Stringy Gravity and the Einstein Double Field Equations

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based on

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Outline

1. **Introduction and motivation**
   - General Relativity
   - Stringy extension?

2. **Stringy Gravity**
   - Review of Double Field Theory as Stringy Gravity
   - Einstein Double Field Equations

3. **Cosmological solutions**
   - Homogeneous and isotropic backgrounds
   - Friedmann Double Field Equations

Introduction

General Relativity is a successful theory of gravity.

- Equivalence Principle: gravity = acceleration; at every spacetime point, $\exists$ local inertial frame in which laws of Physics are invariant.
- Geometry $\Leftrightarrow$ Matter; expressed via Einstein's equations

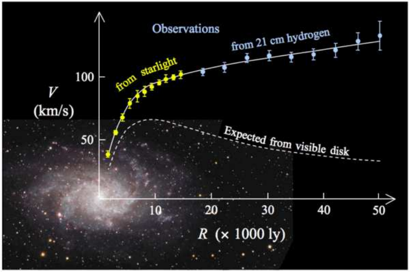
$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \ .$$

Motivation: “dark universe"

- GR accurately describes astrophysical/cosmological phenomena: perihelion precession, gravitational lensing, Big Bang cosmology...
- However, some results cannot be explained by GR + visible matter alone, eg. galaxy rotation curves.
- Kepler/Newton/GR: orbital velocity

$$V^2 = \frac{GM}{R} ,$$

does not match observations.



Broadly, two classes of solutions to such problems:

1. GR is correct, but there is additional dark matter, dark energy, . . .
2. Theory of gravity should be modified for appropriate $R/(MG)$.

Motivation: string theory

In GR, the metric $g_{\mu\nu}$ is the only gravitational field.
In string theory, the closed-string massless sector always includes:

- the metric, $g_{\mu\nu}$;
- an antisymmetric 2-form potential, $B_{\mu\nu}$;
- the dilaton, ϕ.

Furthermore, these fields transform into each other under the stringy symmetry of T-duality: e.g. $R_{\mathrm{IIA}} \sim 1/R_{\mathrm{IIB}}$, momentum $\leftrightarrow$ winding.

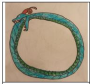
Natural stringy extension of General Relativity:

Consider $\{g_{\mu\nu}, B_{\mu\nu}, \phi\}$ as the fundamental gravitational multiplet.

This is the idea of Stringy Gravity.

Uroboros spectrum

- In Stringy Gravity, the additional degrees of freedom $B_{\mu\nu}$ and ϕ augment gravity beyond GR, allowing new types of solutions.
- E.g. $D = 4$, spherical, static case: Stringy Gravity has 4 free parameters (c.f. 1 parameter in GR, the Schwarzschild mass).
- Gravity is modified at "short" distances (Ko, Park, Suh; 2017); best expressed in terms of the dimensionless variable $R/(MG)$.
- Anomalous behavior of large astrophysical objects corresponds to this parameter range, as very large $M \Rightarrow$ small $R/(MG) \lesssim 10^7$.

	Electron ($R \simeq 0$)	Proton	Hydrogen Atom	Billiard Ball	Earth	Solar System ($1\mathrm{AU}/M_\odot G$)	Milky Way (visible)	Galaxy Cluster	Universe ($M \propto R^3$)
$R/(MG)$	0^+	7.1×10^{38}	2.0×10^{43}	2.4×10^{26}	1.4×10^{9}	1.0×10^{8}	1.5×10^{6}	$\sim 10^{5}$	0^+

'Uroboros' spectrum of the dimensionless Radial variable normalized by Mass in natural units.
The orbital speed of rotation curves is also dimensionless, and depends on the single variable, $R/(MG)$.

A brief introduction to Double Field Theory

- Stringy Gravity can be realized using Double Field Theory (DFT).
- In Double Field Theory (Hull, Zwiebach; 2009) we describe D-dim. physics using $D+D$ coordinates, $x^A = (\tilde{x}_\mu, x^\nu)$, $A = 1, \ldots, D+D$.
- $\exists$ an $\mathbf{O}(D,D)$ T-duality gauge symmetry; doubled vector indices are raised and lowered using the $\mathbf{O}(D,D)$-invariant metric:
  $$\mathcal{J}_{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$
- $\exists$ twofold local Lorentz symmetry: $\mathbf{Spin}(1, D-1) \times \mathbf{Spin}(D-1, 1)$, with local metrics $\eta_{pq} = \mathrm{diag}(-++\cdots+)$, $\bar{\eta}_{\bar{p}\bar{q}} = \mathrm{diag}(+--\cdots-)$.
- Equivalence relation: $x^A \sim x^A + \Delta^A(x)$, for $\Delta^A \sim \partial^A = (\partial_\nu, \tilde{\partial}^\mu)$.
- This is equivalent to the section condition: $\partial_A \partial^A = 2\,\partial_\mu \tilde{\partial}^\mu = 0$.
- Natural choice: $\tilde{\partial}^\nu = 0$. Thus the D coordinates $\{\tilde{x}_\mu\}$ are gauged, and their gauge orbits correspond to points in the resulting D-dimensional spacetime which is spanned by $\{x^\nu\}$.

Field content of Double Field Theory

- The basic fields of Double Field Theory are: $\{d, \mathcal{H}_{AB}\}$, the DFT dilaton and the symmetric $\mathbf{O}(D, D)$ metric, respectively.
- After imposing $\tilde{\partial}^{\mu} = 0$, these fields reduce to the closed-string massless sector, $\{g_{\mu\nu}, B_{\mu\nu}, \phi\}$, e.g. $e^{-2d} \simeq e^{-2\phi}\sqrt{-g}$.
- Can construct "semi-covariant" derivatives, e.g. $\nabla_A = \partial_A + \Gamma_A$, as well as the fully covariant DFT Ricci tensor $S_{p\bar{q}}$ and scalar $S_{(0)}$.
- $\nexists$ normal coordinates where $\Gamma_{ABC} = 0 \Rightarrow$ no equivalence principle! ($B_{\mu\nu}$ sources string; EP does not apply to extended objects.)
- On D-dim. Riemannian backgrounds ($\tilde{\partial}^{\mu} = 0$), reduces to e.g.

$$S_{(0)} = R + 4\Box\phi - 4\partial_{\mu}\phi\partial^{\mu}\phi - \tfrac{1}{12}H_{\lambda\mu\nu}H^{\lambda\mu\nu} \ ,$$

where $H_{\lambda\mu\nu} = \nabla_{[\lambda}B_{\mu\nu]}$.

This gives the spacetime Lagrangian for Stringy Gravity.

Stringy energy-momentum tensor

We will now consider DFT as Stringy Gravity coupled to matter $\{\Upsilon_a\}$. The action is

$$\int_\Sigma e^{-2d}\left[\,\frac{1}{16\pi G}S_{(0)} + L_{\rm matter}(\Upsilon_a)\,\right],$$

where the integral is performed over a D-dimensional section Σ.

Note: $\mathbf{O}(D,D)$ invariance $\Rightarrow$ proper distance, geodesic motion, etc. have a natural covariant definition in string (Jordan) frame.

The resulting equations of motion are

$$S_{p\bar{q}} = 8\pi G K_{p\bar{q}}\,, \qquad S_{(0)} = 8\pi G T_{(0)}\,, \qquad \frac{\delta L_{\rm matter}}{\delta\Upsilon_a} \equiv 0\,.$$

Here the stringy energy-momentum tensor has components

$$K_{p\bar{q}} := \frac{1}{2}\left(V_{Ap}\frac{\delta L_{\rm matter}}{\delta \bar{V}_A{}^{\bar{q}}} - \bar{V}_{A\bar{q}}\frac{\delta L_{\rm matter}}{\delta V_A{}^p}\right)\,, \quad T_{(0)} := e^{2d}\times\frac{\delta\left(e^{-2d}L_{\rm matter}\right)}{\delta d}\,,$$

where V and $\bar{V}$ are DFT vielbeins: $\mathcal{H}_A{}^B = V_{Ap}V^{Bp} + \bar{V}_{A\bar{p}}\bar{V}^{B\bar{p}}$.

Einstein Double Field Equations

Analogously to GR, we can define the stringy Einstein curvature tensor which is covariantly conserved,

$$G_{AB} = 4V_{[A}{}^{p}\bar{V}_{B]}{}^{\bar{q}}S_{p\bar{q}} - \tfrac{1}{2}\mathcal{J}_{AB}S_{(0)}\ , \qquad \mathcal{D}_A G^{AB} = 0 \qquad \text{(off-shell)}\ .$$

This implies that the energy-momentum tensor can be written similarly,

$$T_{AB} := 4V_{[A}{}^{p}\bar{V}_{B]}{}^{\bar{q}}K_{p\bar{q}} - \tfrac{1}{2}\mathcal{J}_{AB}T_{(0)}\ , \qquad \mathcal{D}_A T^{AB} \equiv 0 \qquad \text{(on-shell)}\ .$$

Hence the Einstein Double Field Equations can be summarized as

$$G_{AB} = 8\pi G T_{AB}\ .$$

Note: unlike in GR, the DFT Ricci tensor is traceless $\Rightarrow$ the $S_{(0)} \propto T_{(0)}$ part is an essential and independent component of the equations.

Riemannian backgrounds

- Riemannian backgrounds: EDFEs reduce to usual closed-string equations, plus source terms from $K_{\mu\nu} = 2e_\mu{}^p \bar{e}_\nu{}^q K_{p\bar{q}}$ and $T_{(0)}$:

$$\begin{aligned} R_{\mu\nu} + 2\nabla_\mu(\partial_\nu\phi) - \tfrac{1}{4}H_{\mu\rho\sigma}H_\nu{}^{\rho\sigma} &= 8\pi G K_{(\mu\nu)} \; ; \\ \nabla^\rho\left(e^{-2\phi}H_{\rho\mu\nu}\right) &= 16\pi G e^{-2\phi}K_{[\mu\nu]} \; ; \\ R + 4\Box\phi - 4\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{12}H_{\lambda\mu\nu}H^{\lambda\mu\nu} &= 8\pi G T_{(0)} \; . \end{aligned}$$

- Asymmetric $K_{\mu\nu}$ possible (e.g. fermions, strings) $\to$ source for H.
- In addition, the conservation laws (on-shell) reduce to

$$\begin{aligned} \nabla^\mu K_{(\mu\nu)} - 2\partial^\mu\phi\, K_{(\mu\nu)} + \tfrac{1}{2}H_\nu{}^{\lambda\mu}K_{[\lambda\mu]} - \tfrac{1}{2}\partial_\nu T_{(0)} &\equiv 0 \; , \\ \nabla^\mu\left(e^{-2\phi}K_{[\mu\nu]}\right) &\equiv 0 \; . \end{aligned}$$

- $D = 4$, spherically symmetric solution: gravity modified at small radius-per-mass, $R/(MG)$ (SA, Cho, Park; 2018).

Homogeneous and isotropic backgrounds

Consider solutions in $D = 4$ which are homogeneous and isotropic.

- Solving the (doubled) Killing equations for the gravitational fields gives the ansatz

$$\mathrm{d}s^2 = -N(t)^2\mathrm{d}t^2 + a(t)^2\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\Omega^2\right] ,$$

$$B_{(2)} = \frac{hr^2}{\sqrt{1-kr^2}}\cos\vartheta\,\mathrm{d}r\wedge\mathrm{d}\varphi , \quad \phi = \phi(t) .$$

- Note: can choose e.g. cosmic gauge where the function $N(t) = 1$; solutions are parametrized by $a(t)$, $\phi(t)$, and h (a constant).
- Similarly, the stringy energy-momentum tensor is constrained as

$$K^{\mu}{}_{\nu} = \mathrm{diag}(K^t{}_t(t), K^r{}_r(t), \ldots, K^r{}_r(t)) , \quad T_{(0)} = T_{(0)}(t) .$$

Energy density and pressure

- Define energy density and pressure as

$$\rho := \left(-K^t{}_t + \frac{1}{2}T_{(0)}\right)e^{-2\phi}, \qquad p := \left(K^r{}_r - \frac{1}{2}T_{(0)}\right)e^{-2\phi}\ .$$

- Why? E.g. demand $\rho \equiv \mathcal{H}$ (Hamiltonian): recall Stringy Gravity matter action is of the form $\int e^{-2d}L_{\rm matter}$, where $e^{-2d} = e^{-2\phi}\sqrt{-g}$; $-K^t{}_t = \pi^a\partial_0\Upsilon_a$, $T_0 = -2L_{\rm matter}$ (if $L_{\rm matter}$ is dilaton-independent).

- One non-trivial conservation law:

$$\dot{\rho} + 3H(\rho + p) + \dot{\phi}T_{(0)}e^{-2\phi} = 0\ ,$$

where $H \equiv \frac{\dot{a}}{a}$ (in cosmic gauge), and $\dot{\{\}} = \frac{\mathrm{d}\{\}}{\mathrm{d}t}$.

Friedmann Double Field Equations

- In the homogeneous and isotropic case, the EDFEs reduce to

$$\frac{8\pi G}{3}\rho e^{2\phi} - \frac{k}{a^2} = H^2 - 2\dot{\phi}H + \frac{2}{3}\dot{\phi}^2 - \frac{h^2}{12a^6}\,,$$

$$\frac{4\pi G}{3}(\rho + 3p)e^{2\phi} = -\dot{H} - H^2 + \ddot{\phi} + \dot{\phi}H - \frac{2}{3}\dot{\phi}^2 - \frac{h^2}{6a^6}\,,$$

$$\frac{4\pi G}{3}\left(2\rho e^{2\phi} - T_{(0)}\right) = -\dot{H} - H^2 + \frac{2}{3}\ddot{\phi}$$

  $\rightarrow$ "Friedmann Double Field Equations".
- Note: 3 FDFEs + 1 conservation law $\Rightarrow$ 3 independent equations.
- Analytic solutions for vacuum, massless scalar, radiation...
  (SA, Cho, Franzmann, Mukohyama, Park; to appear)
- For $\dot{\phi} = \ddot{\phi} = 0$, $h = 0$, recover usual cosmology in string frame.
- New terms suppressed at late times; contribute at early times
  $\Rightarrow$ new, expanded framework for early-universe cosmology.

Summary

- Stringy Gravity considers the closed string massless sector $\{g_{\mu\nu}, B_{\mu\nu}, \phi\}$ to be the fundamental gravitational multiplet $\Rightarrow (D^2+1)$ degrees of freedom, thus richer spectrum.
- We studied Double Field Theory as Stringy Gravity in the presence of matter. Imposing on-shell energy-momentum conservation gives the Einstein Double Field Equations,

$$G_{AB} = 8\pi G T_{AB} \, .$$

- $D = 4$: spherically symmetric solutions suggest a modification to gravity at small $R/(MG)$, while homogeneous and isotropic solutions allow for new early-universe cosmology in string frame.

Further work: possibility of de Sitter solutions; generating (almost) scale-invariant curvature perturbations; dilaton stabilization; . . .