Better be careful with cosmological frames

Mario Herrero-Valea

Laboratory of particle physics and cosmology
École Polytechnique Fédérale de Lausanne

M. H-V. arXiv:1602.06962
Kevin Falls, M. H-V. arXiv:1809.xxxx

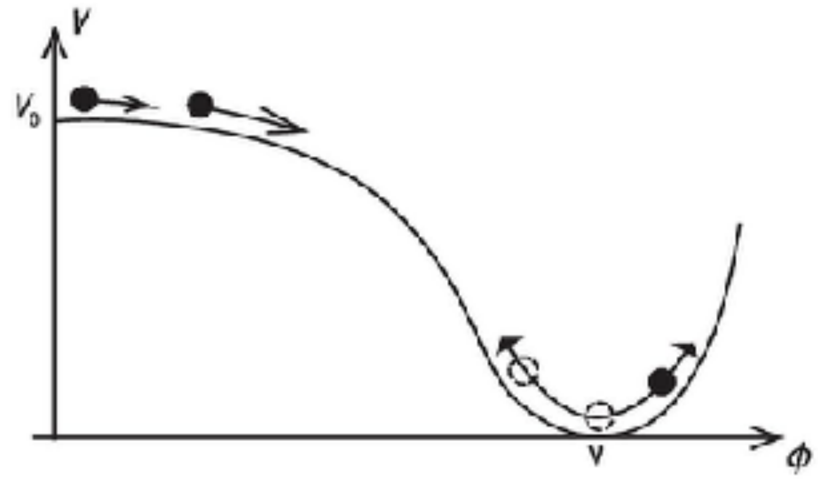
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Inflation models are popular and diverse

$$\mathcal{L} = \sqrt{g}\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi) + F(\phi, R)\right)$$

$$F(\phi, R) \sim \alpha R + \beta R\phi^2 + \dots$$

$$g_{\mu\nu}, \phi \quad \rightarrow \tilde{g}_{\mu\nu}, \tilde{\phi}$$



Jordan frame

$$\mathcal{L}_J = \sqrt{g}\left(-\xi\phi^2 R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{\lambda}{4!}\phi^4\right)$$

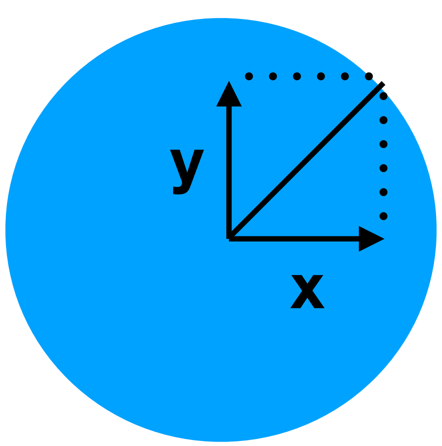
$$Q = \{g_{\mu\nu}, \phi\}$$

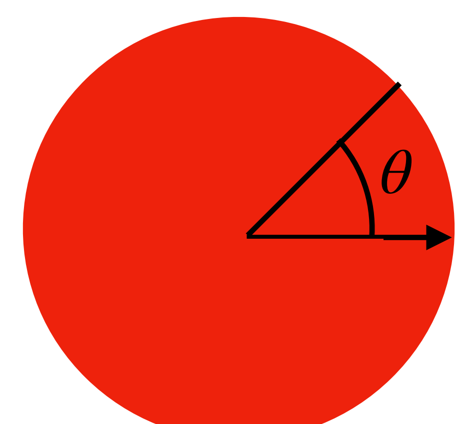
Einstein frame

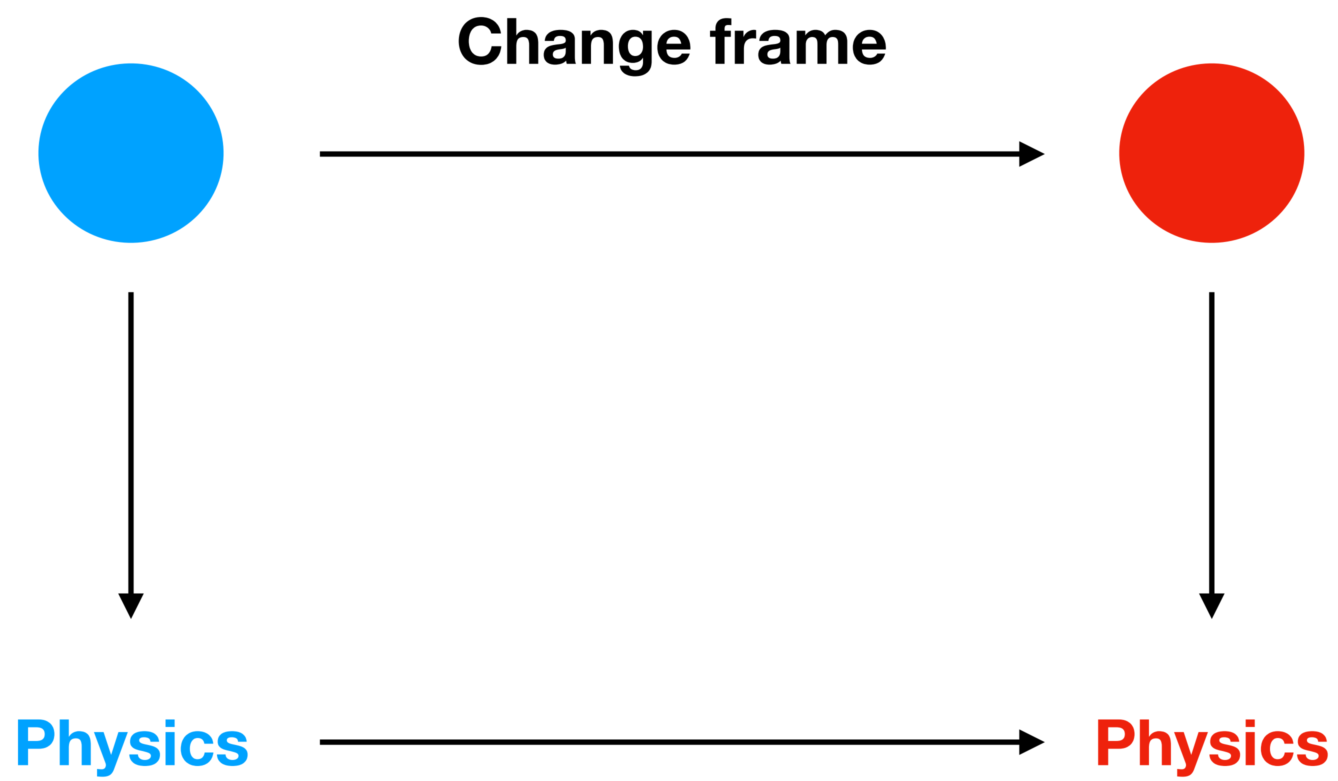
$$\mathcal{L}_E = \sqrt{\tilde{g}}\left(-M_p^2\tilde{R} + \frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} + \frac{\lambda}{4!}\frac{M_p^4}{\xi^2}\right)$$

$$\tilde{Q} = \{\tilde{g}_{\mu\nu}, \tilde{\phi}\}$$

$$\tilde{g}_{\mu\nu} = \frac{\xi\phi^2}{M_p}g_{\mu\nu}, \qquad \tilde{\phi} = M_p\sqrt{\frac{1}{\xi} - 12\log\left(\frac{\phi}{m}\right)}$$







Is this diagram commutative?

It is clear that classical physics is the same

Stationary trajectories —> Stationary trajectories

$$\tilde{Q} = \tilde{Q}(Q) \longrightarrow \frac{\delta S}{\delta Q} = \frac{\delta \tilde{Q}}{\delta Q}\frac{\delta \tilde{S}}{\delta \tilde{Q}}$$

In quantum field theory things are more tricky

$$\mathbf{Z} = \int [DQ] e^{iS(Q)} \longrightarrow \langle \Phi \,|\, \Psi \rangle = \ldots$$

It is clear that classical physics is the same

Stationary trajectories —> Stationary trajectories

$$\tilde{Q} = \tilde{Q}(Q) \longrightarrow \frac{\delta S}{\delta Q} = \frac{\delta \tilde{Q}}{\delta Q}\frac{\delta \tilde{S}}{\delta \tilde{Q}}$$

In quantum field theory things are more tricky

$$\mathbf{Z} = \int [DQ] e^{iS(Q)} \longrightarrow \langle \Phi | \Psi \rangle = \ldots$$

on-shell states $\equiv \frac{\delta S}{\delta Q} = 0$ **Vilkovisky, 1984**

It is clear that classical physics is the same

Stationary trajectories —> Stationary trajectories

$$\tilde{Q} = \tilde{Q}(Q) \longrightarrow \frac{\delta S}{\delta Q} = \frac{\delta \tilde{Q}}{\delta Q}\frac{\delta \tilde{S}}{\delta \tilde{Q}}$$

In quantum field theory things are more tricky

$$\mathbf{Z} = \int [DQ] e^{iS(Q)} \longrightarrow \langle \Phi | \Psi \rangle = \ldots$$

on-shell states $\equiv \frac{\delta S}{\delta Q} = 0$ **Vilkovisky, 1984**

$$\Gamma[Q] = S[Q] + \frac{i\hbar}{2} \log \left[\det \left(\frac{\delta^2 S}{\delta Q}^2 \right) \right] + O(\hbar^2)$$

$$\tilde{\Gamma}[\tilde{Q}] = \tilde{S}[\tilde{Q}] + \frac{i\hbar}{2} \log \left[\det \left(\frac{\delta^2 \tilde{S}}{\delta \tilde{Q}^2} + \left(\frac{\delta \tilde{Q}}{\delta Q} \right)^2 \frac{\delta^2 Q}{\delta \tilde{Q}^2} \frac{\delta \tilde{S}}{\delta \tilde{Q}} \right) \right] + O(\hbar^2)$$

BUT, there might be more transitions in one of the sides!

Jordan frame

$$\mathscr{L}_J = \sqrt{g}\left(-\xi\phi^2 R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{\lambda}{4!}\phi^4\right)$$

$$Q = \{g_{\mu\nu}, \phi\}$$

Einstein frame

$$\mathscr{L}_E = \sqrt{\tilde{g}}\left(-M_p^2\tilde{R} + \frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} + \frac{\lambda}{4!}\frac{M_p^4}{\xi^2}\right)$$

$$\tilde{Q} = \{\tilde{g}_{\mu\nu}, \tilde{\phi}\}$$

$$\tilde{g}_{\mu\nu} = \frac{\xi\phi^2}{M_p}g_{\mu\nu}, \qquad \tilde{\phi} = M_p\sqrt{\frac{1}{\xi} - 12\log\left(\frac{\phi}{m}\right)}$$

Diffeomorphisms
+
Scale invariance

$$g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$$
$$\phi \to \Omega^{-1}\phi$$

Diffeomorphisms
+
Shift symmetry

$$\tilde{\phi} \to \tilde{\phi} + c$$

Scale invariance is <u>anomalous</u>

$$\langle\mathscr{A}\rangle \neq 0 \longrightarrow \Upsilon = \mathscr{A}\,|0\rangle \longrightarrow \langle 0\,|\,\Upsilon\rangle \neq 0$$

Shift symmetry is not anomalous

$$\langle\mathscr{A}\rangle = 0$$

Jordan frame

$$\mathscr{L}_J = \sqrt{g}\left(-\xi\phi^2 R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{\lambda}{4!}\phi^4\right)$$

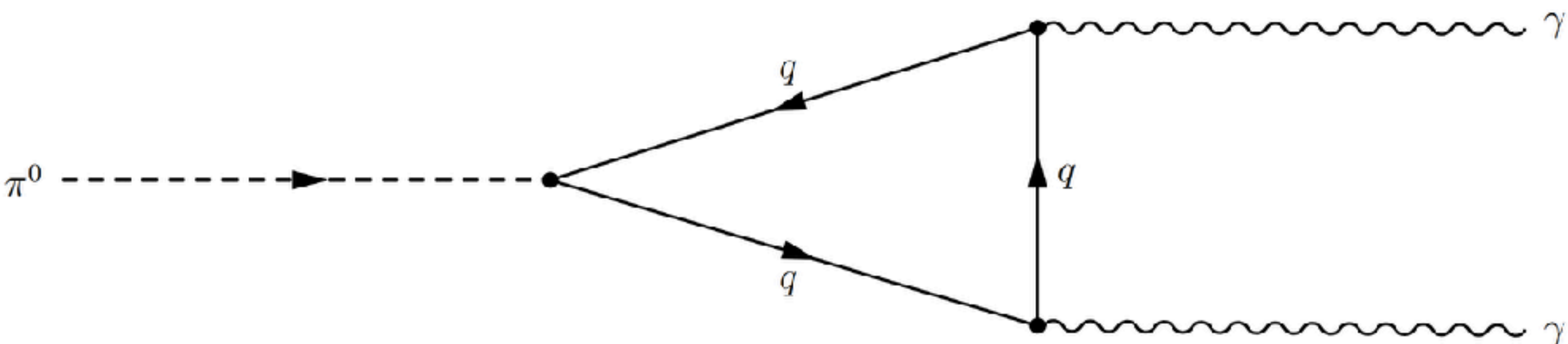
$$Q = \{g_{\mu\nu}, \phi\}$$

Einstein frame

$$\mathscr{L}_E = \sqrt{\tilde{g}}\left(-M_p^2\tilde{R} + \frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} + \frac{\lambda}{4!}\frac{M_p^4}{\xi^2}\right)$$

$$\tilde{Q} = \{\tilde{g}_{\mu\nu}, \tilde{\phi}\}$$

$$\tilde{g}_{\mu\nu} = \frac{\xi\phi^2}{M_p}g_{\mu\nu}, \qquad \tilde{\phi} = M_p\sqrt{\frac{1}{\xi} - 12\log\left(\frac{\phi}{m}\right)}$$



$$\mathbf{Z} = \int [DQ] e^{iS(Q)}$$

$$\mathbf{Z} = \int [DQ] e^{iS(Q)} \qquad \tilde{\mathbf{Z}} = \int [D\tilde{Q}] e^{i\tilde{S}(\tilde{Q})}$$

$$[D\tilde{Q}] \neq \frac{\delta \tilde{Q}}{\delta Q} [DQ]$$

Measure depends on geometry!!

$$\mathbf{Z} = \int [DQ] e^{iS(Q)}$$

$$\mathbf{Z} = \int [DQ] e^{iS(Q)} \qquad \tilde{\mathbf{Z}} = \int [D\tilde{Q}] e^{i\tilde{S}(\tilde{Q})}$$

$$[D\tilde{Q}] \neq \frac{\delta \tilde{Q}}{\delta Q} [DQ]$$

Measure depends on geometry!!

$$[DQ] \sim \det G \qquad G^{\mu\nu}\partial_\mu \partial_\nu$$

for the scalar $\rightarrow g^{\mu\nu}\partial_\mu\partial_\nu$

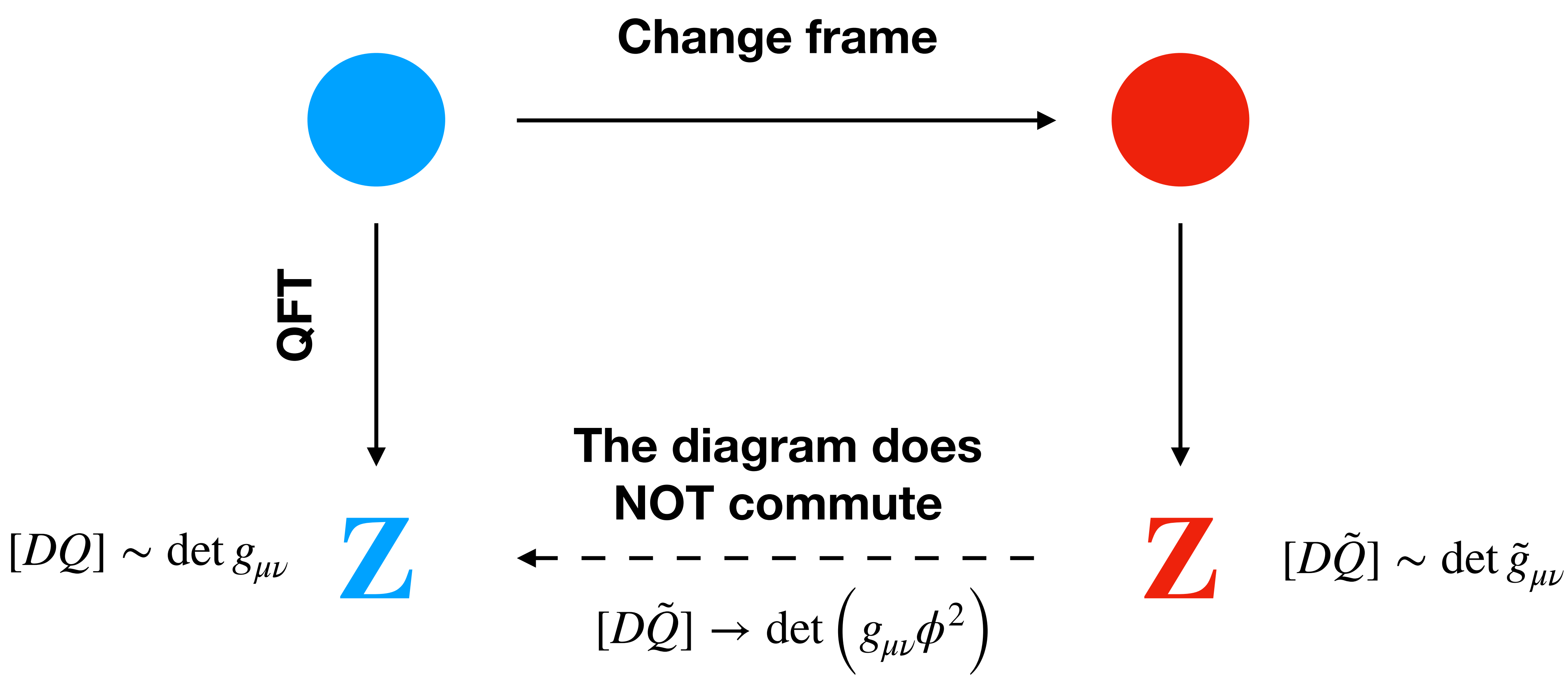
But $g_{\mu\nu}$ **transforms with a field dependent piece** $\rightarrow \det G$ **is field dependent**

$$\tilde{g}_{\mu\nu} = \frac{\xi \phi^2}{M_p} g_{\mu\nu}, \qquad \tilde{\phi} = M_p \sqrt{\frac{1}{\xi} - 12} \log\left(\frac{\phi}{m}\right)$$

$$\mathbf{Z} = \int [DQ] e^{iS(Q)}$$

$$\mathbf{Z} = \int [DQ] e^{iS(Q)} \qquad \tilde{\mathbf{Z}} = \int [D\tilde{Q}] e^{i\tilde{S}(\tilde{Q})}$$

$$[D\tilde{Q}] \neq \frac{\delta \tilde{Q}}{\delta Q} [DQ]$$



Jordan frame

$$\mathscr{L}_J = \sqrt{g}\left(-\xi\phi^2 R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{\lambda}{4!}\phi^4\right)$$

$$Q = \{g_{\mu\nu}, \phi\}$$

$$\langle \mathscr{A} \rangle \neq 0 \longrightarrow \Upsilon = \mathscr{A}\,|0\rangle \longrightarrow \langle 0|\,\Upsilon\rangle \neq 0$$

Einstein frame

$$\mathscr{L}_E = \sqrt{\tilde{g}}\left(-M_p^2\tilde{R} + \frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} + \frac{\lambda}{4!}\frac{M_p^4}{\xi^2}\right)$$

$$\tilde{Q} = \{\tilde{g}_{\mu\nu}, \tilde{\phi}\}$$

$$\tilde{g}_{\mu\nu} = \frac{\xi\phi^2}{M_p} g_{\mu\nu}, \qquad \tilde{\phi} = M_p\sqrt{\frac{1}{\xi} - 12\log\left(\frac{\phi}{m}\right)}$$

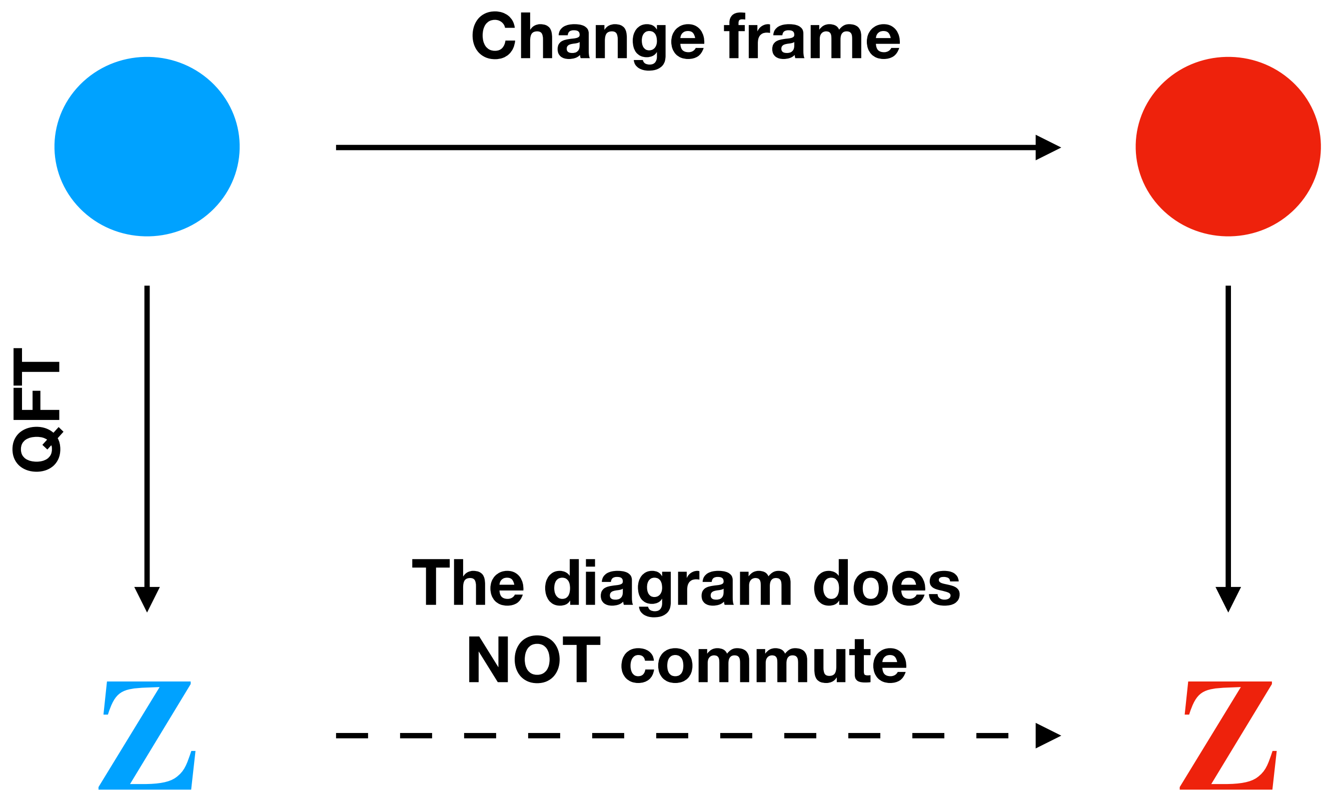
$$\langle \mathscr{A} \rangle = 0$$

The integration measure generates a new term which cancel the anomaly!

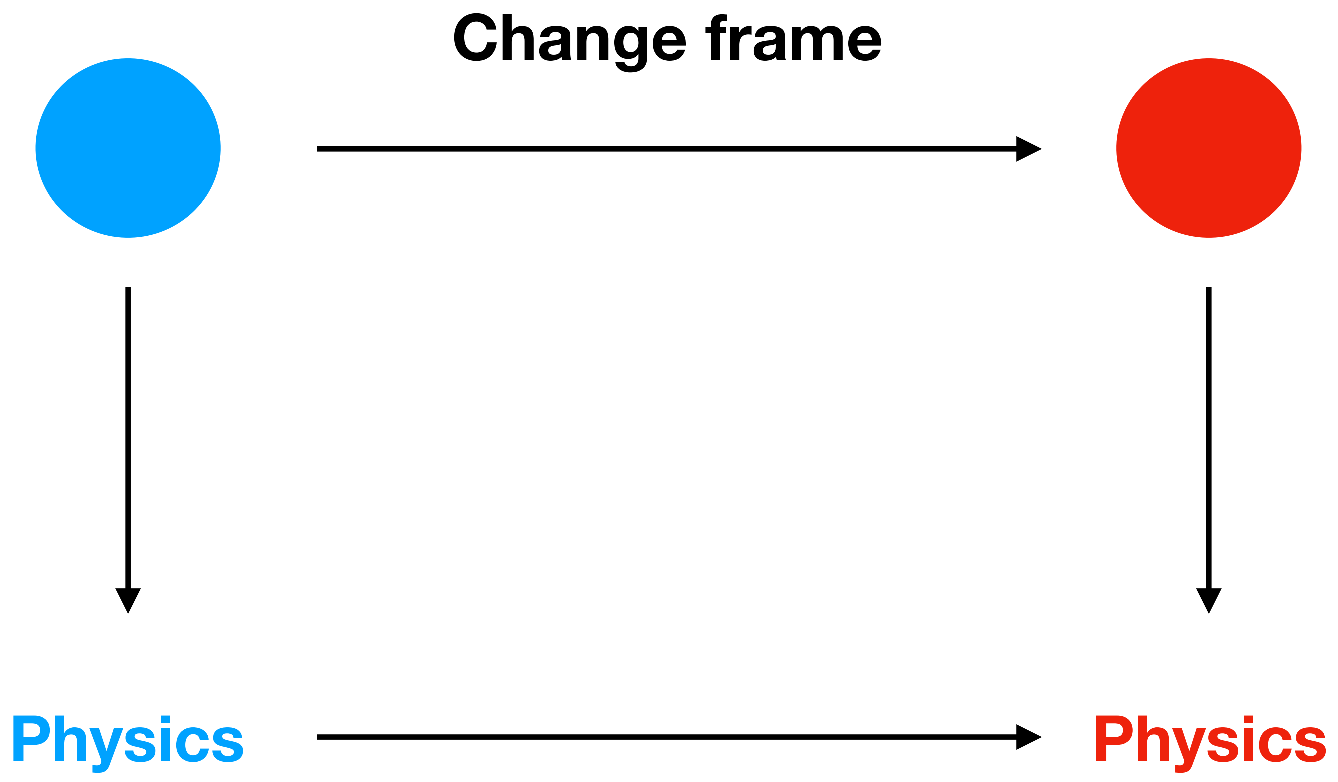
$$\Gamma[Q] = S[Q] + \frac{i\hbar}{2}\log\left[\det\left(\frac{\delta^2 S}{\delta Q}^2\right)\right] + \mathscr{B} + O(\hbar^2), \qquad \delta\mathscr{B} = -\,\mathscr{A}$$

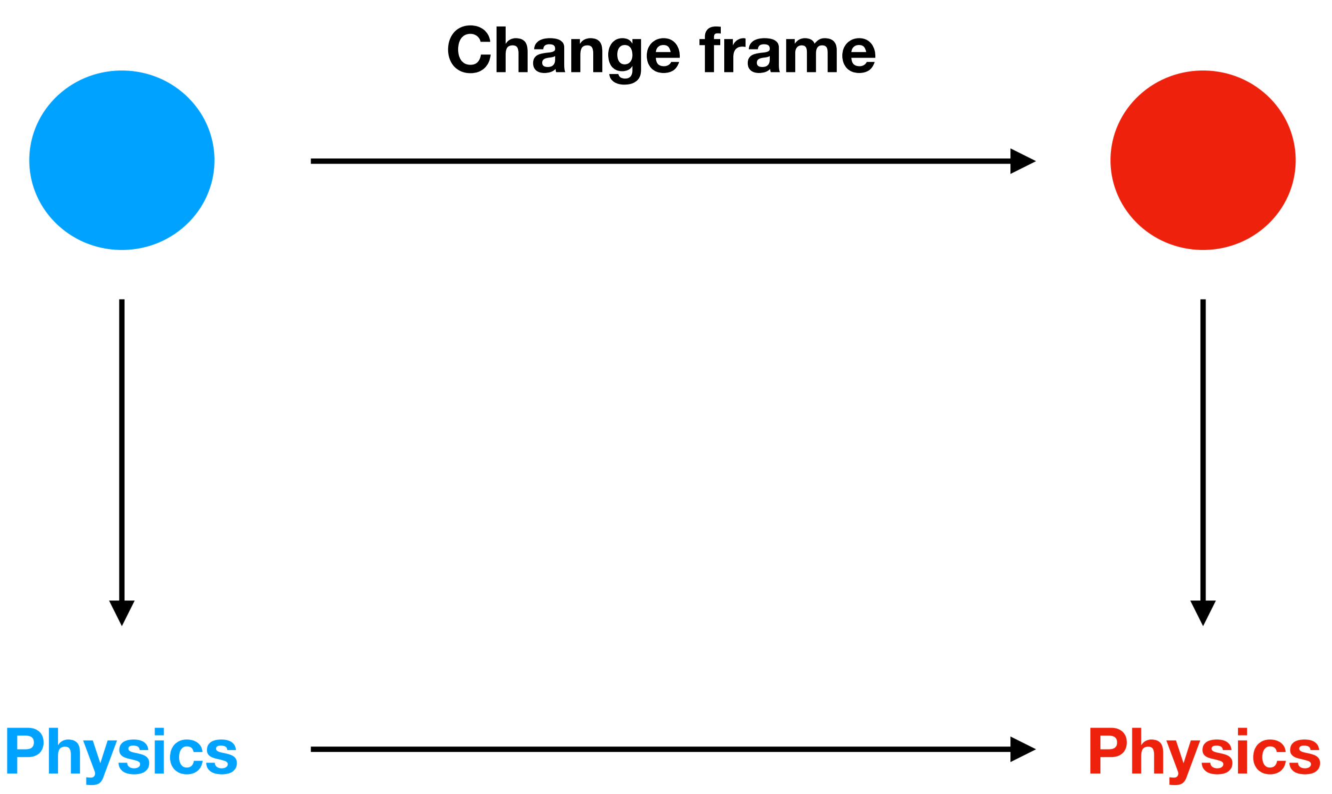
$$\mathscr{B} \sim \int d^4x\sqrt{g}\,\frac{\phi}{m}(R_{\mu\nu\alpha\beta})^2$$

Physics is the same if we take all pieces into account



Change frame
QFT
The diagram does
NOT commute
Z
Z


Change frame
Physics
Physics



For this to be true we need to take into account the non-trivial transformation of the integration measure

Quantum Field Theory cares about frames!!

There might be important effects in inflation models

$$\mathscr{B} \sim \int d^4x \sqrt{g}\, \frac{\phi}{m} (R_{\mu\nu\alpha\beta})^2$$

Summarising

- The integration measure on the path integral depends on the background fields
- One of the background fields is the metric. Measure depends on geometry
- Under a frame re-parameterisation, the measure picks up non-trivial pieces
- A new term in the finite part of the effective action is generated

QFT (with gravity) remembers the original variables!

Better be careful with frames when doing cosmology

Thank you!