The no-boundary proposal in loop quantum cosmology

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(with Martin Bojowald) arXiv:1809.XXXXX

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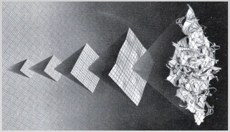
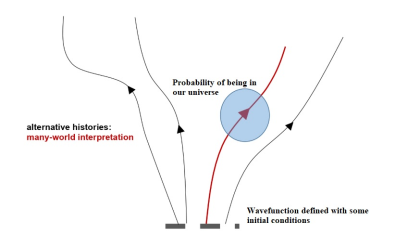
Why quantum cosmology?

"... it appears that quantum theory would have to modify not only Maxwellian electrodynamics, but also the new theory of gravitation."
– Albert Einstein, 1916.



→ Need to look for indirect probes for quantum gravity in early universe cosmology when very high energy scales were naturally reached ⇒ Quantum cosmology.



Ingredients for quantum cosmology

→ Quantum cosmology entails treating the universe as a quantum system.

→ Two parts of the final theory:

- The Hamiltonian (or action) determines the dynamics ⇒ Corrections from quantum gravity?
- The quantum state of the universe ⇒ Initial conditions Set by some 'topological' principle?

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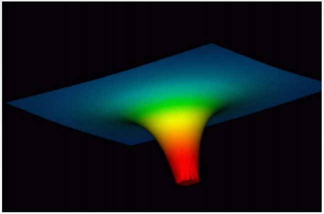
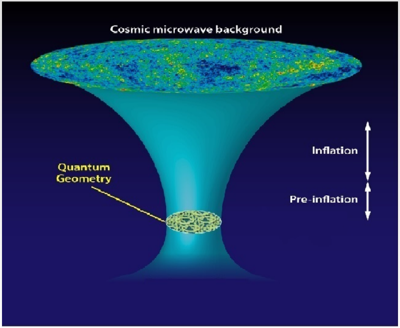


Credit: Pablo Laguna



The no-boundary proposal

→ Restrict to minisuperspace, spatially closed cosmologies with a cosmological constant or a single scalar field.

→ Wavefunction specified by the value of the 3−metric and spatial field configuration on a final spacelike surface $\Sigma \Rightarrow \Psi = \Psi[h_{ab}, \chi]$

→ Saddle-point approximation [J. Hartle & S. Hawking, 1983]

$$\Psi[h_{ab}, \chi] := \int^{(h,\chi)} \mathcal{D}[g]\mathcal{D}[\varphi] e^{-S[g,\varphi]/\hbar} \approx e^{-S_{\rm ext}[h_{ab},\chi]/\hbar}$$

→ No-boundary saddle-points: Extrema of the action (generally complex but Euclidean for the simplest cases), with (h_{ab}, χ) on the boundary at late times and are regular everywhere else.

→ Quantum completion for inflation ⇒ Principle for setting initial conditions for cosmological perturbations.

→ For minisuperspace models, this implies the boundary conditions $a(0) = 0, \;\; \dot{\varphi}(0) = 0$. (Regularity at the South Pole)

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Loop quantum gravity corrections

→ Basic continuum quantities of spatial geometry, such as areas and volumes, are represented by operators with discrete spectra.
An infinitesimal change of these quantities in time — or, more geometrically, the extrinsic curvature of space — no longer has a linear and local expression in space but is instead exponentiated and extended one-dimensionally, along an eponymous loop.[A. Ashtekar, M. Bojowald, T. Thiemann ...]

→ For a cosmological model, they imply two main corrections:

- Holonomy modifications: No operator for extrinsic curvature $\dot{a}$ or the Hubble parameter $\dot{a}/a \Rightarrow$ Well-defined operators only for $SU(2)$ holonomy matrix elements, which are periodic functions such as $\dot{a} \to \sin(\ell(a)\dot{a})/\ell(a)$ with $\ell(a) \sim l_P/a$.
- Inverse-volume corrections: Using $\hat{h}^{-1}[\hat{h}, \sqrt{\hat{a}}] = -\frac{1}{2}\hbar\widehat{\ell a^{-1/2}}$ (where $\hat{h} = \widehat{\exp(i\ell p_a)}$) to get $a^{-1} = f(a)/a$ with $f(a)$ some quantum correction function which goes to 1 for large a. The small-a behaviour eliminates the divergence of a direct inverse at $a = 0$.

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Loop corrections in the no-boundary proposal

→ In the path integral form for the no-boundary proposal, this implies replacing the Einstein-Hilbert action by an effective LQC action, which includes the said corrections.

→ In the canonical picture, instead of solving the standard WDW operator, one solves a "difference" equation in LQC ⇒ Quantum geometry corrections imply a modified Hamiltonian constraint in $\hat{\mathcal{H}}_{\rm LQC}\,\Psi = 0$. Still need boundary conditions for specific solutions.
Naturally, the Friedmann equation is also modified in LQC as a result.

→ The role played by modified constraints crucial in LQG ⇒ They result in deformed gauge transformations. Since background is modified, covariant perturbatons imply an effective line-element $\mathrm{d}s_\beta^2 = -\beta N^2 \mathrm{d}t^2 + a(t)^2 \mathrm{d}\Omega_k$ where $\beta(a, \dot{a})$ changes sign at large curvature resulting in dynamical signature change.

→ South-Pole regularity conditions modified for LQC –
EH: $a(0) = 0, \quad \dot{a}(0) = 1 \Leftrightarrow$ LQC: $a(0) = 0, \quad \dot{a}(0) = 0$

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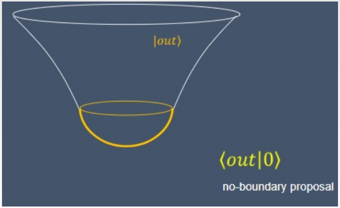
Hartle-Hawking proposal

→ For minisuperspace cosmologies, in the saddle-point approximation, the no-boundary wavefunction simplifies

$$\Psi^{\mathrm{HH}}[\tilde{a},\chi] \approx e^{-S_{\mathrm{E}}^{\mathrm{EH}}[\tilde{a},\chi]/\hbar}$$

→ For simplest models, say with only a cosmological constant, our (Lorentzian) universe tunnels from nothing via an Euclidean region.

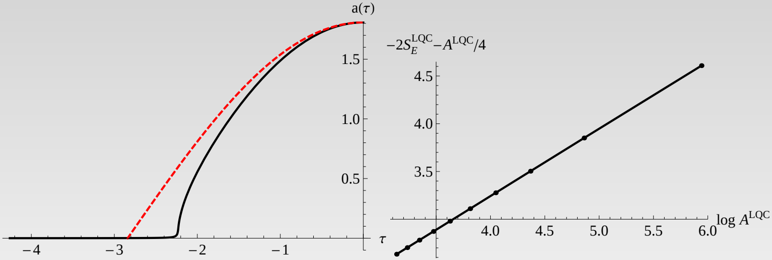
→ Friedmann equation: $\dot{a}^2 = -\mathcal{V}(a)$ and on-shell action $S_{\mathrm{E}}^{\mathrm{EH}} = -\frac{3\pi}{2}\int_0^{\tilde{a}} a\sqrt{|\mathcal{V}(a)|}$.
The nucleation probability of a universe $\mathcal{P} \simeq e^{-2S_{\mathrm{E}}^{\mathrm{LQC}}}$.



Pure de Sitter

[S.B. & D.-h. Yeom, 2018]

$$-\dot{a}^2 = \mathcal{V} := \frac{8\pi a^2}{3} f^2(a) \left[\frac{\rho}{f(a)} - \rho_1\right] \left[\frac{\rho_2 - \frac{\rho}{f(a)}}{\rho_c}\right]$$



→ A typical solution $a(\tau)$ for some numerical values of Λ & l_{Pl}.

→ $-2S_{\rm E}^{\rm LQC} \simeq \frac{\mathcal{A}}{4} + c + d \log \mathcal{A}, \quad d > 0$ where $\mathcal{A} = 4\pi\tilde{a}^2$

→ LQC correction rather small ⇒ There is a potential barrier for both EH ($-\dot{a}^2 \sim -1 + \Lambda a^2$) and LQC scenarios.

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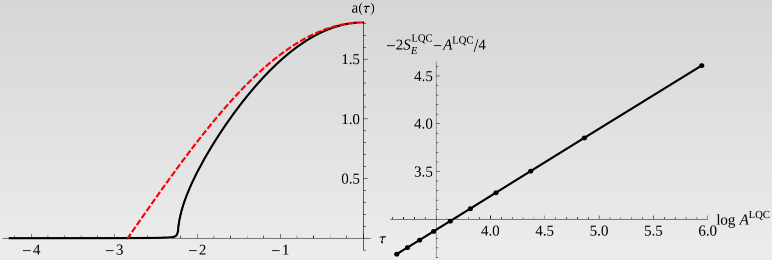

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Massless scalar field

→ Usual KG equation $\ddot{\varphi} + 3H\dot{\varphi} = 0 \Rightarrow \dot{\varphi} = 0$ and non-dynamical solution. In EH theory, no way to get interesting solutions.

→ Modified equations of motion [S.B. & D.-h. Yeom, 2018]

$$\mathcal{V} = \frac{8\pi G}{3} a^2 f^2(a) \left[\frac{a^6 \pi}{4\sqrt{3}\gamma^3 l_{Pl}^6} \left(\frac{\rho}{\rho_c}\right) \left(\frac{g(a)}{f(a)}\right) - \rho_1 \right] \left[\frac{1}{\rho_c} \left(\rho_2 - \frac{a^6 \pi}{4\sqrt{3}\gamma^3 l_{Pl}^6} \left(\frac{\rho}{\rho_c}\right) \left(\frac{g(a)}{f(a)}\right) \right) \right]$$

$$\ddot{\varphi} - \left(\frac{\dot{B}(a)}{B(a)}\right) \dot{\varphi} = 0 \quad \text{Classically, } B(a) \sim a^{-3} \;\; \& \;\; B(a) \sim a^{12} \text{ in QG regime}$$

→ New instantonic solutions for NBWF ⇔ New physical interpretations for LQC

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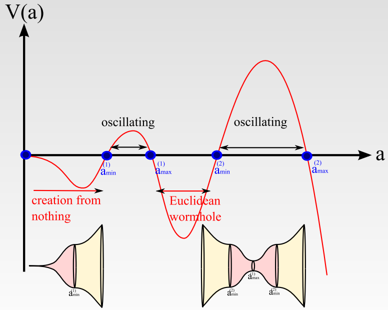

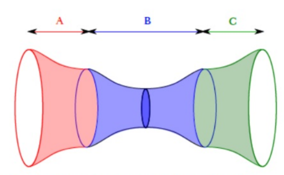

Creation of two universes via an Euclidean wormhole

Loops rescue the no-boundary proposal

→ Euclidean $\int^{h} \mathcal{D}[g] e^{-S_E/\hbar}$ (compact Euclidean 4-geometries bounded by h) vs. Lorentzian $\int_{\emptyset}^{h} \mathcal{D}[g] e^{iS/\hbar}$ (Lorentzian 4-geometries interpolating between a vanishing initial 3-geometries and h). [HH, 1983; A. Vilenkin, 1982]

→ Euclidean path integral diverges for $\Lambda > 0$ for all contours of the lapse ⇒ Lorentzian path integral can be made well-defined by applying Piecard-Lefshetz theory to yield a convergent integral by deforming the lapse contour.

→ Unsuppressed runaway perturbations on the final 3-geometry due to an inverse Gaussian weighting for perturbations ⇒ Old problem of the scale factor having wrong-sign kinetic term. [J. Feldbrugge, J.-l. Lehners & N. Turok, 2017]

→ Dynamical signature change in makes these inverted Gaussians have the correct sign for having a Bunch-Davies state at the onset of inflation. [M. Bojowald & S.B., 2018]

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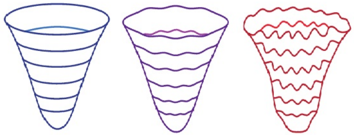

Photo from FLT (arXiv:1705.00192)

Timeless stability of perturbations

→ The mode equation

$$\ddot{v} \approx \frac{1}{4}\left((n-2\epsilon)(n+2)+\epsilon(\epsilon+2)-\beta\frac{N^2\ell(\ell+2)}{c^2}\right)\frac{v}{t^2},$$

and its solution is $v_+ = v_1 t^{\frac{1}{2}(1+\gamma)}$ where

$$\gamma = \sqrt{1+n(n+2)-\beta\frac{\ell(\ell+2)N^2}{c^2}}$$

→ For EH, $\beta = 1, n = 0$, γ and the solutions $v_\pm$ have branch cuts on the real N-axis ⇒ The action evaluated on the regular solution v_+ is equal to $S_+(v_1) = \frac{1}{4}N^{-1}(\gamma - 1)v_1^2$ and has a negative imaginary part above the branch cut. This result leads to a Gaussian with positive exponent in the path integral of perturbations.

→ With dynamical signature change, that is $\beta < 0$, γ is always real for real N. Its branch cuts in the complex plane are now on the imaginary N-axis where they do not affect the Lorentzian path integral ⇒ The action S_+ is always real and finite and does not lead to unbounded contributions to the path integral.

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Summary

→ Conclusions: Fruitful confluence between different approaches to quantum cosmology.

- Loops provide necessary quantum geometry corrections which expands the solution space for the no-boundary proposal.
- The no-boundary wave function is necessary to discover new physical phenomenon in loops which cannot be probed otherwise.
- Remarkable similarity in dynamical signature-change coming loops and the Euclidean (generally, complex) phase in the Hartle-Hawking proposal.

→ Looking ahead:

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