

Aspects of Dark Matter Axion Clumps

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Cosmo18 IBS, August 30 2018

QCD-Axion

$$\Delta\mathcal{L}_{qcd} \sim \theta G_{\mu\nu} \tilde{G}^{\mu\nu}$$

$$|\theta| \lesssim 10^{-10}$$

QCD-Axion

$$\Delta\mathcal{L}_{qcd} \sim \theta G_{\mu\nu} \tilde{G}^{\mu\nu} \quad |\theta| \lesssim 10^{-10}$$

(Peccei-Quinn, Weinberg, Wilczek)


$$\Delta\mathcal{L}_a \sim \frac{\phi}{f_a} G_{\mu\nu} \tilde{G}^{\mu\nu} + \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

Periodic Potential; Expanded:

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \dots$$

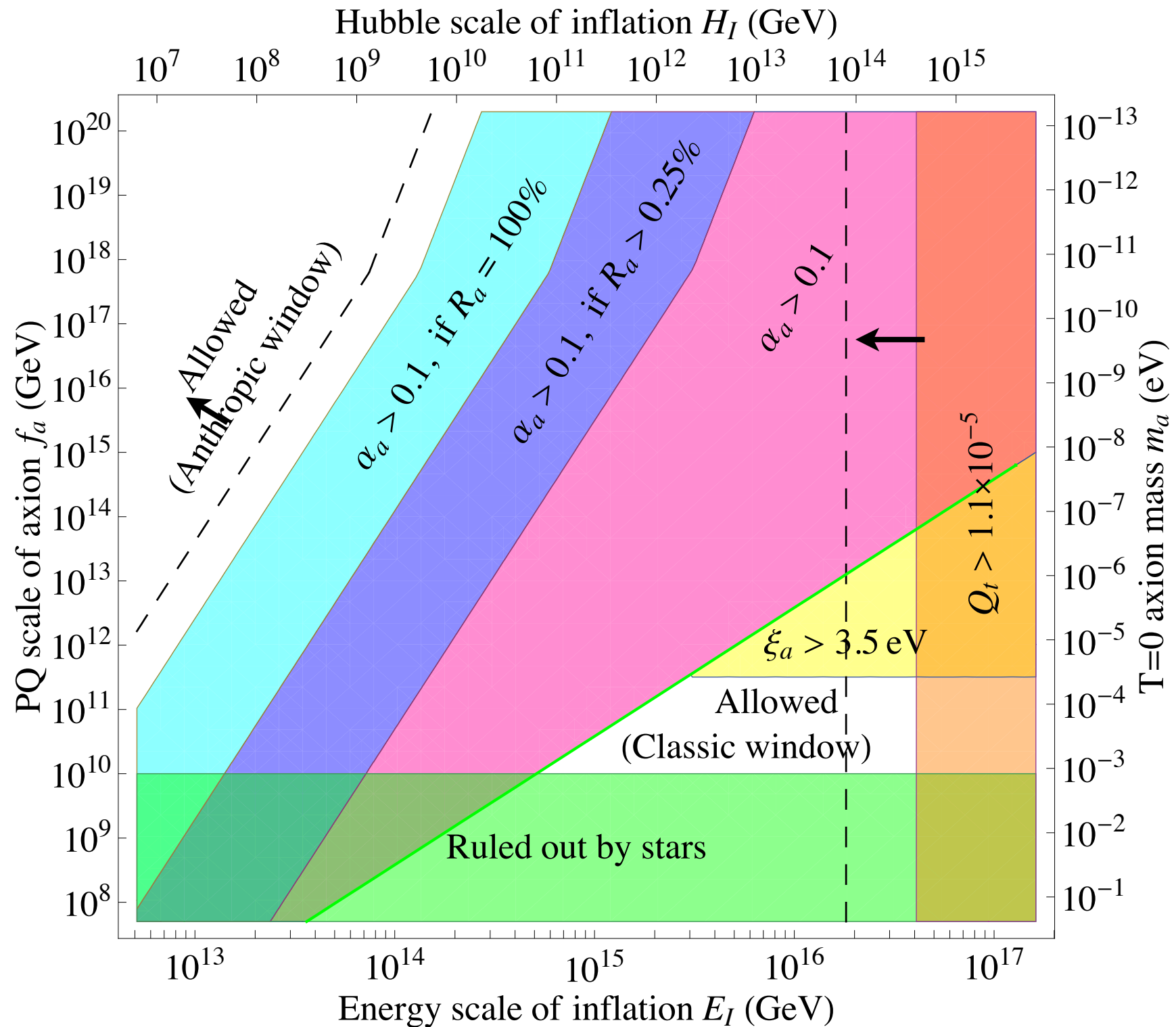
Axion mass:

$$m_a \sim \frac{\Lambda_{qcd}^2}{f_a}$$

(Attractive) Self-Coupling:

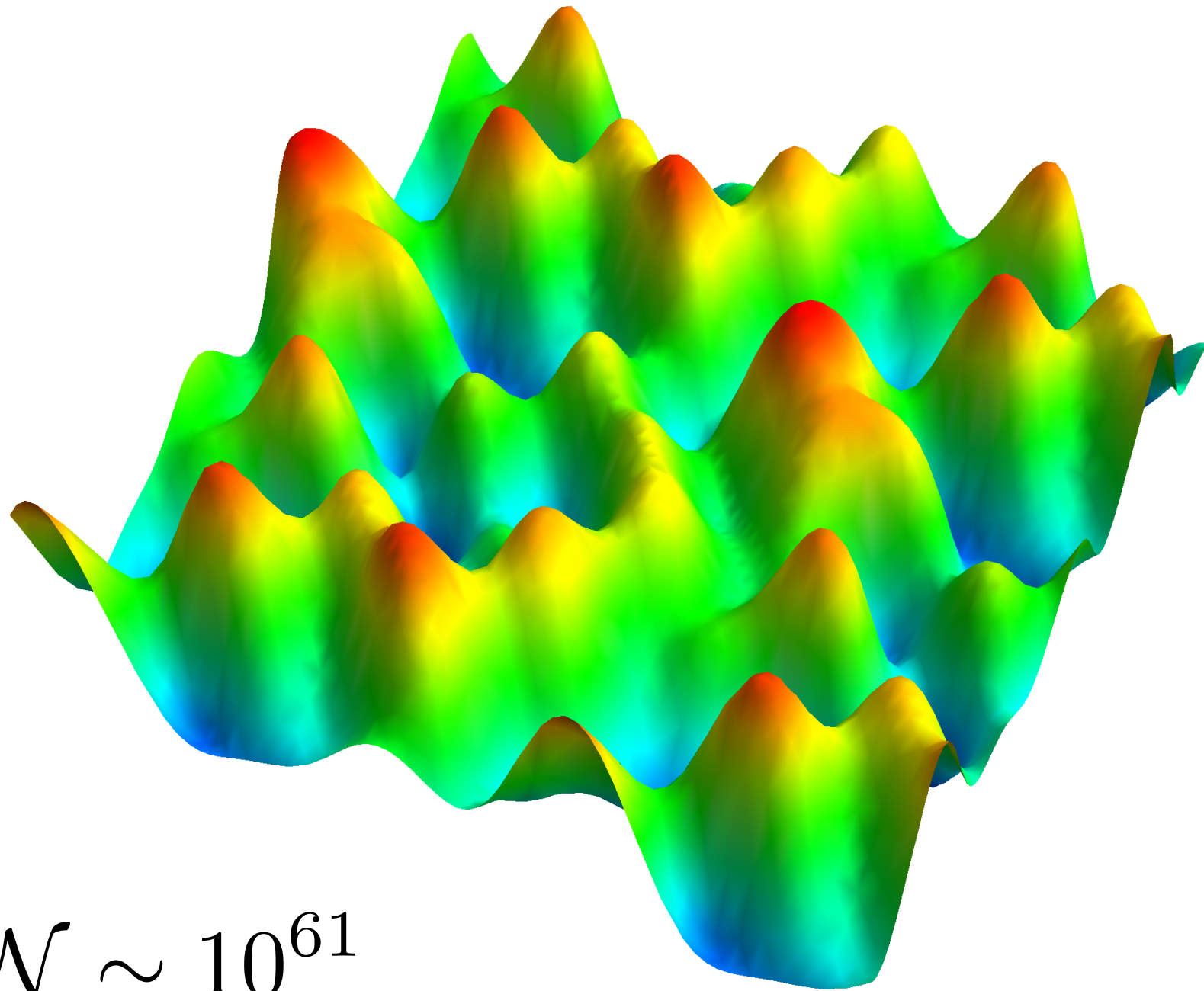
$$\lambda \sim -\frac{\Lambda_{qcd}^4}{f_a^4}$$

QCD-Axion Allowed Windows



Focus on Classic Window

In Classic Window; Axion Initial Distribution



$$\mathcal{N} \sim 10^{61}$$

Consider Non-Relativistic Behavior $\phi \rightarrow \psi$

Hamiltonian

$$\hat{H} = \hat{H}_{\text{kin}} + \hat{H}_{\text{int}} + \hat{H}_{\text{grav}}$$

$$\hat{H}_{\text{kin}} = \int d^3x \frac{1}{2m} \nabla \hat{\psi}^\dagger \cdot \nabla \hat{\psi}$$

$$\hat{H}_{\text{int}} = \int d^3x \frac{\lambda}{16m^2} \hat{\psi}^\dagger \hat{\psi}^\dagger \hat{\psi} \hat{\psi}$$

$$\hat{H}_{\text{grav}} = -\frac{Gm^2}{2} \int d^3x \int d^3x' \frac{\hat{\psi}^\dagger(\mathbf{x}) \hat{\psi}^\dagger(\mathbf{x}') \hat{\psi}(\mathbf{x}) \hat{\psi}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}$$

Number Density

$$\hat{n}(\mathbf{x}) = \hat{\psi}^\dagger(\mathbf{x}) \hat{\psi}(\mathbf{x})$$

(For rigorous treatment: Namjoo, Guth, Kaiser 2017)

Gravitational Thermalization?

Equation of Motion

$$i \dot{\psi} = -\frac{1}{2m} \nabla^2 \psi + \frac{\lambda}{8m^2} |\psi|^2 \psi - Gm^2 \psi \int d^3x' \frac{|\psi(\mathbf{x}')|^2}{|\mathbf{x} - \mathbf{x}'|}$$

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Interaction Rate of Modes

$$\Gamma_k \sim \frac{8\pi G m^2 n_{\text{ave}}}{k^2}$$

Collective rate from coherent Bosons

Much greater than dilute bosons

$$\Gamma \sim \frac{G^2 m^2 n_{\text{ave}}}{v^3}$$

- Sikivie, Yang (2009), Erken, Sikivie, Tam, Yang (2011)

Gravitational Thermalization?

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Interaction Rate of Modes

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Collective rate from coherent Bosons

Thermalization $\Gamma_k > H$ at late times (BEC)

- Sikivie, Yang (2009), Erken, Sikivie, Tam, Yang (2011)

Axion BEC Literature

- Sikivie, Yang (2009)
- Erken, Sikivie, Tam, Yang (2011)
- Chavanis (2012)
- Banik, Sikivie (2013)
- Davidson, Elmer (2013)
- Nouri, Saikawa, Sato, Yamaguchi (2014)
- Vega, Sanchez (2014)
- Li, Rindler-Daller, Shapiro (2014)
- Berges, Haeckel (2014)
- Banik, Christopherson, Sikivie, Todarello (2015)
- Davidson (2015)
-

Classical Description of BEC Phase Transition

Free Theory

$$F[\psi] = \int \frac{d^3 k}{(2\pi)^3} \left[\frac{k^2}{2m} - \mu(T) \right] |\psi_k|^2$$

Number

$$\langle N \rangle = \frac{\int \mathcal{D}\psi N[\psi] \exp(-F[\psi]/T)}{\int \mathcal{D}\psi \exp(-F[\psi]/T)}$$

Density

$$n_{\text{th}} = \int \frac{d^3 k}{(2\pi)^3} \frac{T}{\frac{k^2}{2m} - \mu(T)}$$

Critical Temperature

$$T_{\text{crit}} = \frac{\pi^2 n_{\text{tot}}}{m k_{\text{UV}}}$$

Classical Description of BEC Phase Transition

While BEC is a very quantum phenomenon
from the PARTICLE point of view

BEC is a very classical phenomenon
from the FIELD point of view

Classical vs Quantum with Interactions

What About Interactions?

Fundamental claim of Sikivie, Todarello, 1607.00949

On time scales $t > \tau = 1/\Gamma$ the classical description breaks down, requiring the full quantum theory, which is the only way to see thermalization

Toy Model

Second Quantized Language

$$\hat{H} = \sum_i \omega_i \hat{a}_i^\dagger \hat{a}_i + \frac{1}{4} \sum_{ijkl} \Lambda_{ij}^{kl} \hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_k \hat{a}_l,$$

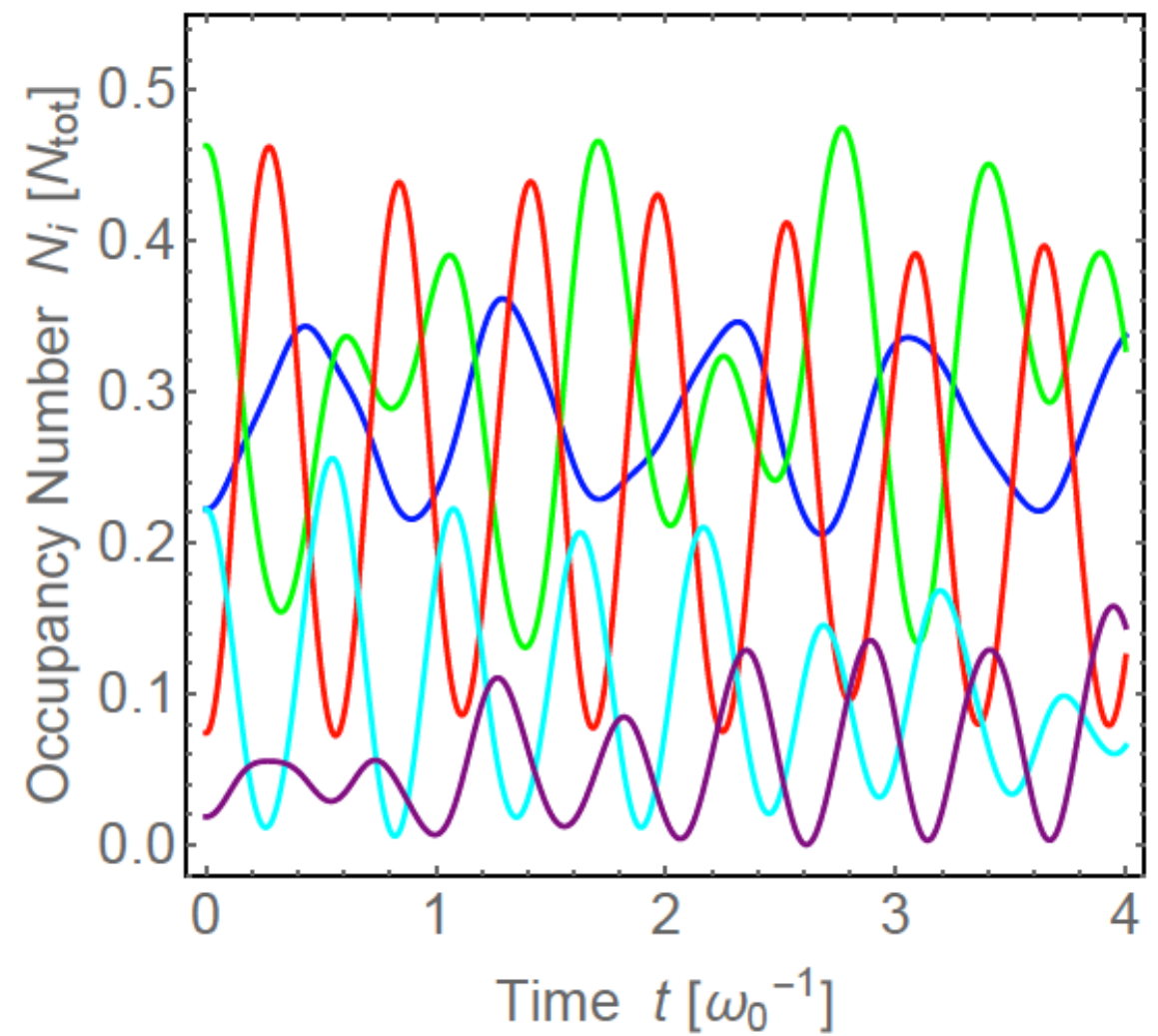
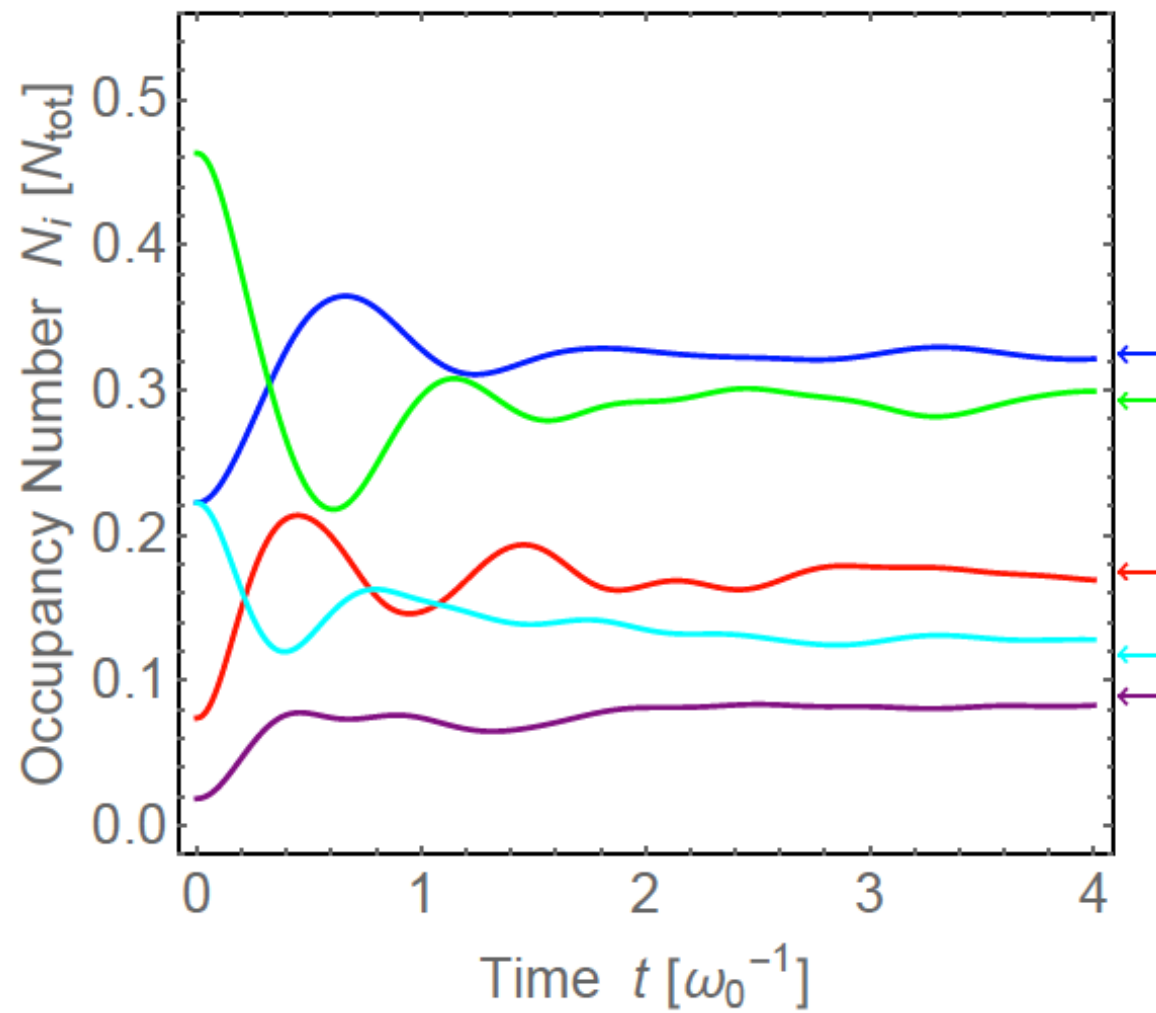
Consider just 5 oscillators for simplicity

Initial quantum state $|\{N_i\}\rangle = |12, 25, 4, 12, 1\rangle$

Initial classical state $a_i = \sqrt{N_i}$

Sikivie, Todarello, 1607.00949

Quantum vs Classical??



Sikivie, Todarello, 1607.00949

Correct Classical Treatment

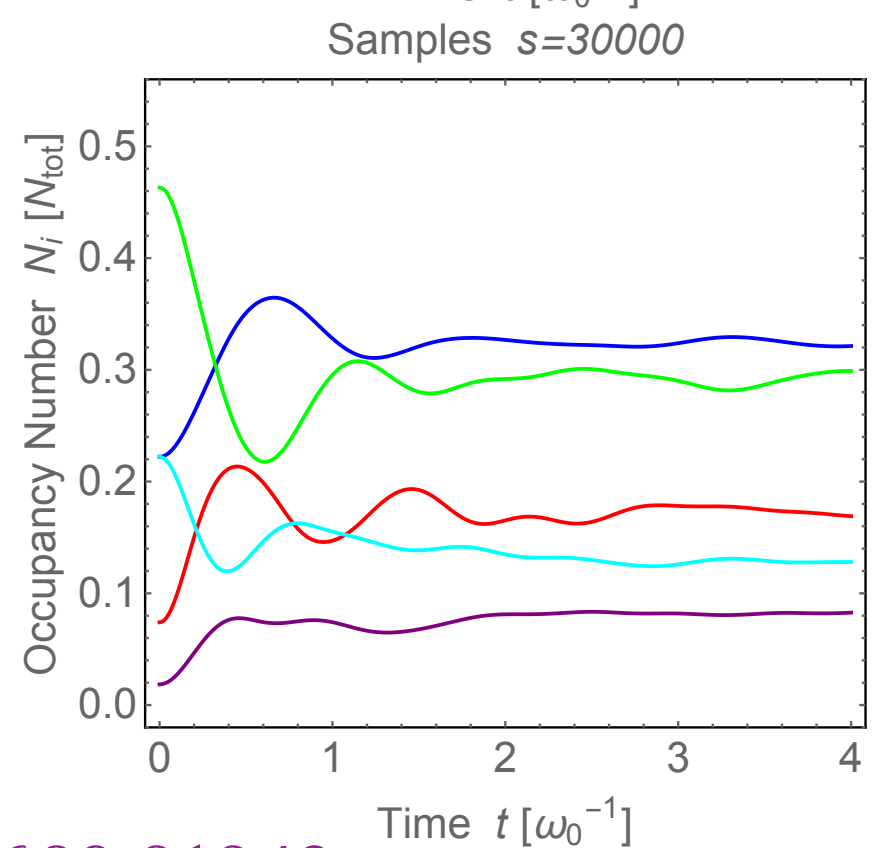
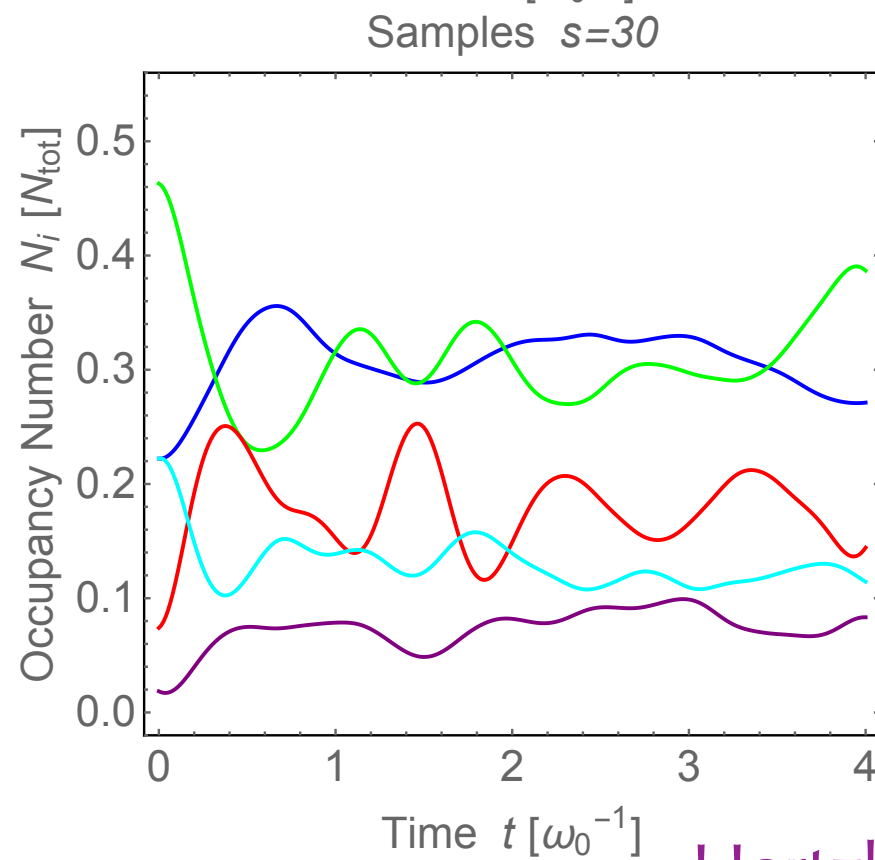
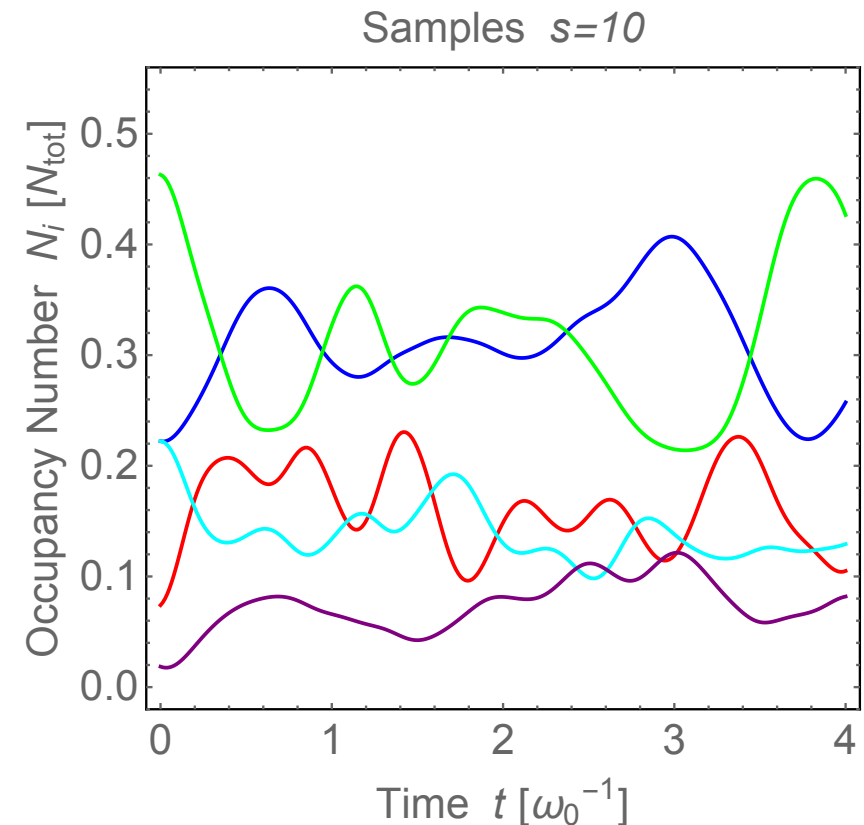
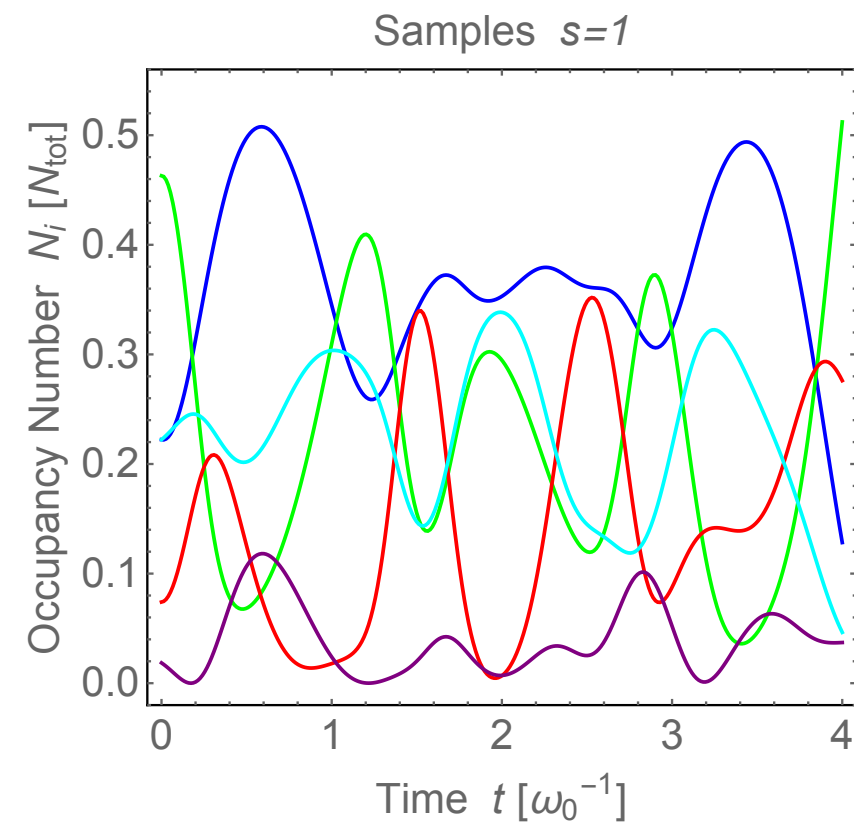
Initial classical state $a_i = \sqrt{N_i} e^{I\theta_i}, \quad \theta_i \in [0, 2\pi)$

Ensemble average over random initial phases

Meaningful comparison

Connects to uncertainty in branch of wavefunction

Correct Classical Treatment



Hertzberg 1609.01342

Implication for Correlation Functions

Implication for Correlation Functions

At high occupancy

$$\langle \{N_i\} | \hat{\psi}^\dagger(\mathbf{x}, t) \hat{\psi}(\mathbf{y}, t) | \{N_i\} \rangle \approx \langle \psi^*(\mathbf{x}, t) \psi(\mathbf{y}, t) \rangle_{ens}$$

Ergodic theorem

$$\langle \psi^*(\mathbf{x}, t) \psi(\mathbf{y}, t) \rangle_{ens} = \frac{1}{V} \int_V d^3 z \psi_\mu^*(\mathbf{x} + \mathbf{z}, t) \psi_\mu(\mathbf{y} + \mathbf{z}, t)$$

Implication for Correlation Functions

Correlation functions of quantum and classical micro-states agree at high occupancy, despite the spread of the quantum wave-function in these chaotic systems

(Note: this is not some trivial consequence of Ehrenfest theorem; more akin to billiard balls which exhibit chaos)

Implication for Axion Dark Matter

Statistically, axions are well described by classical field theory, after all

What is the BEC?

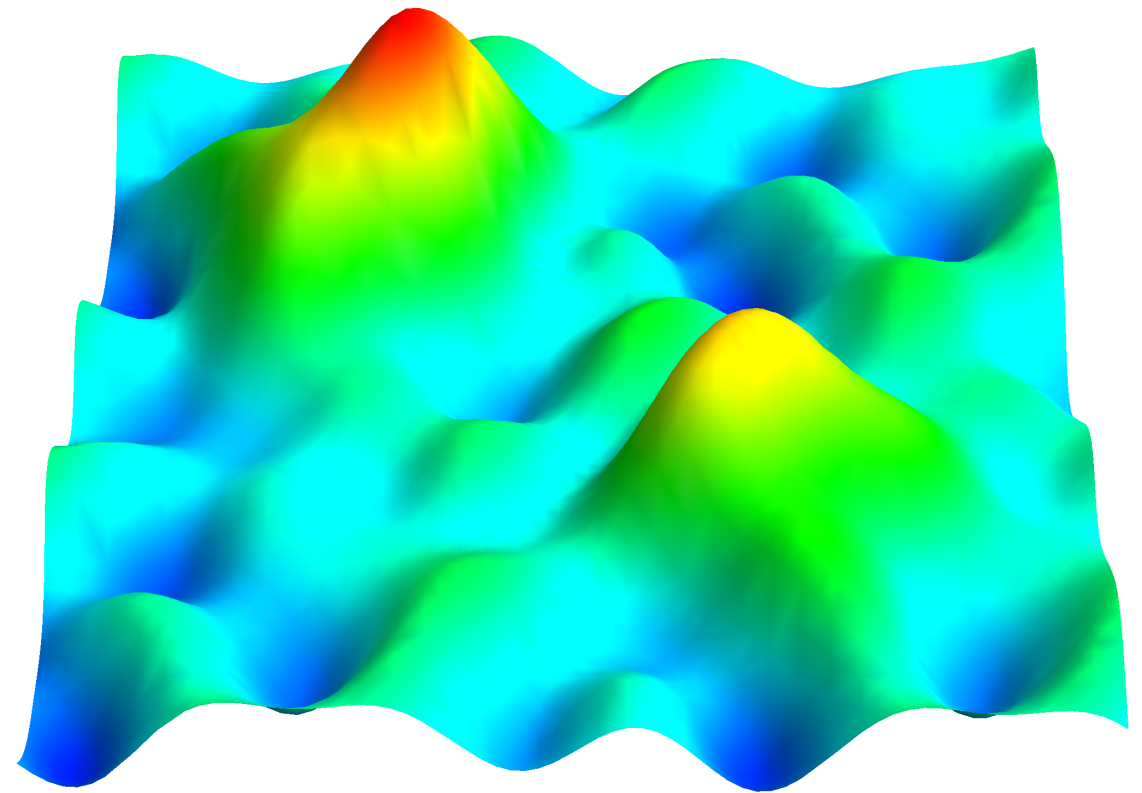
Implication for Axion Dark Matter

Statistically, axions are well described by classical field theory, after all

What is the BEC?

Small clumps
(miniclusters)

that may populate
the galaxy



Axion Clumps in Detail

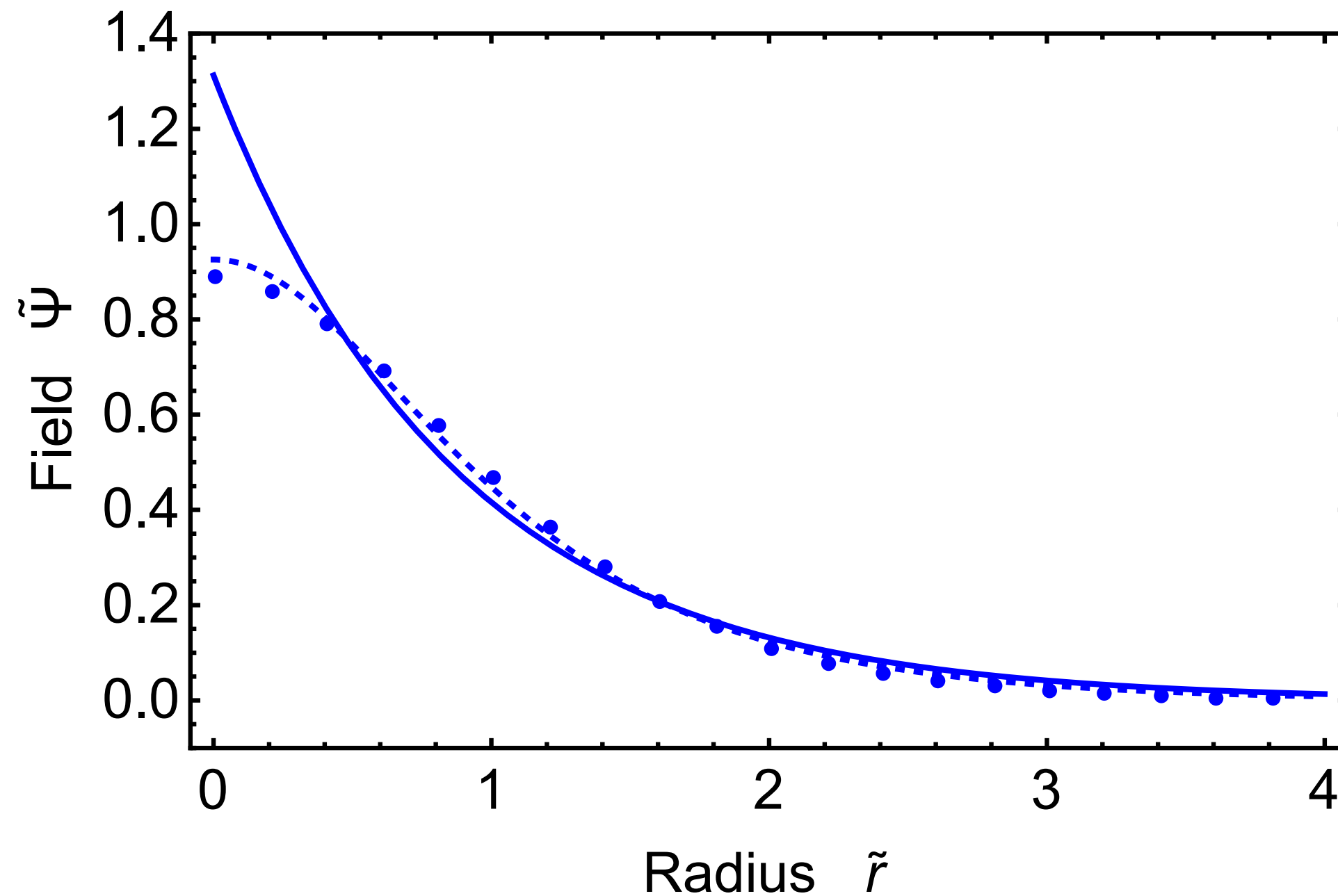
Return to Non-Relativistic Classical Field Theory

Equation of Motion

$$i \dot{\psi} = -\frac{1}{2m} \nabla^2 \psi + \frac{\lambda}{8m^2} |\psi|^2 \psi - Gm^2 \psi \int d^3x' \frac{|\psi(\mathbf{x}')|^2}{|\mathbf{x} - \mathbf{x}'|}$$

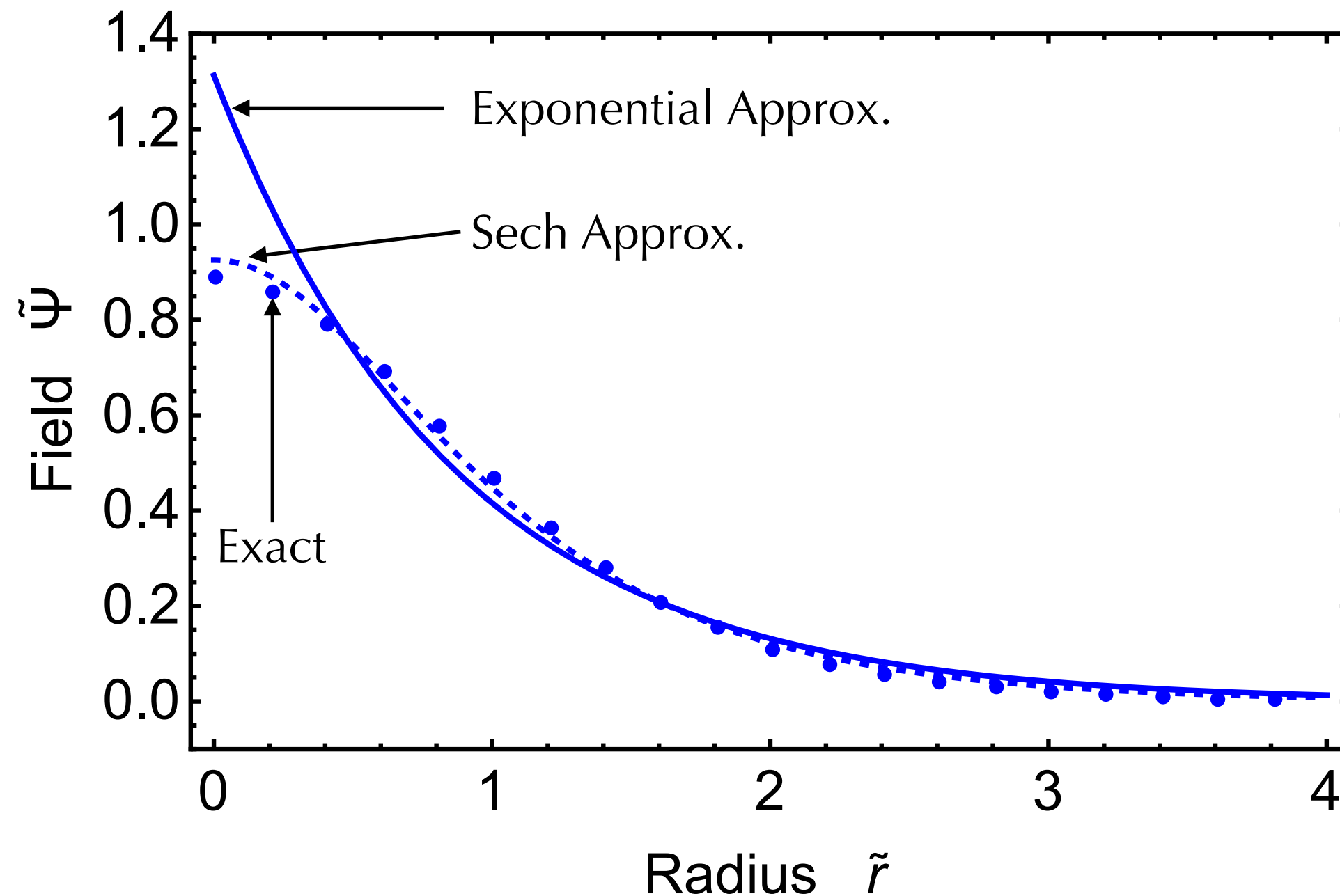
$$(\lambda < 0)$$

Clump Solutions (BEC) at fixed N

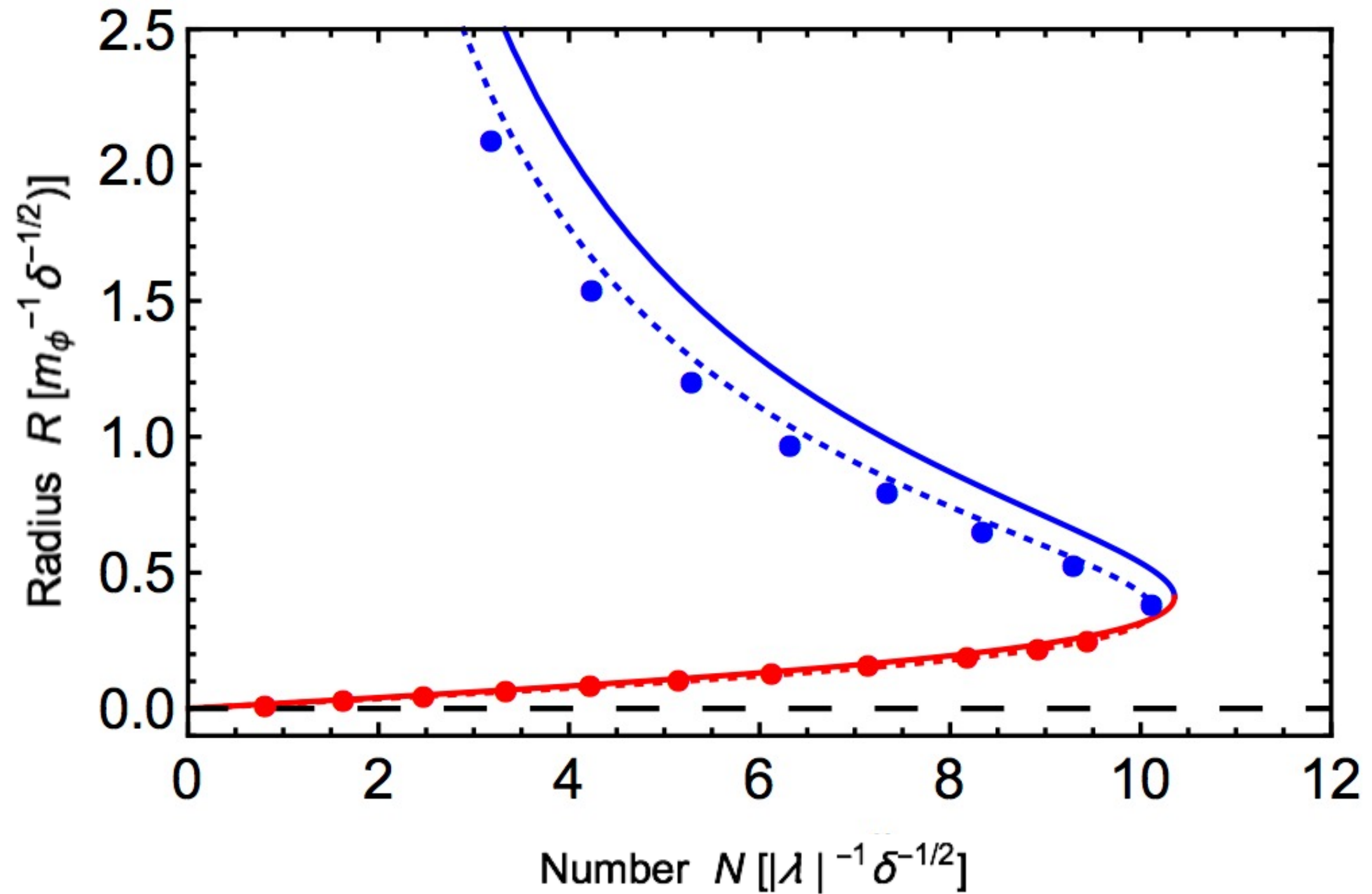


Schiappacasse, Hertzberg 1710.04729

Clump Solutions (BEC) at fixed N

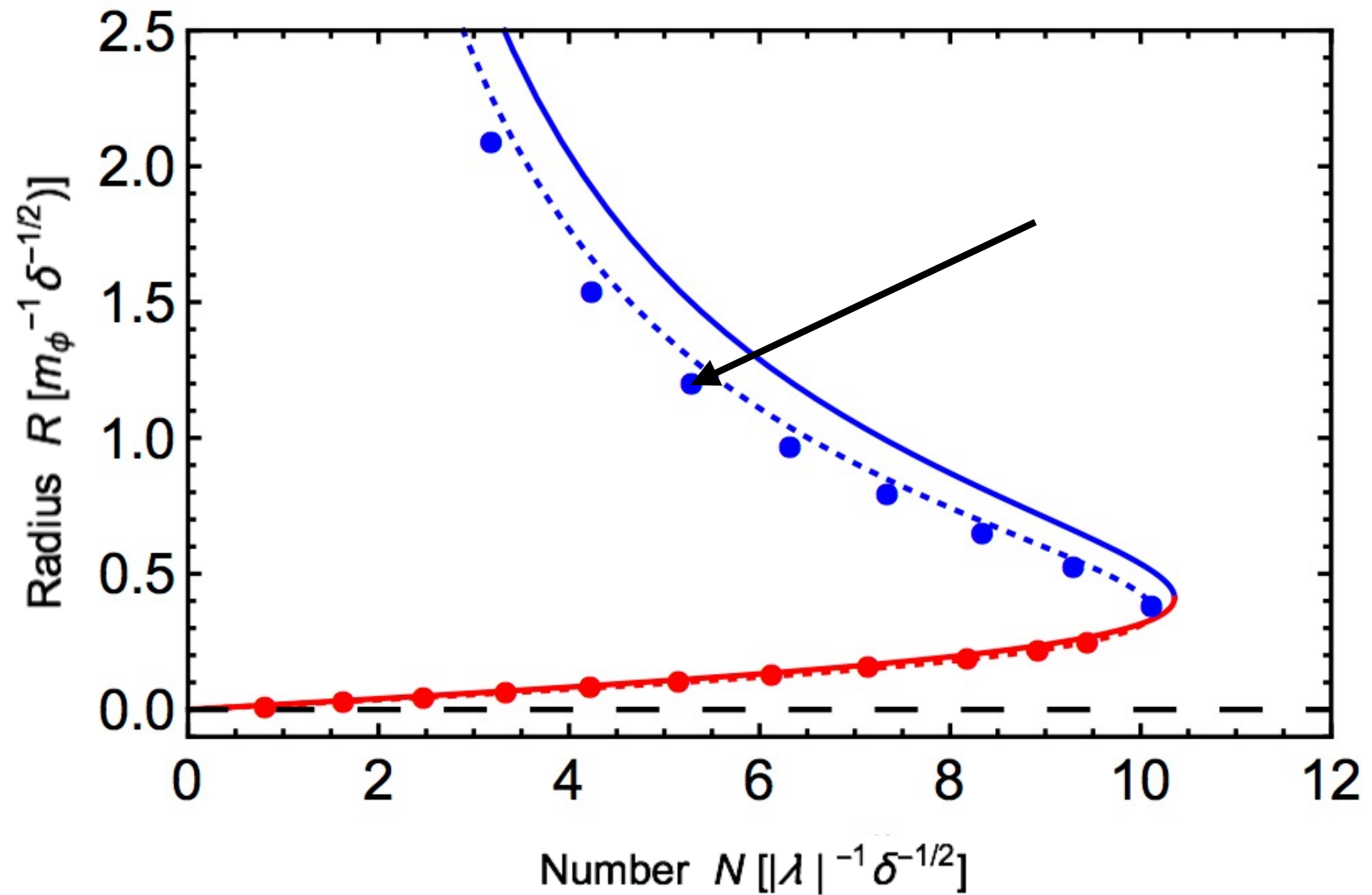


Two Branches of Solutions



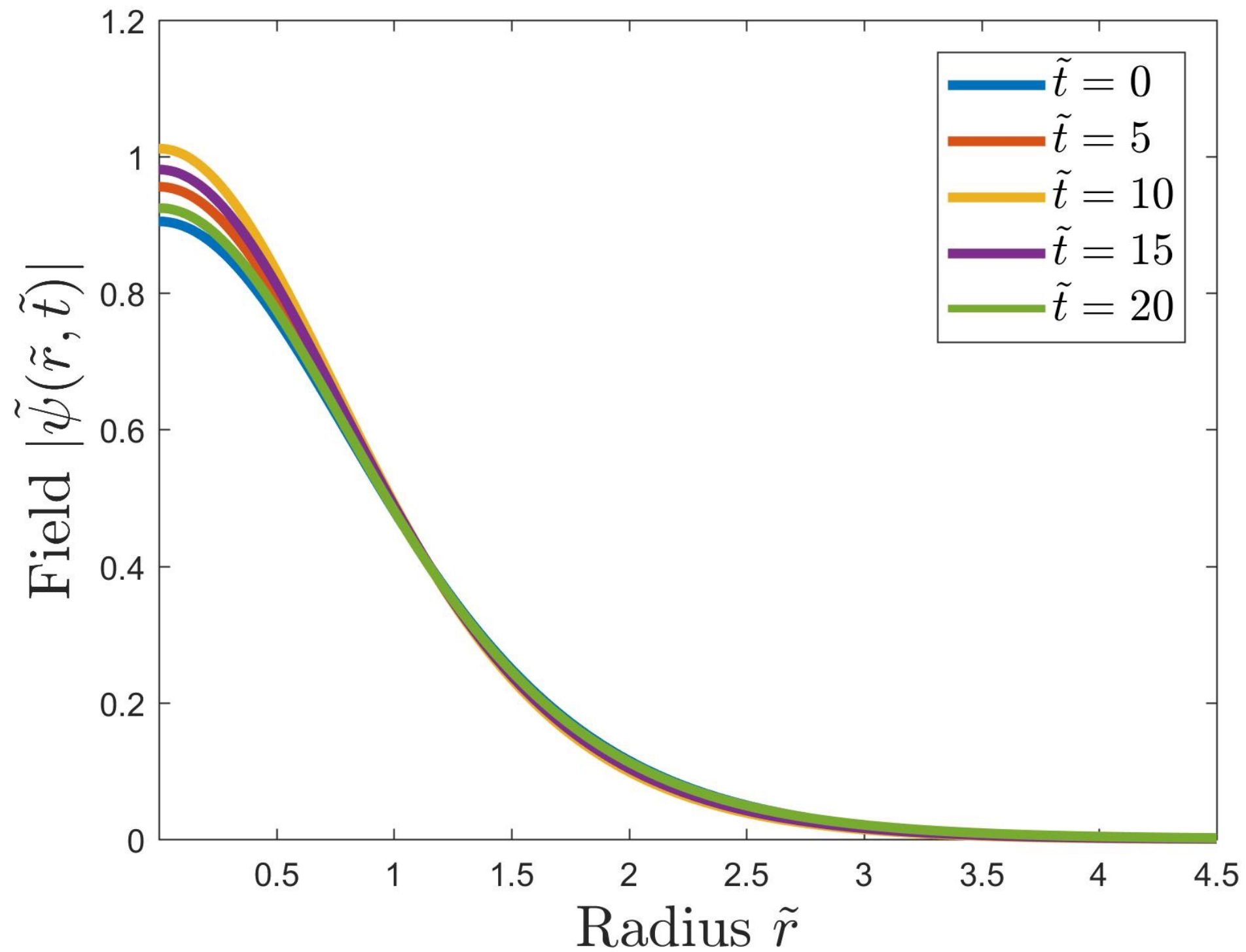
Schiappacasse, Hertzberg 1710.04729

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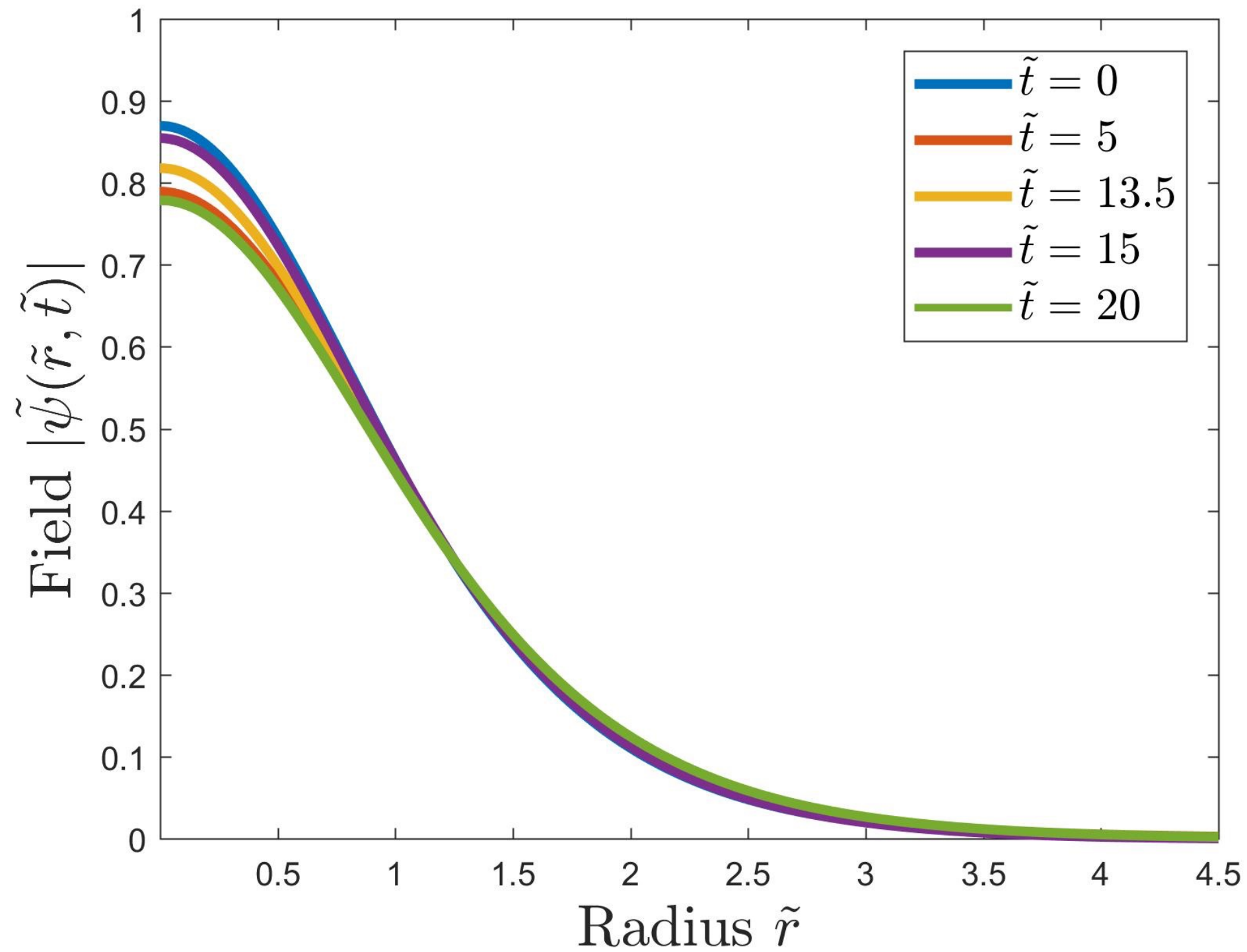
Schiappacasse, Hertzberg 1710.04729

Perturbing Upper Branch



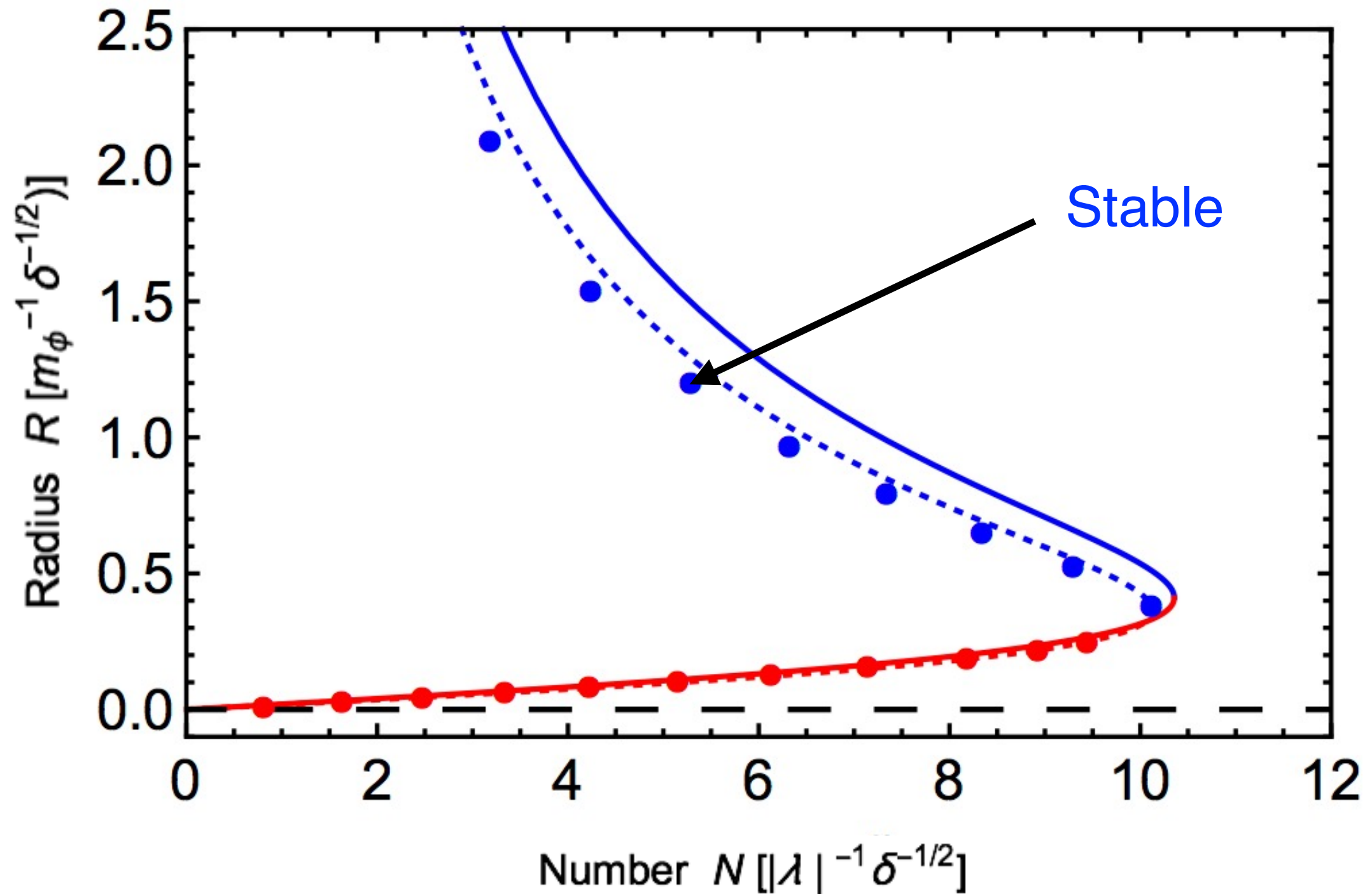
Schiappacasse, Hertzberg 1710.04729

Perturbing Upper Branch



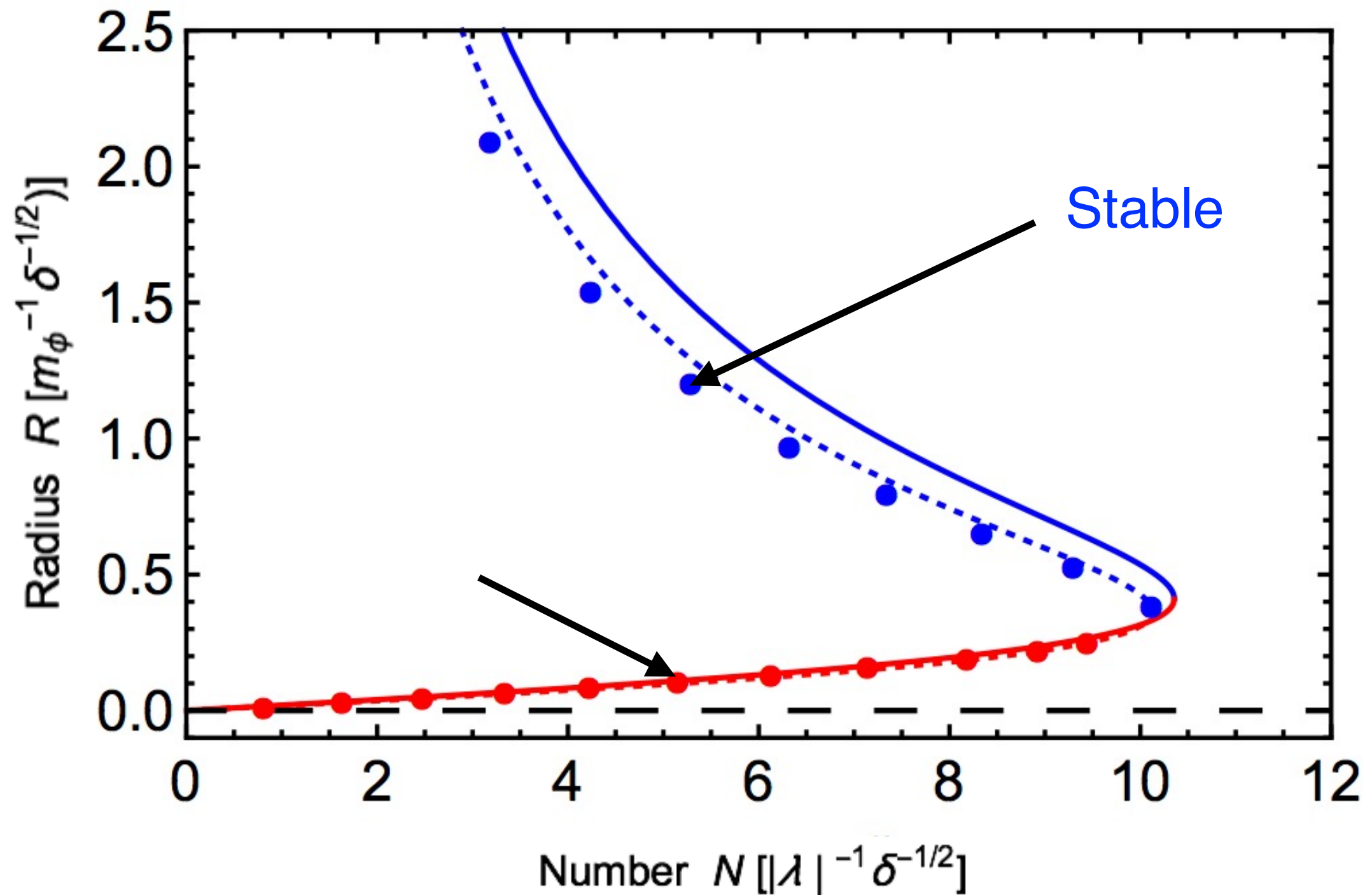
Schiappacasse, Hertzberg 1710.04729

Two Branches of Solutions



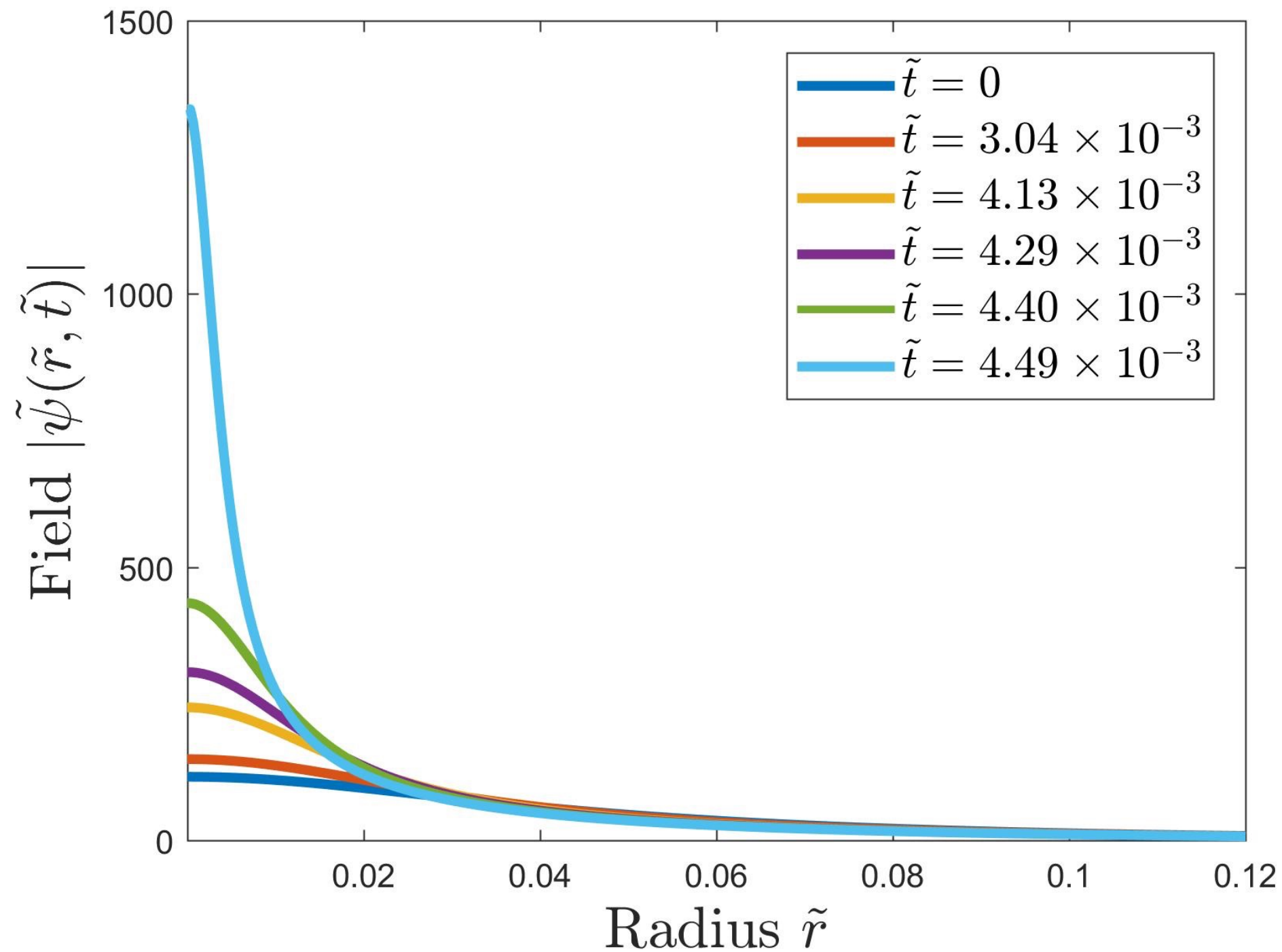
Schiappacasse, Hertzberg 1710.04729

Two Branches of Solutions



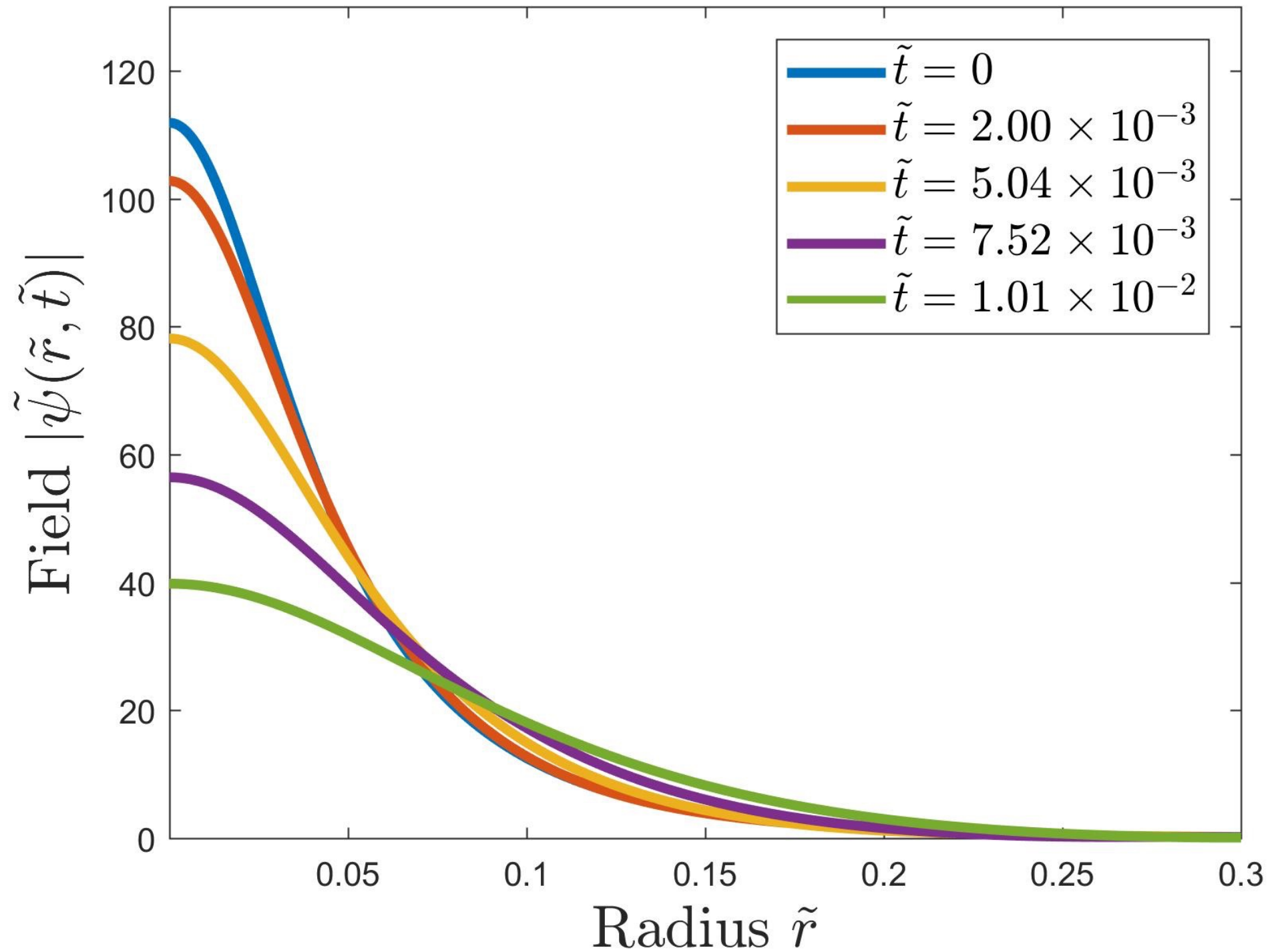
Schiappacasse, Hertzberg 1710.04729

Perturbing Lower Branch



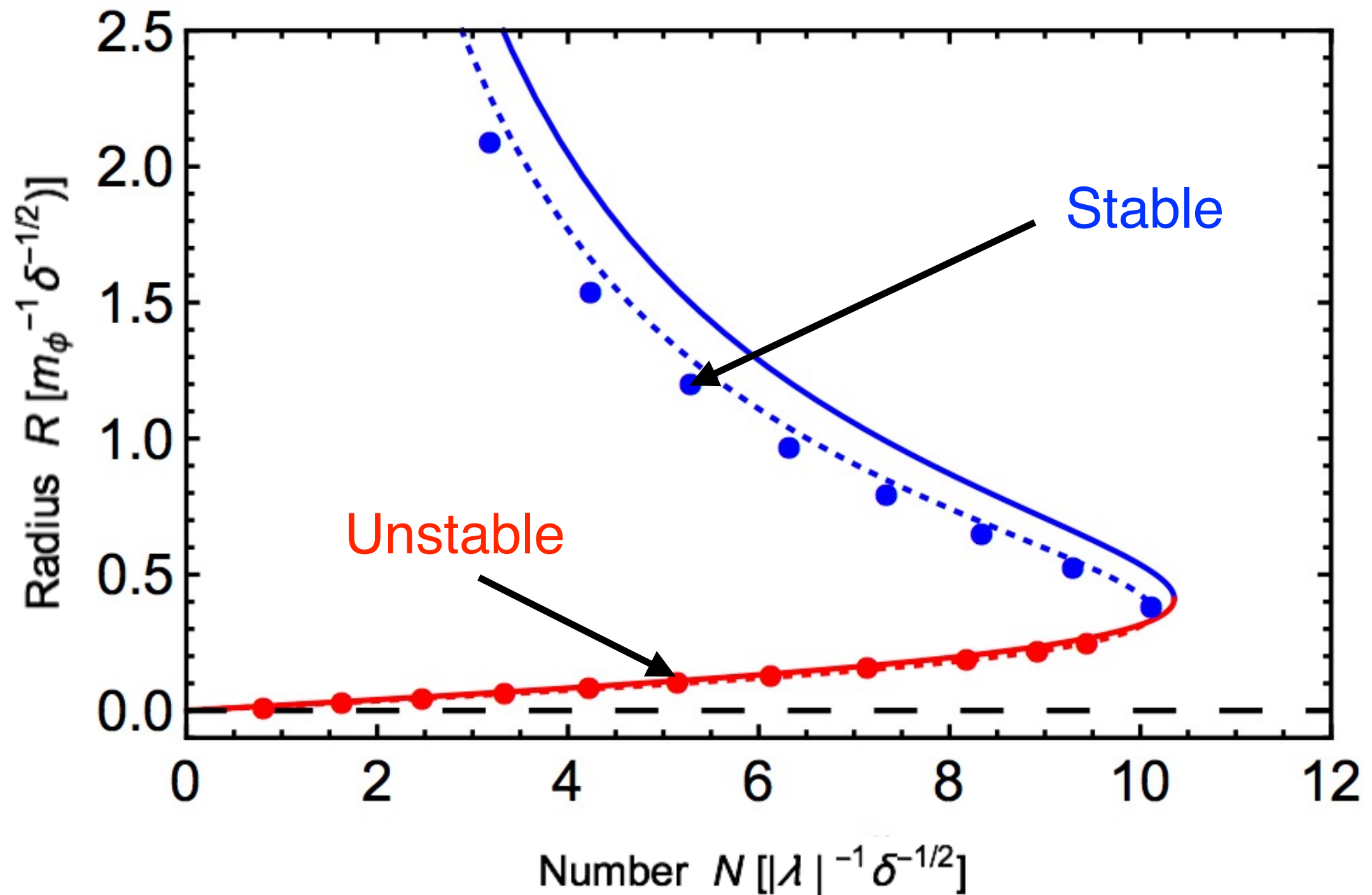
Schiappacasse, Hertzberg 1710.04729

Perturbing Lower Branch



Schiappacasse, Hertzberg 1710.04729

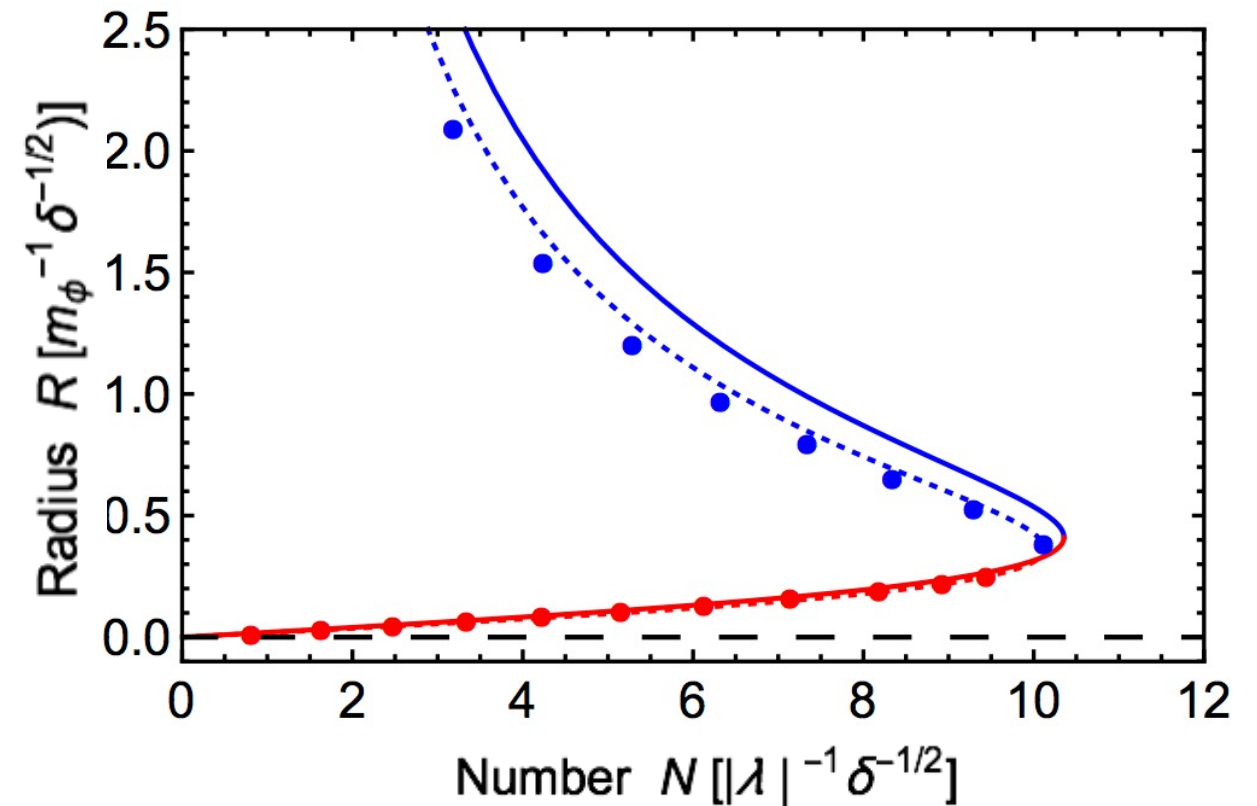
Two Branches of Solutions



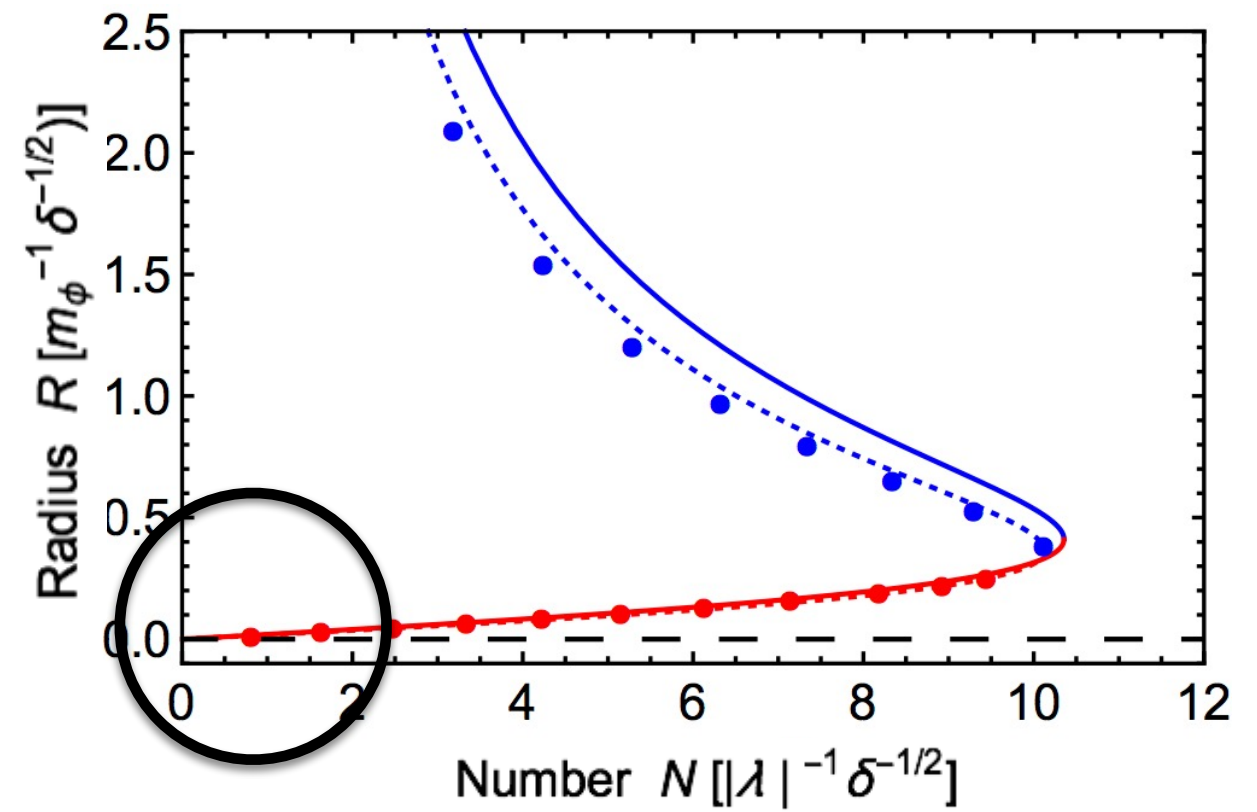
Two Branches of Solutions

$$\begin{aligned}
 N_{max} &= \frac{f_a}{m^2 \sqrt{G}} \tilde{N}_{max} \sim 8 \times 10^{59} (\tilde{m}^{-2} \tilde{f}_a), \\
 M_{max} &= N_{max} m \sim 1.4 \times 10^{19} \text{ kg } (\tilde{m}^{-1} \tilde{f}_a), \\
 R_{90,min} &= \frac{a (\tilde{R}_{90}/\tilde{R})}{b N_{max} G m^3} \sim 130 \text{ km } (\tilde{m}^{-1} \tilde{f}_a^{-1}),
 \end{aligned}$$

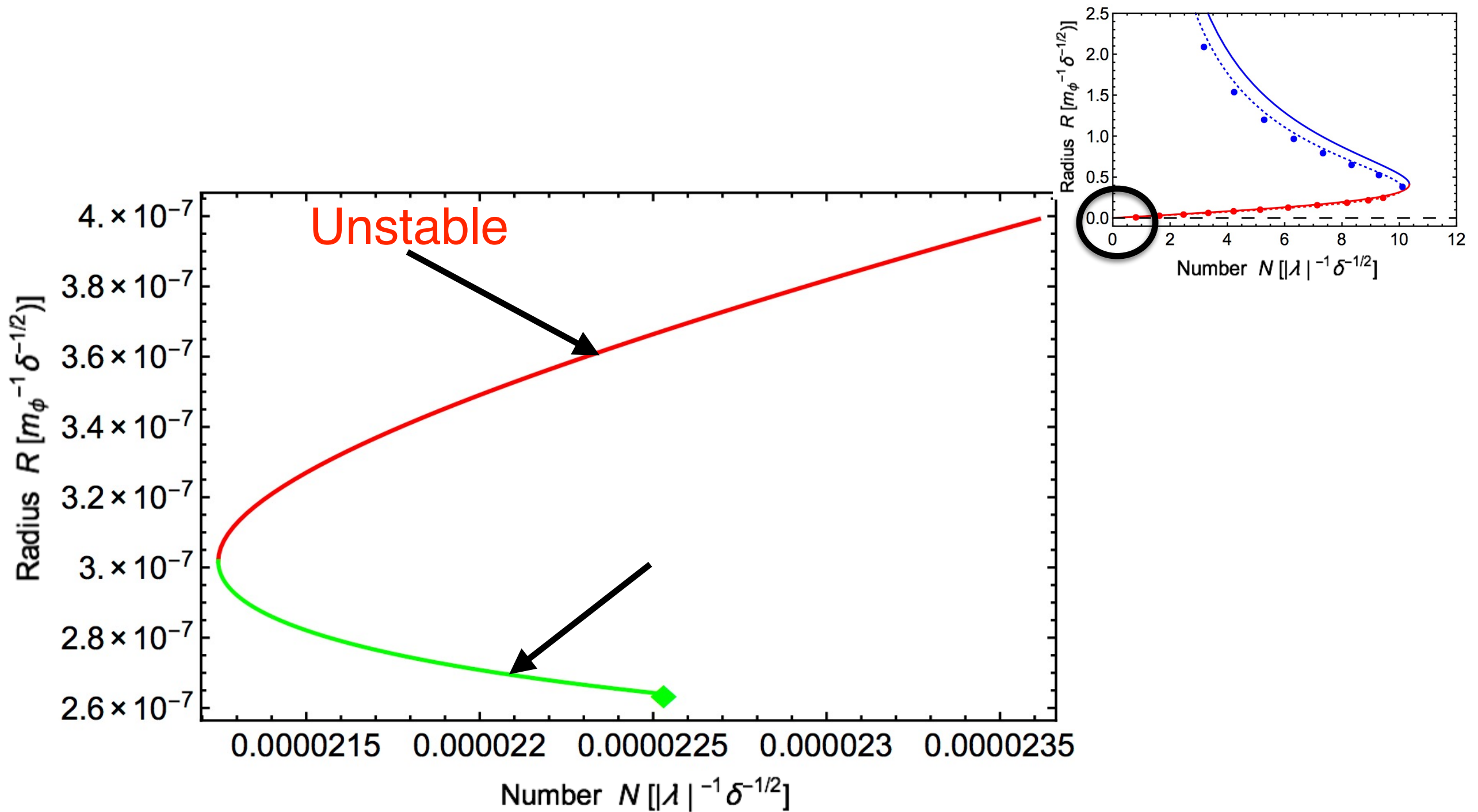
where $\tilde{f}_a \equiv f_a / (6 \times 10^{11} \text{ GeV})$ and $\tilde{m} \equiv m / (10^{-5} \text{ eV})$.



Relativistic Branch (Axiton)

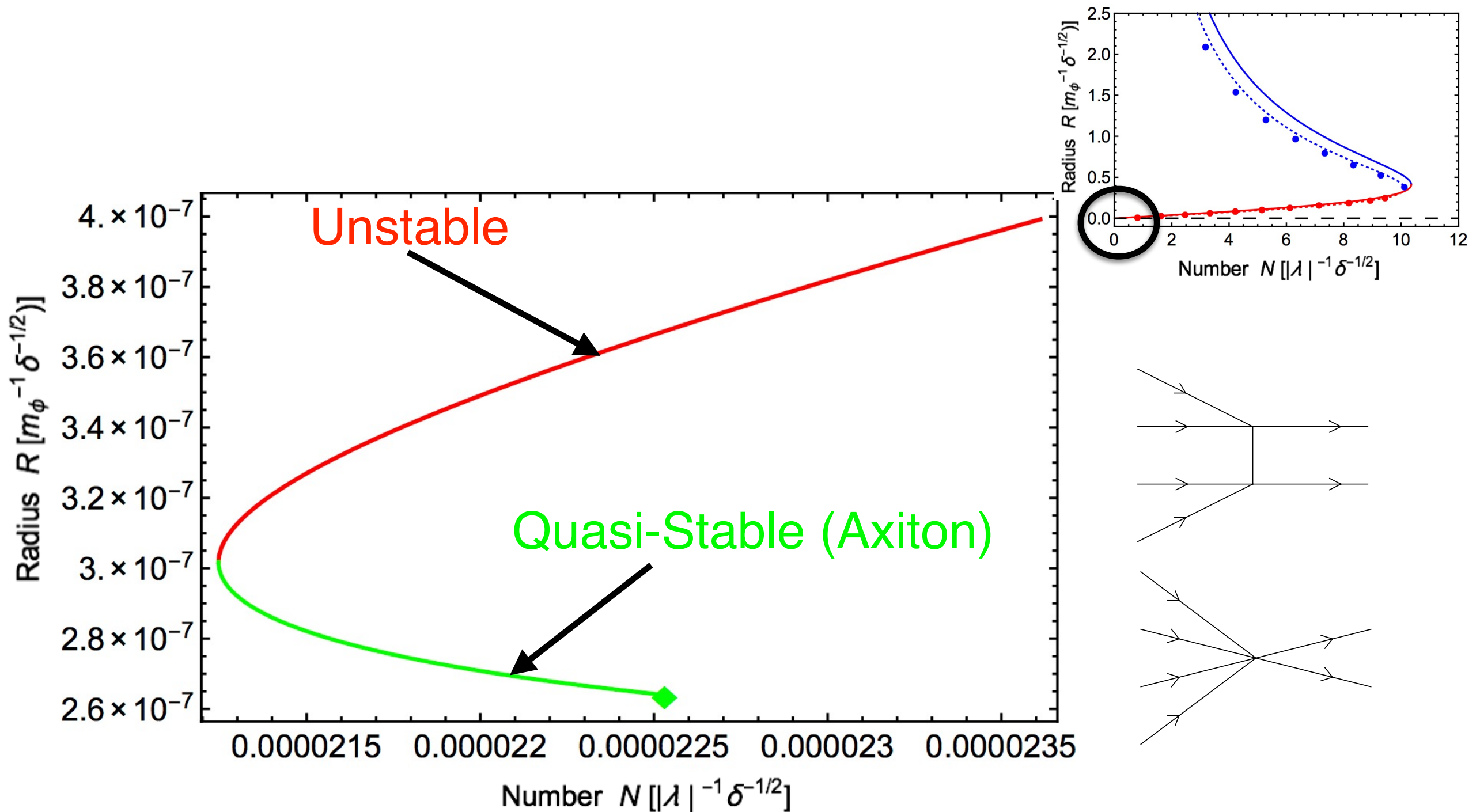


Relativistic Branch (Axiton)



Kolb, Tkachev astro-ph/9311037, Schiappacasse, Hertzberg 1710.04729,
Visinelli, Baum, Redondo, Freese, Wilczek 1710.08910

Relativistic Branch (Axiton)

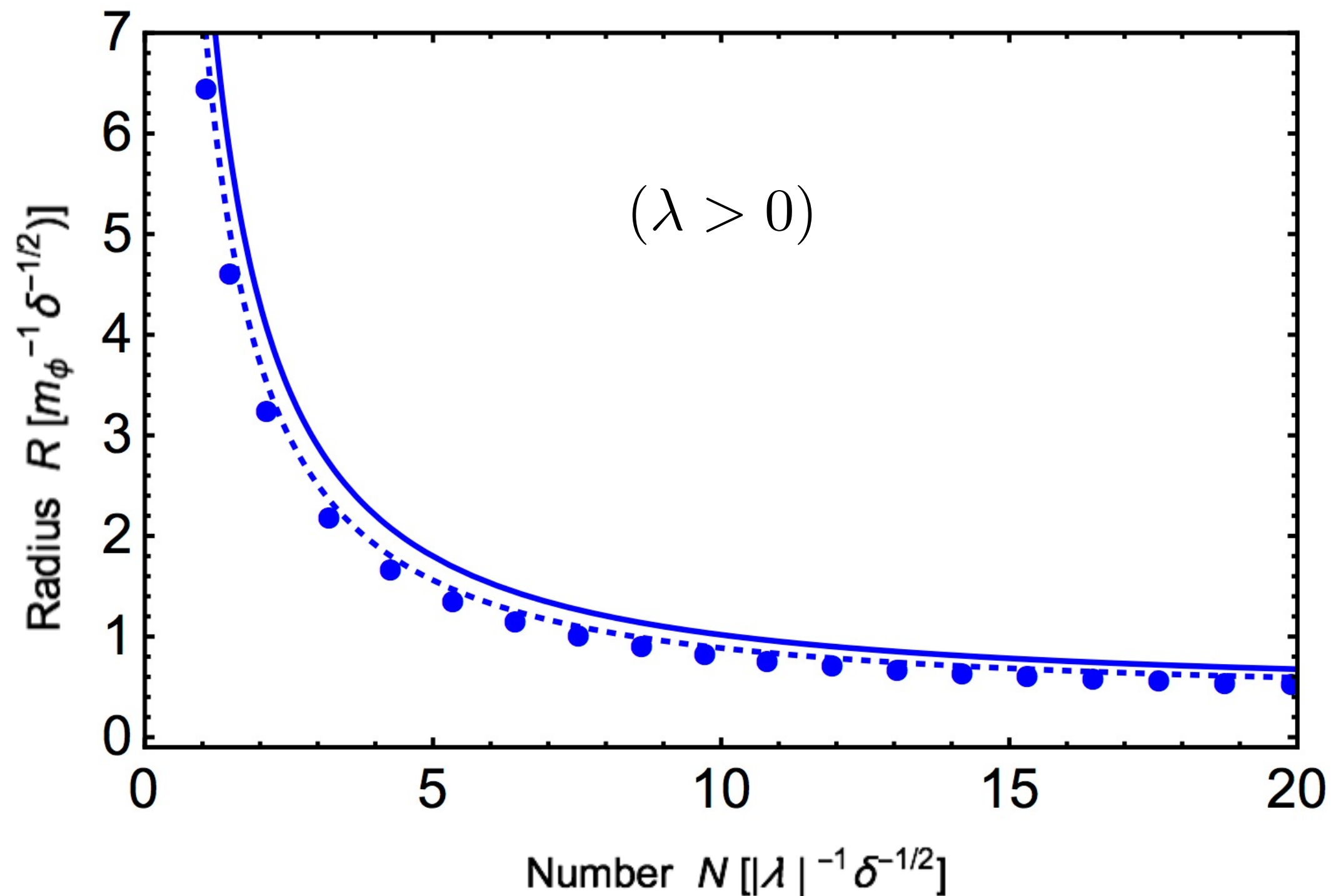


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Repulsive Self Interactions

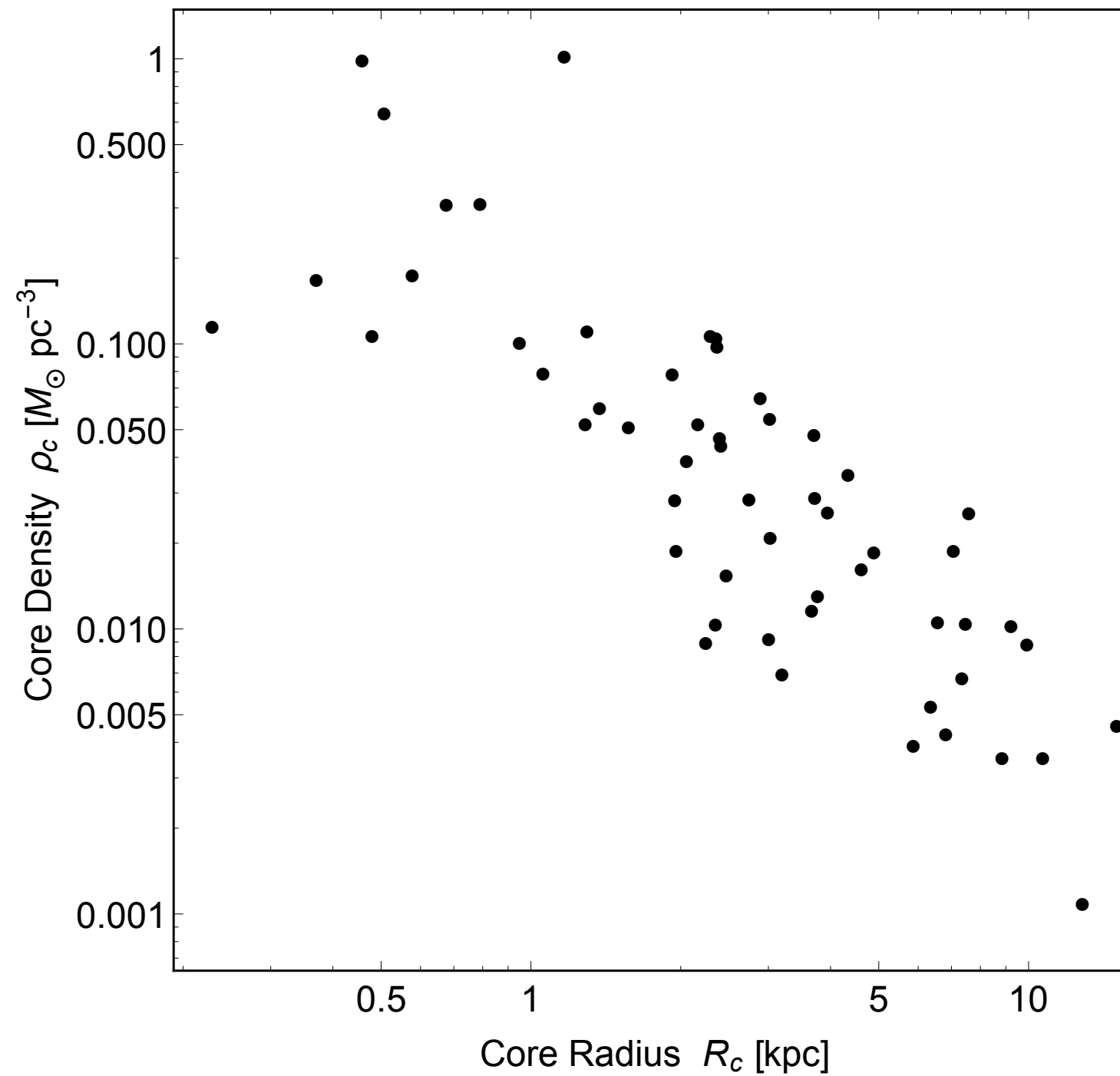
(see; Fan 2016)

Repulsive Self Interaction (Axion-Like Particle)



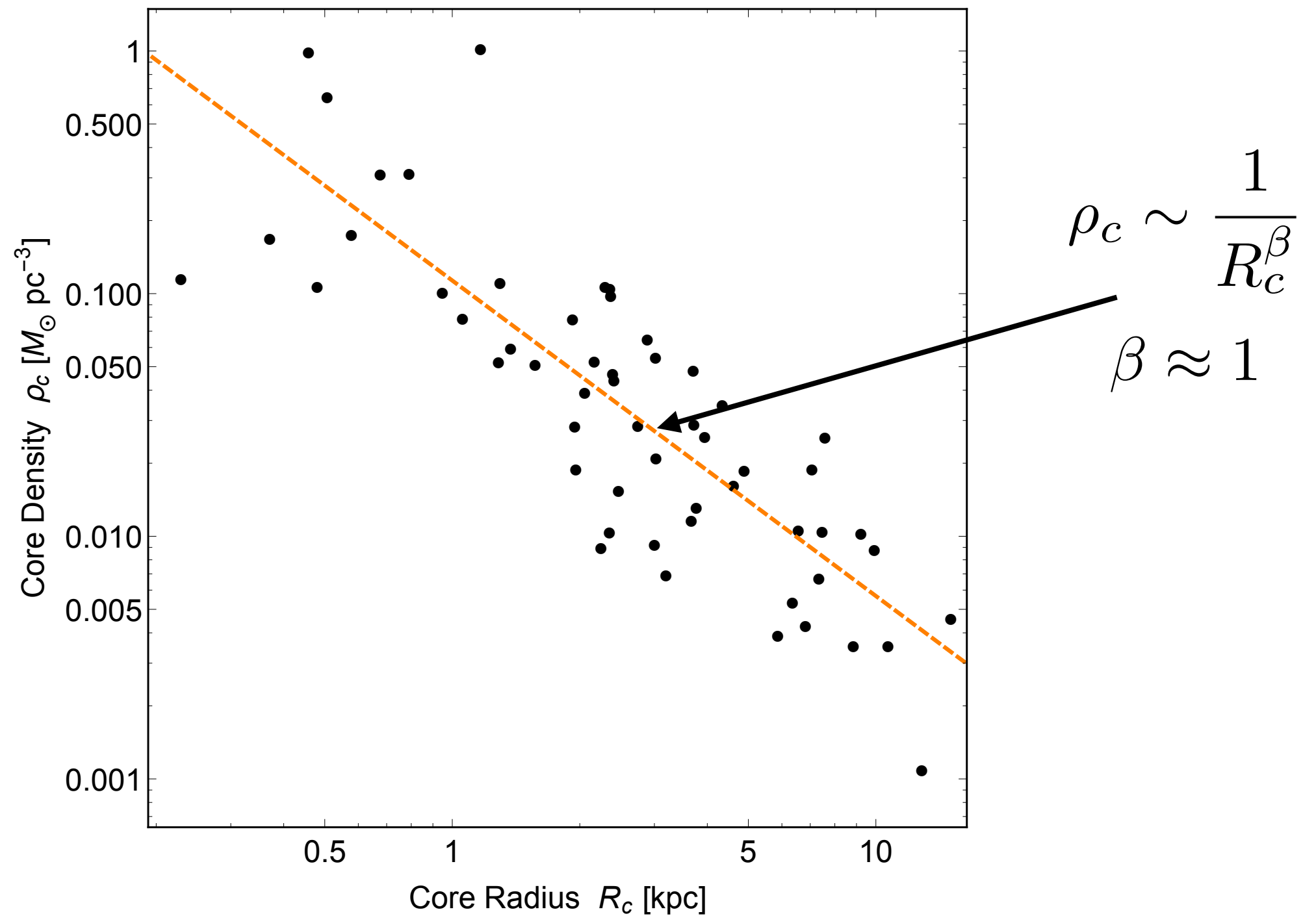
Implications for Fuzzy Dark Matter

Core Density Vs Core Radius (Data)



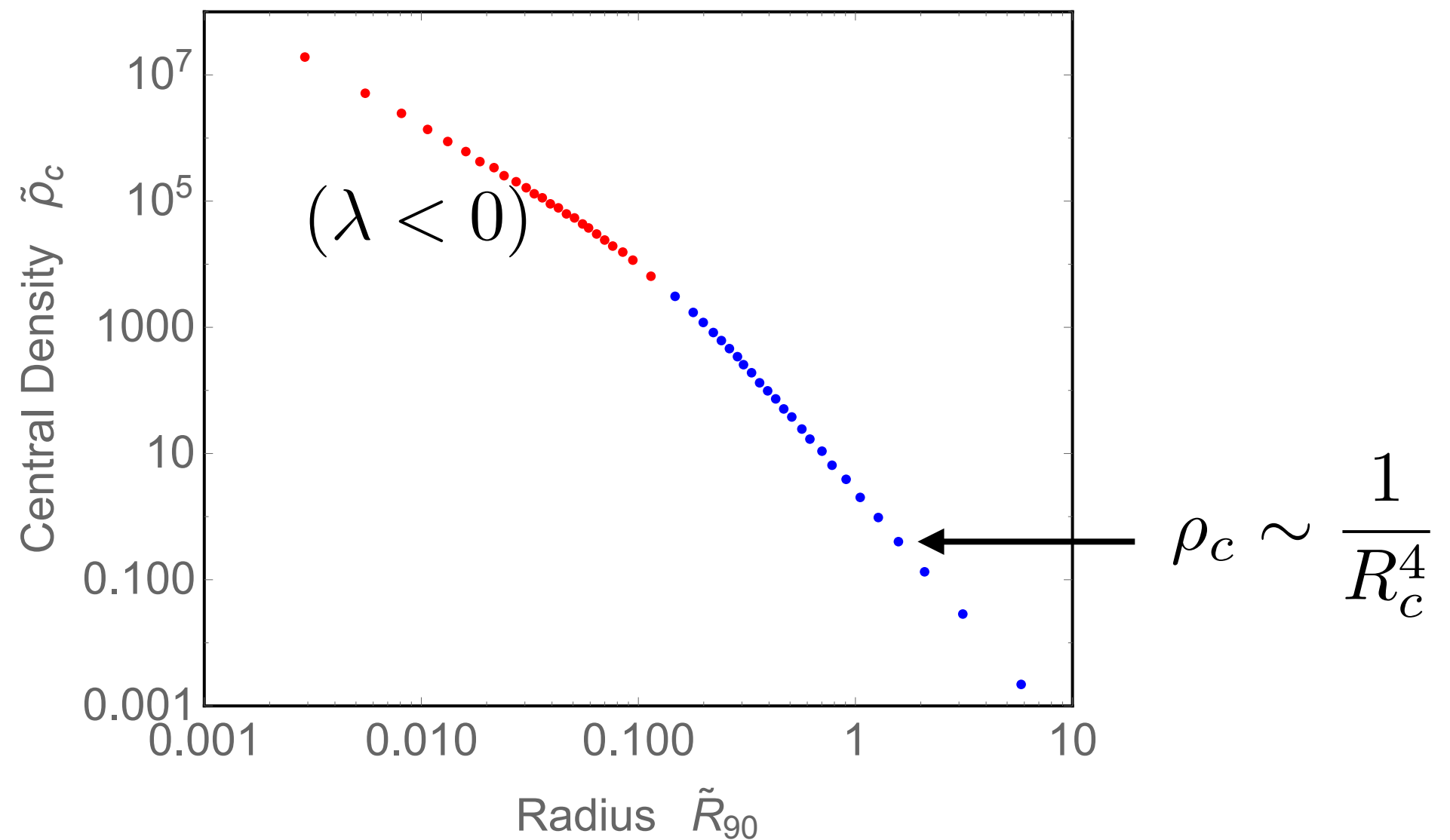
e.g., see Rodriguez et al 1701.02698

Core Density Vs Core Radius (Data)

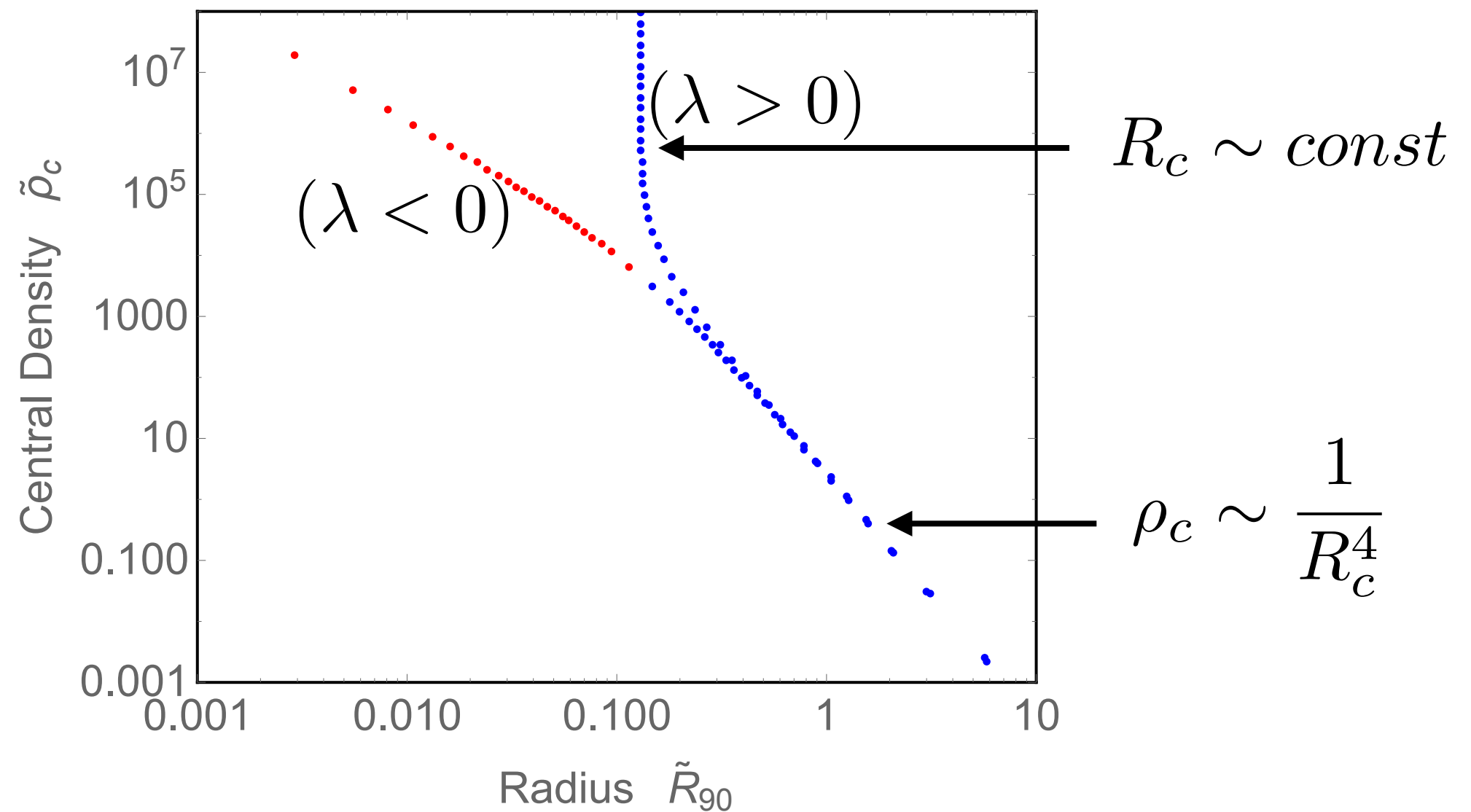


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Core Density Vs Core Radius (Light Scalar in BEC)



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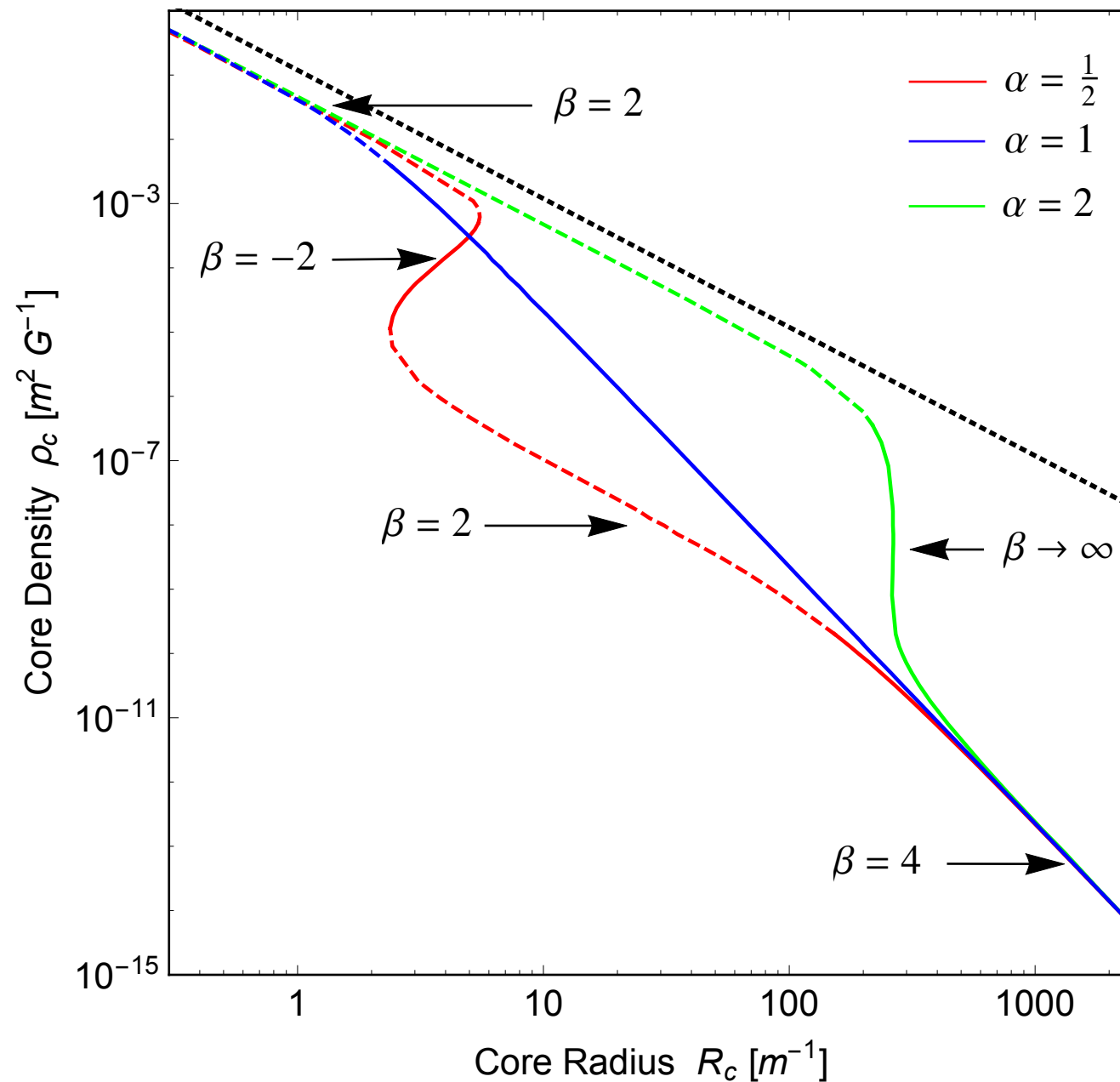


Core Density Vs Core Radius (Light Scalar in BEC)

Extension to general potentials,
polytropes, full relativistic

$$V(\phi) \propto ((1 + \phi^2/F^2)^\alpha - 1)$$

Solid = Stable
Dashed = Unstable



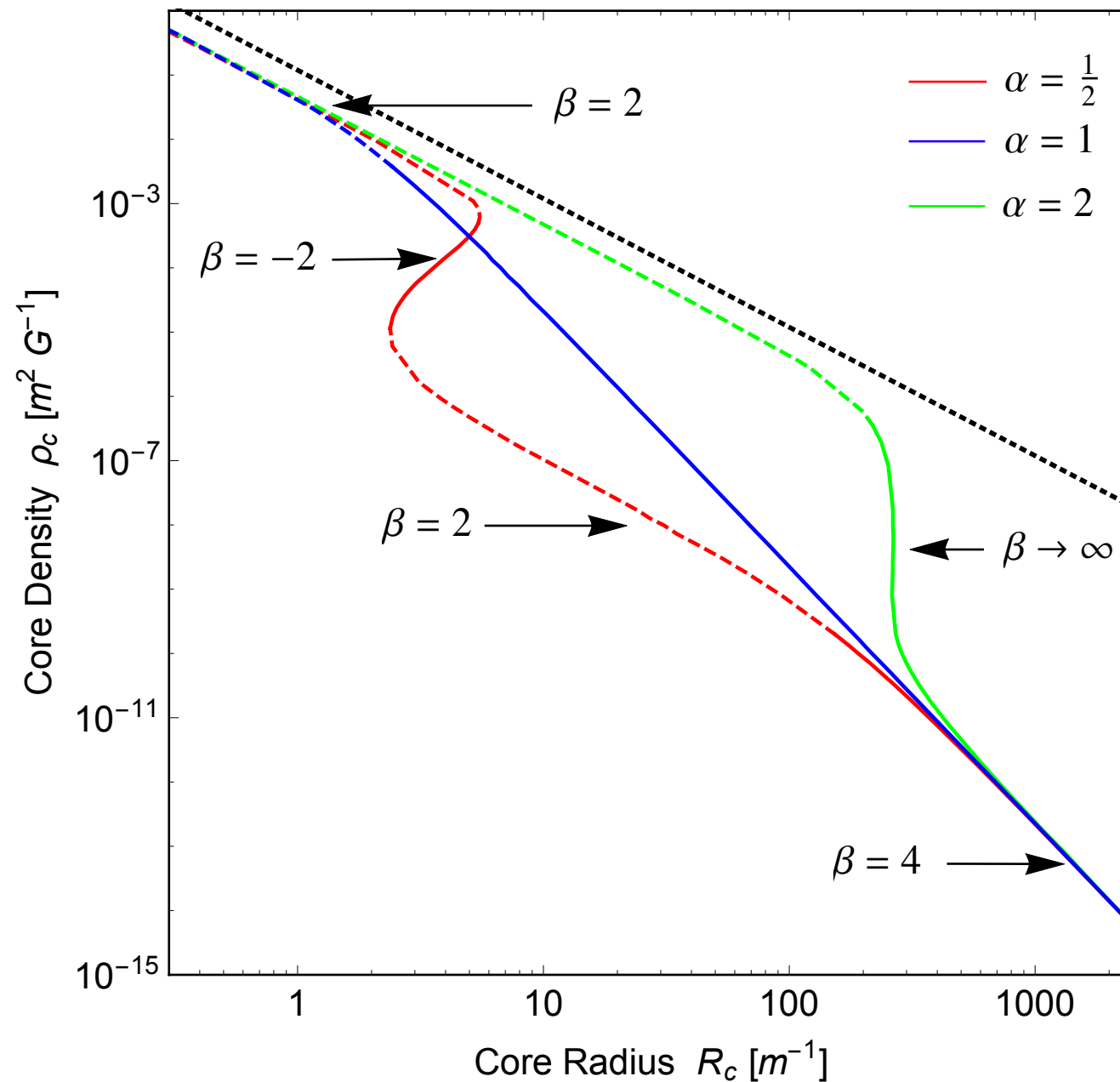
$$\rho_c \sim \frac{1}{R_c^\beta}$$

Core Density Vs Core Radius (Light Scalar in BEC)

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$$V(\phi) \propto ((1 + \phi^2/F^2)^\alpha - 1)$$

Solid = Stable
Dashed = Unstable



$$\rho_c \sim \frac{1}{R_c^\beta}$$

Never obtain $\beta \sim 1$
and stable

Axion Clump Resonance into Photons

Consider Axion to Photon Coupling

Photon Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{g_{a\gamma}}{4}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Equation of motion

$$\ddot{\mathbf{A}} - \nabla^2 \mathbf{A} + g_{a\gamma} \partial_t \phi \nabla \times \mathbf{A} = 0$$

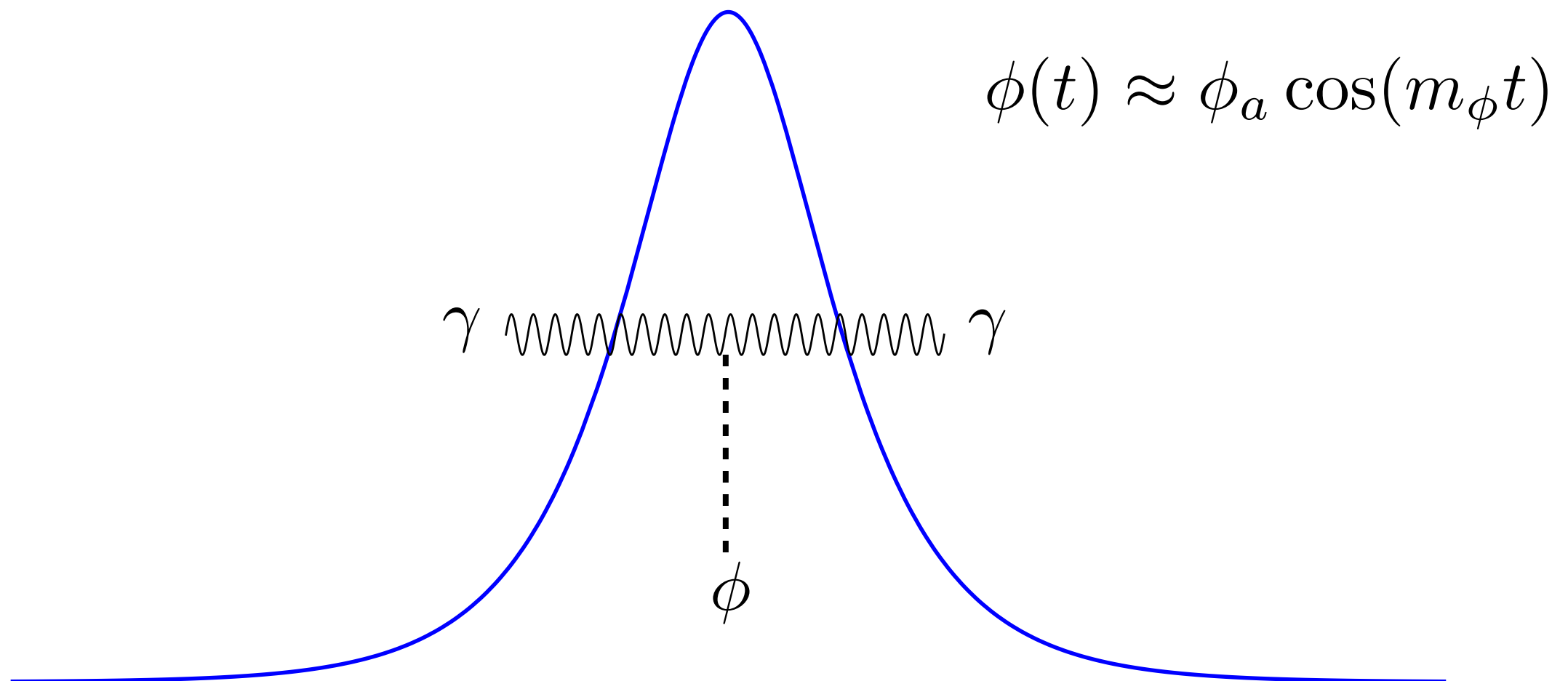
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Homogeneous Axion Field

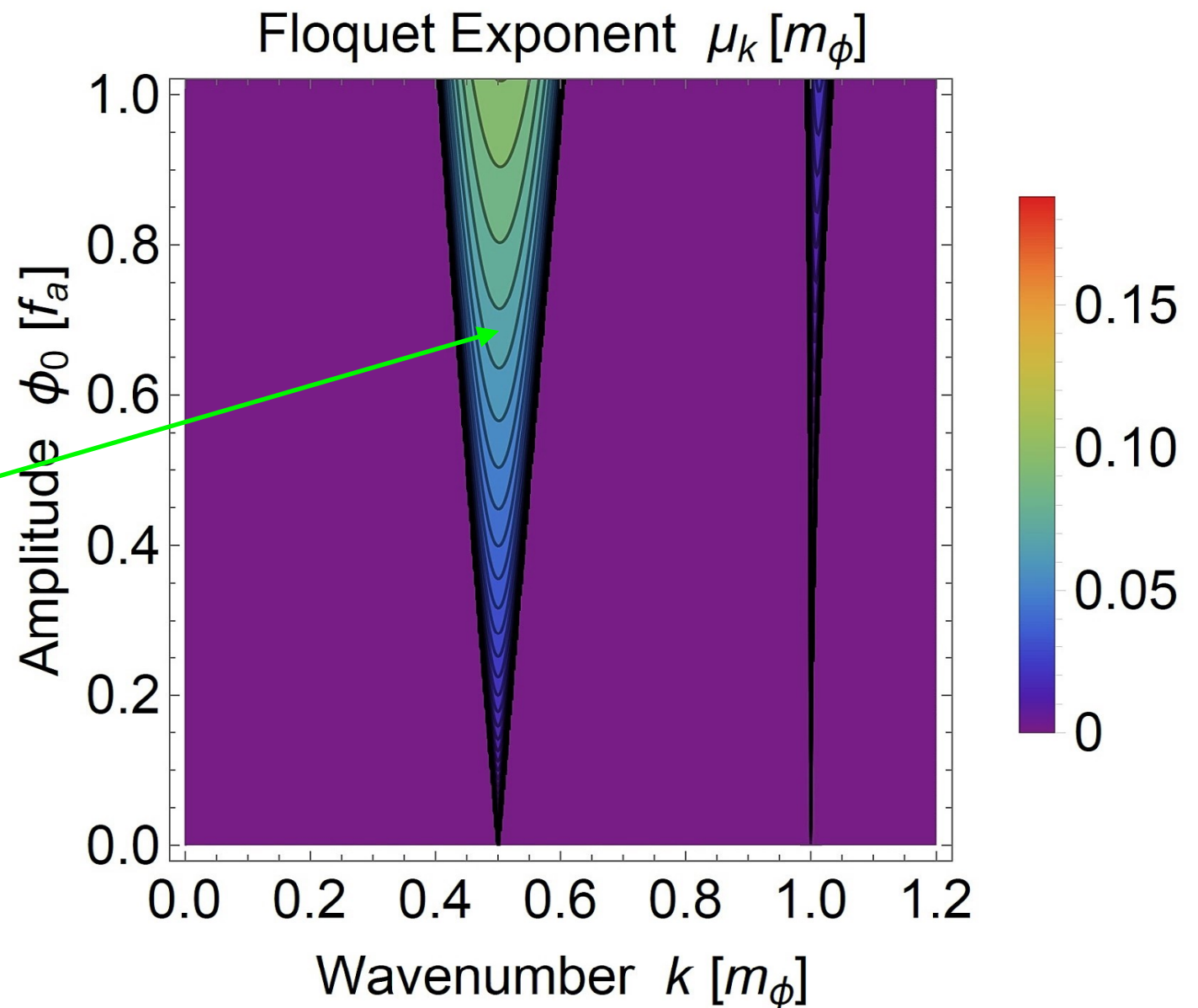
Mathieu Equation

$$\ddot{\mathbf{A}}_k^T + k^2 \mathbf{A}_k^T + g_{a\gamma} k \partial_t \phi(t) \mathbf{A}_k^T = 0$$

Parametric resonance
always present

$$k \approx \frac{m_a}{2}$$

$$\mu_H^* \approx \frac{1}{4} g_{a\gamma} m_\phi \phi_a$$



e.g., Yoshimura 1996

Comment on Photon Plasma Mass

In plasma, the photon acquires an effective mass $\omega_p^2 = \frac{4\pi\alpha n_e}{m_e}$

In early universe, this is huge; preventing resonance

Comment on Photon Plasma Mass

In plasma, the photon acquires an effective mass $\omega_p^2 = \frac{4\pi\alpha n_e}{m_e}$

In early universe, this is huge; preventing resonance

Clumps in halo: $\omega_p^2 \approx \frac{n_e}{0.03 \text{ cm}^{-3}} (6 \times 10^{-12} \text{ eV})^2$

Negligibly small; allowing for resonance

Inhomogeneous (Spherical) Axion Clump

Decomposition into vector spherical harmonics

$$\mathbf{A}(\mathbf{r}, t) = \sum_{lm} \int \frac{d^3 k}{(2\pi)^3} [a_{lm}(k, t) \mathbf{N}_{lm}(k, \mathbf{r}) + b_{lm}(k, t) \mathbf{M}_{lm}(k, \mathbf{r})]$$

where

$$\mathbf{M}_{lm}(k, \mathbf{r}) = i \frac{j_l(kr)}{\sqrt{l(l+1)}} \nabla \times [Y_{lm}(\theta, \varphi) \mathbf{r}]$$

$$\mathbf{N}_{lm}(k, \mathbf{r}) = \frac{i}{k} \nabla \times \mathbf{M}_{lm}$$

Inhomogeneous (Spherical) Axion Clump

Instability channel $l = 1, m = 0, \quad b_{10} = -i a_{10}$

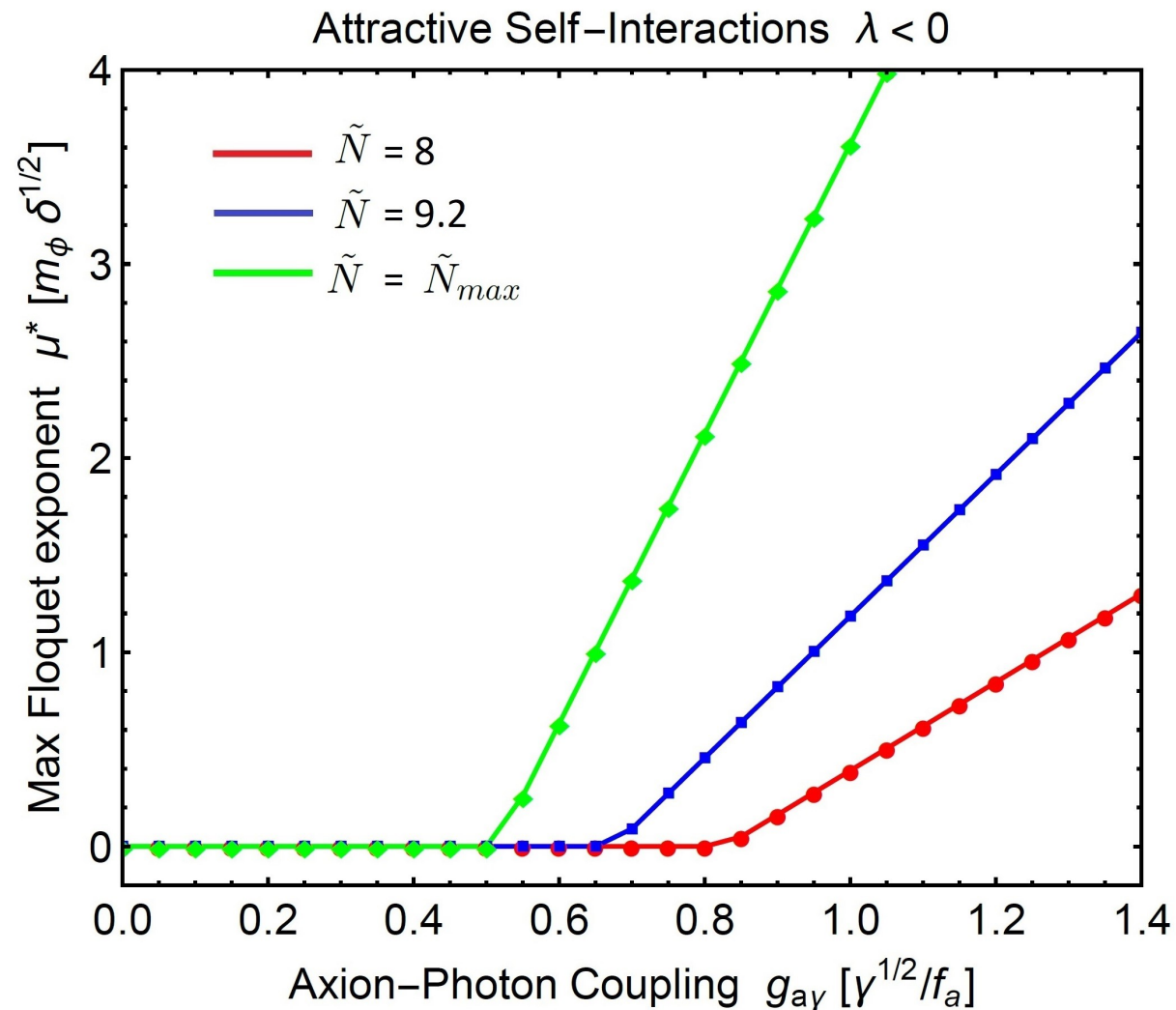
$$\ddot{a}_{10}(k, t) + k^2 a_{10}(k, t) + g_{a\gamma} k \int \frac{dk'}{(2\pi)} \partial_t \tilde{\phi}(k - k') a_{10}(k', t) = 0$$

Inhomogeneous (Spherical) Axion Clump

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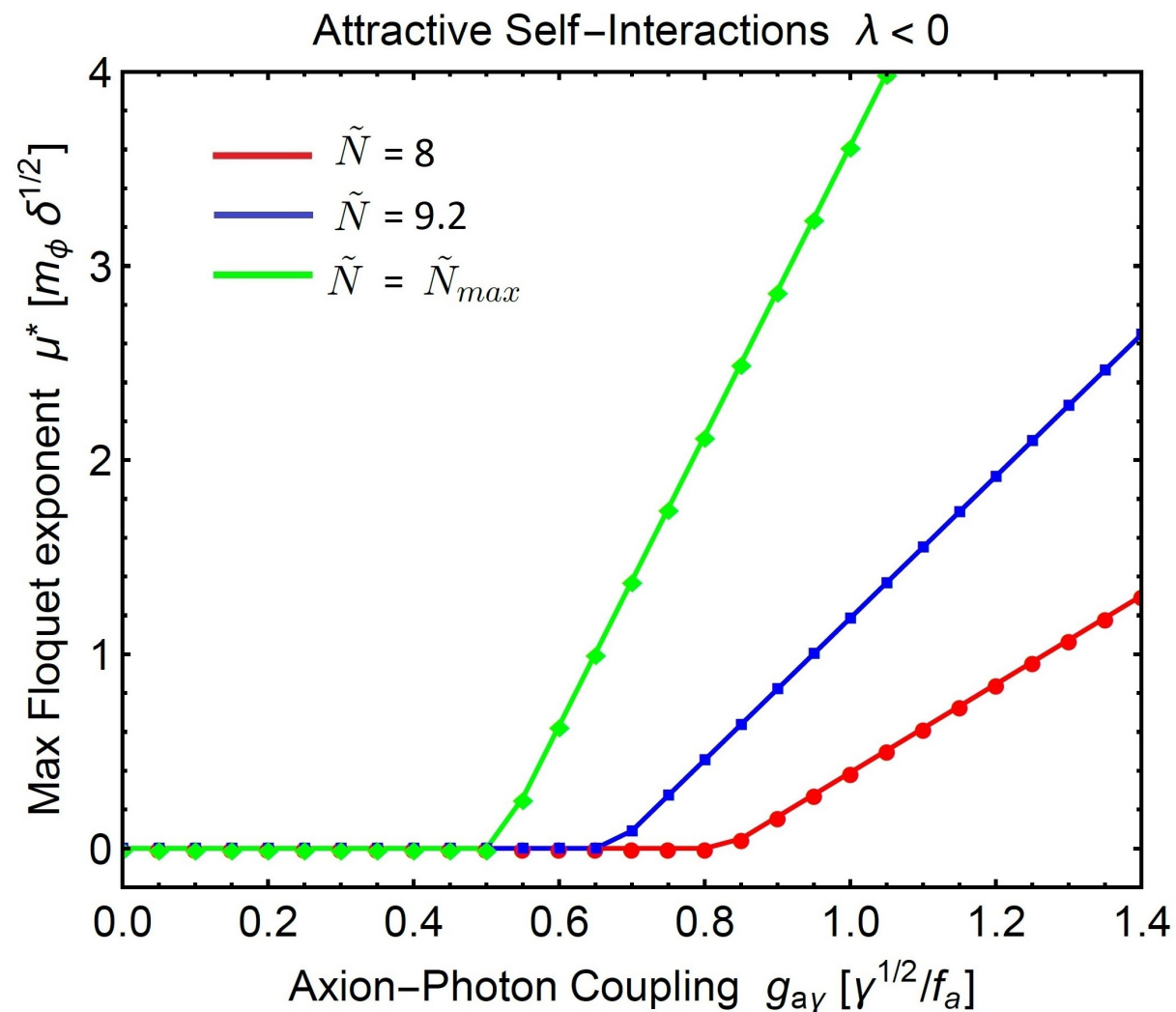
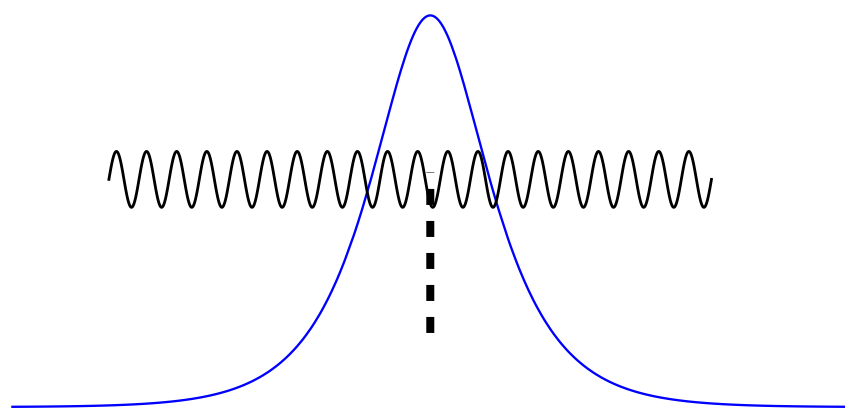
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$$\mu_{hom} > \mu_{esc}$$



Resonance Condition (Spherical) Axion Clump

$$\boxed{g_{a\gamma} > \frac{0.3}{f_a}} \quad (\lambda < 0)$$

No resonance for standard QCD axion-photon coupling $g_{a\gamma} \sim \frac{\alpha}{f_a}$

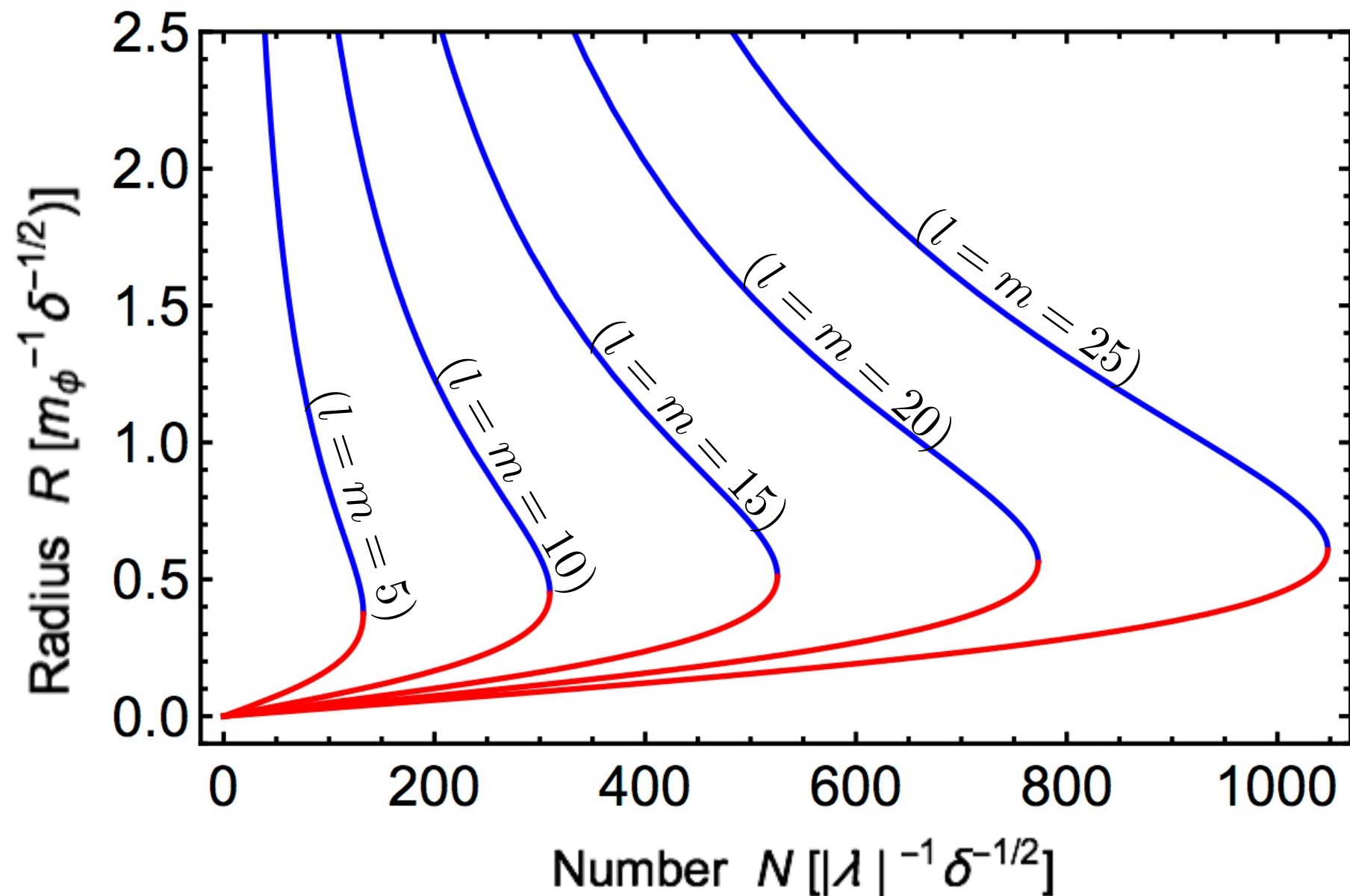
Allowed for models with enhanced couplings, decay to hidden sectors,
or repulsive interactions

↓
(see; Fan 2016)

↓
(see; Daido, Takahashi, Yokozaki 2018)

Including Angular Momentum

Two Branches of Solutions (with Angular Momentum)



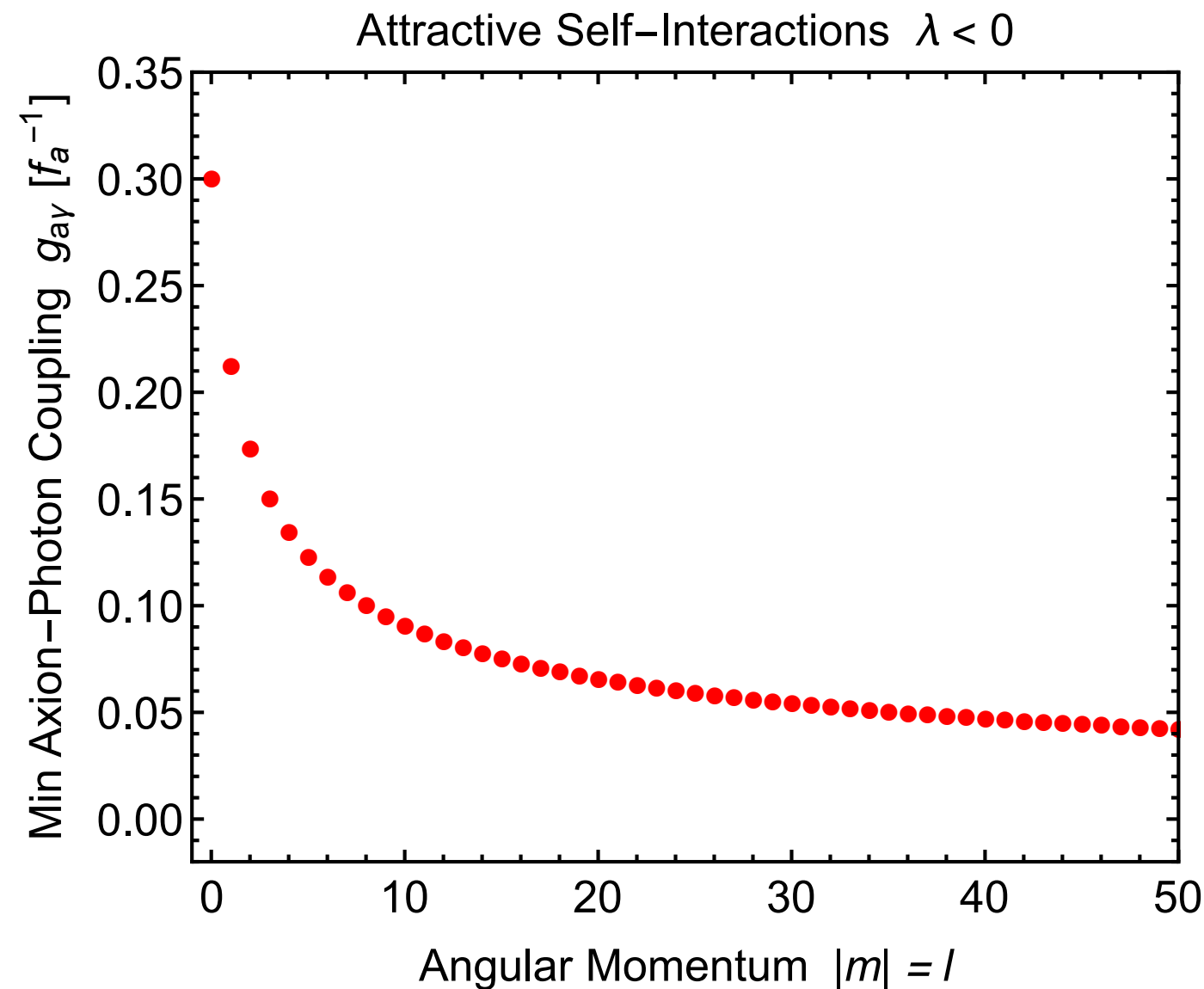
High angular momentum allows higher amplitude at core,
which helps for resonance into photons

Hertzberg, Schiappacasse 1804.07255

Resonance Condition (Non-Spherical) Axion Clump

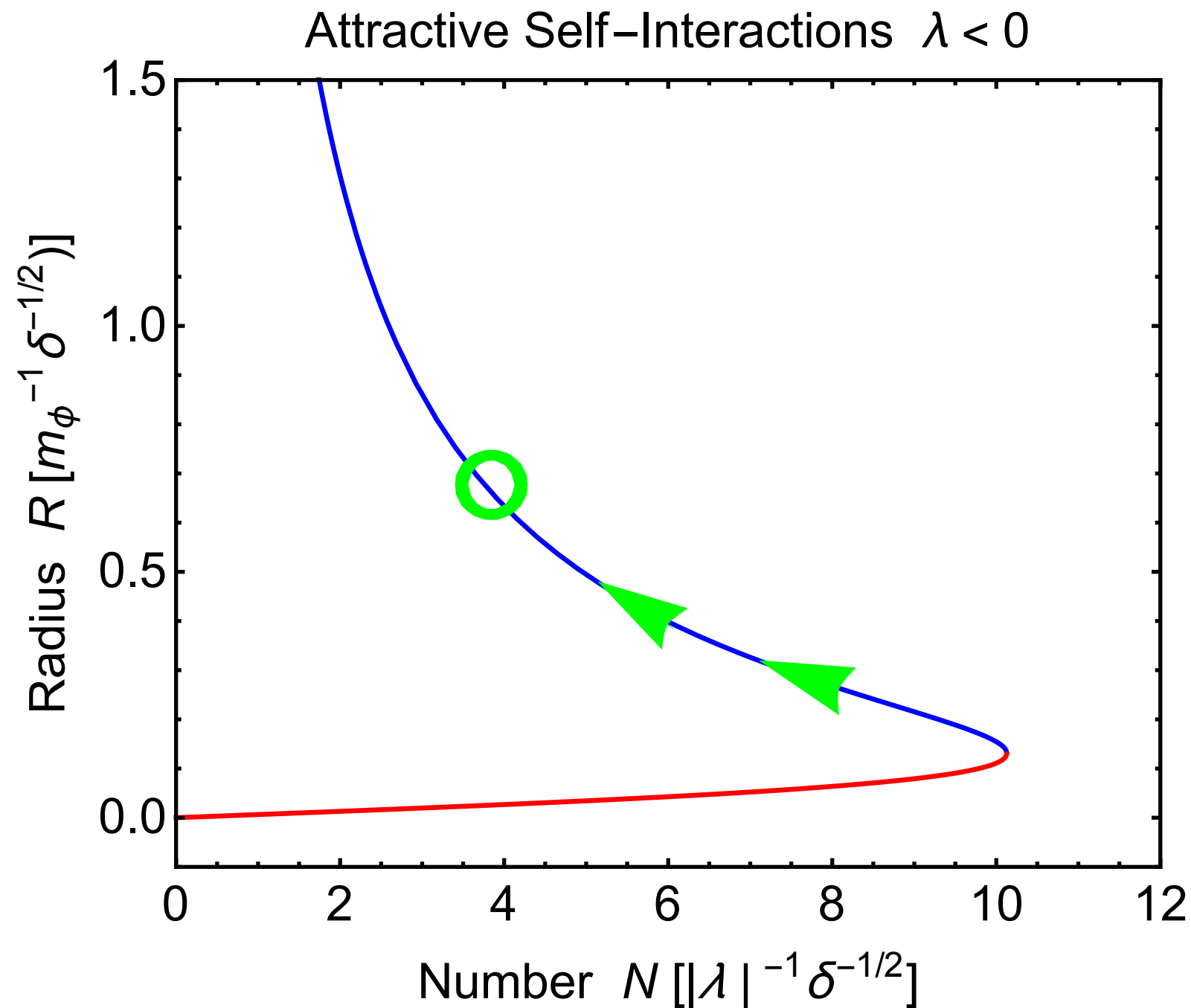
$$g_{a\gamma} > \frac{0.3}{f_a \sqrt{l+1}}$$

$$(\lambda < 0)$$



Resonance allowed for standard QCD axion-photon couplings, with high angular momentum

Astrophysical Consequences



(i) Mass Pile Up

(ii) Late Time Mergers;
Radio-wave Bursts

$$\lambda_{EM} = \frac{2\pi}{k} \approx \frac{4\pi}{m_a}$$

$$= \mathcal{O}(1) \text{ meters}$$

Thank you