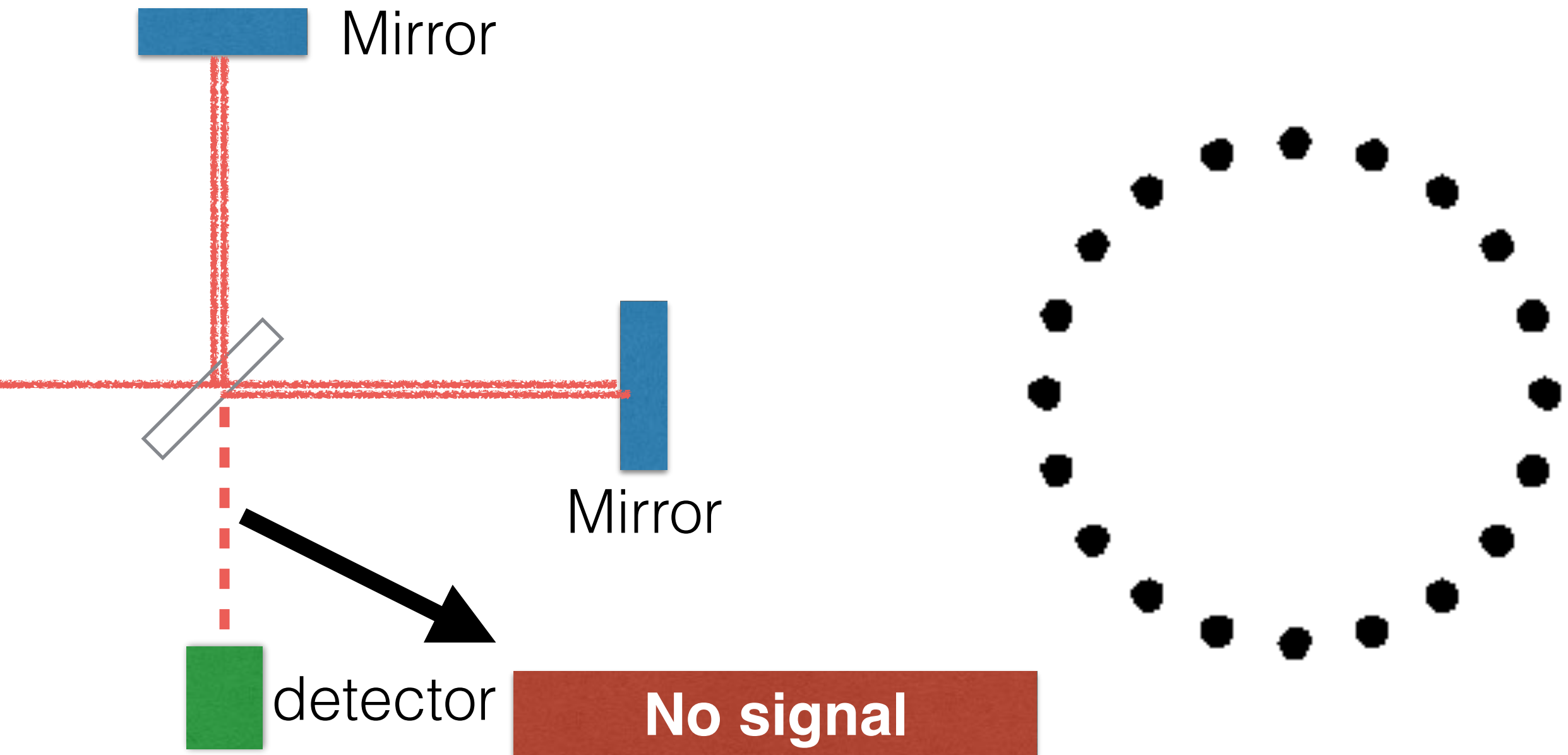
$$d\ell^2 = d\mathbf{x}^2 = \sum_{ij} \delta_{ij} dx^i dx^j$$



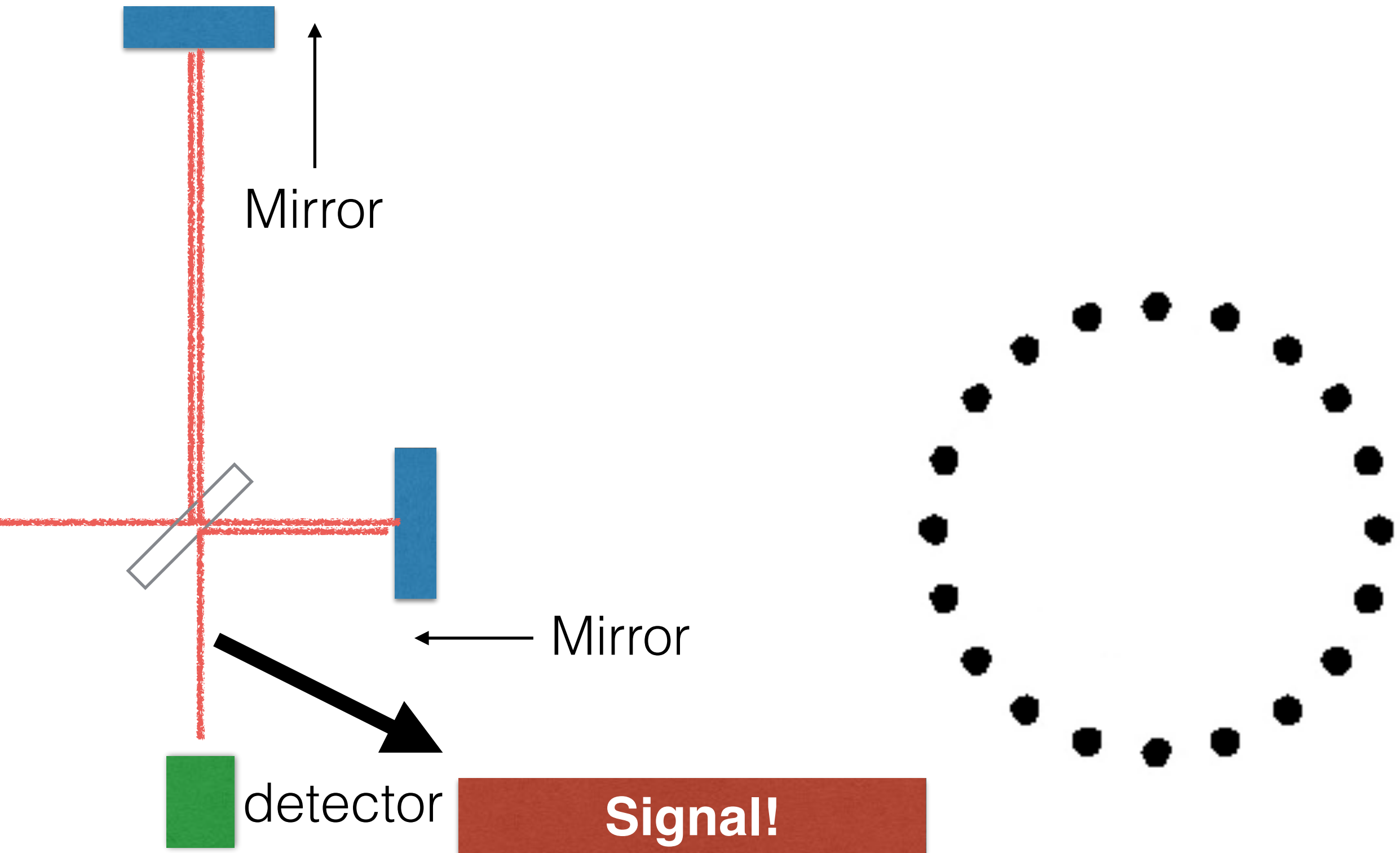
$$d\ell^2 = \sum_{ij} (\delta_{ij} + \text{hatched } h_{ij}) dx^i dx^j$$



Laser Interferometer



Laser Interferometer



LIGO detected GW from a binary blackholes, with the wavelength of thousands of kilometres

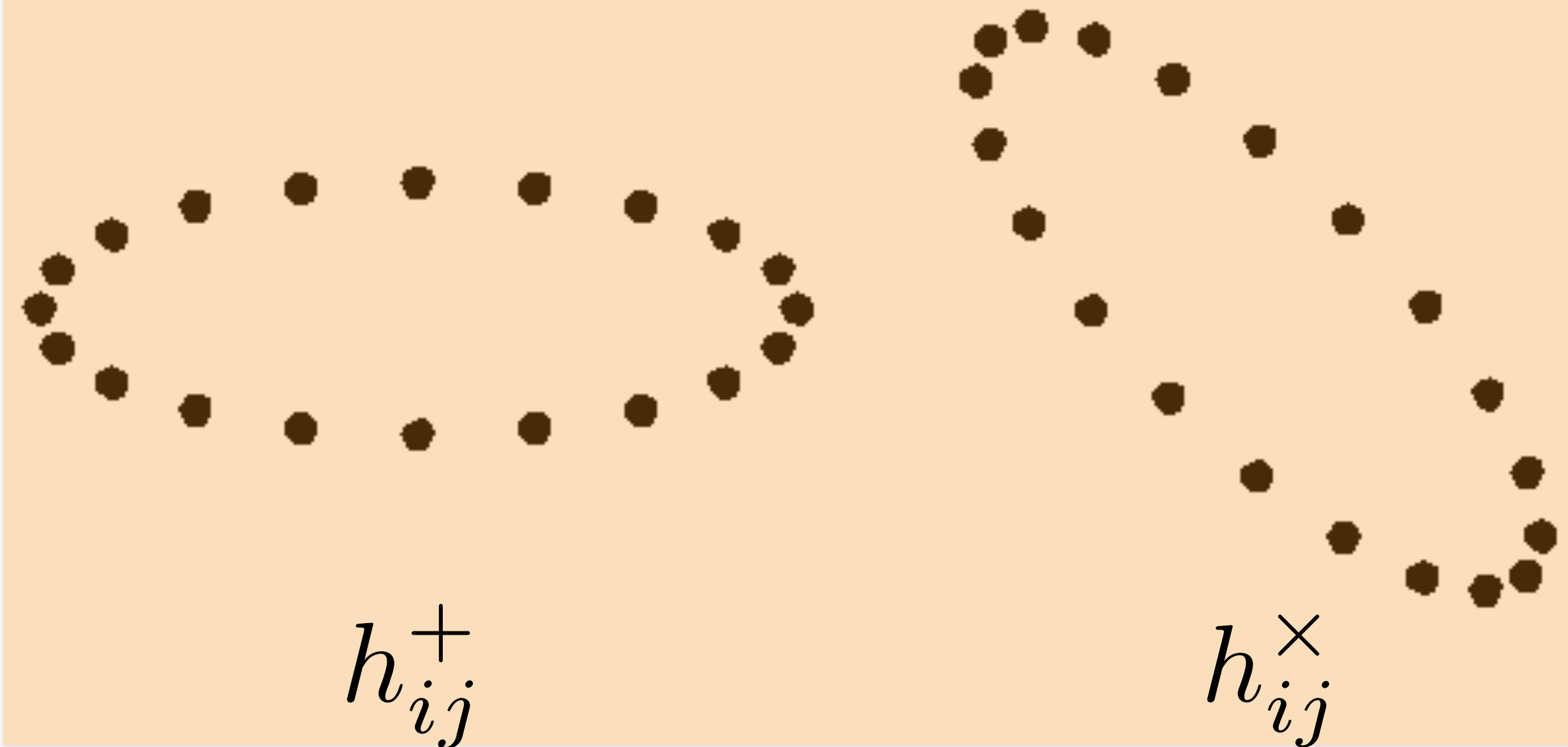
But, the primordial GW affecting the CMB has a wavelength of **billions of light-years!!** How do we find it?

Detecting GW by CMB

Isotropic electro-magnetic fields

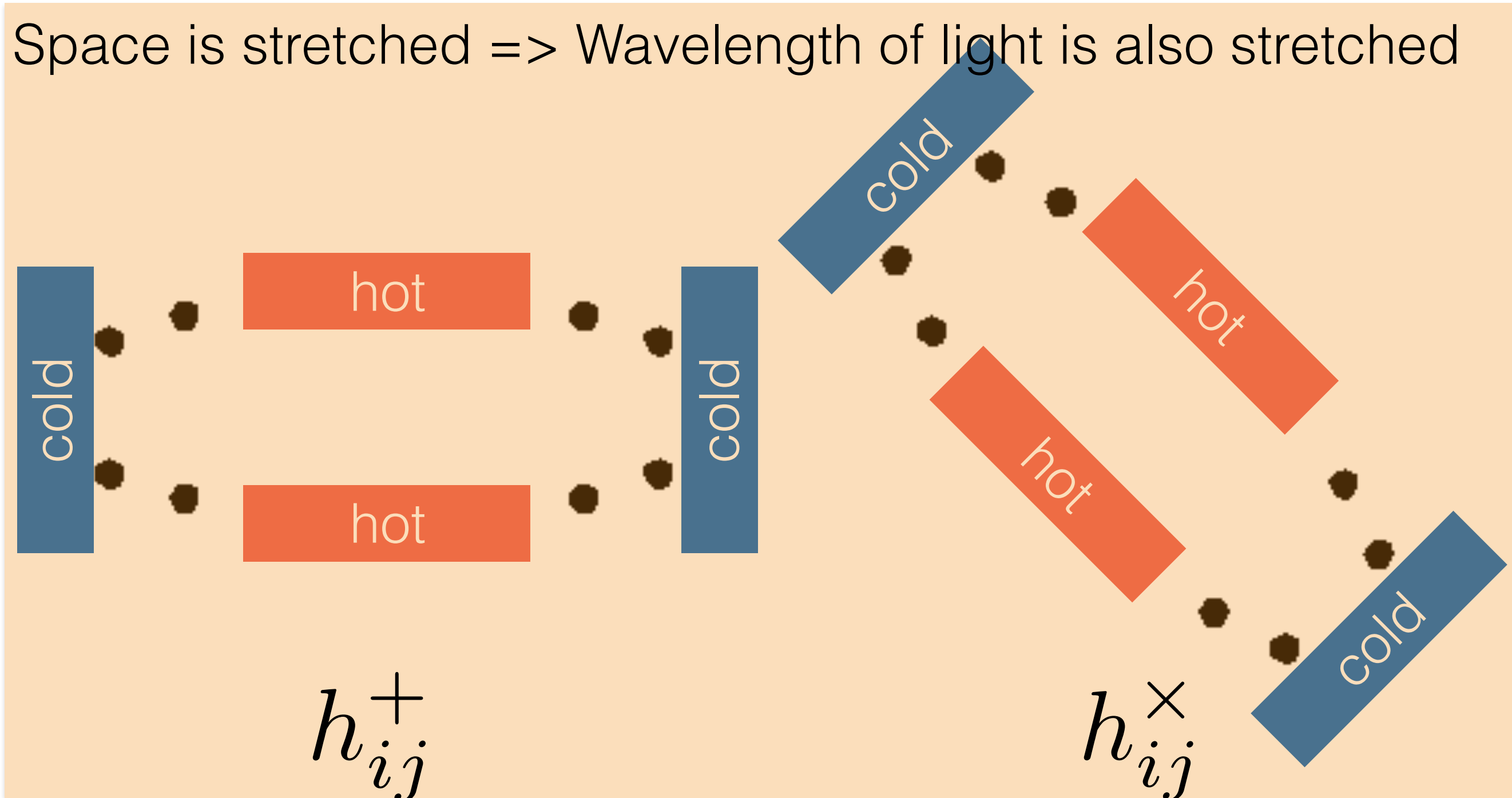
Detecting GW by CMB

GW propagating in isotropic electro-magnetic fields



Detecting GW by CMB

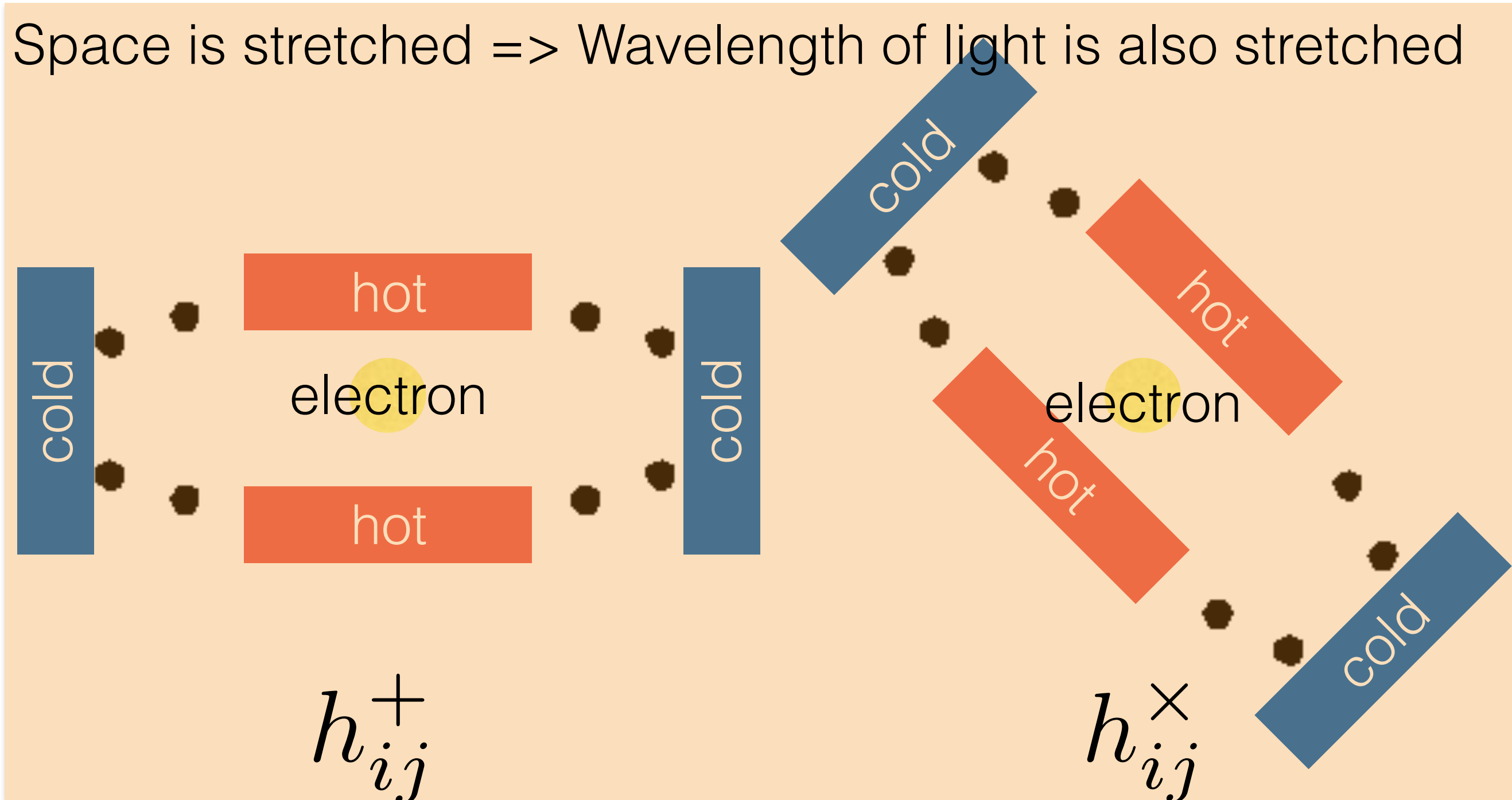
Space is stretched => Wavelength of light is also stretched



Detecting GW by CMB

Polarisation

Space is stretched => Wavelength of light is also stretched



Detecting GW by CMB

Polarisation

Space is stretched => Wavelength of light is also stretched

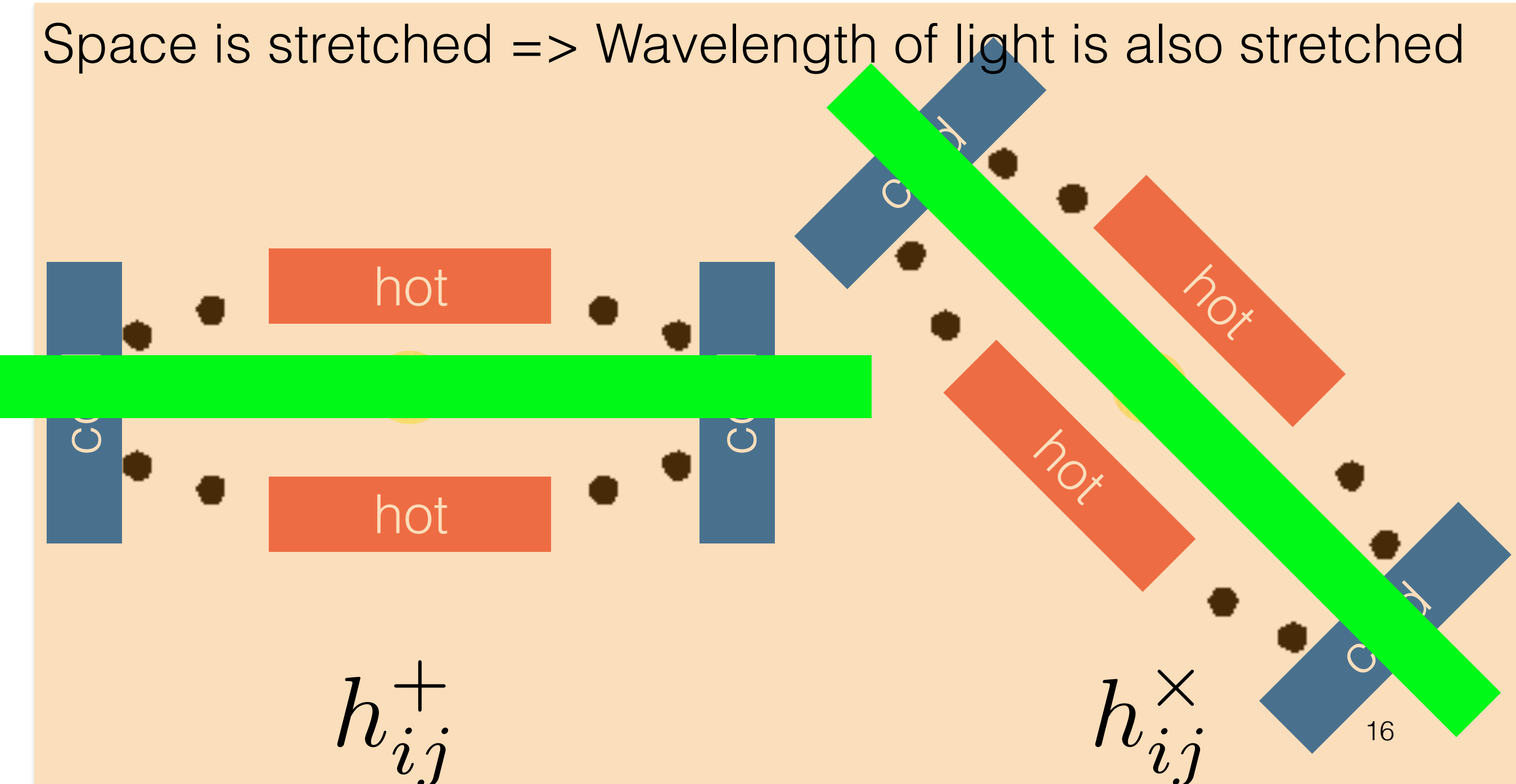


Photo Credit: TALEX



horizontally polarised

Photo Credit: TALEX



Gravitational waves as the quantum vacuum fluctuation in spacetime

- Quantising the gravitational waves in de Sitter space in **vacuum**

$$\square h_{ij} = 0$$

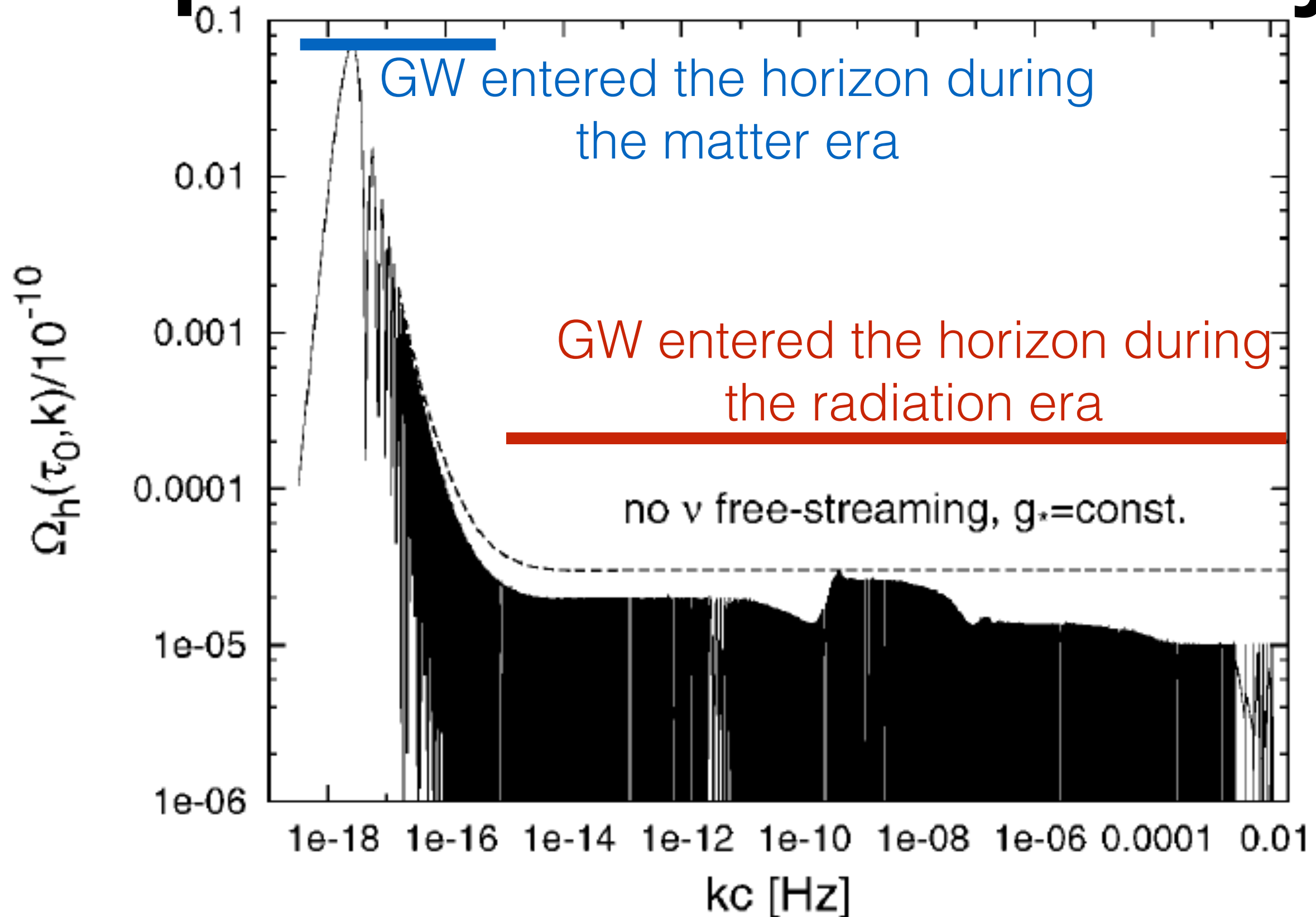
gives

$$k^3 \langle h_{ij}(\mathbf{k}) h^{ij*}(\mathbf{k}') \rangle$$

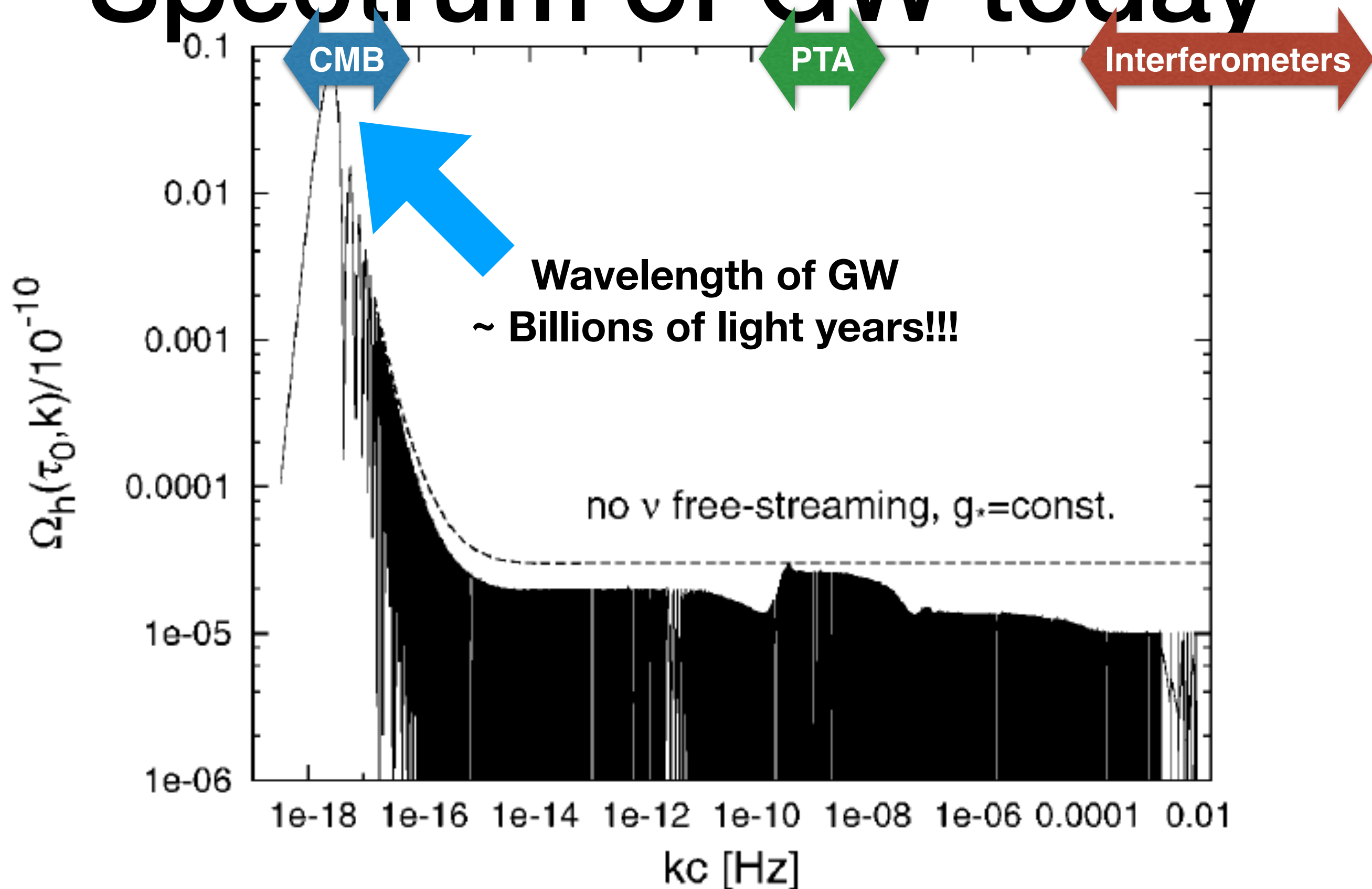
$$= (2\pi)^3 \delta_D(\mathbf{k} - \mathbf{k}') \frac{8}{M_{\text{pl}}^2} \left(\frac{H}{2\pi} \right)^2$$

scale-invariant spectrum

Theoretical energy density Spectrum of GW today



Theoretical energy density Spectrum of GW today



But, wait a minute...

Are GWs from vacuum fluctuation in spacetime, or from sources?

$$\square h_{ij} = -16\pi G \pi_{ij}$$

- **Homogeneous solution**: “GWs from vacuum fluctuation”
- **Inhomogeneous solution**: “GWs from sources”
 - Scalar and vector fields cannot source tensor fluctuations at linear order (possible at non-linear level; **see Marco Peloso’s talk**)
 - SU(2) gauge field can!

Maleknejad & Sheikh-Jabbari (2013); Dimastrogiovanni & Peloso (2013);
Adshead, Martinec & Wyman (2013); Obata & Soda (2016); ...

Important Message

$$\square h_{ij} = -16\pi G \pi_{ij}$$

- Do not take it for granted if someone told you that detection of the primordial gravitational waves would be a signature of “quantum gravity”!
- Only the homogeneous solution corresponds to the vacuum tensor metric perturbation. **There is no *a priori* reason to neglect an inhomogeneous solution!**
- Contrary, we have several examples in which detectable B-modes are generated by **sources** [U(1) and SU(2)]

Experimental Strategy

Commonly Assumed So Far

1. Detect CMB polarisation in multiple frequencies, to make sure that it is from the CMB (i.e., Planck spectrum)
2. Check for scale invariance: Consistent with a scale invariant spectrum?
 - Yes => Announce discovery of the vacuum fluctuation in spacetime
 - No => WTF?

New Experimental Strategy: New Standard!

1. Detect CMB polarisation in multiple frequencies, to make sure that it is from the CMB (i.e., Planck spectrum)
 2. Consistent with a scale invariant spectrum?
 3. Parity violating correlations consistent with zero?
 4. Consistent with Gaussianity?
- If, and **ONLY IF** Yes to **all** => Announce discovery of the vacuum fluctuation in spacetime

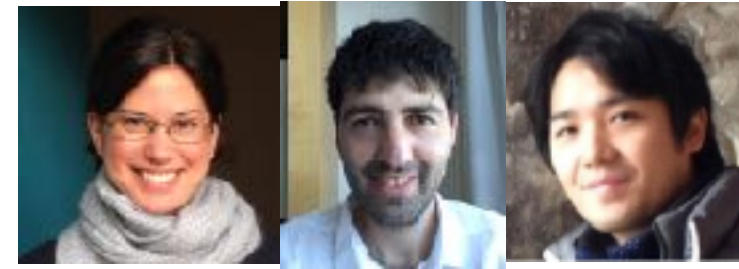
If not, you may have just discovered new physics during inflation!

2. Consistent with a scale invariant spectrum?
 3. Parity violating correlations consistent with zero?
 4. Consistent with Gaussianity?
- If, and **ONLY IF** Yes to **all** => Announce discovery of the vacuum fluctuation in spacetime

Further Remarks

- “*Guys, you are complicating things too much!*”
- **No.** These sources (eg., gauge fields) should be ubiquitous in a high-energy universe. They have every right to produce GWs if they are around
- Sourced GWs with $r \gg 0.001$ can be phenomenologically more attractive than the vacuum GW from the large-field inflation [requiring super-Planckian field excursion]. Better radiative stability, etc
- Rich[er] phenomenology: Better integration with the Standard Model; reheating; baryon synthesis via leptogenesis, etc. **Testable using many more probes!**

GW from Axion-SU(2) Dynamics



$$\mathcal{L} = \mathcal{L}_{GR} + \mathcal{L}_\phi + \mathcal{L}_\chi - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{\lambda \chi}{4f} F_{\mu\nu}^a \tilde{F}^{a\mu\nu}$$

- ϕ : inflaton field => Just provides quasi-de Sitter background
[I don't want to touch this sector because I don't understand inflaton]
- χ : pseudo-scalar “axion” field. Spectator field (i.e., negligible energy density compared to the inflaton)
- Field strength of an SU(2) field A_ν^a :

$$F_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g\epsilon^{abc} A_\mu^b A_\nu^c$$

A well-defined set up:

Axion-SU(2) gauge field dynamics in a given de-Sitter background.

Everything is calculable!

- ϕ : inflaton field => Just provides quasi-de Sitter background
[I don't want to touch this sector because I don't understand inflaton]
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Background and Perturbation



A. Maleknejad
(MPA)

- In an inflating background, the SU(2) field has an **isotropic** background solution:

$$A_i^a = [\text{scale factor}] \times Q \times \delta_i^a$$

$$Q \equiv (-f \partial_\chi U / 3g\lambda H)^{1/3}$$

U: axion potential

- Perturbations contain a tensor (spin-2) mode (as well as S&V)

$$\delta A_i^a = t_{ai} + \dots$$

$$t_{ii} = \partial_a t_{ai} = \partial_i t_{ai} = 0$$

Scenario

- The $SU(2)$ field contains tensor, vector, and scalar components
- The tensor components are amplified strongly by a coupling to the axion field
- Only one helicity is amplified $\Rightarrow$ GW is **chiral** (well-known result, also for $U(1)$; see Marco's talk)
- New result: **GWs sourced by this mechanism are strongly non-Gaussian!**

Agrawal, Fujita & EK, PRD, 97, 103526 (2018)

Gravitational Waves

- Defining canonically-normalised circular polarisation modes as

$$\psi_{L,R} \equiv (aM_{\text{Pl}}/2)(h_+ \pm ih_\times)$$

- The equations of motion for L and R modes are

$$\square \psi_{L,R} \neq 0$$

Gravitational Waves

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- The equations of motion for L and R modes are ($x \equiv k/aH$)

$$\partial_x^2 \psi_{R,L} + \left(1 - \frac{2}{x^2}\right) \psi_{R,L} = \frac{2\sqrt{\epsilon_E}}{x} \partial_x t_{R,L} + \frac{2\sqrt{\epsilon_B}}{x^2} (m_Q \mp x) t_{R,L}$$

spin-2 field spin-2 field

$$\left(\begin{array}{l} m_Q \equiv gQ/H = \text{a few} \\ \epsilon_B \equiv g^2 Q^4 / (H M_{\text{Pl}})^2 \ll 1 \\ \epsilon_E \equiv (H\dot{Q} + \dot{Q})^2 / (\dot{H} M_{\text{Pl}})^2 \ll 1 \end{array} \right.$$

Spin-2 Field from SU(2)

- The equations of motion for L and R modes of SU(2) are

$$\partial_x^2 t_{R,L} + \left[1 + \frac{2}{x^2} (m_Q \xi \mp x(m_Q + \xi)) \right] t_{R,L} \\ = -\frac{2\sqrt{\epsilon_E}}{x} \partial_x \psi_{R,L} + \frac{2}{x^2} [(m_Q \mp x) \sqrt{\epsilon_B} + \sqrt{\epsilon_E}] \psi_{R,L}$$

the minus sign gives an instability -> exponential amplification of t_R !

$$\left(\begin{array}{l} \xi \equiv \frac{\lambda}{2fH} \dot{\chi} \simeq m_Q + \frac{1}{m_Q} \\ m_Q \equiv gQ/H = \text{a few} \\ \epsilon_B \equiv g^2 Q^4 / (H M_{\text{Pl}})^2 \ll 1 \\ \epsilon_E \equiv (H\dot{Q} + \dot{Q})^2 / (\dot{H} M_{\text{Pl}})^2 \ll 1 \end{array} \right.$$

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the minus sign gives an instability -> exponential amplification of t_R !

- The produced gravitational waves are totally chiral!**
- The solution (when all the parameters are constant and the terms on the right hand side are ignored):

$$t_R(x) = \frac{1}{\sqrt{2k}} i^\beta W_{\beta,\alpha}(-2ix) \quad \begin{pmatrix} \alpha \equiv -i\sqrt{2m_Q\xi - 1/4} \\ \beta \equiv -i(m_Q + \xi) \end{pmatrix}$$

[Whittaker function]

Gravitational Waves

- Defining canonically-normalised circular polarisation modes as

$$\psi_{L,R} \equiv (aM_{\text{Pl}}/2)(h_+ \pm ih_\times)$$

- The equations of motion for L and R modes are ($x \equiv k/aH$)

$$\partial_x^2 \psi_{R,L} + \left(1 - \frac{2}{x^2}\right) \psi_{R,L} = \frac{2\sqrt{\epsilon_E}}{x} \partial_x t_{R,L} + \frac{2\sqrt{\epsilon_B}}{x^2} (m_Q \mp x) t_{R,L}$$

- Inhomogeneous solution:

$$\lim_{x \rightarrow 0} \psi_R^{(s)}(x) = \frac{1}{\sqrt{2kx}} \left[\mathcal{F}_E \sqrt{\epsilon_E} + \mathcal{F}_B \sqrt{\epsilon_B} \right]$$

$\mathcal{F}_E, \mathcal{F}_B$: some complicated functions

Power Spectrum!

$$\mathcal{P}_h^{(s)}(k) = \frac{H^2}{\pi^2 M_{\text{Pl}}^2} \left| \sqrt{2kx} \lim_{x \rightarrow 0} \psi_R^{(s)}(x) \right|^2 = \frac{\epsilon_B H^2}{\pi^2 M_{\text{Pl}}^2} \mathcal{F}^2$$

$$\mathcal{F}^2 \equiv \left| \mathcal{F}_B + \sqrt{\epsilon_E / \epsilon_B} \mathcal{F}_E \right|^2 \approx \exp(3.6 m_Q)$$

- This exponential dependence on m_Q makes it possible to have $\mathbf{P_{sourced} \gg P_{vacuum} = (2/\pi^2)H^2/M_{Pl}^2}$
- **New Paradigm**

Phenomenology

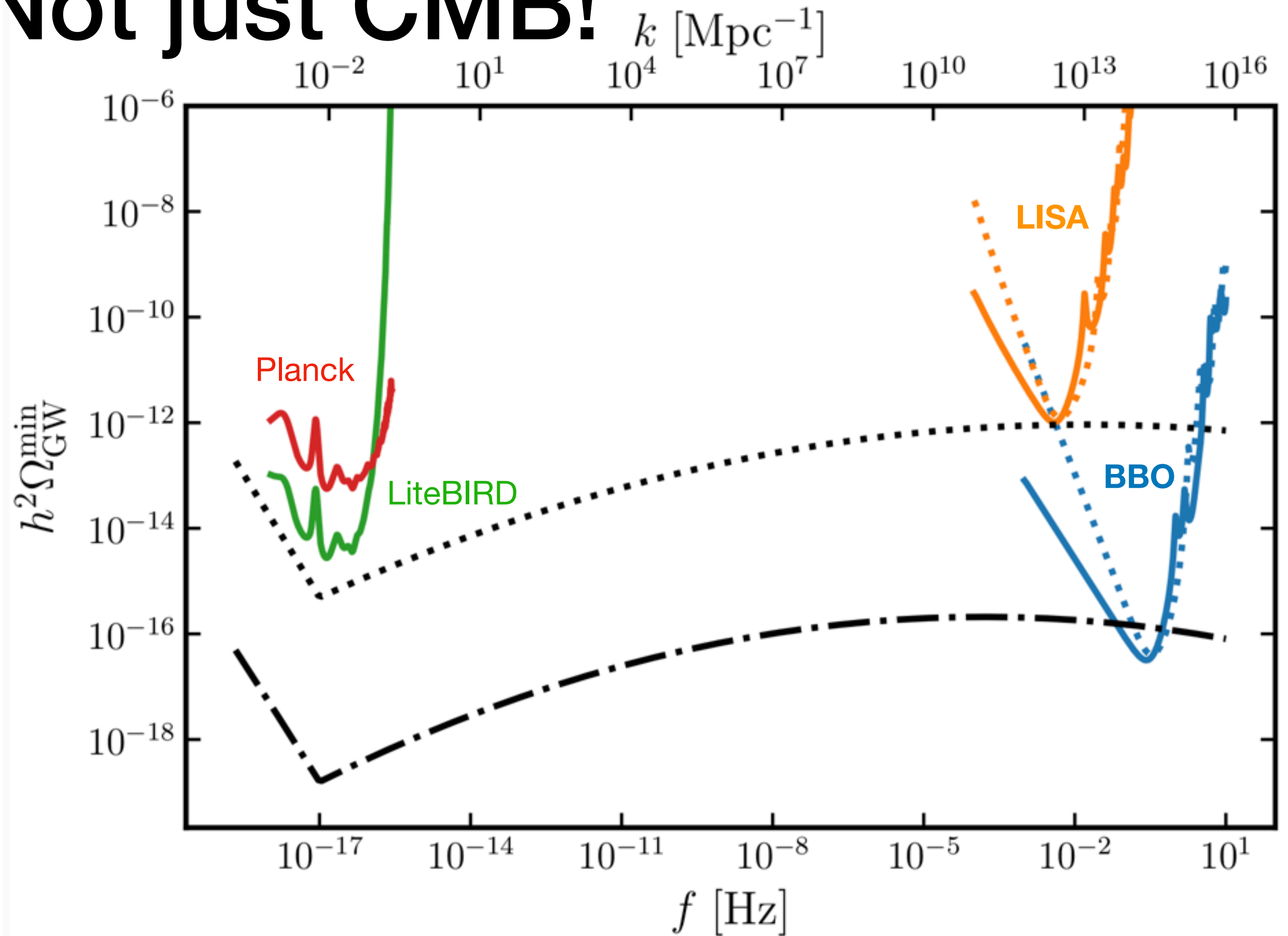
$$\partial_x^2 t_{R,L} + \left[1 + \frac{2}{x^2} (m_Q \xi \mp x(m_Q + \xi)) \right] t_{R,L} = \dots$$

the minus sign gives an instability -> exponential amplification of t_R !

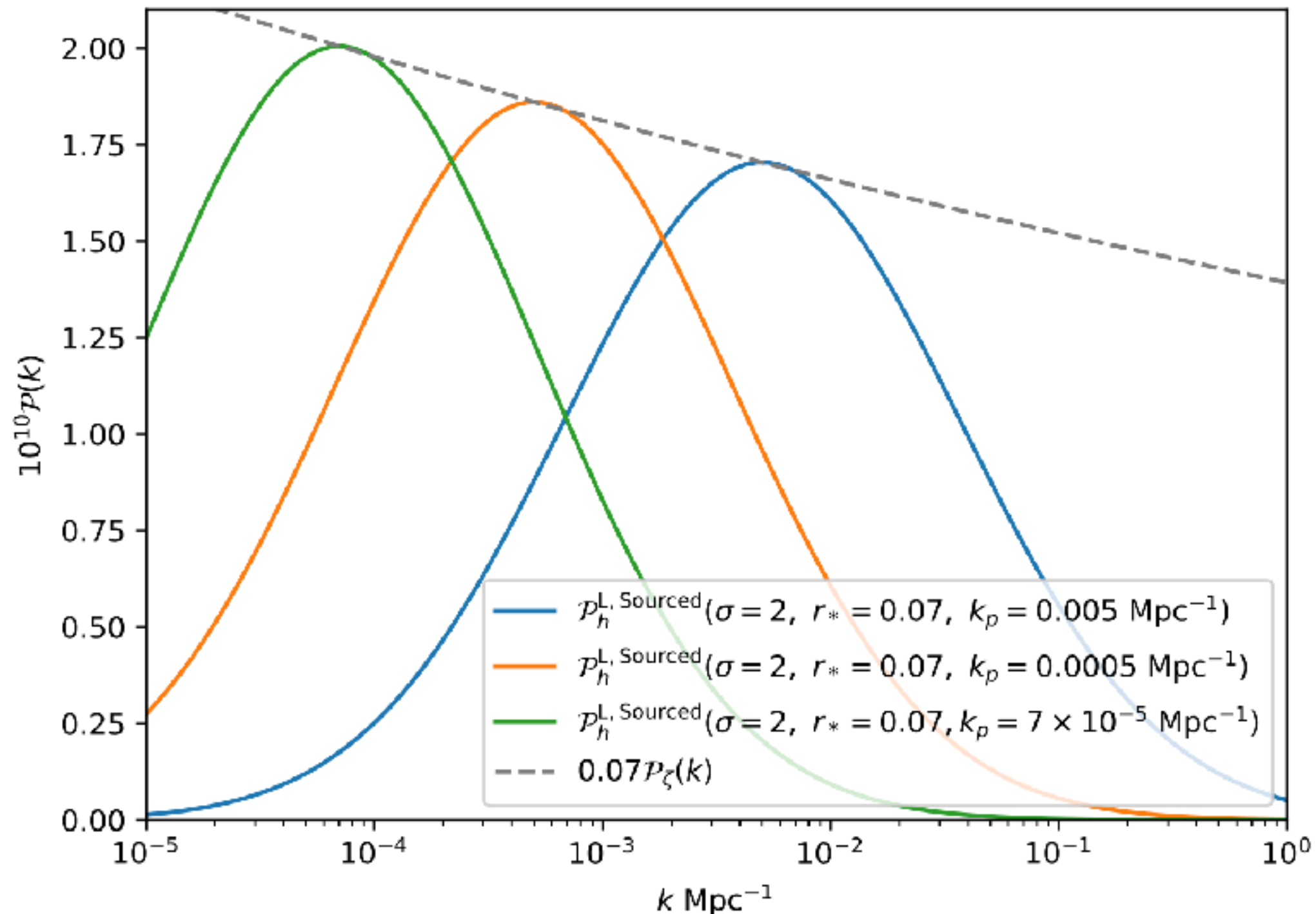
$$\left(\begin{array}{l} \xi \equiv \frac{\lambda}{2fH} \dot{\chi} \simeq m_Q + \frac{1}{m_Q} \\ m_Q \equiv gQ/H = \text{a few} \end{array} \right.$$

- The scale-dependence of the produced tensor modes is determined by how m_Q changes with time
- E.g., Axion rolling faster towards the end of inflation: BLUE TILTED power spectrum! Therefore...

Not just CMB!



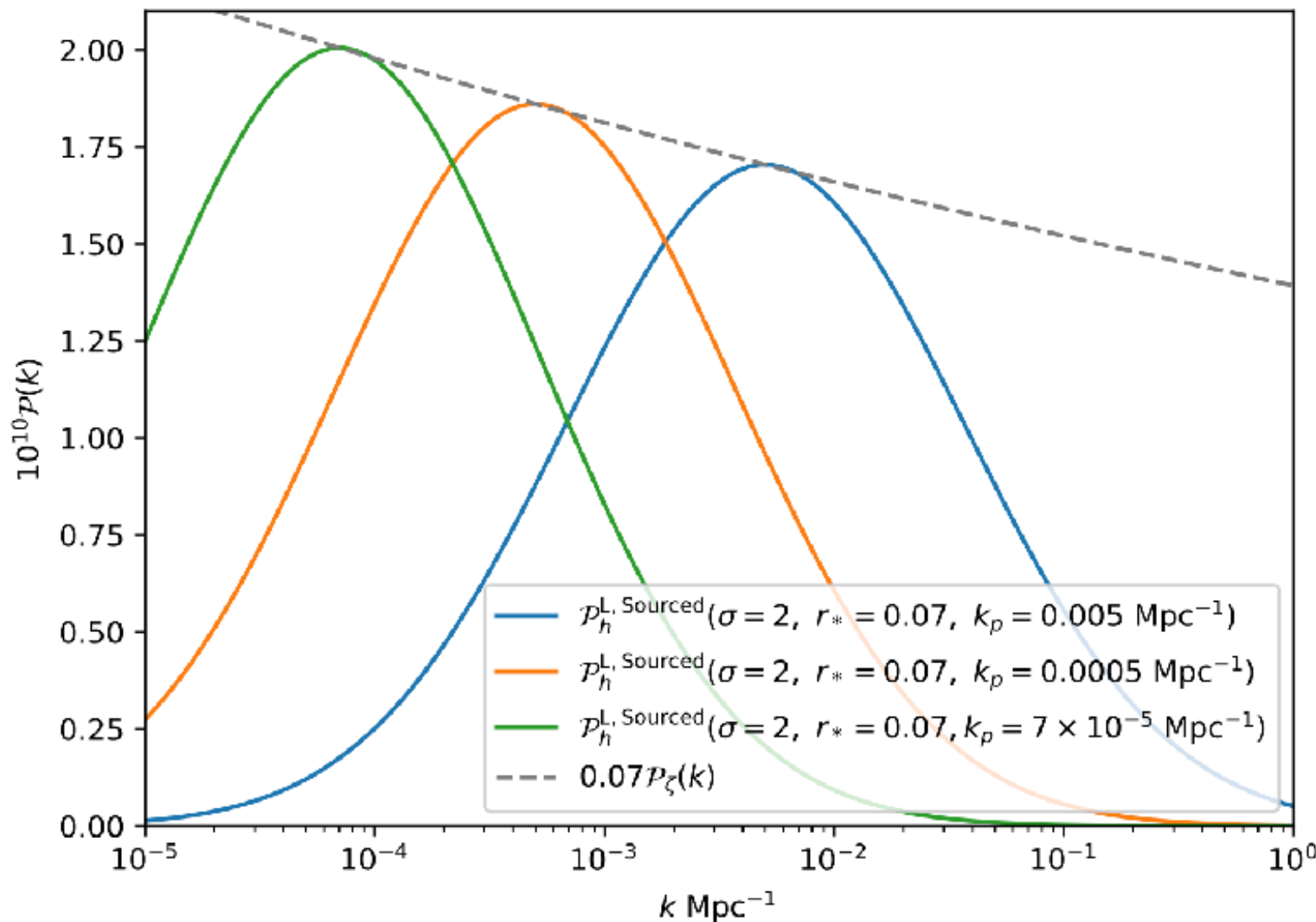
Example Tensor Spectra



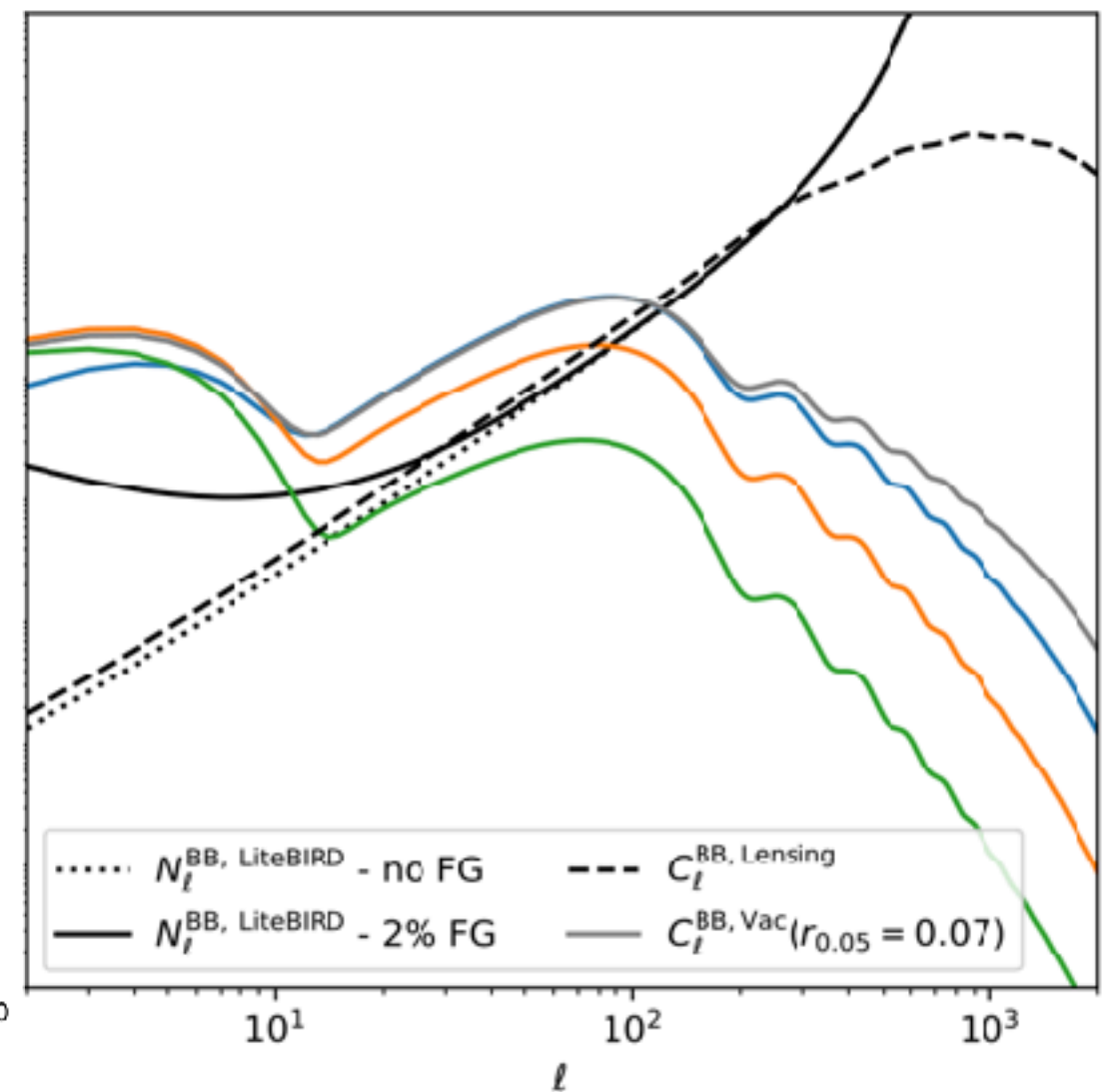
- Sourced tensor spectrum can also be bumpy

Example Tensor Spectra

Tensor Power Spectrum, $P(k)$

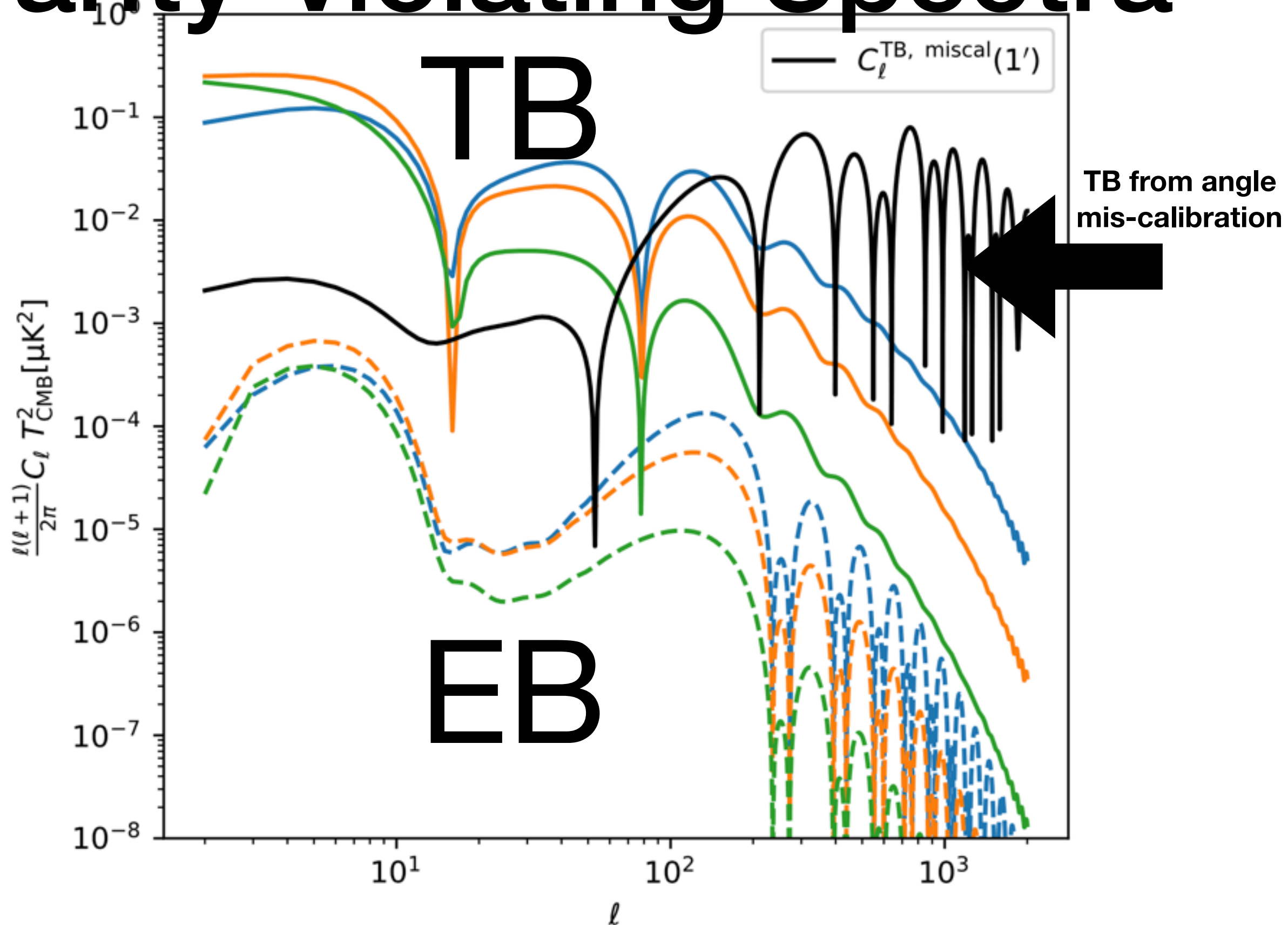


B-mode CMB spectrum, C_l^{BB}



- The B-mode power spectrum still looks rather normal

Parity-violating Spectra



- Angle mis-calibration can be distinguished easily!

Large bispectrum in GW from SU(2) fields



Aniket Agrawal
(MPA)

$$\frac{B_h^{RRR}(k, k, k)}{P_h^2(k)} \approx \frac{25}{\Omega_A}$$



Tomo Fujita
(Kyoto)

$$\langle \hat{h}_R(\mathbf{k}_1) \hat{h}_R(\mathbf{k}_2) \hat{h}_R(\mathbf{k}_3) \rangle = (2\pi)^3 \delta \left(\sum_{i=1}^3 \mathbf{k}_i \right) B_h^{RRR}(k_1, k_2, k_3)$$

- $\Omega_A \ll 1$ is the energy density fraction of the gauge field
- **B_h/P_h^2 is of order unity for the vacuum contribution**
[Maldacena (2003); Maldacena & Pimentel (2011)]
- ***Gaussianity offers a powerful test of whether the detected GW comes from the vacuum or sources***

NG generated at the tree level

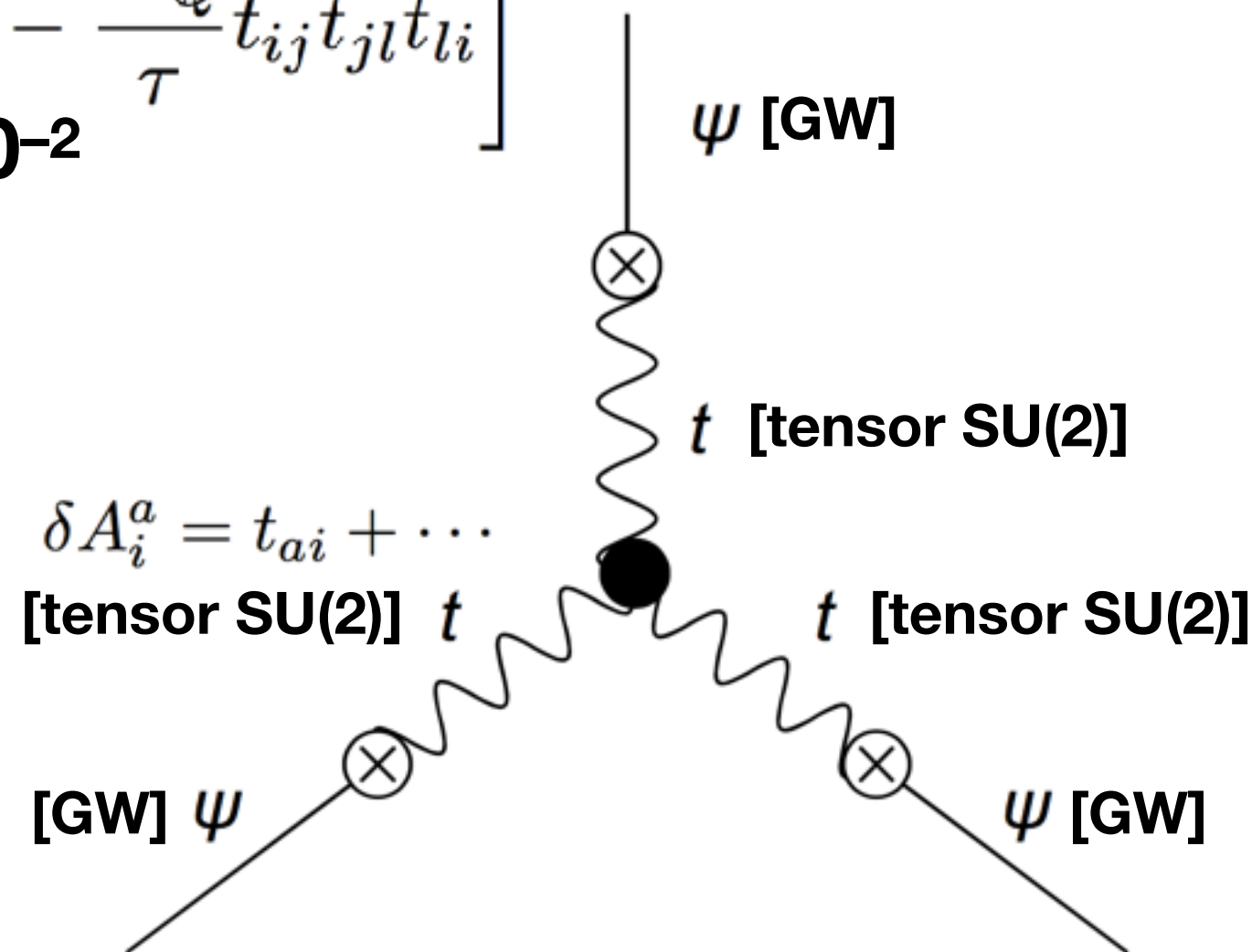
$$L_3^{(i)} = c^{(i)} \left[\epsilon^{abc} t_{ai} t_{bj} \left(\partial_i t_{cj} - \frac{m_Q^2 + 1}{3m_Q \tau} \epsilon^{ijk} t_{ck} \right) \right.$$

$$\left. - \frac{m_Q}{\tau} t_{ij} t_{jl} t_{li} \right] \\ c^{(i)} = g = m_Q^2 H / \sqrt{\epsilon_B} M_{\text{Pl}} \sim \mathbf{10^{-2}}$$

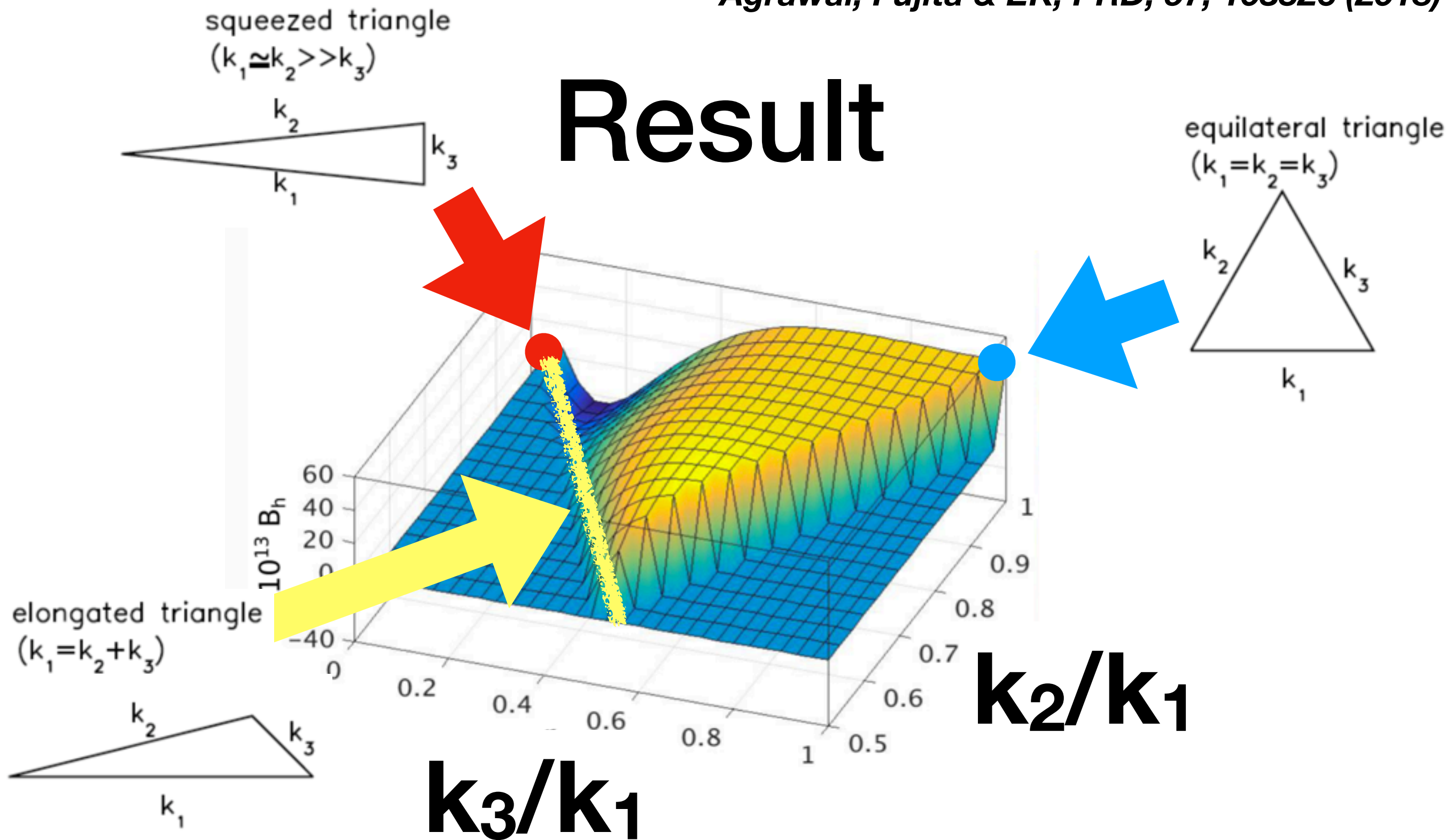
$$\epsilon_B \equiv \frac{g^2 Q^4}{H^2 M_{\text{Pl}}^2} \simeq \frac{2\Omega_A}{1 + m_Q^{-2}} \ll 1$$

$$m_Q \equiv gQ/H \quad [\mathbf{m_Q \sim a\ few}]$$

- This diagram generates second-order equation of motion for GW



Result



- This shape is similar to, but not exactly the same as, what was used by the Planck team to look for tensor bispectrum

Current Limit on Tensor NG

- The Planck team reported a limit on the tensor bispectrum in the following form:

$$f_{\text{NL}}^{\text{tens}} \equiv \frac{B_h^{+++}(k, k, k)}{F_{\text{scalar}}^{\text{equil.}}(k, k, k)}$$

- The denominator is the **scalar** equilateral bispectrum template, giving $F_{\text{scalar}}^{\text{equil.}}(k, k, k) = (18/5)P_{\text{scalar}}^2(k)$
- The current 68%CL constraint is $f_{\text{NL}}^{\text{tens}} = 400 \pm 1500$

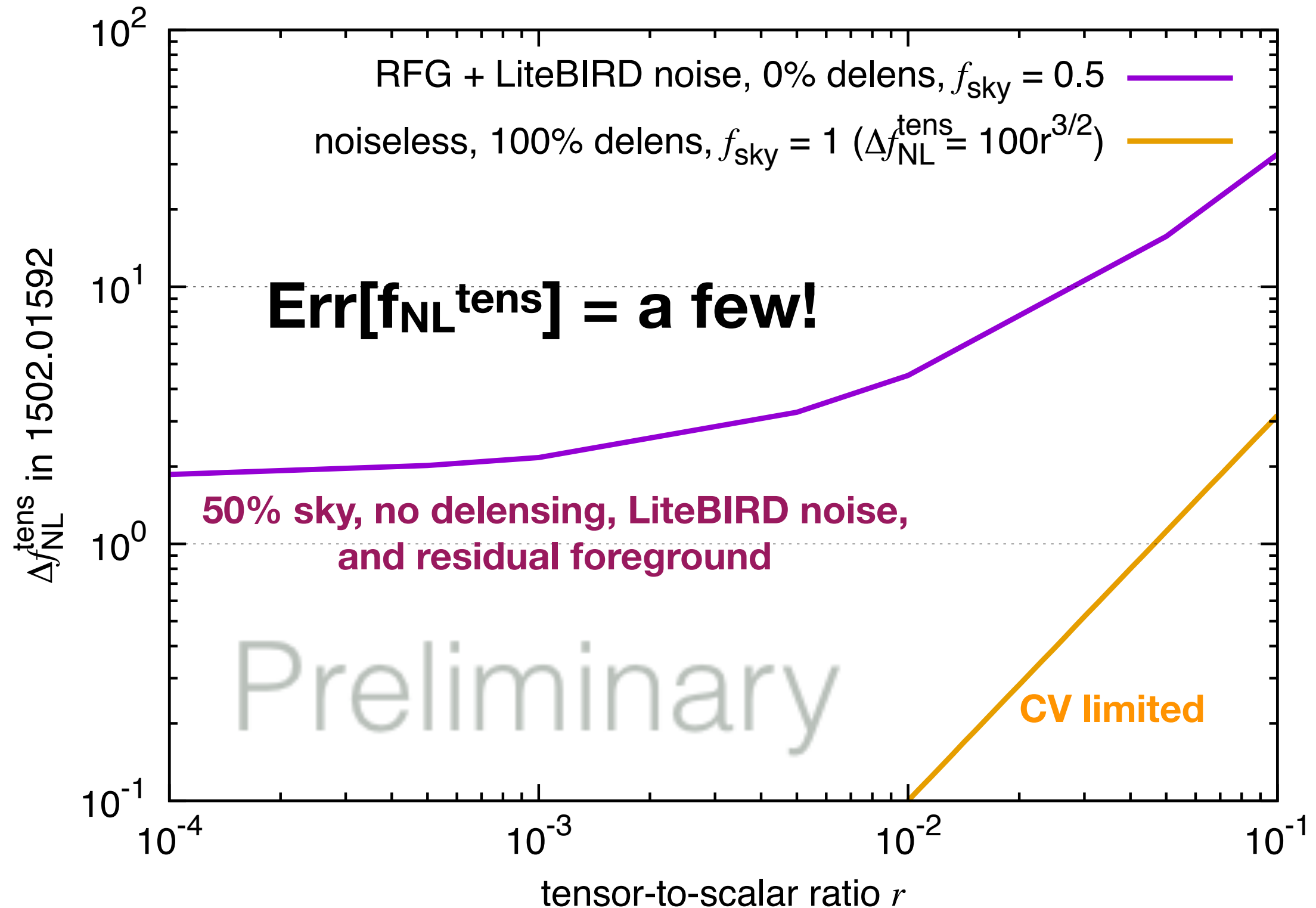
SU(2), confronted

- The SU(2) model of Dimastrogiovanni et al. predicts:

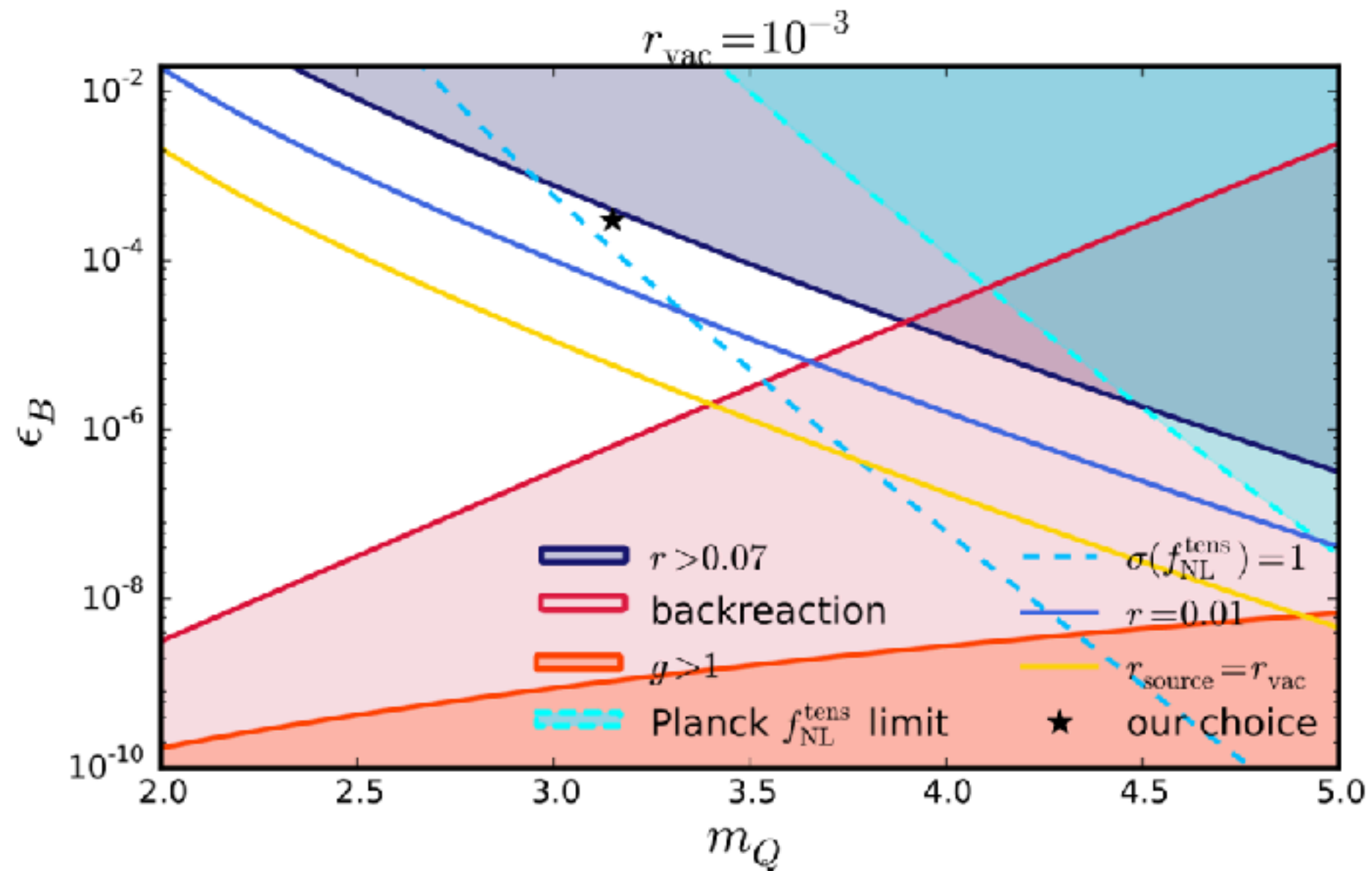
$$f_{\text{NL}}^{\text{tens}} \approx \frac{125}{18\sqrt{2}} \frac{r^2}{\epsilon_B} \approx 2.5 \frac{r^2}{\Omega_A}$$

- The current 68%CL constraint is $f_{\text{NL}}^{\text{tens}} = 400 \pm 1500$
 - This is already constraining!

LiteBIRD would nail it!



Parameter Scan



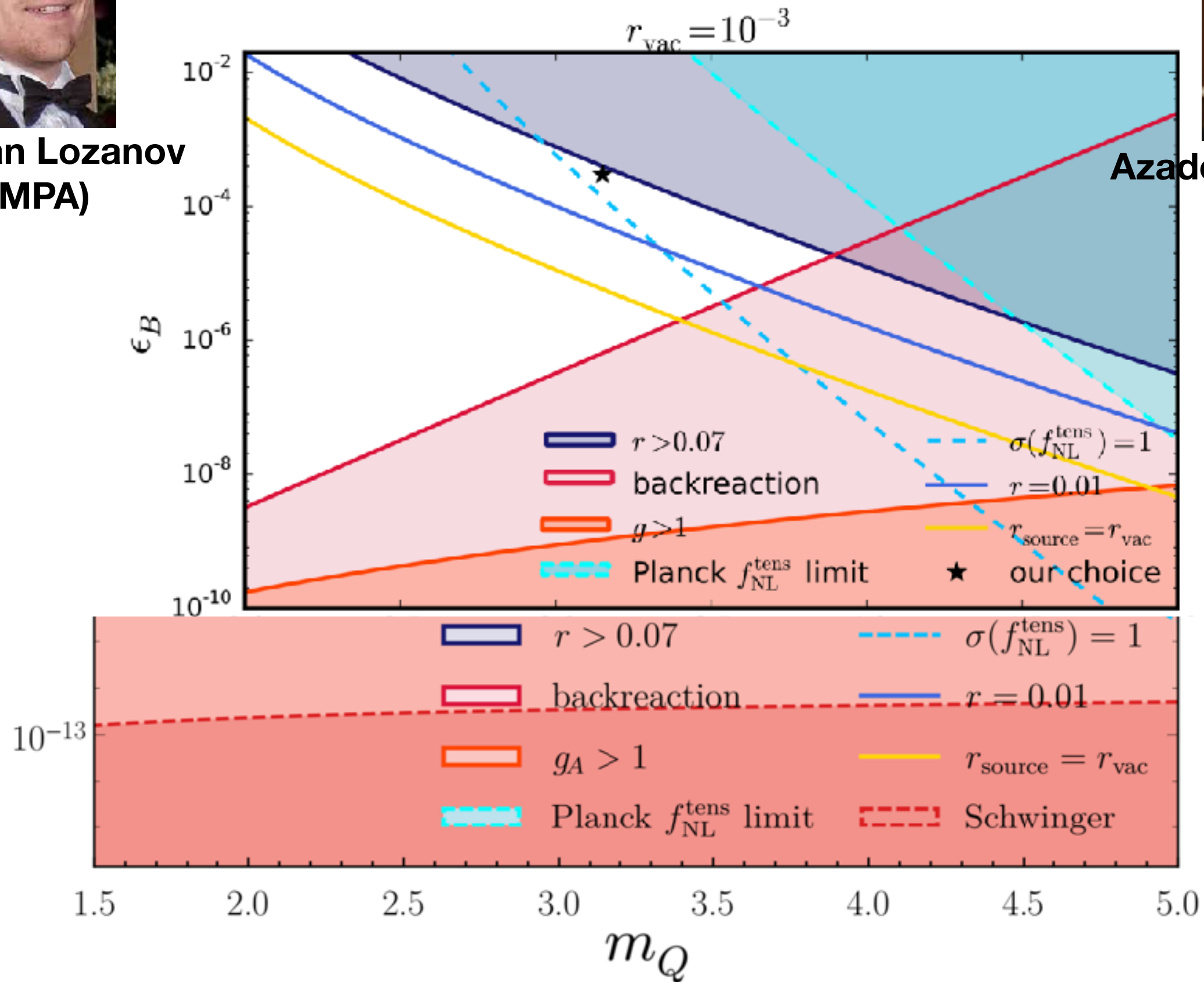
Schwinger Effect



Kaloian Lozanov
(MPA)



Azadeh Maleknejad
(MPA)



Contribution to ζ

$$\zeta = \frac{\sum_i \delta\rho_i}{3 \sum_i (\rho_i + P_i)} \approx \frac{\Omega_\phi \delta\rho_\phi / \rho_\phi + \Omega_\chi \delta\rho_\chi / \rho_\chi + \Omega_A \delta\rho_A / \rho_A}{2\epsilon}$$

- Most important channel: $A \rightarrow \chi \rightarrow \phi \rightarrow \zeta$

- Power spectrum of χ : $k^3 P_{\delta\chi}^{tt} / H^2 \approx g^2 (\Lambda/2)^2 e^{8m_Q}$
[$\Lambda = \lambda Q/f$]

This is
big!

- Power spectrum of ϕ :

$$k^3 P_{\delta\phi}^{tt} / H^2 \approx \epsilon_\phi \epsilon_\chi g^2 (\Lambda/2)^2 e^{8m_Q} \approx 7.5 \times 10^{-3}$$

$$\epsilon_\phi = 10^{-4}, \epsilon_\chi = 10^{-8}, g = 10^{-2}, \text{ and } m_Q = 3.45$$

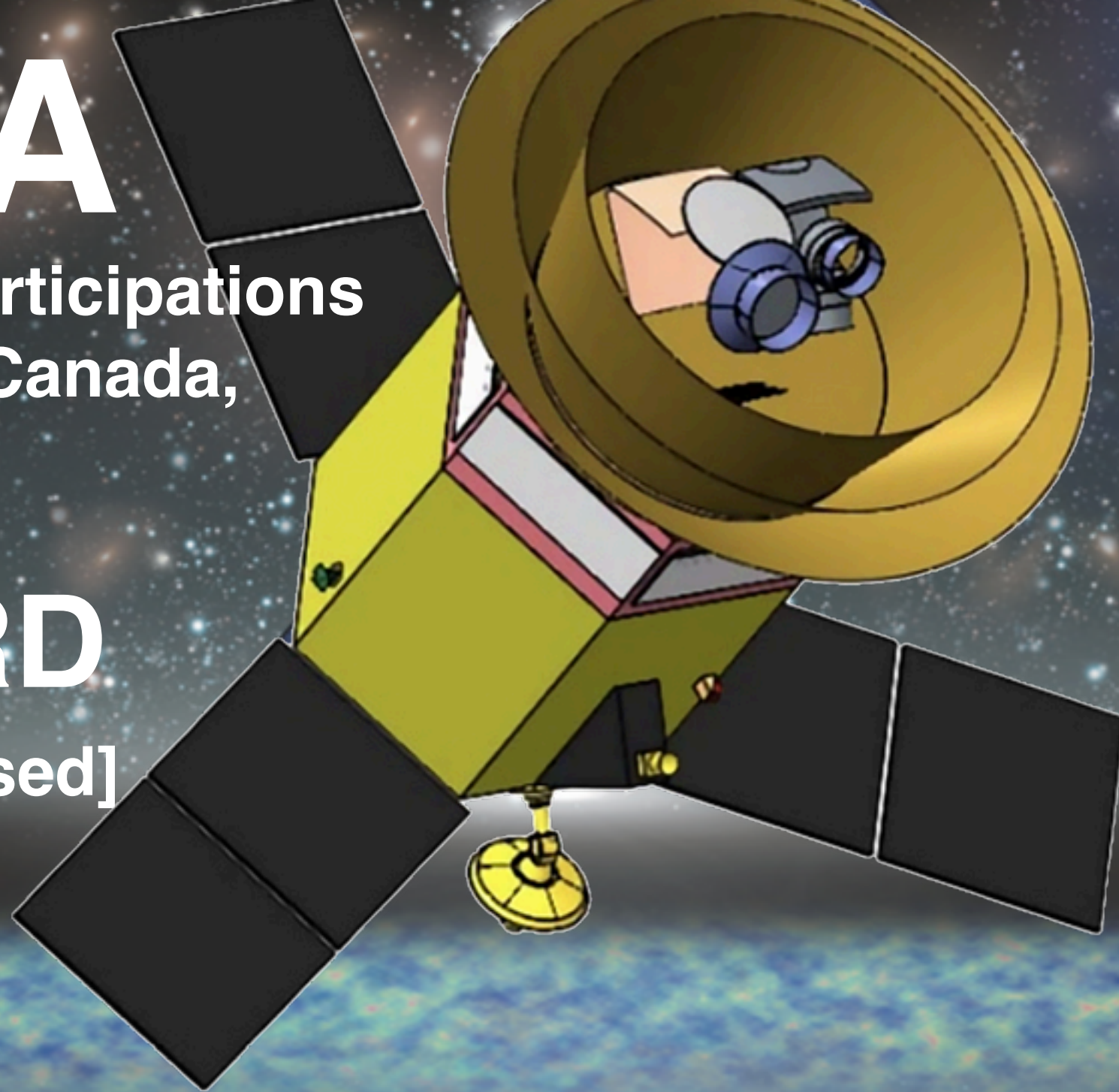
This is
small!

JAXA

+ possible participations
from USA, Canada,
Europe

LiteBIRD

2027– [proposed]



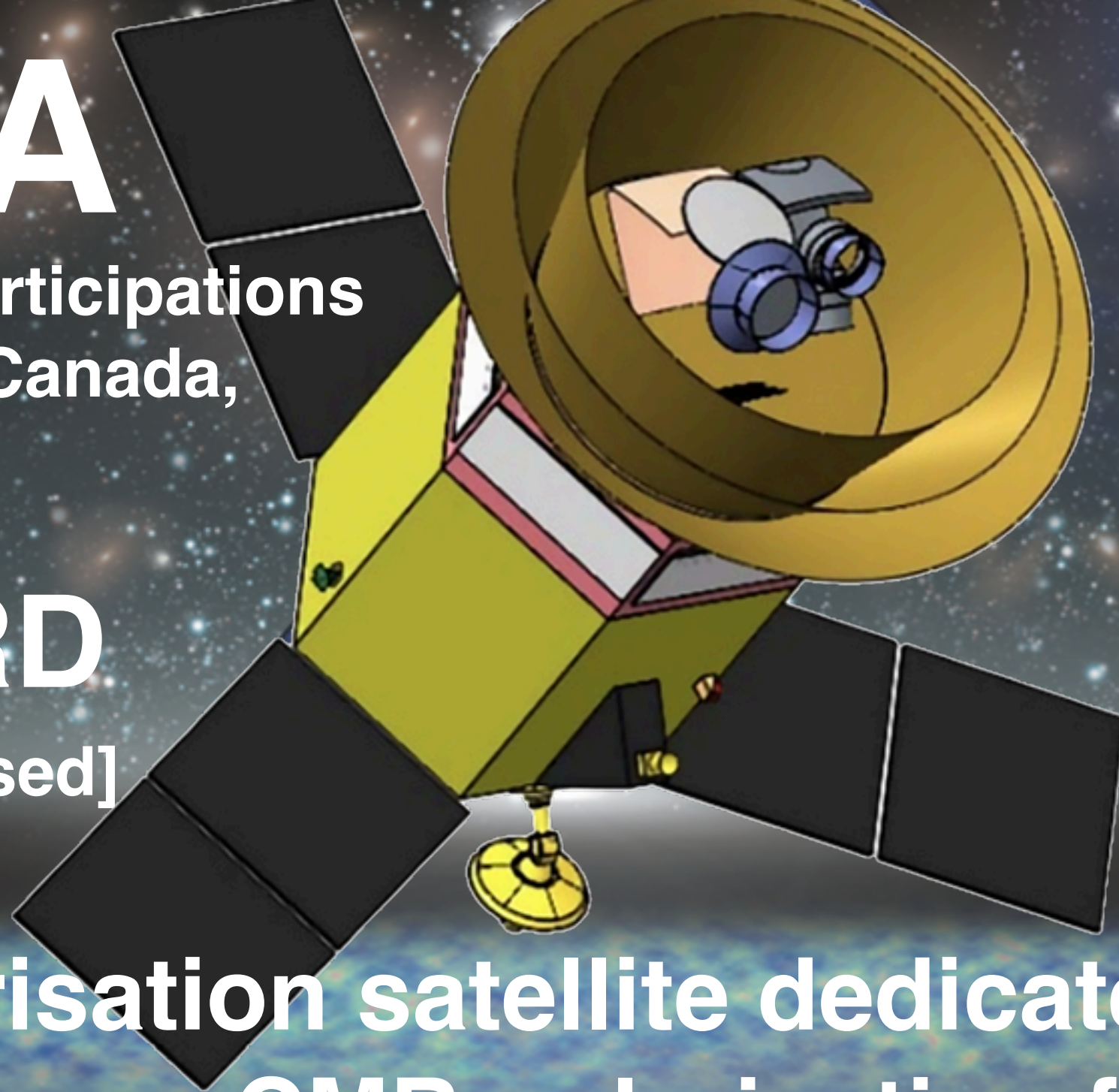
Target: $\delta r < 0.001$ (68%CL)

JAXA

+ possible participations
from USA, Canada,
Europe

LiteBIRD

2027– [proposed]



**Polarisation satellite dedicated to
measure CMB polarisation from
primordial GW, with a few thousand
super-conducting detectors in space**

JAXA

+ possible participations
from USA, Canada,
Europe

LiteBIRD

2027– [proposed]

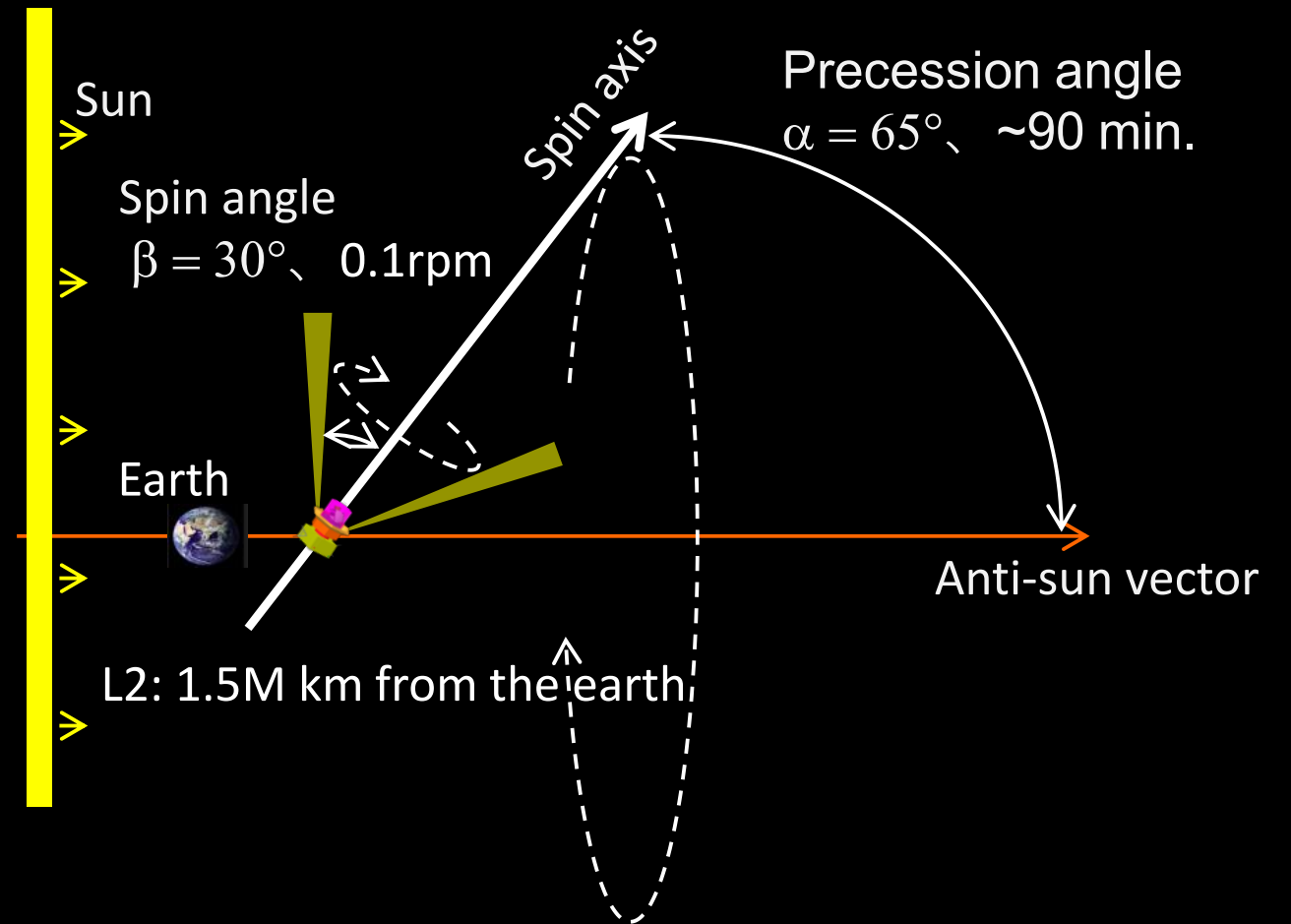


Down-selected by JAXA as
one of the two missions
competing for a launch in mid 2020's

Observation Strategy



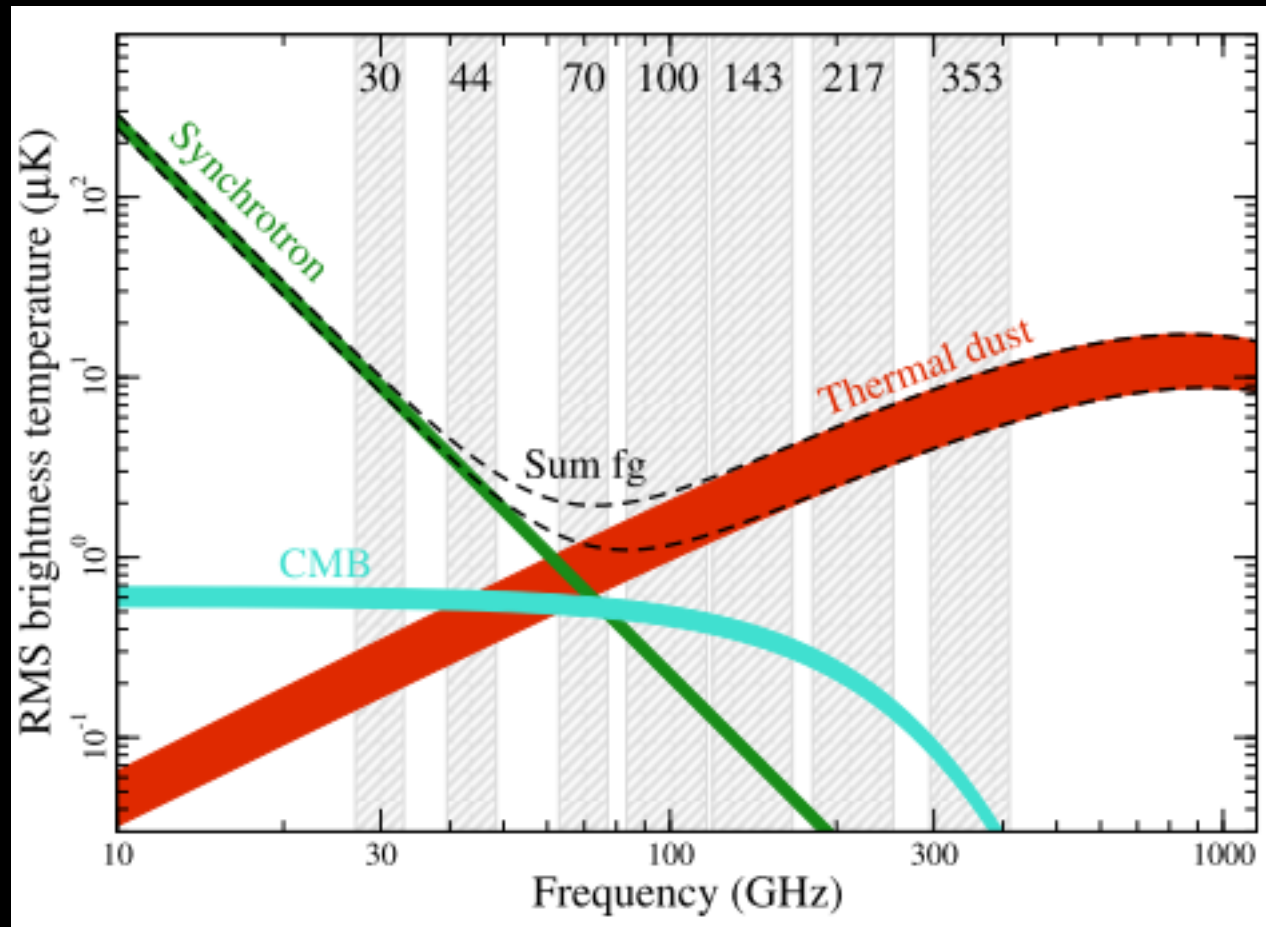
JAXA H3 Launch Vehicle (JAXA)



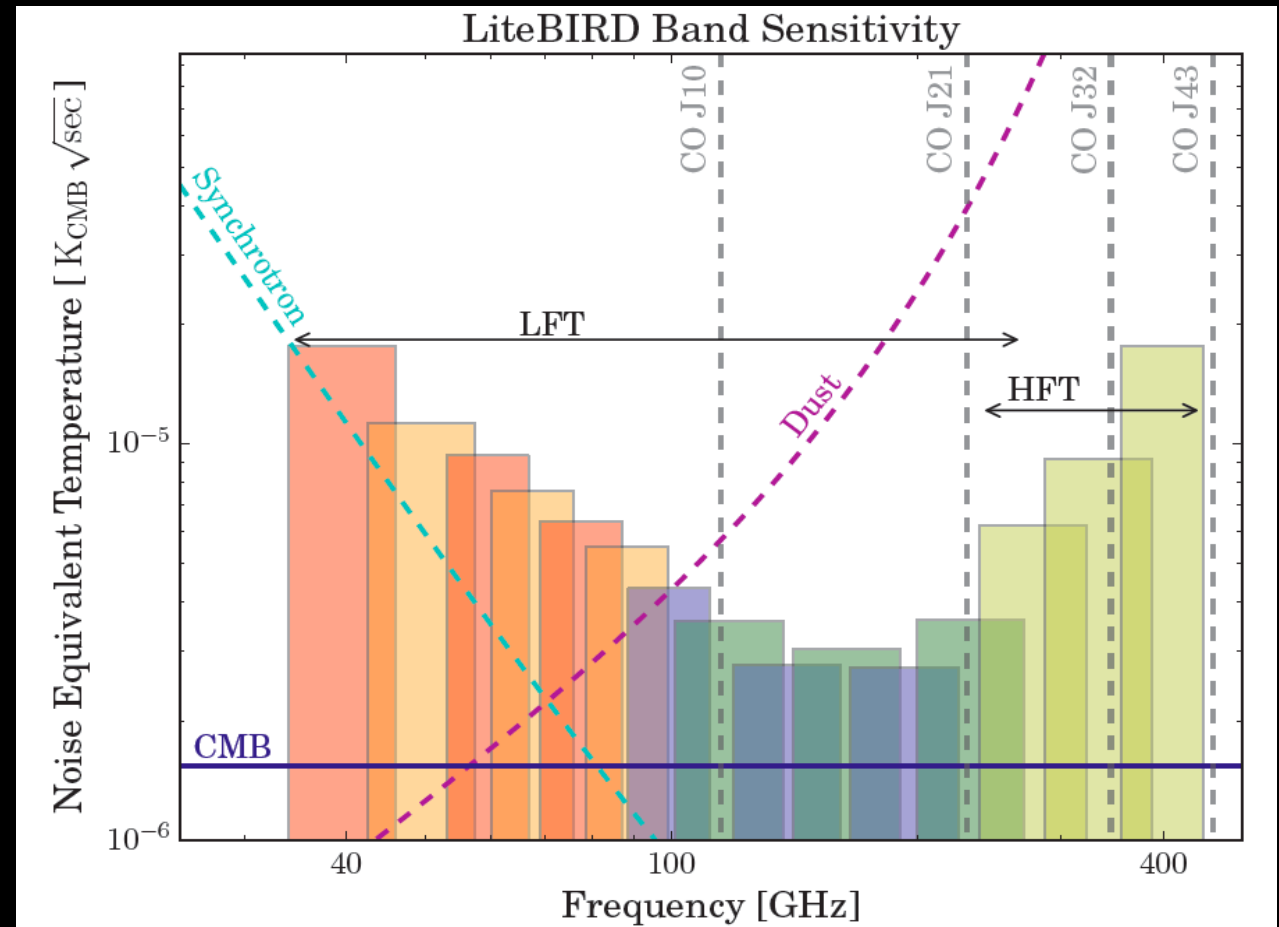
- Launch vehicle: **JAXA H3**
- Observation location: Second Lagrangian point (**L2**)
- Scan strategy: **Spin and precession, full sky**
- Observation duration: **3-years**
- Proposed launch date: **Mid 2020's**

Slide courtesy Toki Suzuki (Berkeley)

Foreground Removal



Polarized galactic emission (Planck X)

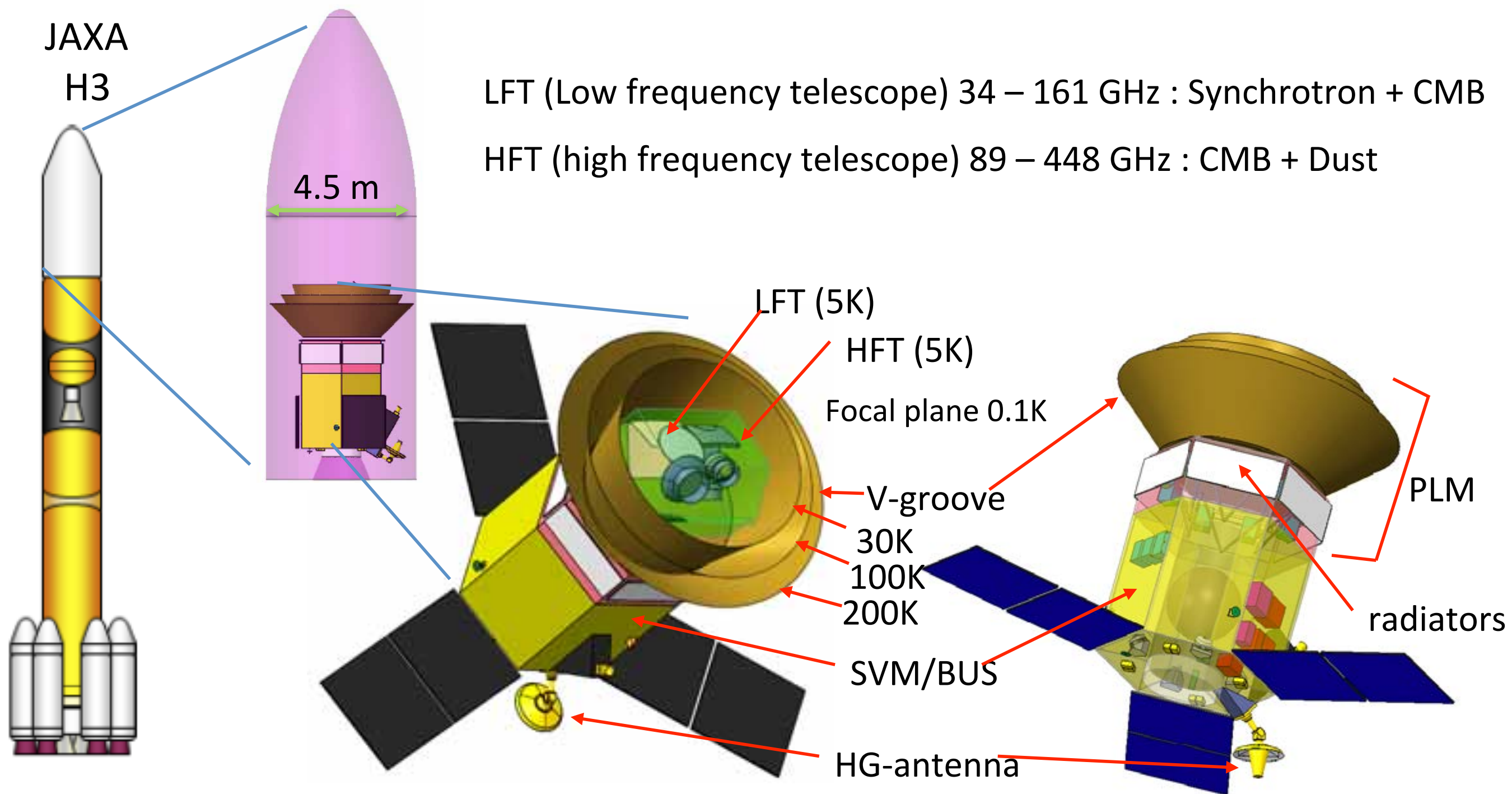
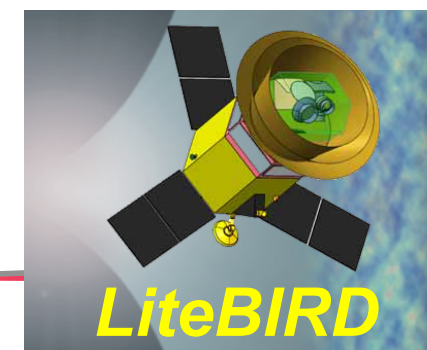


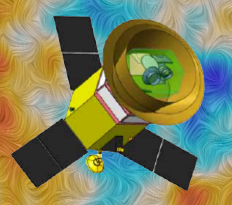
LiteBIRD: 15 frequency bands

- Polarized foregrounds
 - Synchrotron radiation and thermal emission from inter-galactic dust
 - Characterize and remove foregrounds
- 15 frequency bands between 40 GHz - 400 GHz
 - Split between Low Frequency Telescope (LFT) and High Frequency Telescope (HFT)
 - LFT: 40 GHz – 235 GHz
 - HFT: 280 GHz – 400 GHz

Slide courtesy Toki Suzuki (Berkeley)

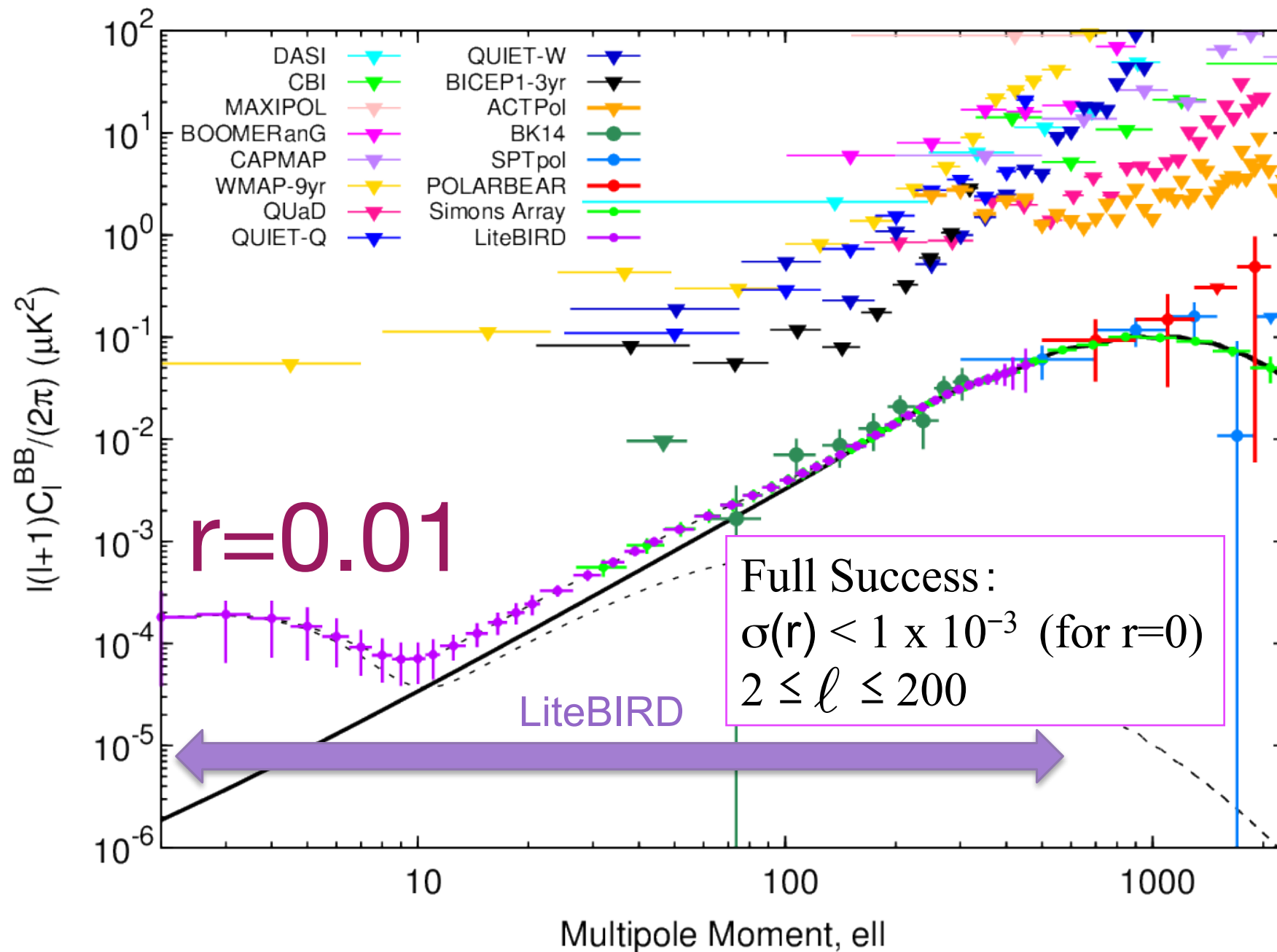
LiteBIRD Spacecraft





The Quest of the Primordial Gravitational Waves

LiteBIRD Expectation



Slide courtesy Ludovic Montier

Summary

- **Next frontier**: Using CMB polarisation to find GWs from inflation. **Critical test of the physics of inflation!**
- With LiteBIRD we plan to reach $r \sim 10^{-3}$, i.e., 100 times better than the current bound
- GW from vacuum or sources? An exciting window to new physics
- Check not only for scale invariant, but also for chirality and **non-Gaussianity**