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Quantum Entanglement in multi-field inflation

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with

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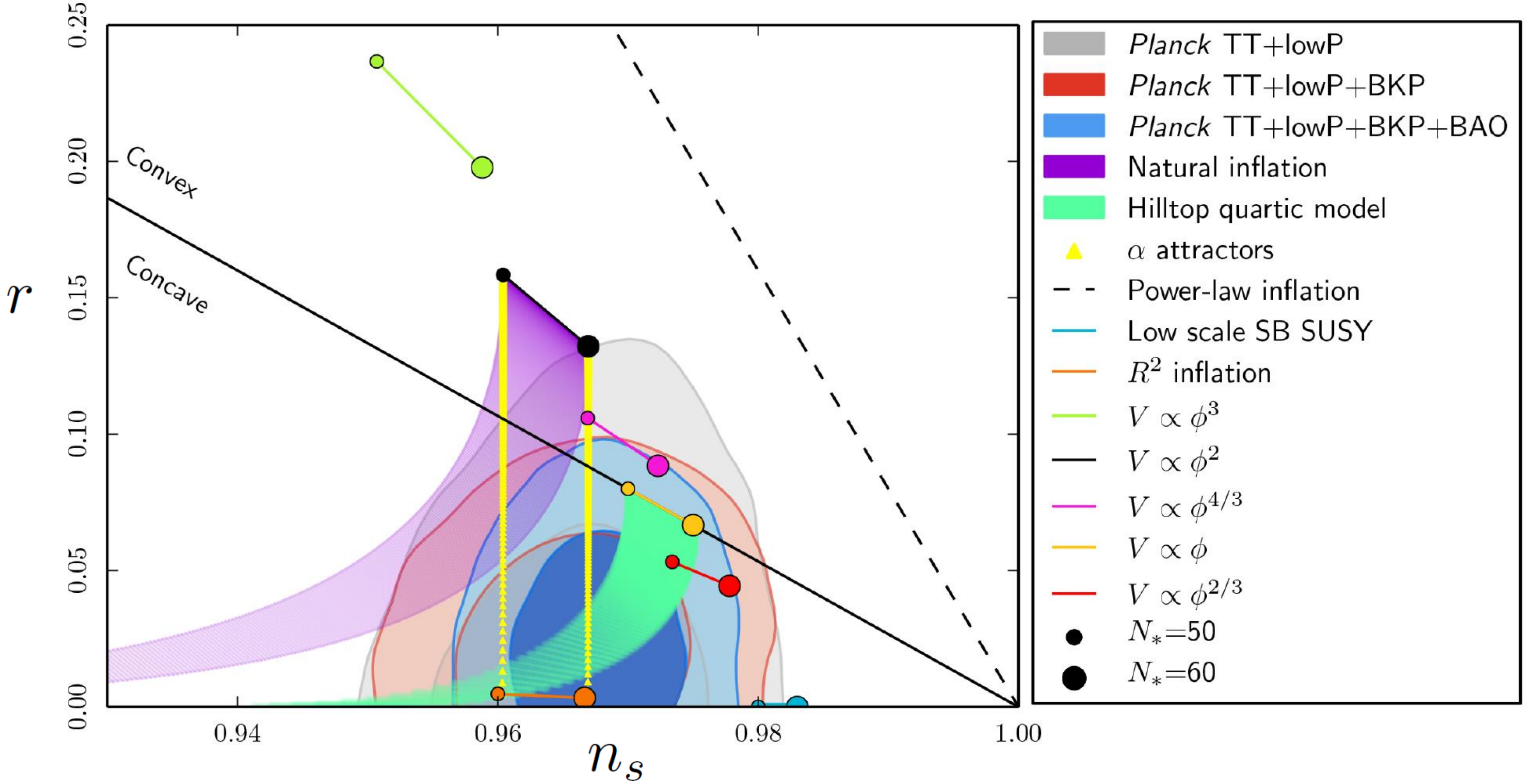
Inflation

Constraints from Planck

Ade et al `16



- Single-field slow-roll inflation is successful phenomenologically

- But it is difficult to specify what is inflaton

New physics ?

Quantum entanglement

- Non-entangled state

$$|\psi\rangle = \mathcal{C}\Big(|0\rangle_\phi + \mathcal{E}|1\rangle_\phi\Big) \otimes \mathcal{C}\Big(|0\rangle_\sigma + \mathcal{E}|1\rangle_\sigma\Big)\,, \qquad |\mathcal{C}|^2(1+|\mathcal{E}|^2) = 1$$

If we cannot observe σ, we must trace out the d.o.f. of σ

$$\rho_\phi = \mathrm{Tr}_\sigma |\psi\rangle\langle\psi| = |\mathcal{C}|^2\Big(|0\rangle_\phi + \mathcal{E}|1\rangle_\phi\Big)\Big({}_\phi\langle 0_{\mathbf{k}}| + \mathcal{E}^*{}_\phi\langle 1_{\mathbf{k}}|\Big)$$

pure-state $\mathrm{Tr}\rho_\phi^2 = 1$ information not lost

- Entangled state

$$|\psi\rangle = \mathcal{C}\Big(|0\rangle_\phi|0\rangle_\sigma + \mathcal{E}|1\rangle_\phi|1\rangle_\sigma\Big) \Rightarrow \rho_\phi = |\mathcal{C}|^2\Big(|0\rangle_\phi\,{}_\phi\langle 0| + |\mathcal{E}|^2|1\rangle_\phi\,{}_\phi\langle 1|\Big)$$

mixed-state $\mathrm{Tr}\rho_\phi^2 < 1$ information lost

Entanglement entropy in de Sitter space Maldacena, Pimentel, `13

Quantum entanglement in multi-field inflation ?

Influence of initial state entanglement

Albrecht, Bolis, Holman, (ABH) `14

- Action

$$S = -\frac{1}{2}\int dt d^3x a^3\left[\delta_{IJ}\partial_\mu\phi^I\partial^\mu\phi^J + M_{IJ}\phi^I\phi^J\right]$$

$$\phi^I = \{\underline{\phi}, \underline{\sigma}\} \qquad M_{IJ} = \mathrm{diag}\{m_\phi^2\,,\ m_\sigma^2\} \qquad a = e^{Ht} = -1/(H\eta)$$

inflaton spectator

fields' perturbations in fixed de Sitter background

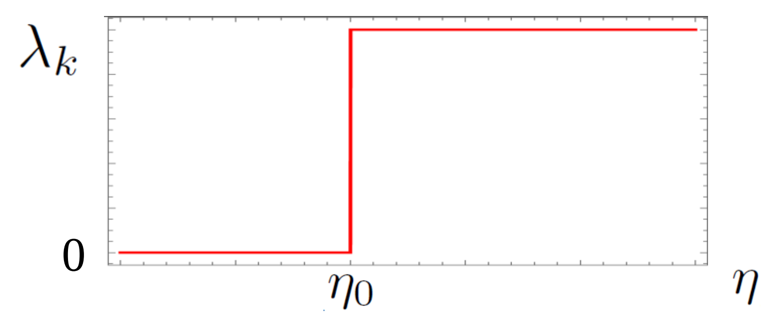
- Gaussian wave function

$$\Psi_{\mathbf{k}}[\{\phi_{\mathbf{k}}\}\,,\,\{\sigma_{\mathbf{k}}\}\,;\eta] = \prod_{\mathbf{k}}\psi_{\mathbf{k}}[\{\phi_{\mathbf{k}}\}\,,\,\{\sigma_{\mathbf{k}}\}\,;\eta]$$

Entanglement

$$\psi_{\mathbf{k}} \propto \exp\left[-\frac{1}{2}\left(A_k(\eta)\phi_{\mathbf{k}}\phi_{-\mathbf{k}} + B_k(\eta)\sigma_{\mathbf{k}}\sigma_{-\mathbf{k}} + C_k(\eta)\left(\phi_{\mathbf{k}}\sigma_{-\mathbf{k}} + \sigma_{\mathbf{k}}\phi_{-\mathbf{k}}\right)\right)\right]$$

switching on entanglement at $\eta = \eta_0$



Evolution of fields' perturbations

- Wave function for each mode

$$\psi_{\mathbf{k}} \propto \exp\left[-\frac{1}{2}\left(A_k(\eta)\phi_{\mathbf{k}}\phi_{-\mathbf{k}} + B_k(\eta)\sigma_{\mathbf{k}}\sigma_{-\mathbf{k}} + C_k(\eta)\left(\phi_{\mathbf{k}}\sigma_{-\mathbf{k}} + \sigma_{\mathbf{k}}\phi_{-\mathbf{k}}\right)\right)\right]$$

- Schrodinger equation for each mode $\quad ' \equiv \frac{d}{d\eta}$

constant

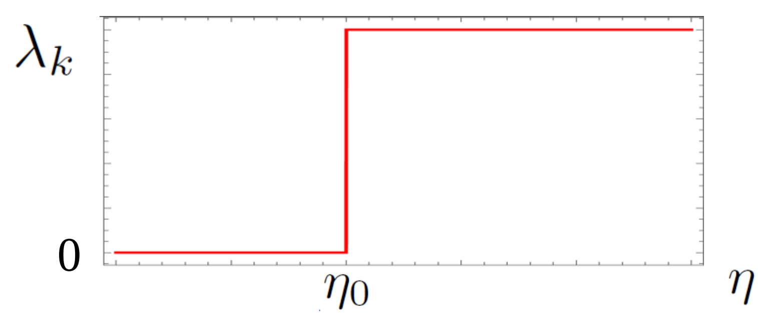
$$A_k = -ia^2\left(\frac{f_k'}{f_k} - \frac{a'}{a}\right), \quad B_k = -ia^2\left(\frac{g_k'}{g_k} - \frac{a'}{a}\right), \quad C_k = \frac{\lambda_k}{f_k g_k}a^2$$

$$f_k'' + \left(k^2 + a^2 m_\phi^2 - \frac{a''}{a}\right)f_k = \frac{C_k^2}{a^4}f_k\,, \quad g_k'' + \left(k^2 + a^2 m_\sigma^2 - \frac{a''}{a}\right)g_k = \frac{C_k^2}{a^4}g_k$$

- ``Initial" condition

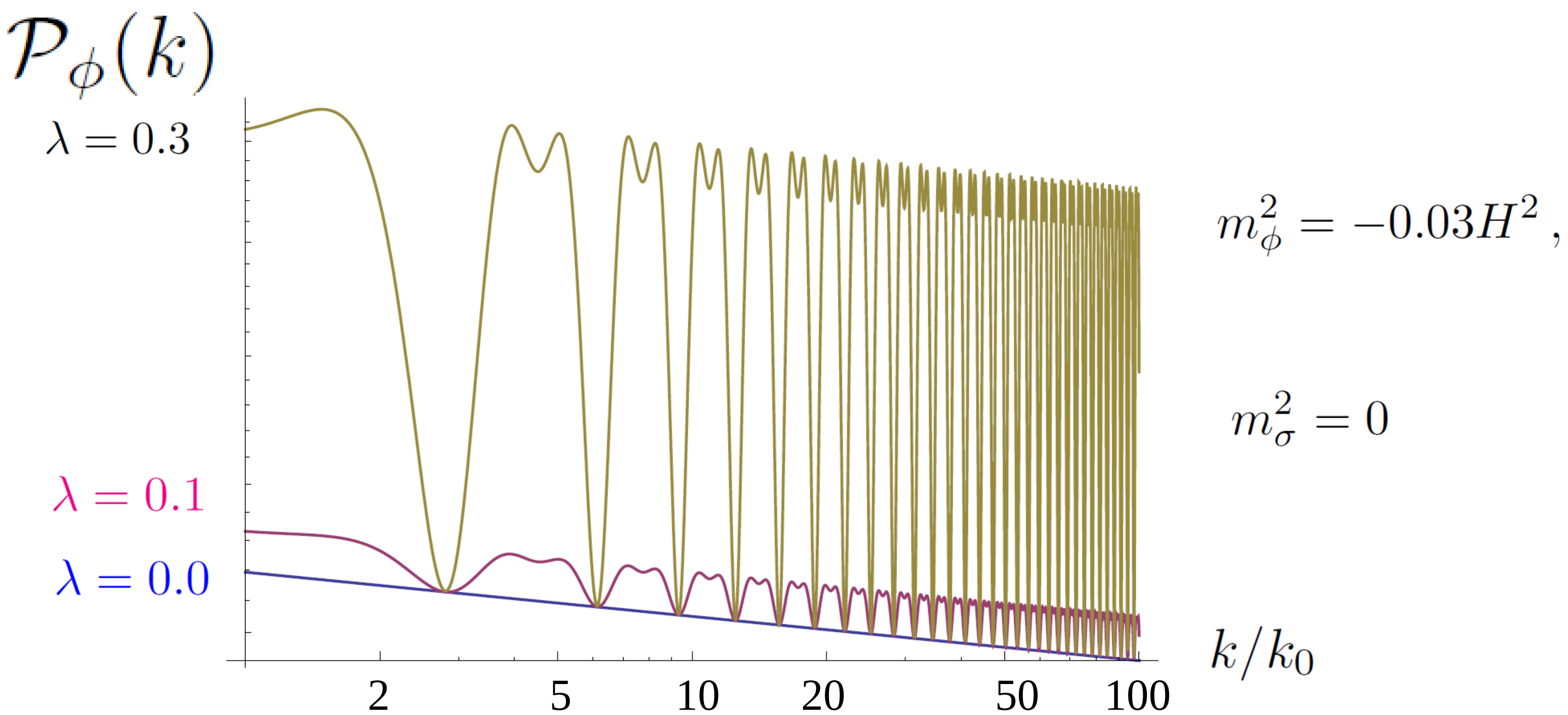
$$f_k(\eta_0) = u_{\phi k}^{\mathrm{BD}\,*}(\eta_0) \qquad g_k(\eta_0) = u_{\sigma k}^{\mathrm{BD}\,*}(\eta_0)$$

Bunch-Davies vacuum



Power spectrum of inflaton perturbations

tracing out the σ degrees of freedom



larger entanglement ⇒ larger oscillation amplitude

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Quantum entanglement multi-field inflation

- Action

$$S = -\frac{1}{2}\int dt d^3x a^3 \Big[G_{IJ}(t)\partial_\mu \phi^I \partial^\mu \phi^J + M_{IJ}(t)\phi^I \phi^J \Big] \qquad \phi^I = \{\phi\,,\ \sigma\}$$

fields' perturbations in fixed de Sitter background

- Sufficiently late time

$$G_{IJ}(t) \to \delta_{IJ}\,, \qquad M_{IJ}(t) \to \mathrm{diag}\{m_\phi^2\,,\ m_\sigma^2\}$$

- Sufficiently early time

$$G_{IJ}(t)\partial_\mu \phi^I \partial^\mu \phi^J \to \delta_{IJ}\partial_\mu \Phi^I \partial^\mu \Phi^J$$

$G_{IJ}(t)$ is diagonalized by another set $\Phi^I = \{\Phi\,,\ S\}$

⇒ We can construct in-vacuum for Φ^I and out-vacuum for ϕ^I

In-vacuum seen from late-time observers

- ``Generalized” Bogoliubov transformation

$$\hat{a}_{\boldsymbol{k}}^{(\phi)} = \alpha_k \hat{a}_{\boldsymbol{k}}^{(\Phi)} + \beta_k (\hat{a}_{-\boldsymbol{k}}^{(\Phi)})^\dagger + \gamma_k \hat{b}_{\boldsymbol{k}}^{(S)} + \delta_k \, (\hat{b}_{-\boldsymbol{k}}^{(S)})^\dagger$$

$$\hat{b}_{\boldsymbol{k}}^{(\sigma)} = \bar{\alpha}_k \hat{a}_{\boldsymbol{k}}^{(\Phi)} + \bar{\beta}_k (\hat{a}_{-\boldsymbol{k}}^{(\Phi)})^\dagger + \bar{\gamma}_k \hat{b}_{\boldsymbol{k}}^{(S)} + \bar{\delta}_k \, (\hat{b}_{-\boldsymbol{k}}^{(S)})^\dagger$$

$$\left[\begin{array}{l} |\alpha_k|^2 - |\beta_k|^2 + |\gamma_k|^2 - |\delta_k|^2 = 1 \\ |\bar{\alpha}_k|^2 - |\bar{\beta}_k|^2 + |\bar{\gamma}_k|^2 - |\bar{\delta}_k|^2 = 1 \\ \alpha_k \bar{\beta}_k - \beta_k \bar{\alpha}_k + \gamma_k \bar{\delta}_k - \delta_k \bar{\gamma}_k = 0 \\ \alpha_k \bar{\alpha}_k{}^* - \beta_k \bar{\beta}_k{}^* + \gamma_k \bar{\gamma}_k{}^* - \delta_k \bar{\delta}_k{}^* = 0 \end{array} \right.$$

- Relation between in-vacuum state and out-vacuum state

$$|0\rangle_{\rm in} = f(\{(\hat{a}_{\boldsymbol{k}}^{(\phi)})^\dagger\}, \{(\hat{a}_{-\boldsymbol{k}}^{(\phi)})^\dagger\}, \{(\hat{b}_{\boldsymbol{k}}^{(\sigma)})^\dagger\}, \{(\hat{b}_{-\boldsymbol{k}}^{(\sigma)})^\dagger\}) \, |0\rangle_{\rm out}$$

$$f_{\boldsymbol{k}} \propto \exp\left[\frac{1}{2}\left(\mathcal{C}_{\phi\phi k} \, (\hat{a}_{-\boldsymbol{k}}^{(\phi)})^\dagger (\hat{a}_{\boldsymbol{k}}^{(\phi)})^\dagger + \mathcal{C}_{\sigma\sigma k} \, (\hat{b}_{-\boldsymbol{k}}^{(\sigma)})^\dagger (\hat{b}_{\boldsymbol{k}}^{(\sigma)})^\dagger + \mathcal{C}_{\phi\sigma k} \left((\hat{a}_{-\boldsymbol{k}}^{(\phi)})^\dagger (\hat{b}_{\boldsymbol{k}}^{(\sigma)})^\dagger + (\hat{a}_{\boldsymbol{k}}^{(\phi)})^\dagger (\hat{b}_{-\boldsymbol{k}}^{(\sigma)})^\dagger \right)\right)\right]$$

$$\mathcal{C}_{\phi\phi k} = -\frac{\bar{\alpha}_k^* \delta_k - \beta_k \bar{\gamma}_k^*}{\alpha_k^* \bar{\gamma}_k^* - \bar{\alpha}_k^* \gamma_k^*}, \qquad \mathcal{C}_{\sigma\sigma k} = \frac{\alpha_k^* \bar{\delta}_k - \bar{\beta}_k \gamma_k^*}{\alpha_k^* \bar{\gamma}_k^* - \bar{\alpha}_k^* \gamma_k^*}, \qquad \mathcal{C}_{\phi\sigma k} = \frac{\alpha_k^* \delta_k - \beta_k \gamma_k^*}{\alpha_k^* \bar{\gamma}_k^* - \bar{\alpha}_k^* \gamma_k^*}$$

In-vacuum and out-vacuum wave function

Equivalent description in Schrodinger picture

- Out-vacuum wave function for each mode

$$\psi_{\boldsymbol{k}} \equiv \langle \phi_{\boldsymbol{k}}, \sigma_{\boldsymbol{k}} | 0 \rangle_{\rm out} \propto \exp\left[\frac{1}{2}\left(\omega_k^\phi\, \phi_{-\boldsymbol{k}}\phi_{\boldsymbol{k}} + \omega_k^\sigma\, \sigma_{-\boldsymbol{k}}\sigma_{\boldsymbol{k}}\right)\right]$$

$$\omega_k^\phi \equiv i a^2 \partial_\eta \ln(u_\phi^{\rm BD\,*}/a), \qquad \omega_k^\sigma \equiv i a^2 \partial_\eta \ln(u_s^{\rm BD\,*}/a)$$

- In-vacuum wave function for each mode

$$\psi_{\boldsymbol{k}} \equiv \langle \phi_{\boldsymbol{k}}, \sigma_{\boldsymbol{k}} | 0 \rangle_{\rm in} \propto \exp\left[\frac{1}{2}\left(\Omega_k^\phi\, \phi_{-\boldsymbol{k}}\phi_{\boldsymbol{k}} + \Omega_k^\sigma\, \sigma_{-\boldsymbol{k}}\sigma_{\boldsymbol{k}} + \Omega_k^{\phi\sigma}\,(\phi_{-\boldsymbol{k}}\sigma_{\boldsymbol{k}} + \sigma_{-\boldsymbol{k}}\phi_{\boldsymbol{k}})\right)\right]$$

this reproduces the state considered in ABH

if $\mathcal{C}_{\phi\phi k} = \mathcal{C}_{\sigma\sigma k} = \beta_k \bar{\delta}_k - \bar{\beta}_k \delta_k = 0 \qquad \mathcal{C}_{\phi\sigma k} = -\lambda_k$

Entanglement ⟺ $\alpha_k^* \delta_k - \beta_k \gamma_k^* \neq 0 \qquad (\mathcal{C}_{\phi\sigma k} \neq 0)$

Contents

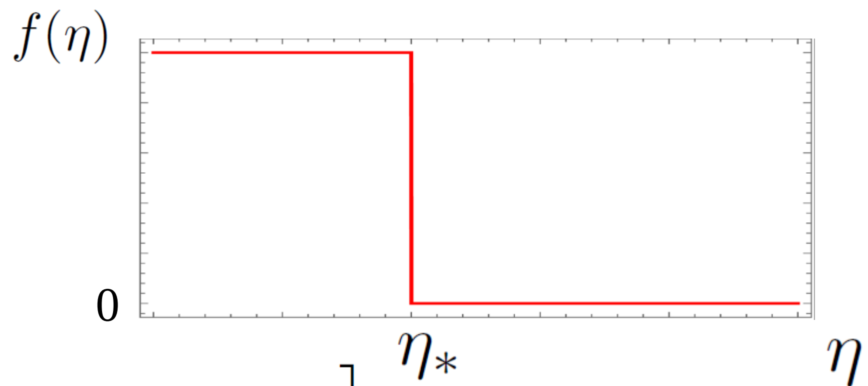
Concrete model with kinetic-mixing

- Model

$$G_{IJ}(\eta) = \begin{pmatrix} 1 & f(\eta) \\ f(\eta) & 1 \end{pmatrix}, \qquad M_{IJ} = \begin{pmatrix} 0 & 0 \\ 0 & m_\sigma^2 \end{pmatrix}$$

$$f(\eta) = f_c \, \Theta(\eta_* - \eta)$$

$$(0 < f_c < 1)$$



- Action at sufficiently early time

$$S = -\frac{1}{2}\int dt d^3x a^3 \left[\delta_{IJ}\partial_\mu \Phi^I \partial^\mu \Phi^J + \tilde{M}_{IJ}(t)\Phi^I\Phi^J\right]$$

$$\Phi^I = \{\Phi\,,\ S\} \qquad \phi^I = \{\phi\,,\ \sigma\} \qquad \phi^I = \mathcal{K}_{IJ}\Phi^J \qquad \tilde{M}_{IJ} = (\mathcal{K}^T M \mathcal{K})_{IJ}$$

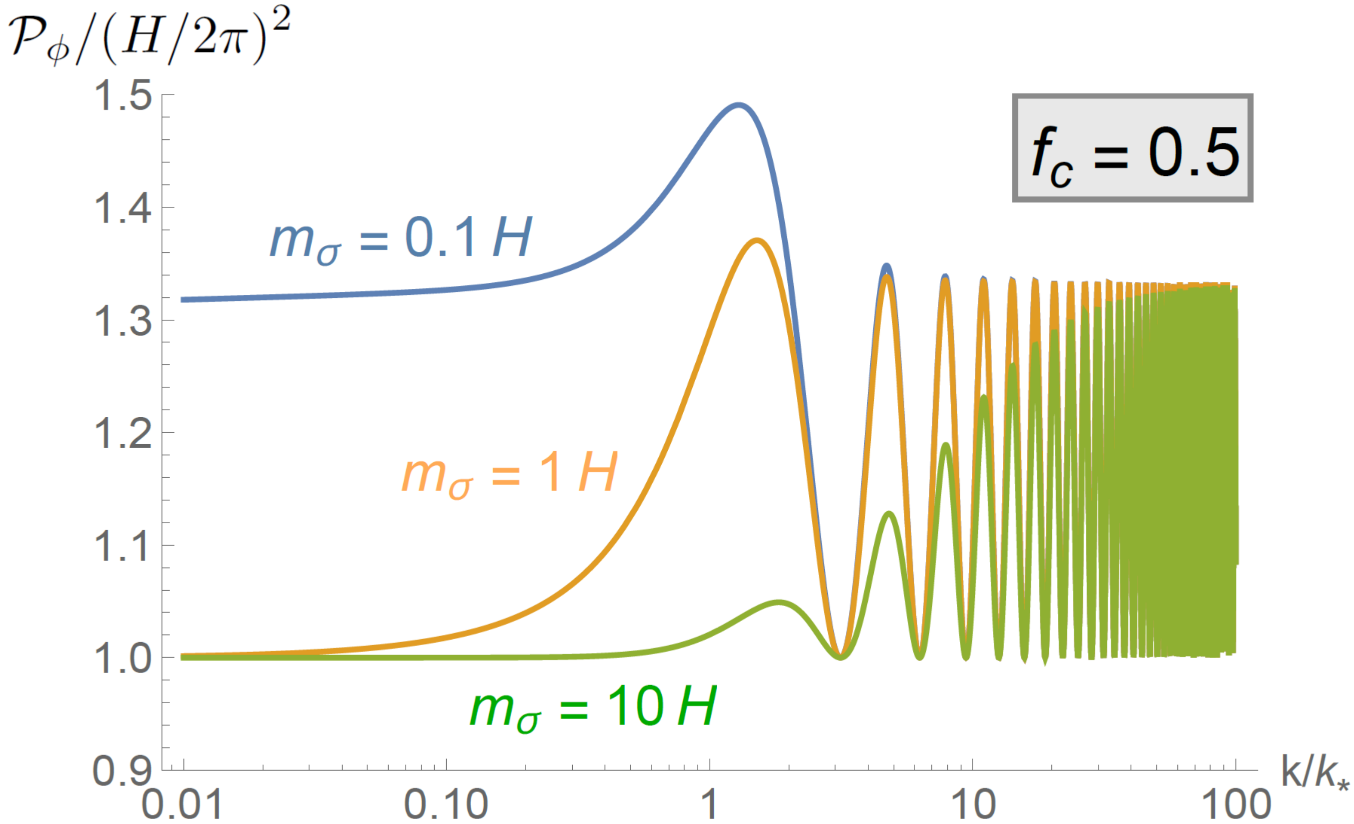
$$\mathcal{K}_{IJ} \equiv \begin{pmatrix} 1 & \frac{-f_c}{\sqrt{1-f_c^2}} \\ 0 & \frac{1}{\sqrt{1-f_c^2}} \end{pmatrix} \qquad \tilde{M}_{IJ} \equiv \begin{pmatrix} 0 & 0 \\ 0 & \frac{m_\sigma^2}{1-f_c^2} \end{pmatrix}$$

→ Generalized Bogoliubov coefficients from matching condition

Power spectrum of inflaton perturbations

$$\mathcal{P}_\phi(k) \to \left(\frac{H}{2\pi}\right)^2 \left(|\alpha_k - \beta_k^*|^2 + |\gamma_k - \delta_k^*|^2\right) \qquad \eta \to 0$$

$= 1$ entanglement



We obtain oscillations from quantum entanglement !!

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Conclusions

- It was known that oscillations are produced in the spectrum of scalar perturbations by an initial entangled state in inflation

$$\psi_{\mathbf{k}} \propto \exp\left[-\frac{1}{2}\left(A_k(\eta)\phi_{\mathbf{k}}\phi_{-\mathbf{k}} + B_k(\eta)\sigma_{\mathbf{k}}\sigma_{-\mathbf{k}} + C_k(\eta)\left(\phi_{\mathbf{k}}\sigma_{-\mathbf{k}} + \sigma_{\mathbf{k}}\phi_{-\mathbf{k}}\right)\right)\right]$$

- We have clarified the condition for entanglement in inflation

$$\hat{a}_{\boldsymbol{k}}^{(\phi)} = \alpha_k \hat{a}_{\boldsymbol{k}}^{(\Phi)} + \beta_k (\hat{a}_{-\boldsymbol{k}}^{(\Phi)})^\dagger + \gamma_k \hat{b}_{\boldsymbol{k}}^{(S)} + \delta_k\,(\hat{b}_{-\boldsymbol{k}}^{(S)})^\dagger \;\Longrightarrow\; \alpha_k^*\delta_k - \beta_k\gamma_k^* \neq 0$$

- We have presented a simple concrete model with entanglement and confirmed the oscillations in the scalar spectrum

$$S = -\frac{1}{2}\int dt d^3x a^3\left[G_{IJ}(t)\partial_\mu\phi^I\partial^\mu\phi^J + M_{IJ}\phi^I\phi^J\right] \quad G_{IJ}(\eta) = \begin{pmatrix} 1 & f(\eta) \\ f(\eta) & 1 \end{pmatrix},$$

$$\phi^I = \{\phi\,,\ \sigma\} \qquad M_{IJ} = \mathrm{diag}\{0\,,\ m_\sigma^2\} \qquad f(\eta) = f_c\,\Theta(\eta_* - \eta)$$

Discussions

- Primordial power spectrum?

de Sitter background ⇨ slow-roll background

inflaton perturbation ⇨ curvature perturbation

- If oscillations appear also in primordial power spectrum, can we distinguish this from the ones by other models?

- Primordial non-Gaussianity?

$$\mathcal{L}_{\rm eff}[\phi,\sigma] = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2 + \rho\dot{\phi}\sigma - \frac{1}{2}\frac{(\partial\phi)^2\sigma}{\Lambda} - \mu\sigma^3$$

Assassi, Baumann, Green, McAllister `14

- Other signature of entanglement ?

infinite violation of Bell inequalities   Kanno, Soda `17

Thank you very much !!