
The measurements on $H(z)$ from DA & BAO and the possible solution for the tension of the local and global $H(z)$

Seokcheon Lee
(Sungkyunkwan Univ)

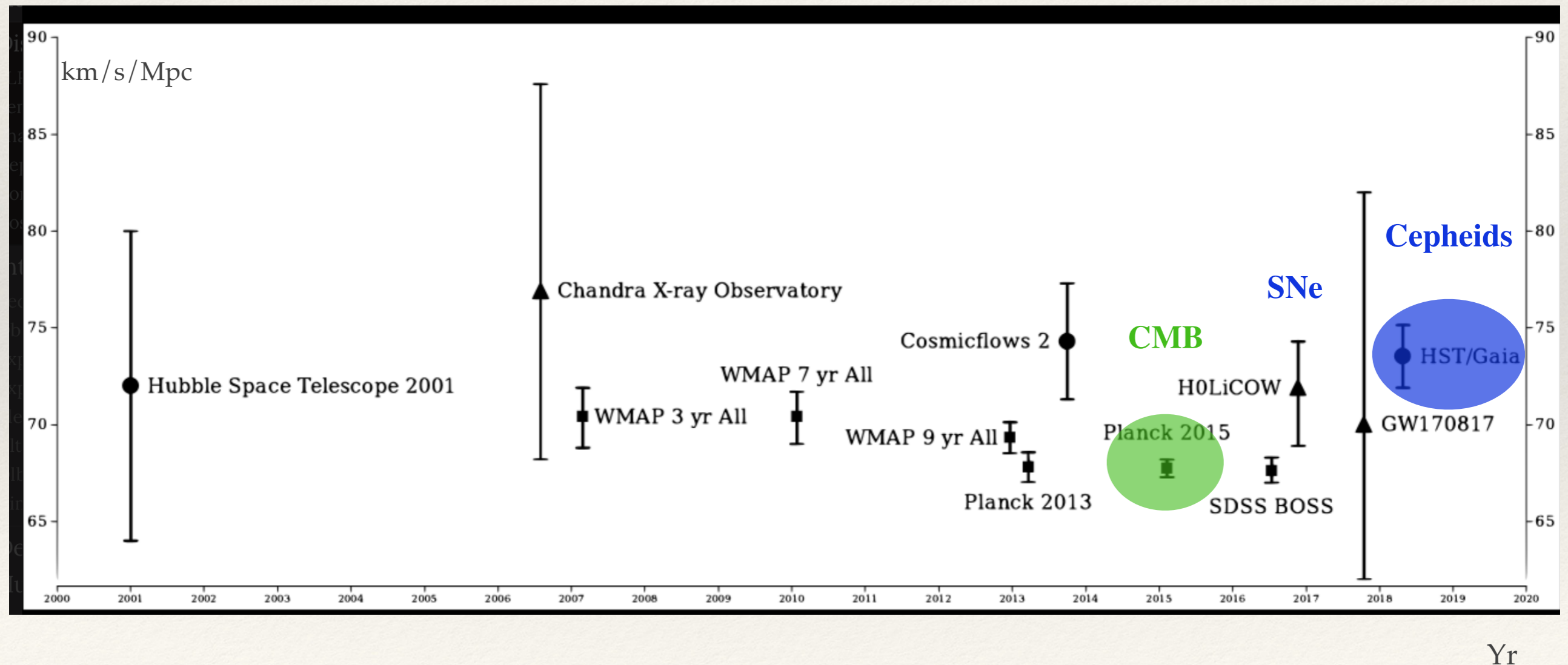
COSMO18
(Aug.27-Aug.31, IBS)

OUTLINE

- ◉ Observations for Hubble parameter
- ◉ Tensions
- ◉ Possible solutions
- ◉ Current measurements
- ◉ Modified gravity as a solution
- ◉ Conclusion

OBSERVATIONS I

- Tension btw local and global measurements of H_0 [km/s/Mpc] (*Refer Peiris, Wechsler, Ashley*)
- **Hubble Space Telescope** (2018, local) $73.52 \pm 1.62 \Leftrightarrow$ **Planck** (2015, global) 67.74 ± 0.46 : **3.8σ tension**



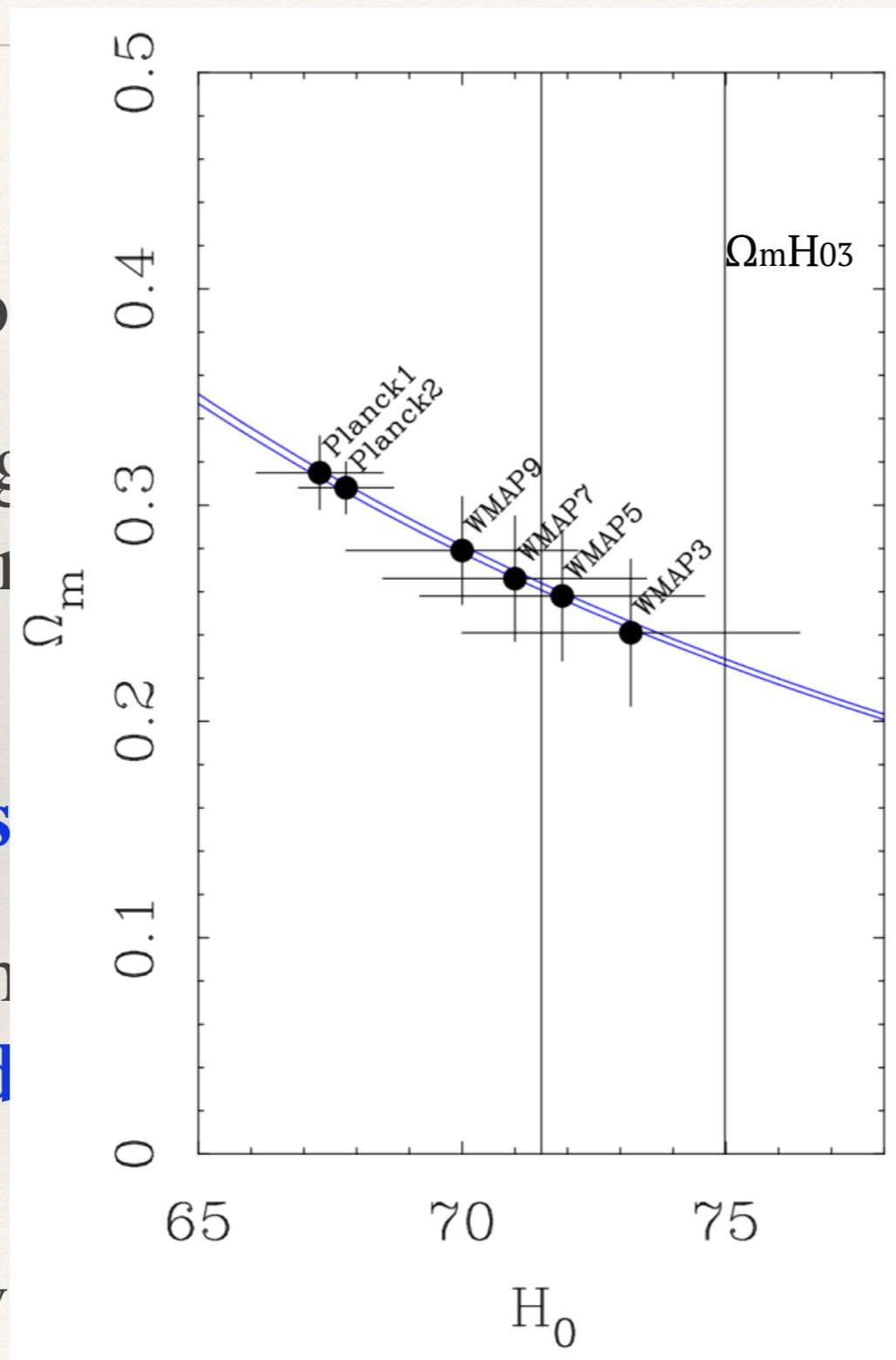
| Date published [◆] | Hubble constant (km/s)/Mpc [◆] | Observer [◆] | Citation | Remarks / methodology |
|-----------------------------|---|---|----------|---|
| 2018-07-18 | 67.66 ± 0.42 | Planck Mission | [15] | Final Planck 2018 results. |
| 2018-04-27 | 73.52 ± 1.62 | Hubble Space Telescope and Gaia | [16][17] | Additional HST photometry of galactic Cepheids with early Gaia parallax measurements. The revised value increases tension with CMB measurements at the 3.8 σ level. Continuation of a collaboration known as Supernovae, H_0 , for the Equation of State of Dark Energy (SHoES). |
| 2018-02-22 | 73.45 ± 1.66 | Hubble Space Telescope | [18][19] | Parallax measurements of galactic Cepheids for enhanced calibration of the distance ladder ; the value suggests a discrepancy with CMB measurements at the 3.7 σ level. The uncertainty is expected to be reduced to below 1% with the final release of the Gaia catalog. SHoES collaboration. |
| 2017-10-16 | 70.0 ^{+12.0} _{-8.0} | The LIGO Scientific Collaboration and The Virgo Collaboration | [20] | Measurement was independent of a cosmic 'distance ladder'; the gravitational-wave analysis of a binary neutron star (BNS) merger GW170817 directly estimated the luminosity distance out to cosmological scales. An estimate of fifty similar detections in the next decade may arbitrate tension of other methodologies. ^[21] Detection and analysis of a neutron star-black hole merger (NSBH) may provide greater precision than BNS could allow. ^[22] |
| 2016-11-22 | 71.9 ^{+2.4} _{-3.0} | Hubble Space Telescope | [23] | Uses time delays between multiple images of distant variable sources produced by strong gravitational lensing . Collaboration known as H_0 Lenses in COSMOGRAIL's Wellspring (H0LiCOW). |
| 2016-07-13 | 67.6 ^{+0.7} _{-0.6} | SDSS-III Baryon Oscillation Spectroscopic Survey | [24] | Baryon acoustic oscillations . An extended survey (eBOSS) began in 2014 and is expected to run through 2020. The extended survey is designed to explore the time when the universe was transitioning away from the deceleration effects of gravity from 3 to 8 billion years after the Big Bang. ^[25] |
| 2016-05-17 | 73.24 ± 1.74 | Hubble Space Telescope | [26] | Type Ia supernova , the uncertainty is expected to go down by a factor of more than two with upcoming Gaia measurements and other improvements. SHoES collaboration. |
| 2015-02 | 67.74 ± 0.46 | Planck Mission | [27][28] | Results from an analysis of <i>Planck</i> 's full mission were made public on 1 December 2014 at a conference in Ferrara , Italy. A full set of papers detailing the mission results were released in |

ISSUES

- ◉ Systematic error somewhere?
- ◉ **CMB** (advantage : its phenomenal sensitivity to cosmic geometry) community : presume the problem with galaxy measurements : **WMAP** (more primordial) **vs** **Planck** (high l : more secondary effects)
- ◉ **Local galaxies** community : CMB value is a non-starter : **cosmological expansion** **vs** **peculiar velocity**
- ◉ Observational methods from **direct measurement of redshifts and distances** **vs** from **multi-parameter fits**

ISSUES

- Systematic error
- CMB** (advantage geometry) cosmological measurements
- Local galaxies**
- Observational measurements at low redshifts and distances



consistency to cosmic expansion with galaxy

is a non-starter

**Measurement of
cosmological parameter fits**

<https://tritonstation.wisc.edu/>

SL (COSMO18. Aug.30)

on-the-hubble-constant/

H₀ tension and MG

POSSIBLE SOLUTIONS

- ◉ Add extra radiation component ($\nu\Lambda$ CDM) : [Wyman et.al 13](#)
- ◉ Using running vacuum model (RVM) : [Sola et.al 17](#)
- ◉ Non-flat Univ : Extension to Vanila concordance model
- ◉ Inhomogeneous Univ (wLTB) : Tenhu et.al 18
- ◉ Modified gravities (frame dependent) : This work (already experience in Li problem)
- ◉ Others

CURRENT MEASUREMENTS

- Differential Age (DA) : 31 data points
- Baryonic Acoustic Oscillation (BAO) : 73 data points

Table 8: 31 Hubble parameter measurements $H(z)$ [km/s/Mpc] and their errors σ_H at redshift z obtained from DA measurements.

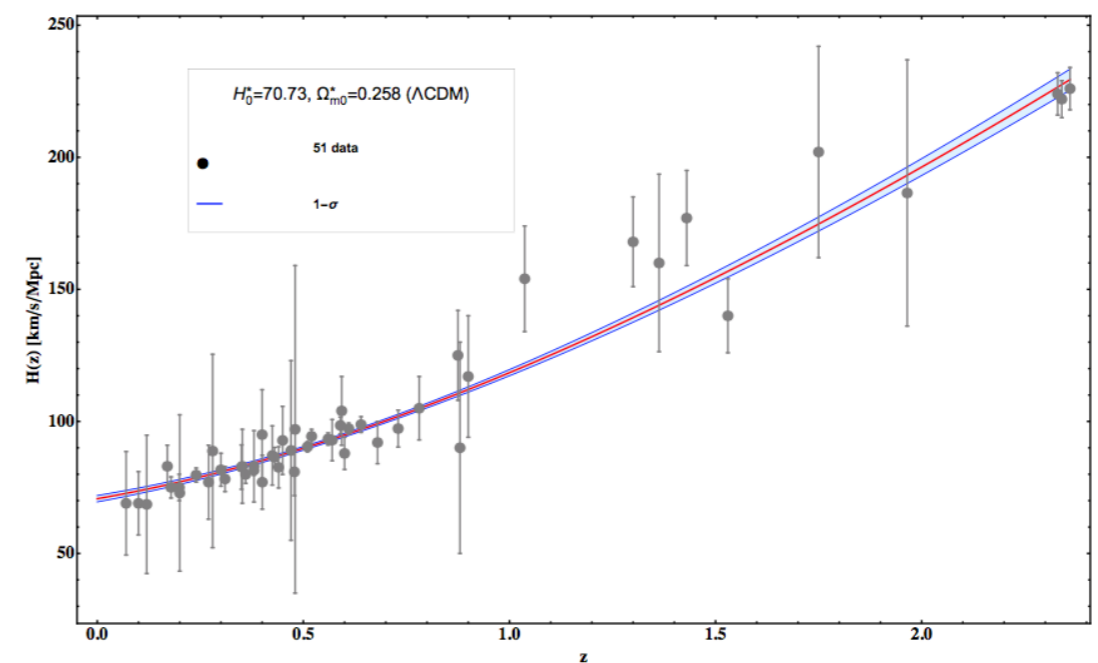
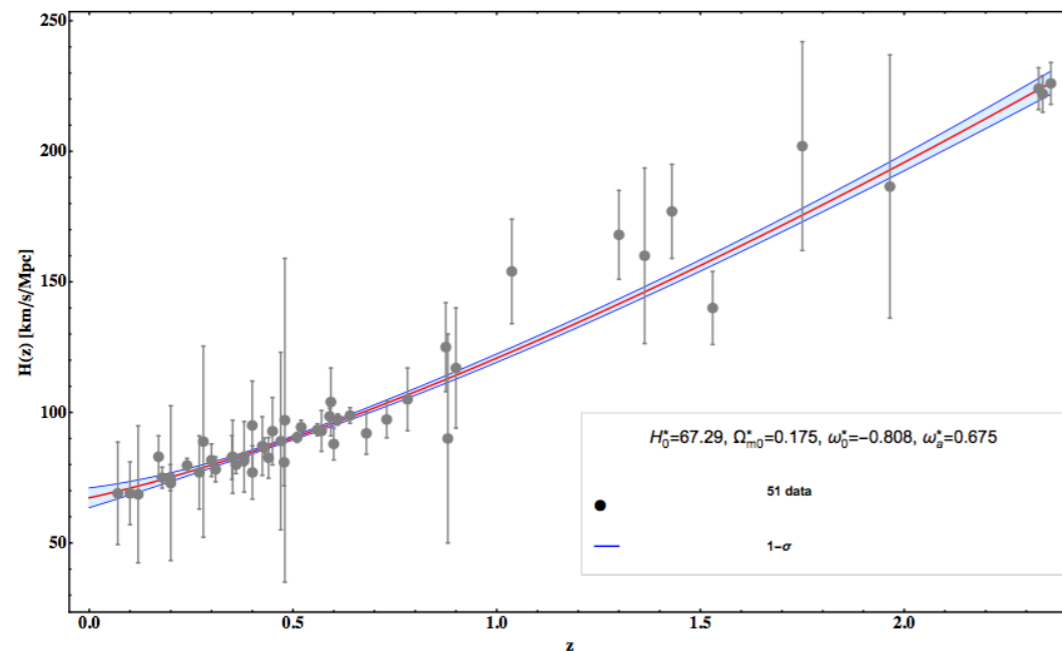
| z | $H(z)$ | σ_H | Ref | z | $H(z)$ | σ_H | Ref |
|--------|--------|------------|-----|--------|--------|------------|-----|
| 0.07 | 69 | 19.6 | [5] | 0.4783 | 80.9 | 9 | [7] |
| 0.1 | 69 | 12 | [3] | 0.48 | 97 | 60 | [3] |
| 0.12 | 68.6 | 26.2 | [5] | 0.5929 | 104 | 13 | [4] |
| 0.17 | 83 | 8 | [3] | 0.6797 | 92 | 8 | [4] |
| 0.1791 | 75 | 4 | [4] | 0.7812 | 105 | 12 | [4] |
| 0.1993 | 75 | 5 | [4] | 0.8754 | 125 | 17 | [4] |
| 0.2 | 72.9 | 29.6 | [5] | 0.88 | 90 | 40 | [3] |
| 0.27 | 77 | 14 | [3] | 0.9 | 117 | 23 | [3] |
| 0.28 | 88.8 | 36.6 | [5] | 1.037 | 154 | 20 | [4] |
| 0.3519 | 83 | 14 | [4] | 1.3 | 168 | 17 | [3] |
| 0.3802 | 83 | 13.5 | [7] | 1.363 | 160 | 33.6 | [6] |
| 0.4 | 95 | 17 | [3] | 1.43 | 177 | 18 | [3] |
| 0.4004 | 77 | 10.2 | [7] | 1.53 | 140 | 14 | [3] |
| 0.4247 | 87.1 | 11.2 | [7] | 1.75 | 202 | 40 | [3] |
| 0.4497 | 92.8 | 12.9 | [7] | 1.965 | 186.5 | 50.4 | [6] |
| 0.47 | 89 | 34 | [8] | | | | |

Table 9: 73 Hubble parameter measurements $H(z)$ [km/s/Mpc] and their errors σ_H at redshift z obtained from BAO measurements.

| z | $H(z)$ | σ_H | Obs | Ω_{m0} | Ref | z | $H(z)$ | σ_H | Obs | Ω_{m0} | Ref |
|------|--|-----------------------------------|---|--------------------------------------|--------------------------------------|---------------|--|-----------------------------------|---|------------------------|--------------------------------------|
| 0.24 | 76.69 78.8 | 2.65 5.6 | DR6 CF DR12 CF | 0.25 0.307 | [9] [23] | 0.52 | 91.97 94.35 89.4 87.5 | 6.88 2.64 5.4 6.8 | DR12 PS DR12 CF DR12 CF DR12 PS | 0.307 | [24] [25] [28] [31] |
| 0.3 | 81.7 | 6.22 | DR7 PS | 0.32 | [18] | 0.56 | 97.30 93.34 96.3 91.7 | 7.20 2.30 5.1 6.1 | DR12 PS DR12 CF DR12 CF DR12 PS | 0.307 | [24] [25] [28] [31] |
| 0.31 | 78.30 78.18 77.5 79.7 | 4.11 4.74 4.1 4.2 | DR12 PS DR12 CF DR12 CF DR12 PS | 0.307 | [24] [25] [28] [31] | 0.57 | 92.4 92.9 102.09 | 4.5 7.8 2.92 | DR9 CF, AP DR9 CF DR12 PS | 0.278 0.274 0.31 | [11] [17] [21] |
| 0.32 | 80.93 75.0 | 4.80 4.0 | DR12 PS DR12 CF | 0.31 0.307 | [21] [25] | 0.59 | 97.07 98.48 96.7 94.3 95.0 | 5.26 3.18 2.7 4.1 3.2 | DR12 PS DR12 CF DR12 CF DR12 CF DR12 PS | 0.307 | [24] [25] [23] [28] [31] |
| 0.34 | 83.80 | 3.36 | DR6 CF | 0.25 | [9] | 0.60 | 87.9 | 6.1 | WiggleZ, AP | 0.27 | [12] |
| 0.35 | 82.1 79.6 82.7 84.4 | 6.86 12.02 8.4 7.0 | DR7 CF DR7 CF DR7 CF DR7 CF | 0.25 0.25 0.25 0.27 | [10] [13] [15] [16] | 0.61 | 97.3 98.9 | 2.1 2.3 | DR12 CF DR12 PS | 0.31 0.31 | [26] [22] |
| 0.36 | 77.20 79.94 81.2 83.5 | 5.31 3.38 5.9 8.9 | DR12 PS DR12 CF DR12 CF DR12 PS | 0.307 | [24] [25] [28] [31] | 0.64 | 97.70 98.82 98.4 96.8 98.0 | 4.58 2.98 3.7 3.5 3.2 | DR12 PS DR12 CF DR12 CF DR12 CF DR12 PS | 0.307 | [24] [25] [23] [28] [31] |
| 0.37 | 74.8 | 6.3 | DR12 CF | 0.307 | [23] | 0.73 | 97.3 | 7.0 | WiggleZ, AP | 0.27 | [12] |
| 0.38 | 81.5 80.7 | 1.9 2.4 | DR12 CF DR12 PS | 0.31 0.31 | [26] [22] | 0.978 | 113.72 | 14.63 | DR14Q PS | 0.314 | [30] |
| 0.40 | 79.72 82.04 82.5 86.0 | 4.31 2.03 4.9 6.5 | DR12 PS DR12 CF DR12 CF DR12 PS | 0.307 | [24] [25] [28] [31] | 1.23 | 131.44 | 12.42 | DR14Q PS | 0.314 | [30] |
| 0.43 | 86.45 | 3.68 | DR6 CF | 0.25 | [9] | 1.52 1.526 | 162 148.11 | 12 12.75 | DR14Q PS DR14Q PS | 0.307 0.314 | [29] [30] |
| 0.44 | 82.6 80.29 84.81 80.8 85.5 | 7.8 2.99 1.83 3.2 2.7 | WiggleZ, AP DR12 PS DR12 CF DR12 CF DR12 PS | 0.27 3.07 3.07 3.07 3.07 | [12] [24] [25] [28] [31] | 1.944 | 172.63 | 14.79 | DR14Q PS | 0.314 | [30] |
| 0.48 | 84.69 87.79 86.2 86.5 | 3.22 2.03 3.6 3.7 | DR12 PS DR12 CF DR12 CF DR12 PS | 0.307 | [24] [25] [28] [31] | 2.3 | 224 | 8 | DR9Q CF | 0.27 | [16] |
| 0.49 | 87.5 | 4.8 | DR12 CF | 0.307 | [23] | 5.233 2.34 | 224 222 | 8 7 | DR12Q CF DR11Q CF | 0.27 0.27 | [27] [20] |
| 0.51 | 90.4 90.8 | 1.9 2.0 | DR12 CF DR12 PS | 0.31 0.31 | [26] [22] | 2.36 | 226 | 8 | DR11Q CF | 0.27 | [19] |

CURRENT MEASUREMENTS

- Differential Age (DA)
- Baryonic Acoustic Oscillation (BAO)



A SOLUTION FROM MG

- ◉ Modified gravities show the additional factor in Hubble parameter in EF
- ◉ $H_{\text{EF}}(z) = F(z) H_{\text{JF}}(z)$
- ◉ This additional factor $F(z)$ gives the different values between H_{EF} and H_{JF} when one adopts same Ω_{m0} , Ω_{r0} , and w

FRAME DEPENDENCES ON $H(z)$

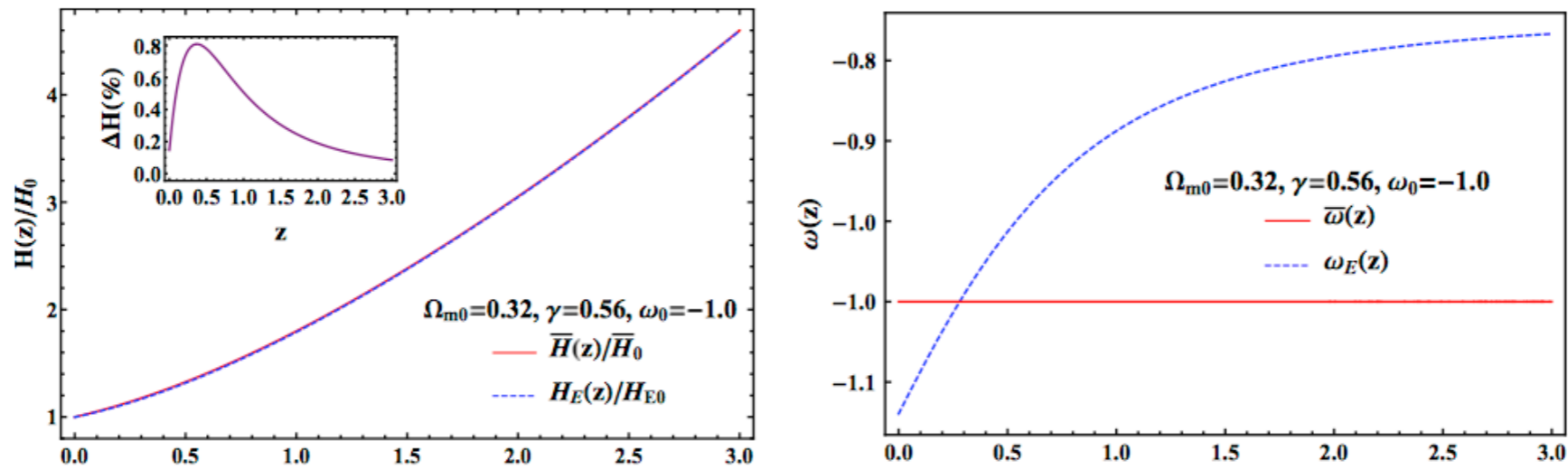


Figure 1: a) Evolutions of the normalized Hubble parameters both on Jordan frame (solid line) and on Einstein frame (dashed line) when $\Omega_{m0} = 0.32, \omega_0 = -1.0$, and $\gamma_0 = 0.56$. b) Evolution of effective equation of states of the dark energy for both frames.

Hyun,Kim
SL : 17

EXPAMPLE :
STG

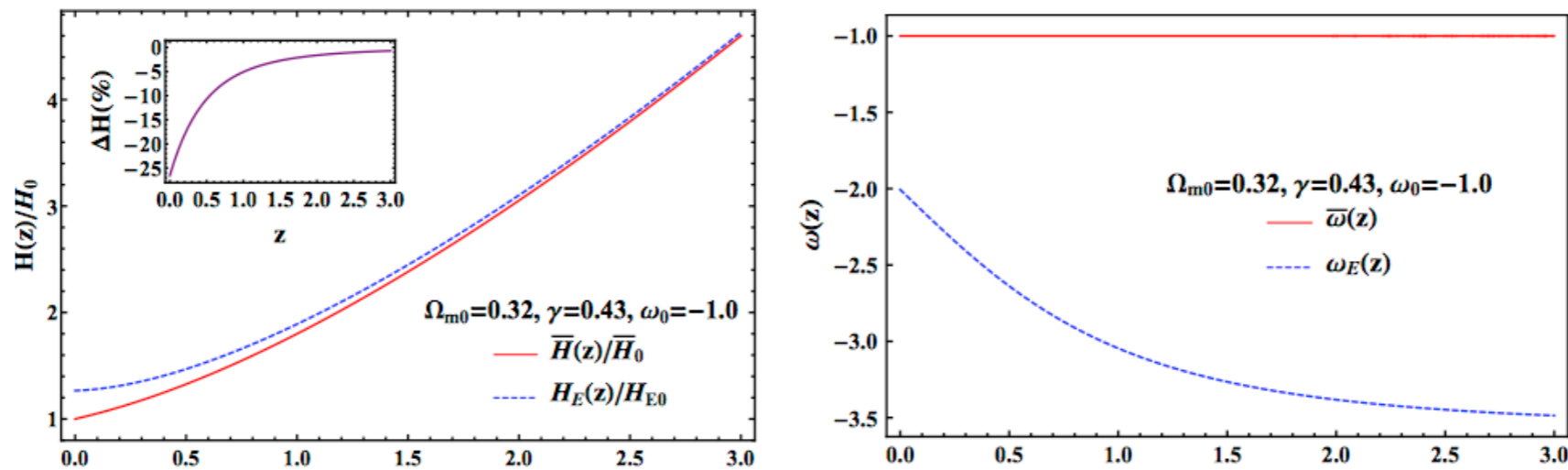


Figure 2: a) Evolutions of the normalized Hubble parameters both on Jordan frame (solid line) and on Einstein frame (dashed line) when $\Omega_{m0} = 0.32, \omega_0 = -1.0$, and $\gamma_0 = 0.43$. b) Evolution of effective equation of states of the dark energy for both frames.

EXTRA

● JF

$$S_J = \frac{1}{16\pi G_*} \int d^4x \sqrt{-\bar{g}} \left[F(\phi) \bar{R} - \bar{g}^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - 2U(\phi) \right] + S_m(\bar{g}_{\mu\nu}, \psi_m)$$

$$\begin{aligned} 3F_0 \bar{H}^2 &= 8\pi G_* \bar{\rho}_m + \frac{1}{2} \bar{H}^2 \phi'^2 - 3\bar{H}^2 F' + 3\bar{H}^2 (F_0 - F) + U, \\ 2F_0 \bar{H} \bar{H}' &= -8\pi G_* \bar{\rho}_m - \bar{H}^2 \phi'^2 - \bar{H}^2 F'' + (\bar{H}^2 - \bar{H} \bar{H}') F' + 2\bar{H} \bar{H}' (F_0 - F), \\ \phi'' + \left(3 + \frac{\bar{H}'}{\bar{H}}\right) \phi' &= 3 \left(2 + \frac{\bar{H}'}{\bar{H}}\right) \frac{F'}{\phi'} - \frac{1}{\bar{H}^2} \frac{U'}{\phi'}, \\ \bar{\rho}'_m + 3\bar{\rho}_m &= 0, \\ \delta''_m + \left(2 + \frac{\bar{H}'}{\bar{H}}\right) \delta'_m - \frac{4\pi G_{\text{eff}} \rho_m}{\bar{H}^2} \delta_m &\simeq 0, \text{ where } G_{\text{eff}} = \frac{G_*}{F} \left(\frac{2F + 4F_{,\phi}^2}{2F + 3F_{,\phi}^2} \right), \end{aligned}$$

$$\bar{g}_{\mu\nu} = F^{-1}(\phi) g_{\mu\nu}$$

● EF

scale factor $a \equiv \sqrt{F} \bar{a}$ and the physical time $dt \equiv \sqrt{F} d\bar{t}$ c

$$S_E = \frac{1}{16\pi G_*} \int d^4x \sqrt{-g} \left[R - 2g^{\mu\nu} \nabla_\mu \varphi \nabla_\nu \varphi - 4V(\varphi) \right] + S_m(g_{\mu\nu}, \psi_m)$$

$$\begin{aligned} 3H^2 &\equiv \left(\frac{1}{a} \frac{da}{dt} \right)^2 = 8\pi G_* \rho + \left(\frac{d\varphi}{dt} \right)^2 + 2V(\varphi) \\ -\frac{3}{a} \frac{d^2 a}{dt^2} &= 4\pi G_* (\rho + 3P) + 2 \left(\frac{d\varphi}{dt} \right)^2 - 2V(\varphi), \end{aligned}$$

CONCLUSIONS

- ◉ The current measurements on $H(z)$ from local and global shows the discrepancies.
- ◉ One should check the systematics in both measurement.
- ◉ If this tension is true, then there are several ways to explain this tension
- ◉ This problem can be solved if one uses MG
- ◉ One needs to investigate the MG effects in other measurements to confirm this solution.

“THANKS!”

(감사합니다)