

Primordial Gravitational Waves induced by Gauss-Bonnet Inflation

Gansukh Tumurtushaa



based on: arXiv:1807.04424 with Seoktae Koh and Bum-Hoon Lee

Contents:

- Introduction
- Review: Inflationary models with the Gauss-Bonnet term
- Gravitational waves induced by a Gauss-Bonnet inflation
- Summary

- According to the latest Planck 2018 results,
 - the spectral tilt of the curvature perturbation:

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n_s = 0.9653 \pm 0.0041 \ (68 \% CL, Planck TT,TE,EE+lowE+lensing+BK14)
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• upper limit on the tensor-to-scalar ratio:

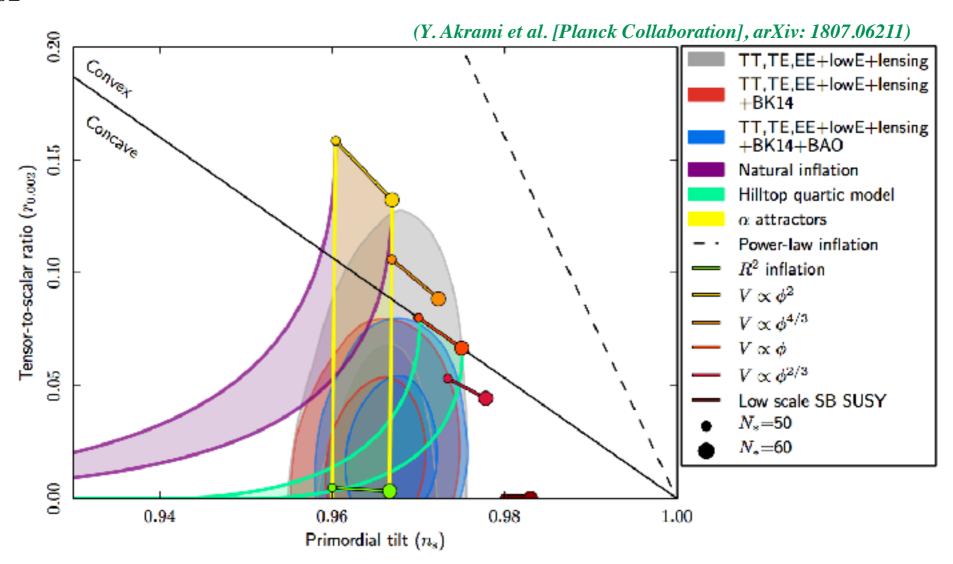
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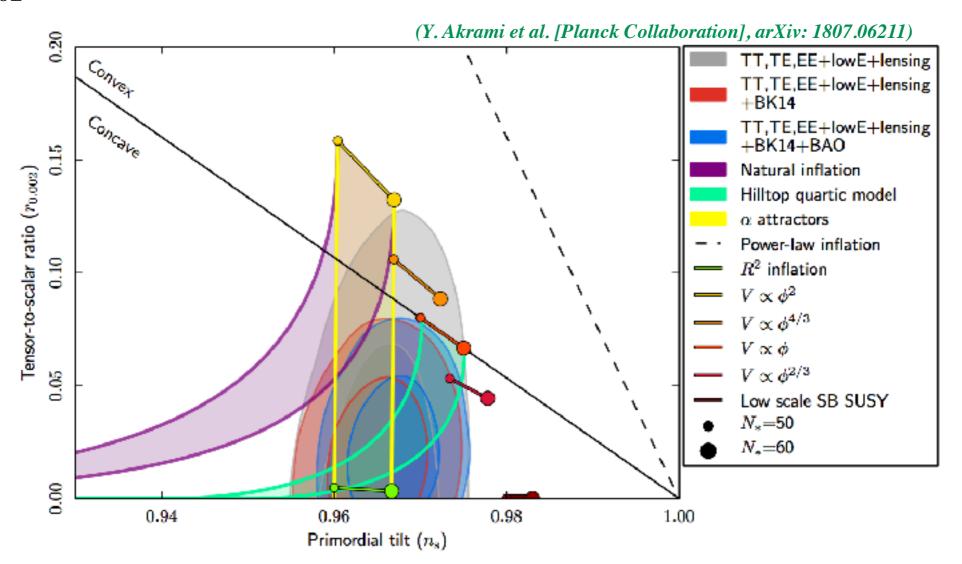


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• Among hundreds of inflation models that are consistent with the observational data, extended models of inflation seem to be more promising! The simplest scenario of inflation is based upon a single field, which is minimally coupled to a gravity, with a flat potential;

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi - V(\phi) \right],$$

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- We consider high order [quantum] corrections to Ricci-scalar, so-called the Gauss-Bonnet term:

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- Thus, the action becomes,

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2} g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi - V(\phi) - \frac{1}{2} \xi(\phi) R_{\text{GB}}^2 \right],$$

• Here, it is worth noting that $\xi(\phi) \neq const$.

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• The background EoM in flat FRW universe: $ds^2 = -dt^2 + a^2 \left(dr^2 + r^2 d\Omega^2 \right),$

$$\begin{split} H^2 &= \frac{\kappa^2}{3} \left(\frac{1}{2} \dot{\phi}^2 + V + 12 \dot{\xi} H^3 \right) , \\ \dot{H} &= -\frac{\kappa^2}{2} \left[\dot{\phi}^2 - 4 \ddot{\xi} H^2 - 4 \dot{\xi} H \left(2 \dot{H} - H^2 \right) \right] , \\ \ddot{\phi} &+ 3 H \dot{\phi} + V_{\phi} + 12 \xi_{\phi} H^2 \left(\dot{H} + H^2 \right) = 0 , \end{split}$$



Thus, we are interested in understanding the effects of this additional term

- to the inflationary observables
- to the primordial GW spectra

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The observable quantities for GB inflation models are derived in our previous work [PRD 90, 063527 (2014)] as,

$$\mathscr{P}_{S}(k) = \mathscr{P}_{S}(k_{*}) \left(\frac{k}{k_{*}}\right)^{n_{S}-1}, \qquad \mathscr{P}_{T}(k) = \mathscr{P}_{T}(k_{*}) \left(\frac{k}{k_{*}}\right)^{n_{T}}$$

$$n_S - 1 \simeq -2\epsilon - \frac{2\epsilon(2\epsilon + \eta) - \delta_1(\delta_2 - \epsilon)}{2\epsilon - \delta_1}, \quad n_T \simeq -2\epsilon, \quad r \simeq 8(2\epsilon - \delta_1),$$

where
$$\epsilon \equiv -\frac{\dot{H}}{H^2}$$
, $\eta \equiv \frac{\ddot{H}}{H\dot{H}}$, $\zeta \equiv \frac{\dddot{H}}{H^2\dot{H}}$, $\delta_1 \equiv 4\kappa^2\dot{\xi}H$, $\delta_2 \equiv \frac{\ddot{\xi}}{\dot{\xi}H}$, $\delta_3 = \frac{\dddot{\xi}}{\dot{\xi}H^2}$.

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class-I:
$$n_T < 0$$
, $n_T \simeq -2\epsilon$ $\epsilon > 0$, $n_T \simeq -2\epsilon$ $\epsilon < 0$, class-II: $n_T > 0$, $n_T \simeq -2\epsilon$

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$$\epsilon = \frac{1}{2\kappa^2} \frac{V_{\phi}}{V} \left(\frac{V_{\phi}}{V} + \frac{4}{3} \kappa^4 \xi_{\phi} V \right)$$

Conditions for the potential and the coupling functions:

$$\begin{cases} \xi_{\phi} > -\frac{3}{4\kappa^4} \frac{V_{\phi}}{V^2}, & \text{for } V_{\phi} > 0, \\ \xi_{\phi} < -\frac{3}{4\kappa^4} \frac{V_{\phi}}{V^2}, & \text{for } V_{\phi} < 0. \end{cases}$$

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Selected models for each category:

Model-I:
$$V(\phi) = \frac{V_0}{\kappa^4} (\kappa \phi)^n$$
, $\xi(\phi) = \xi_0 (\kappa \phi)^{-n}$,

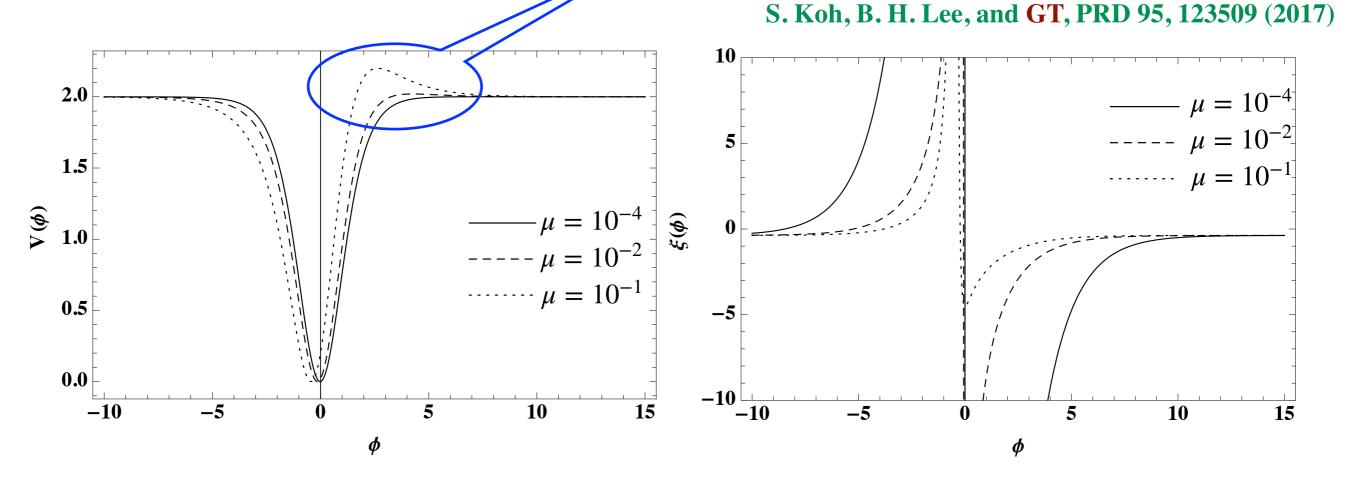
$$\textit{Model-II:} \quad V(\phi) = \frac{1}{\kappa^4} \left[\tanh\left(\kappa\phi\right) + \sqrt{\mu} \operatorname{sech}\left(\kappa\phi\right) \right]^2 \,, \quad \xi(\phi) = \frac{3 \left[\sinh^2(\kappa\phi) - \frac{1}{\sqrt{\mu}} \sinh(\kappa\phi) \right]}{4 \left[\sqrt{\mu} + \sinh\left(\kappa\phi\right) \right]^2} \,,$$

S. Koh, B. H. Lee, and **GT**, PRD 95, 123509 (2017)

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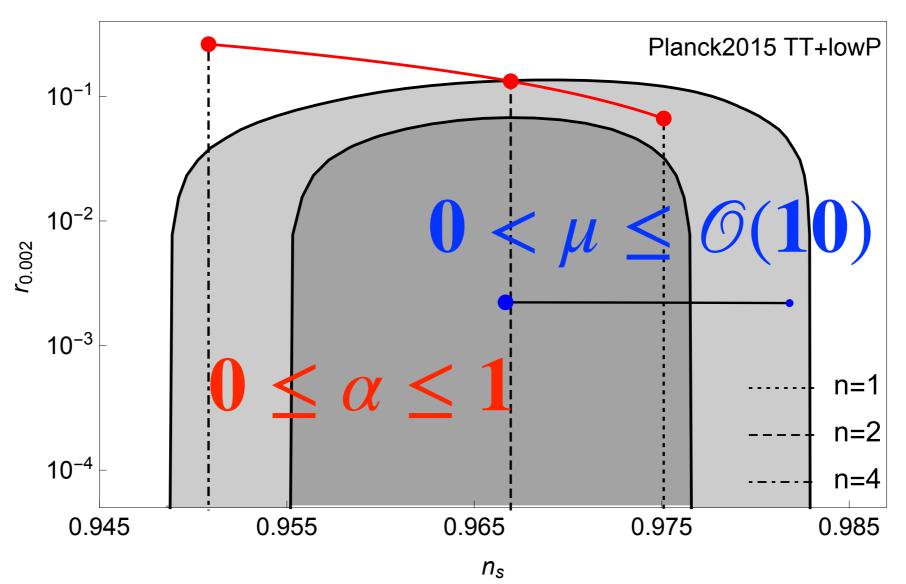
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• These models predict the inflationary tensor power spectrum to have both $n_T < 0$, and $n_T > 0$.

Primordial GW spectrum

• The pGWs are described by the tensor part of the pert. metric

$$ds^{2} = a^{2}(\tau) \left[-d\tau^{2} + (\delta_{ij} + h_{ij}) dx^{i} dx^{j} \right],$$

The strength of GW is characterized by their energy spectrum

$$\Omega_{GW}(k) = \frac{1}{\rho_{\text{crit}}} \frac{d\rho_{GW}}{d\ln k},$$

$$\rho_{GW} = \frac{M_p^2}{4} \int d\ln k \left(\frac{k}{a}\right)^2 \frac{k^3}{\pi^2} \sum_{\lambda} \langle h_{\lambda,k}^{\dagger} h_{\lambda,k} \rangle.$$

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$$P_T \equiv \frac{k^3}{\pi^2} \sum_{\lambda} \langle h_{\lambda,k}^{\dagger} h_{\lambda,k} \rangle = \mathcal{T}^2(k) \mathcal{P}_T(k).$$

$$\mathcal{P}_T(k) = \mathcal{P}_T(k_*) \left(\frac{k}{k_*}\right)^{n_T + \frac{\alpha_T}{2} \ln(k/k_*)},$$

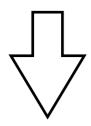
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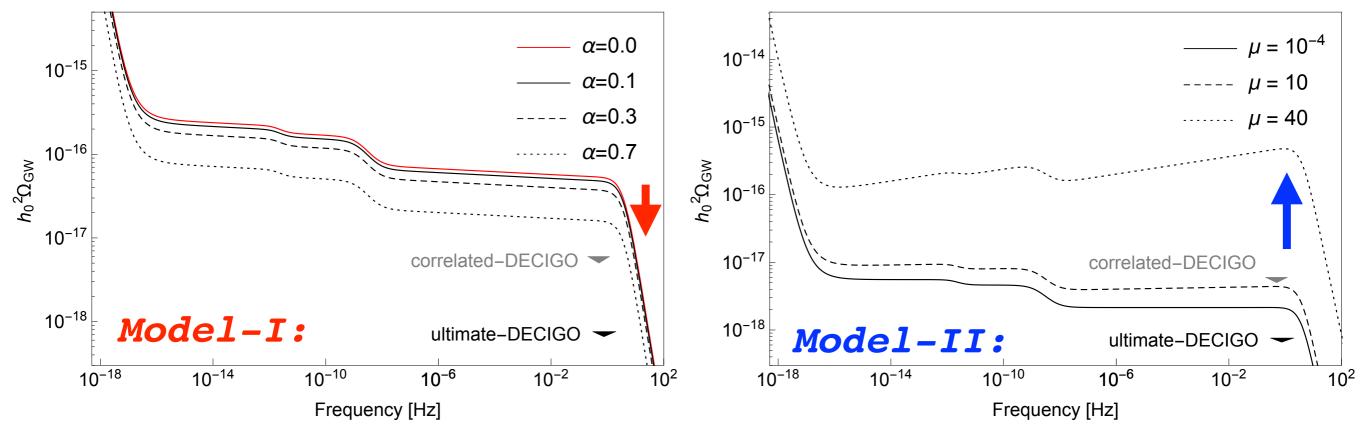
$$h_{\lambda,k}'' + 2\frac{z_T'}{z_T}h_{\lambda,k}' + k^2c_T^2h_{\lambda,k} = 0,$$

$$h_0^2 \Omega_{GW} = \frac{3h_0^2}{32\pi^2 H_0^2 \tau_0^4 f^2} \Omega_m^2 \mathcal{T}_1^2 \left(\frac{f}{f_{\text{eq}}}\right) \mathcal{T}_2^2 \left(\frac{f}{f_{\text{th}}}\right) r \mathcal{P}_S \left(\frac{f}{f_*}\right)^{n_T + \frac{\alpha_T}{2} \ln(f/f_*)},$$

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$$V(\phi) = \frac{V_0}{\kappa^4} (\kappa \phi)^n , \quad \xi(\phi) = \xi_0 (\kappa \phi)^{-n} , \quad \text{with} \quad \alpha \equiv \frac{4}{3} V_0 \xi_0$$

$$V(\phi) = \frac{1}{\kappa^4} \left[\tanh(\kappa \phi) + \sqrt{\mu} \operatorname{sech}(\kappa \phi) \right]^2 , \quad \xi(\phi) = \frac{3 \left[\sinh^2(\kappa \phi) - \frac{1}{\sqrt{\mu}} \sinh(\kappa \phi) \right]}{4 \left[\sqrt{\mu} + \sinh(\kappa \phi) \right]^2} ,$$



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$$Model-I:$$

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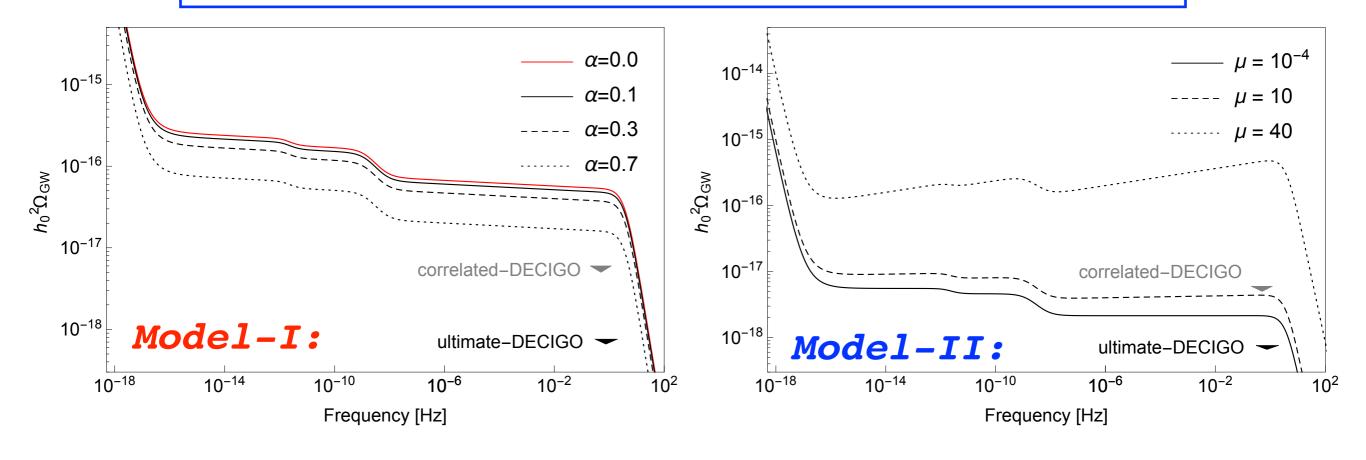
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$$\mathcal{T}_{1}^{2}\left(\frac{k}{k_{\text{eq}}}\right) = 1 + 1.65\left(\frac{k}{k_{\text{eq}}}\right) + 1.92\left(\frac{k}{k_{\text{eq}}}\right)^{2}, \qquad \mathcal{T}_{2}^{2}\left(\frac{k}{k_{\text{th}}}\right) = \left[1 + \gamma\left(\frac{k}{k_{\text{th}}}\right)^{\frac{3}{2}} + \sigma\left(\frac{k}{k_{\text{th}}}\right)^{2}\right]^{-1},$$

Once the pGWs from inflationary origin are detected:

$$k_{\text{th}} = 1.7 \times 10^{13} \,\text{Mpc}^{-1} \left(\frac{g_{*s}(T_{\text{th}})}{106.75}\right)^{\frac{1}{6}} \left(\frac{T_{\text{th}}}{10^6 \,\text{GeV}}\right)$$

Constraints on reheating parameters

- By assuming,
 - constant equation-of-state during reheating, $\omega_{\mathsf{th}} = const$,
- no entropy production after the end of reheating we calculate the duration of reheating and the thermalization temperature at the end of reheating,

$$N_{\rm th} = \frac{4}{3\omega_{\rm th} - 1} \left[\ln \left(\frac{k}{a_0 T_0} \right) + \frac{1}{3} \ln \left(\frac{11 g_{*s}}{43} \right) + \frac{1}{4} \ln \left(\frac{30 \lambda_{\rm end}}{\pi^2 g_*} \right) + \frac{1}{4} \ln \left(\frac{V_{\rm end}}{H_*^4} \right) + N_* \right].$$

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 where $\lambda_{
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m end}}$

• In our numerical study, we consider following models:

$$\textit{Model-I:} \qquad V(\phi) = \frac{V_0}{\kappa^4} (\kappa \phi)^n \,, \quad \xi(\phi) = \xi_0 (\kappa \phi)^{-n} \,, \quad \square \!\!\!\! > \quad \alpha \equiv \frac{4}{3} V_0 \xi_0$$

Model-II:
$$V(\phi) = \frac{1}{\kappa^4} \left[\tanh(\kappa\phi) + \sqrt{\mu} \operatorname{sech}(\kappa\phi) \right]^2, \quad \xi(\phi) = \frac{3 \left[\sinh^2(\kappa\phi) - \frac{1}{\sqrt{\mu}} \sinh(\kappa\phi) \right]}{4 \left[\sqrt{\mu} + \sinh(\kappa\phi) \right]^2},$$

$$N_{\text{th}} = \frac{V_0}{\kappa^4} (\kappa \phi)^n, \quad \xi(\phi) = \xi_0 (\kappa \phi)^{-n}, \quad \text{with} \quad \alpha = \frac{4}{3} V_0 \xi_0$$

 n_{S}

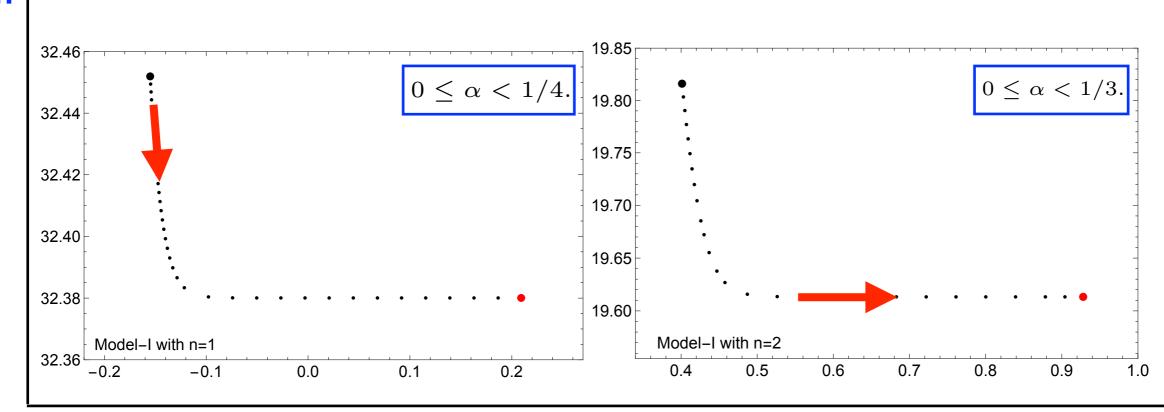
From solid to dotted: $\omega_{th} = -1/3; 0; 1/4$ and 1.

$$\mbox{Model-I:} \ V(\phi) = \frac{V_0}{\kappa^4} (\kappa \phi)^n \,, \quad \xi(\phi) = \xi_0 (\kappa \phi)^{-n} \,, \qquad \mbox{with} \quad \alpha \equiv \frac{4}{3} V_0 \xi_0 \,. \label{eq:Vphi}$$

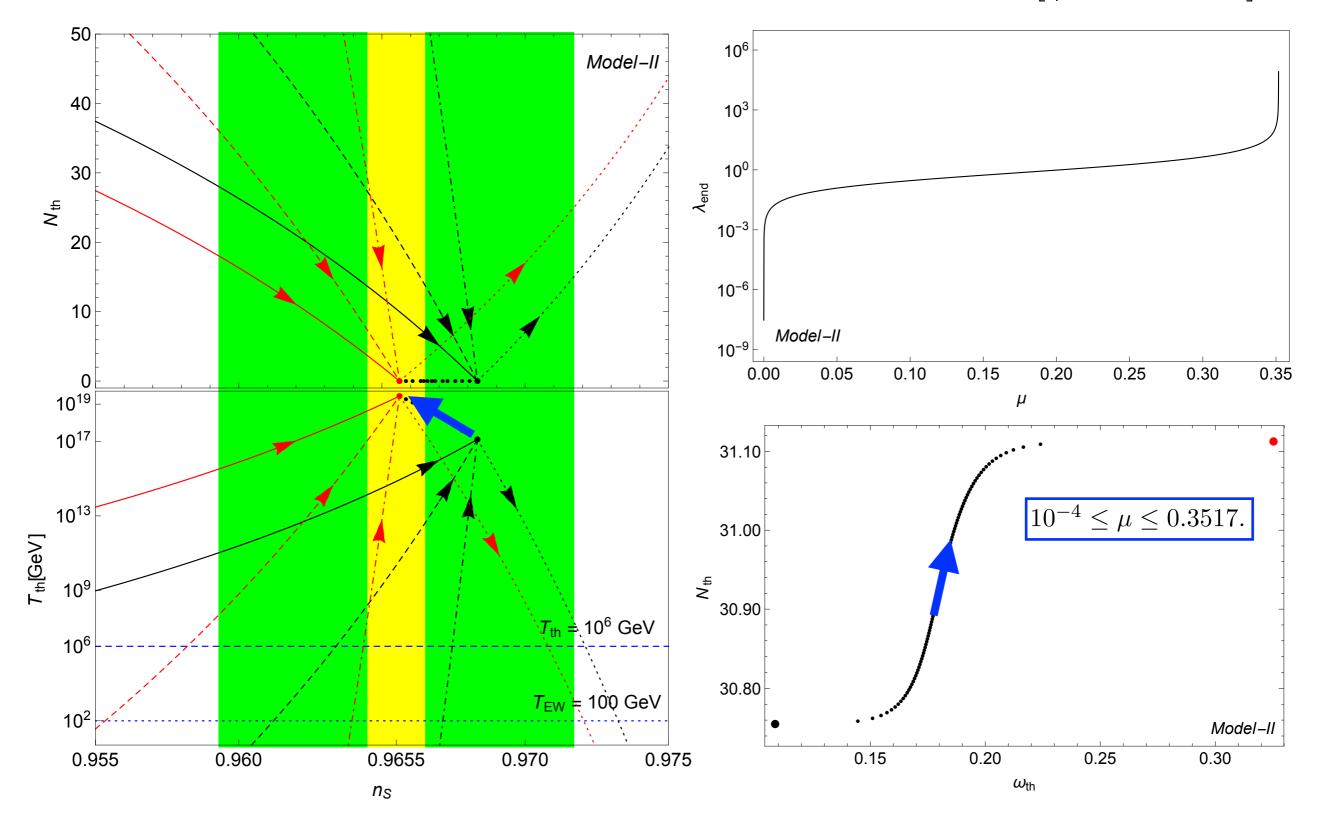
$$N_{\rm th} = \frac{4}{3\omega_{\rm th} - 1} \left[-60.0085 + \frac{1}{4} \ln \left(\frac{3\lambda_{\rm end}}{100\pi^2} \right) + \frac{1}{4} \ln \left(\frac{V_{\rm end}}{H_*^4} \right) + N_* \right]$$

$$\lambda_{\mathrm{end}} = \frac{6}{6-2\epsilon-\delta_1(5-2\epsilon+\delta_2)}\Big|_{\phi=\phi_{\mathrm{end}}} > 0; \quad 0 \leq \alpha \leq 1 \quad \square > \quad 0 \leq \alpha < \frac{n}{2n+2}.$$

$$N_{\mathsf{th}}$$
 for T_{th} =10 6 GeV and n_s =0.9655:



Model-II:
$$V(\phi) = \frac{1}{\kappa^4} \left[\tanh(\kappa\phi) + \sqrt{\mu} \operatorname{sech}(\kappa\phi) \right]^2, \quad \xi(\phi) = \frac{3 \left[\sinh^2(\kappa\phi) - \frac{1}{\sqrt{\mu}} \sinh(\kappa\phi) \right]}{4 \left[\sqrt{\mu} + \sinh(\kappa\phi) \right]^2},$$





We are interested in understanding the effects of this additional term

- to the inflationary observables
- to the primordial GW spectra

SUMMARY:

- Inflationary models with a Gauss-Bonnet term are consistent with observational data,
- These models predict both red- and blue-tilted inflationary tensor power spectrum,
- Primordial GW spectrum suppresses (enhances) for $n_T < 0$ ($n_T > 0$),
- Once pGWs are detected (DECIGO), T_{th} can be determined hence the other parameter of reheating including N_{th} and W_{th} can also be determined.
- Tth significantly increases in the presence of the GB term
- Moreover, reheating can be used as an additional constraint to inflationary models!



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THANK YOU FOR YOUR KIND ATTENTION!