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Inflationary universes born out of the highly inhomogeneous initial condition

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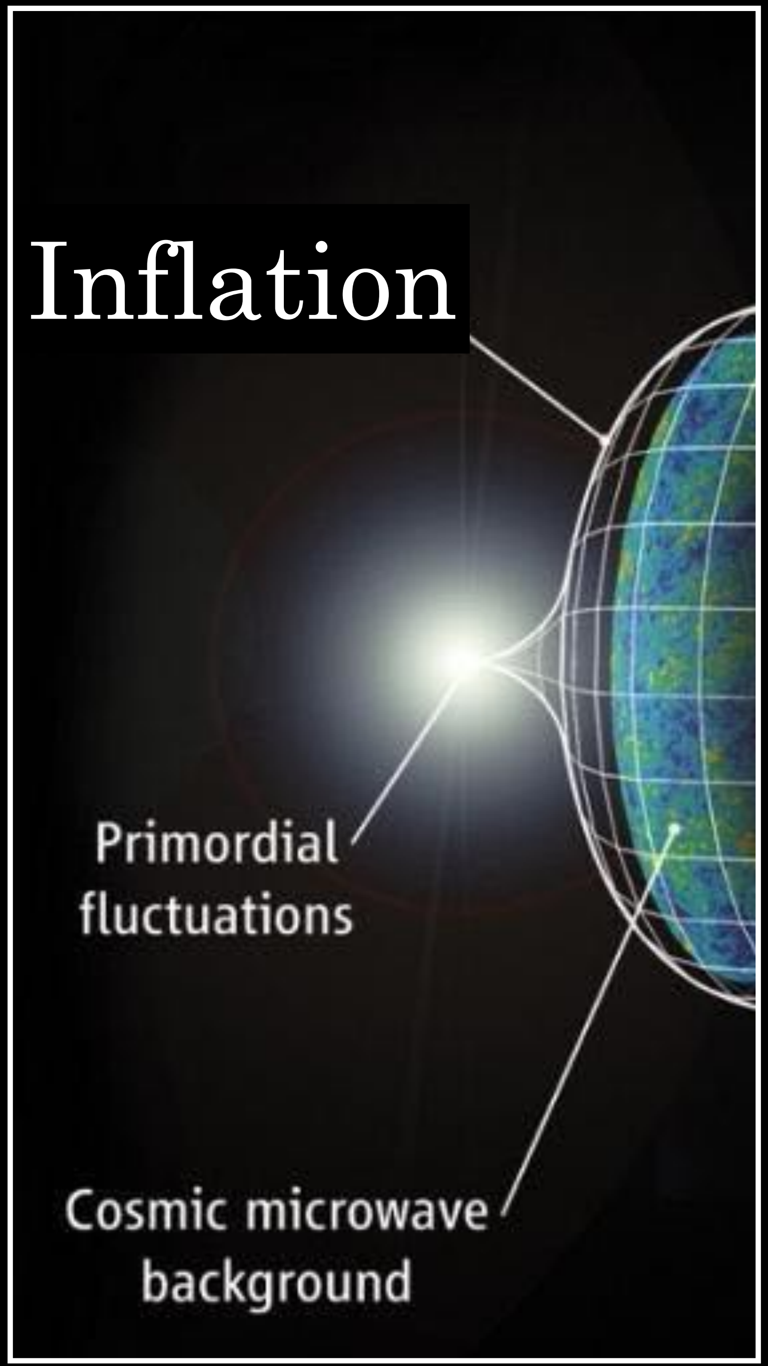
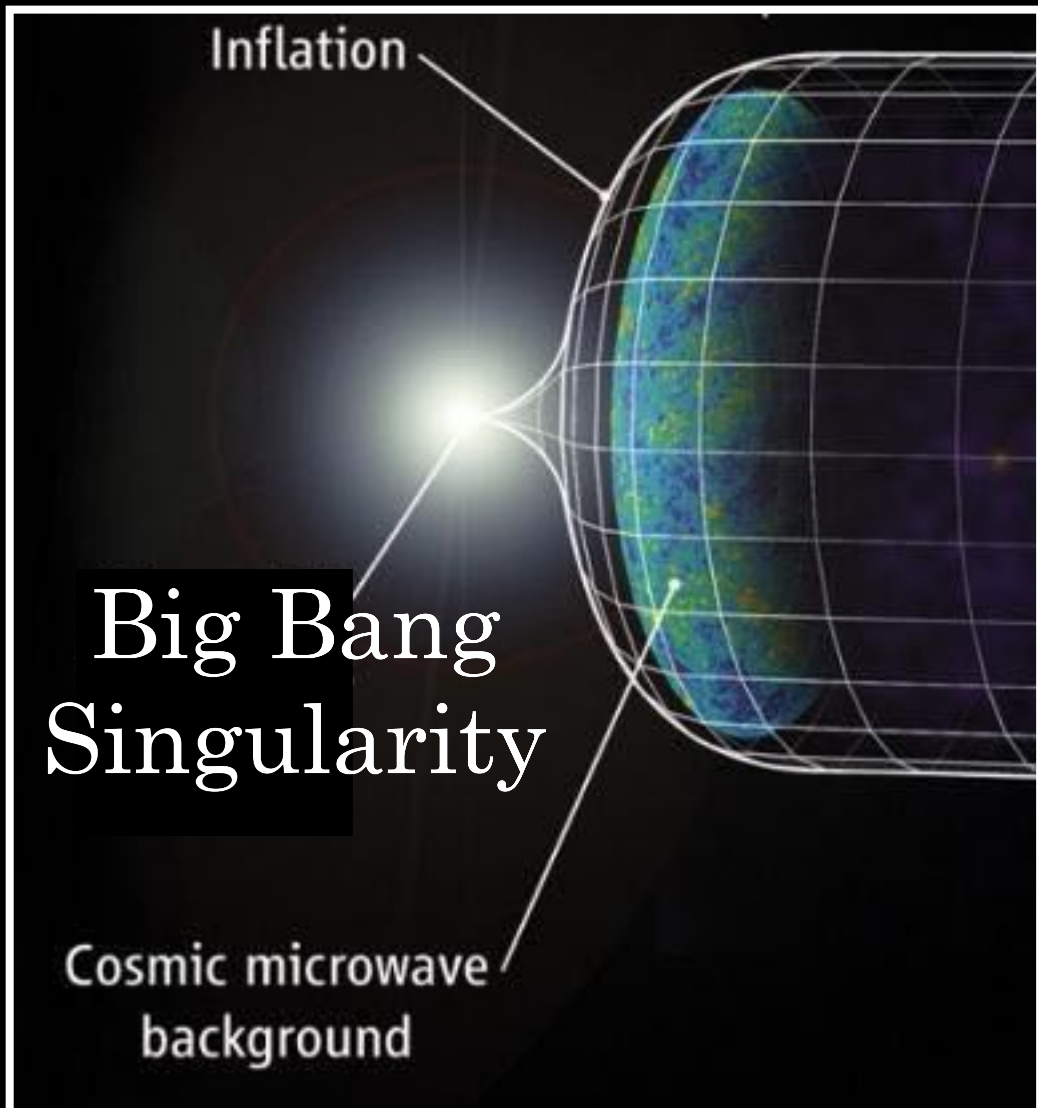


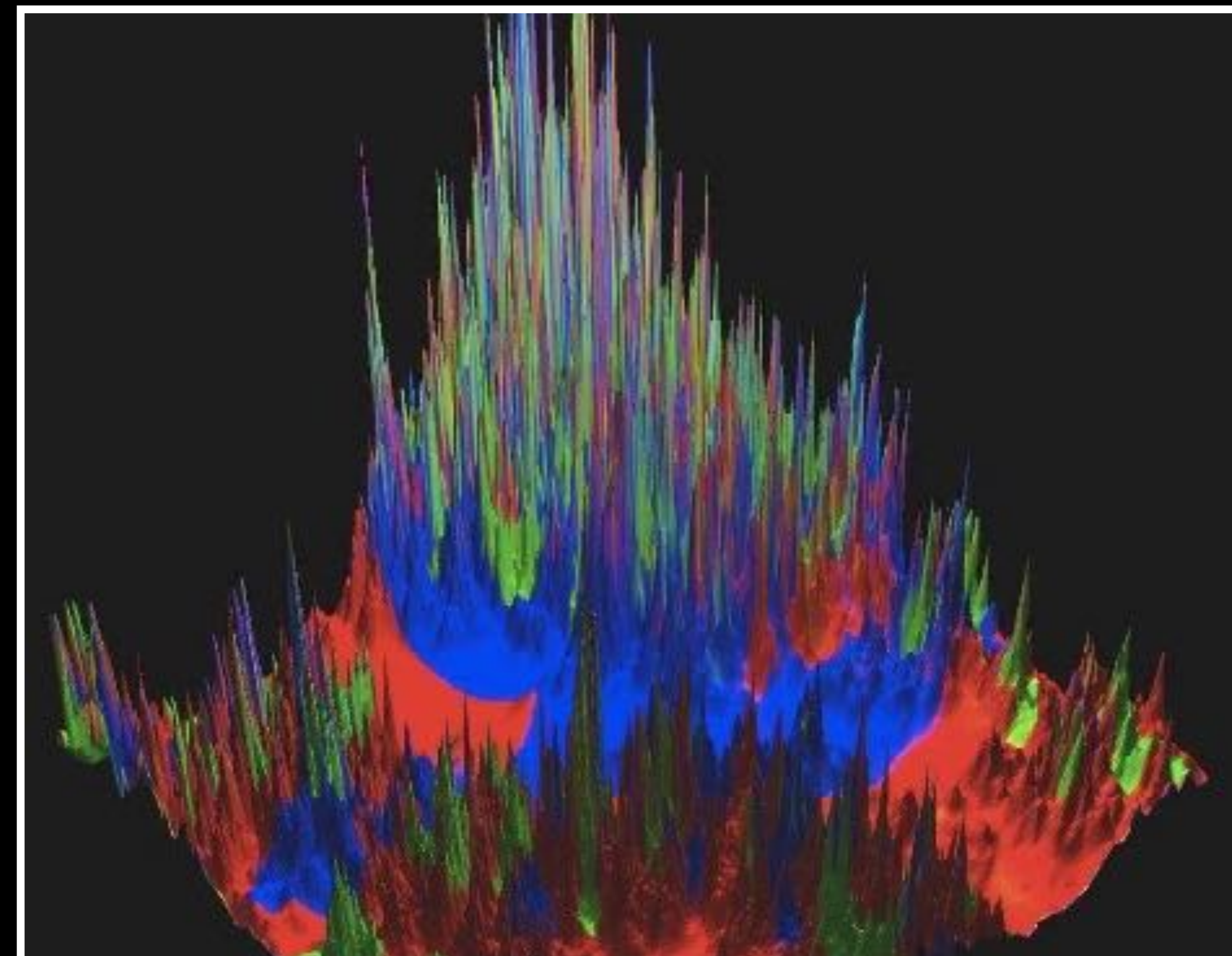
Fine-tuning problems

(i.e. horizon problem, flatness problem, monopole problem)

Problems in the inflationary paradigm



A. Borde (1987), A. Vilenkin (1992),
A. Borde + (1994), A. Borde + (2003)



$$S = \int d^4x \sqrt{-g} \frac{1}{-16\pi G} R + \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - V(\varphi)$$

$V(\varphi)$ is dominant over $\dot{\varphi}^2$ and $(\nabla\varphi)^2$

➡ cosmic expansion

Otherwise, NO inflation?

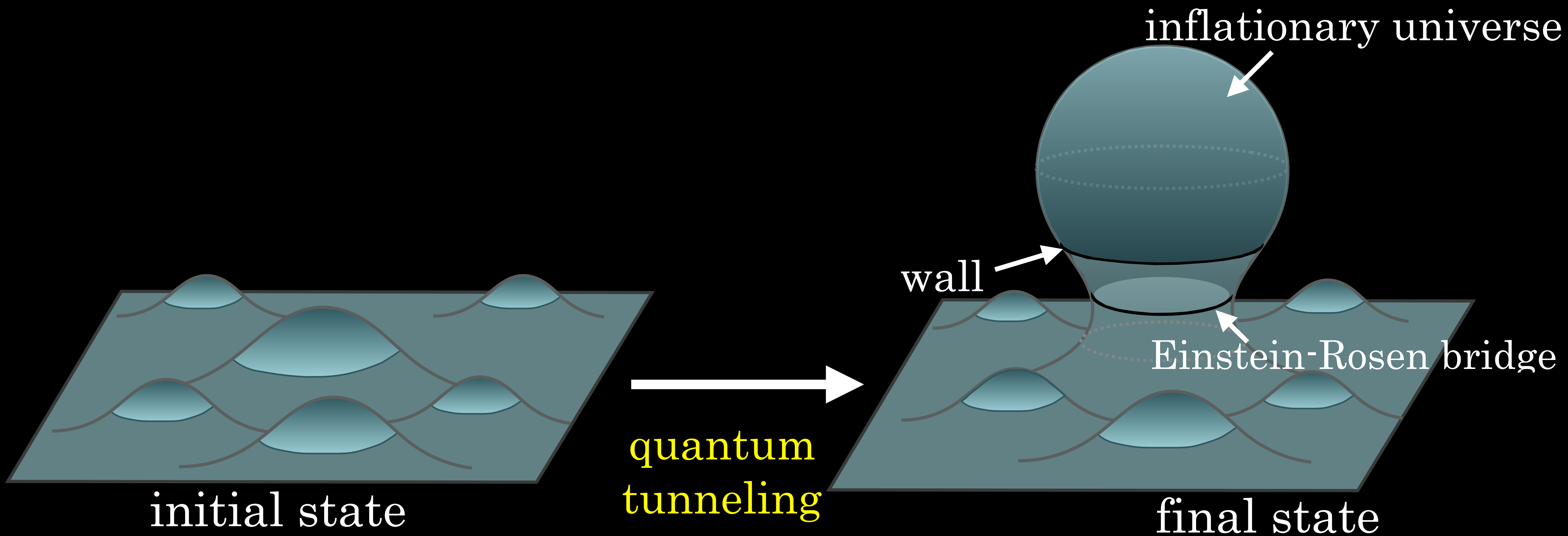
R. Brandenberger (2016) [review]

Main proposal

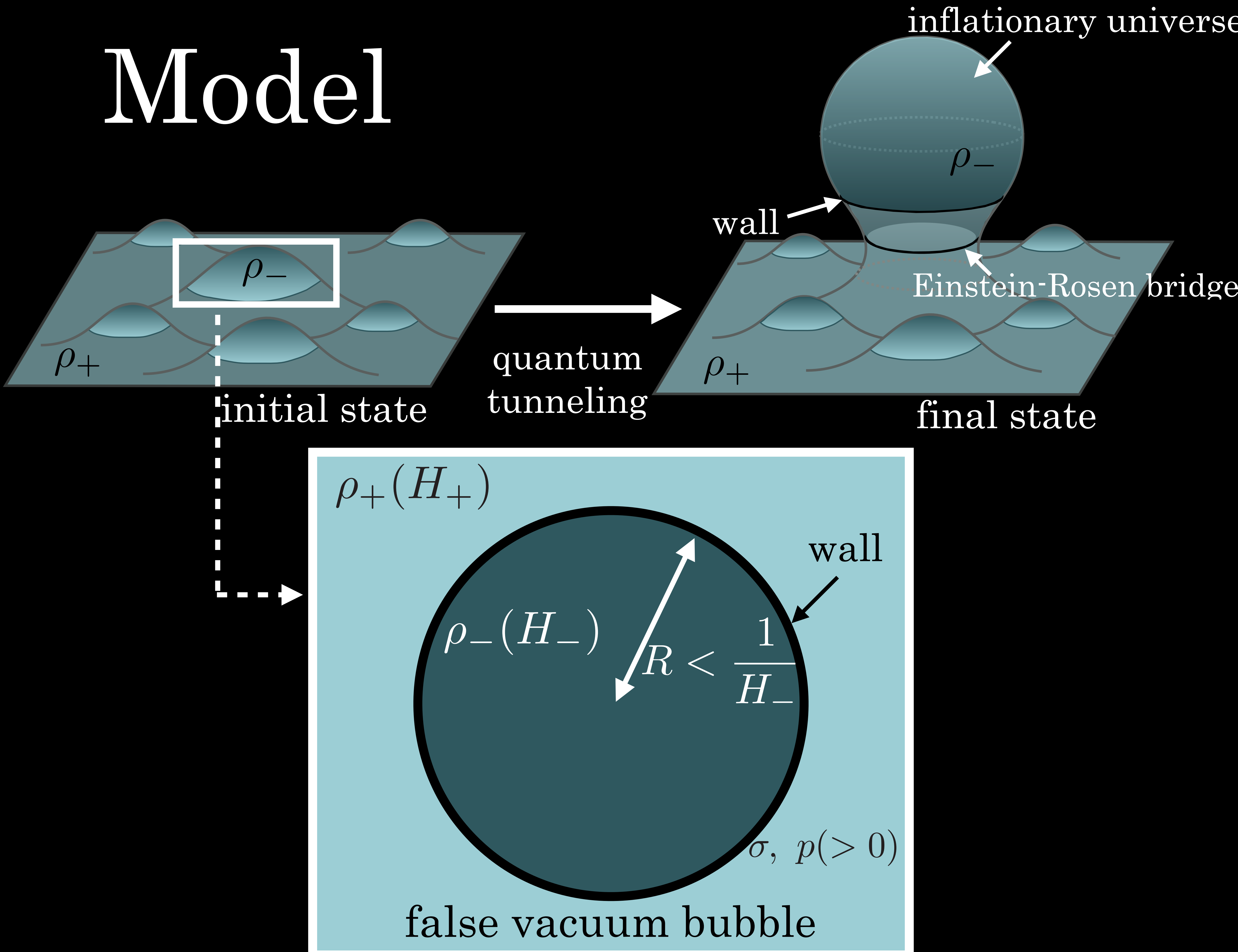
Quantum effect on inhomogeneous space plays an essential role in the early universe?

NO initial singularity

highly inhomogeneous initial condition (NO fine-tuning)

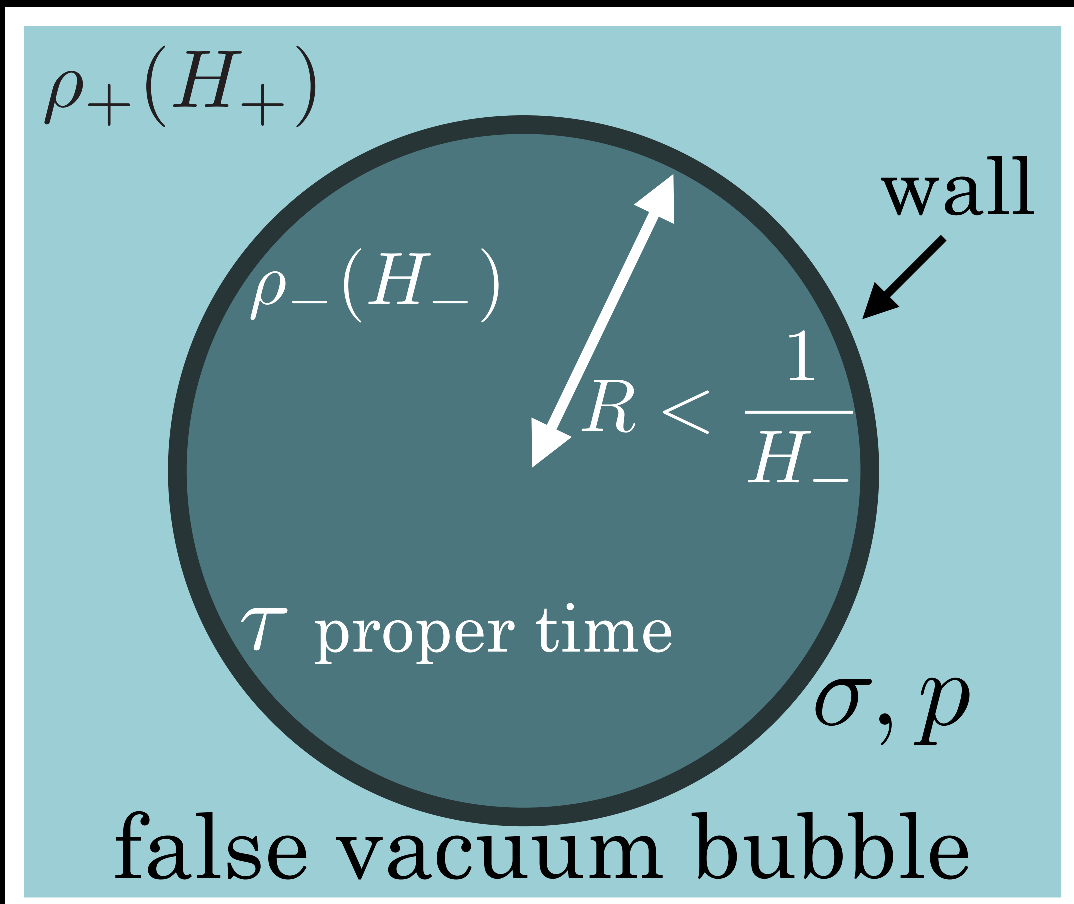


Model



inflationary universe
ρ_-
wall
Einstein-Rosen bridge
ρ_-
ρ_+
initial state
quantum
tunneling
ρ_+
final state
$\rho_+(H_+)$
wall
$\rho_-(H_-)$
$R < \frac{1}{H_-}$
$\sigma, \ p(> 0)$
false vacuum bubble

Model



metric inside and outside of the wall

$$ds^2 = -f_\pm(r_\pm)dt_\pm^2 + f_\pm^{-1}(r_\pm)dr_\pm^2 + r_\pm^2 d\Omega_2^2$$

$$f_+(r) \equiv 1 - \frac{2GM}{r} - H_+^2 r^2 \qquad f_-(r) \equiv 1 - H_-^2 r^2$$

Israel junction conditions

$$K_{ab}^{(+)} - K_{ab}^{(-)} = -8\pi G \left(S_{ab} - \frac{1}{2} h_{ab} \mathrm{Tr}(S_{ab}) \right)$$

$a, b = t,\ \theta,\ \varphi$

$K_{ab}^{(\pm)}$ extrinsic curvature

S_{ab} EMT of the wall

h_{ab} induced metric on the wall

$w \equiv p/\sigma \qquad \beta_\pm \equiv \sqrt{f_\pm + \dot{R}^2}$

$\epsilon_\pm \equiv \mathrm{sign}[K_{\theta\theta}^{(\pm)}]$

$(a, b) = (\theta, \theta)$ or (φ, φ)

$$\epsilon_- \beta_- - \epsilon_+ \beta_+ = 4\pi G \sigma(R) R$$

$(a, b) = (t, t)$

$$\frac{d}{dR}(\epsilon_- \beta_- - \epsilon_+ \beta_+) = -4\pi G \sigma(R)(1 + 2w)$$

Israel junction conditions

$$K_{ab}^{(+)} - K_{ab}^{(-)} = -8\pi G \left(S_{ab} - \frac{1}{2} h_{ab} \mathrm{Tr}(S_{ab}) \right)$$

$(a,b) = (\theta,\theta)$ or (φ,φ)

$$\epsilon_- \beta_- - \epsilon_+ \beta_+ = 4\pi G \sigma(R) R$$

$(a,b) = (t,t)$

$$\frac{d}{dR}(\epsilon_- \beta_- - \epsilon_+ \beta_+) = -4\pi G \sigma(R)(1+2w)$$

$$\sigma(R) \propto R^{-2(1+w)}$$

$$\sigma(R) = m^{-1-2w} R^{-2(1+w)}$$

m typical energy scale of the wall

$$\left(\frac{dz}{d\tau'} \right)^2 + V(z) = E$$

$$E \equiv -\frac{4\gamma^2}{(2GMH_-)^{2/3}(1-h^2)^{1/3}}$$

$$V(z) \equiv -\frac{4\gamma^2}{1-h^2} z^2 - z^{4w} \left(1 - z^3 + \frac{\gamma^2}{z^{1+4w}} \right)^2$$

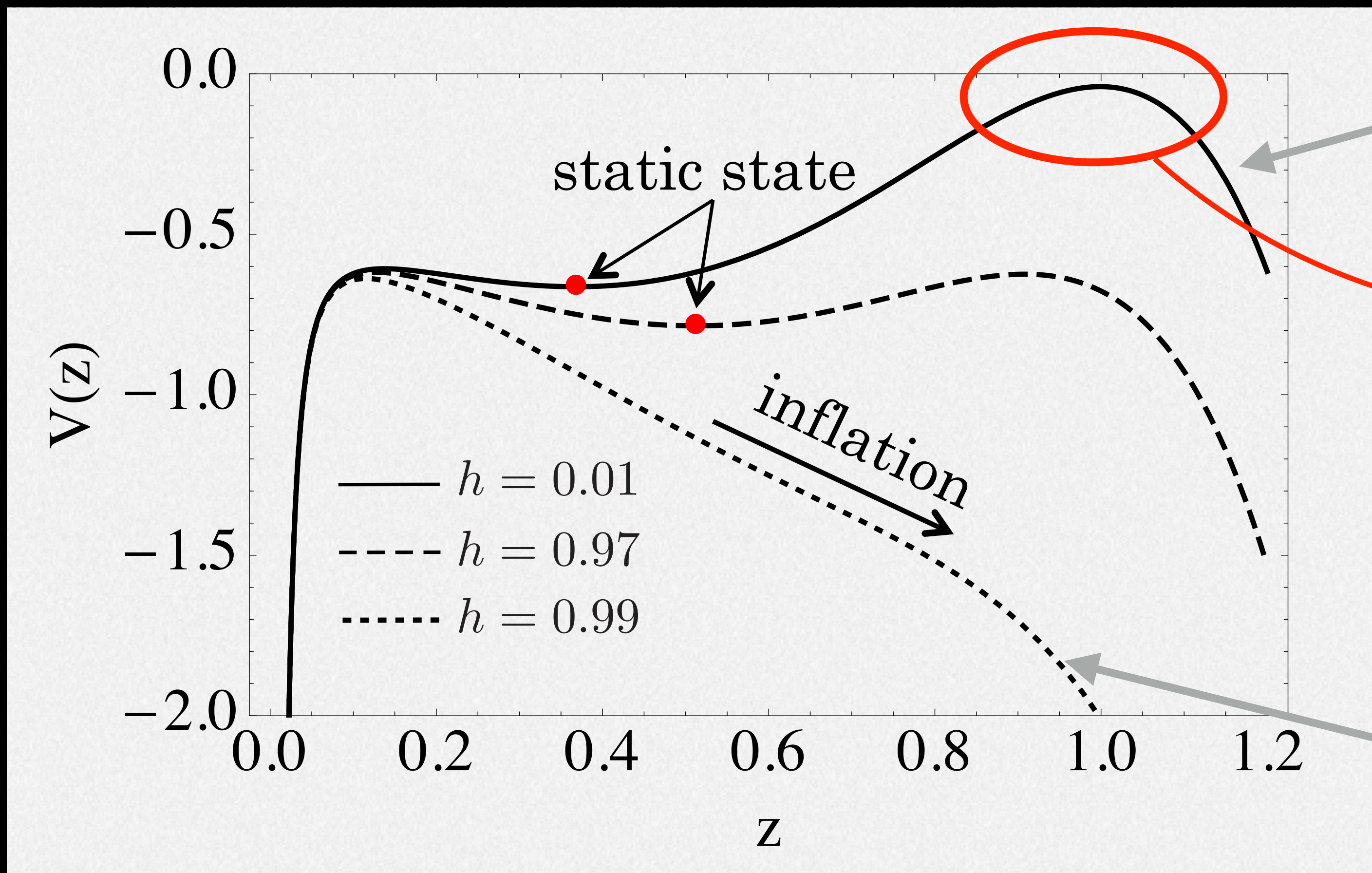
$$h \equiv H_+/H_- \quad \gamma \equiv \frac{4\pi m H_- (H_-/m)^{2w}(1-h^2)^{(1+4w)/6}}{M_{\mathrm{Pl}}^2 (2GMH_-)^{2(1+w)/3}}$$

non-dimensional variables

$$z \equiv \left(\frac{1-h^2}{2GMH_-} \right)^{1/3} H_- R$$

$$\tau' \equiv \frac{\sqrt{1-h^2}}{2\gamma} H_- \tau$$

Inhomogeneity and inflation



inhomogeneous

potential barrier

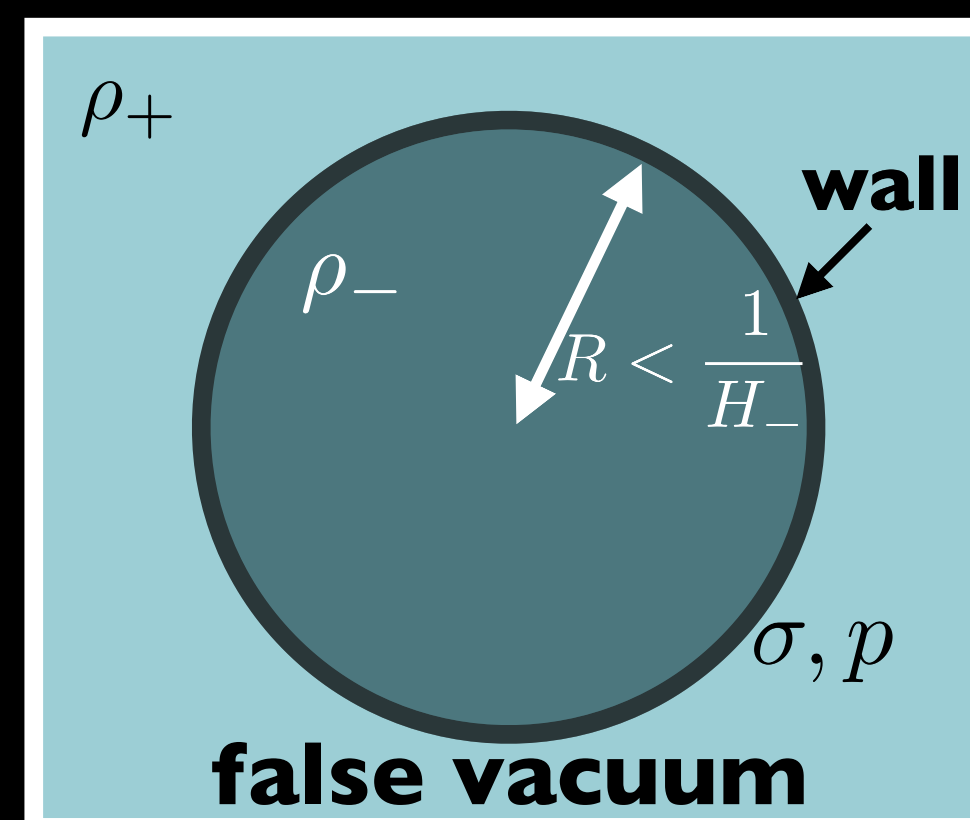
separating the inflationary phase and the static state

homogeneous

(non-dimensional radius)

$h \equiv H_+/H_-$

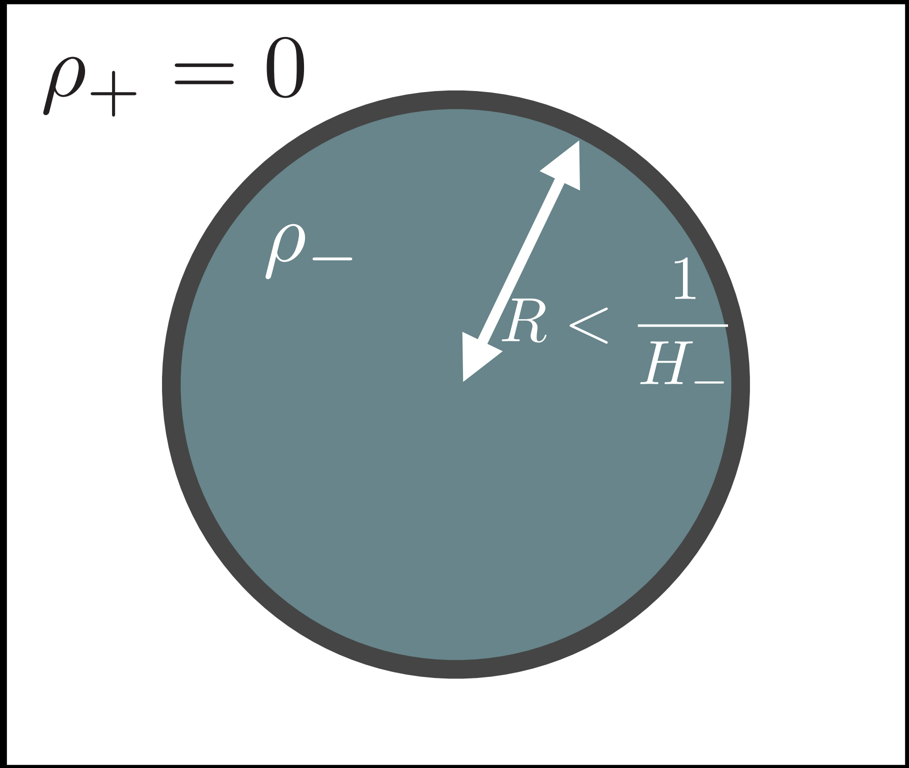
$$\left(\frac{dz}{d\tau'}\right)^2 + V(z) = E$$

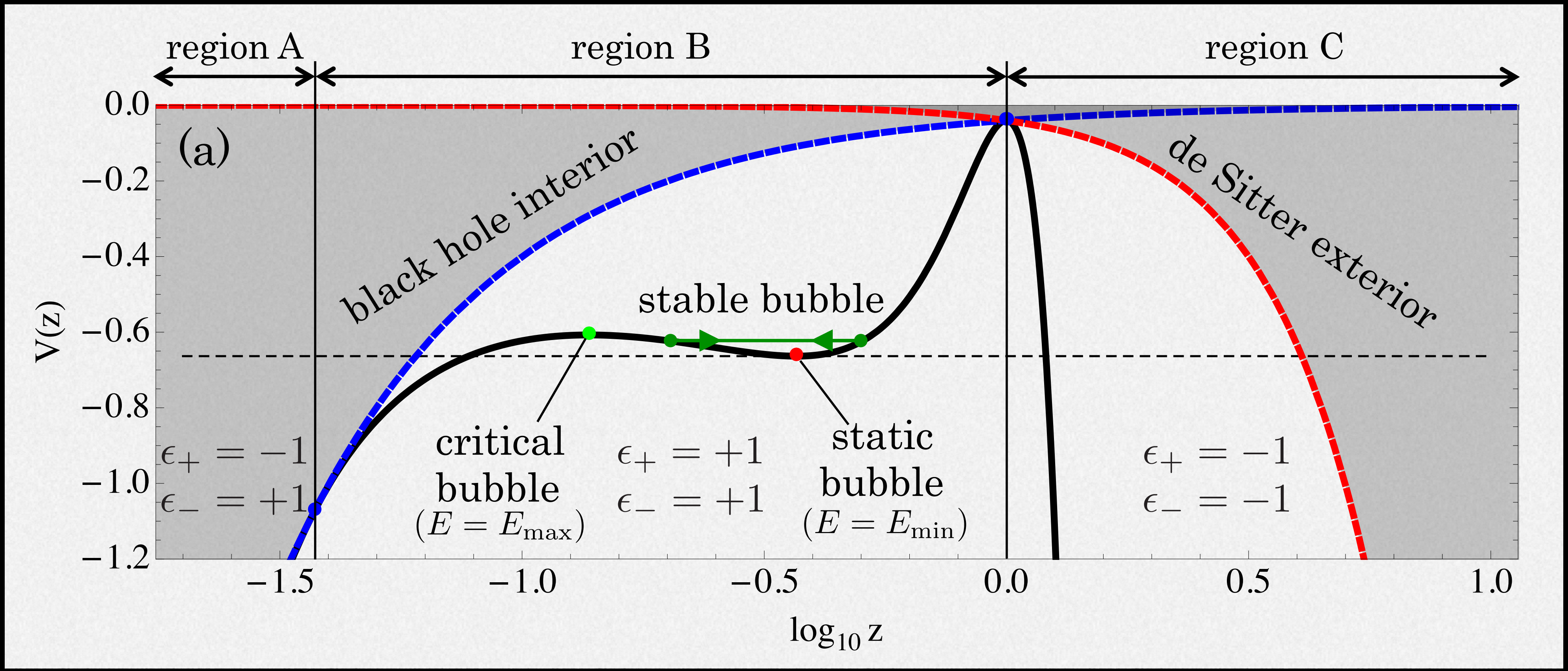


Most conservative situation

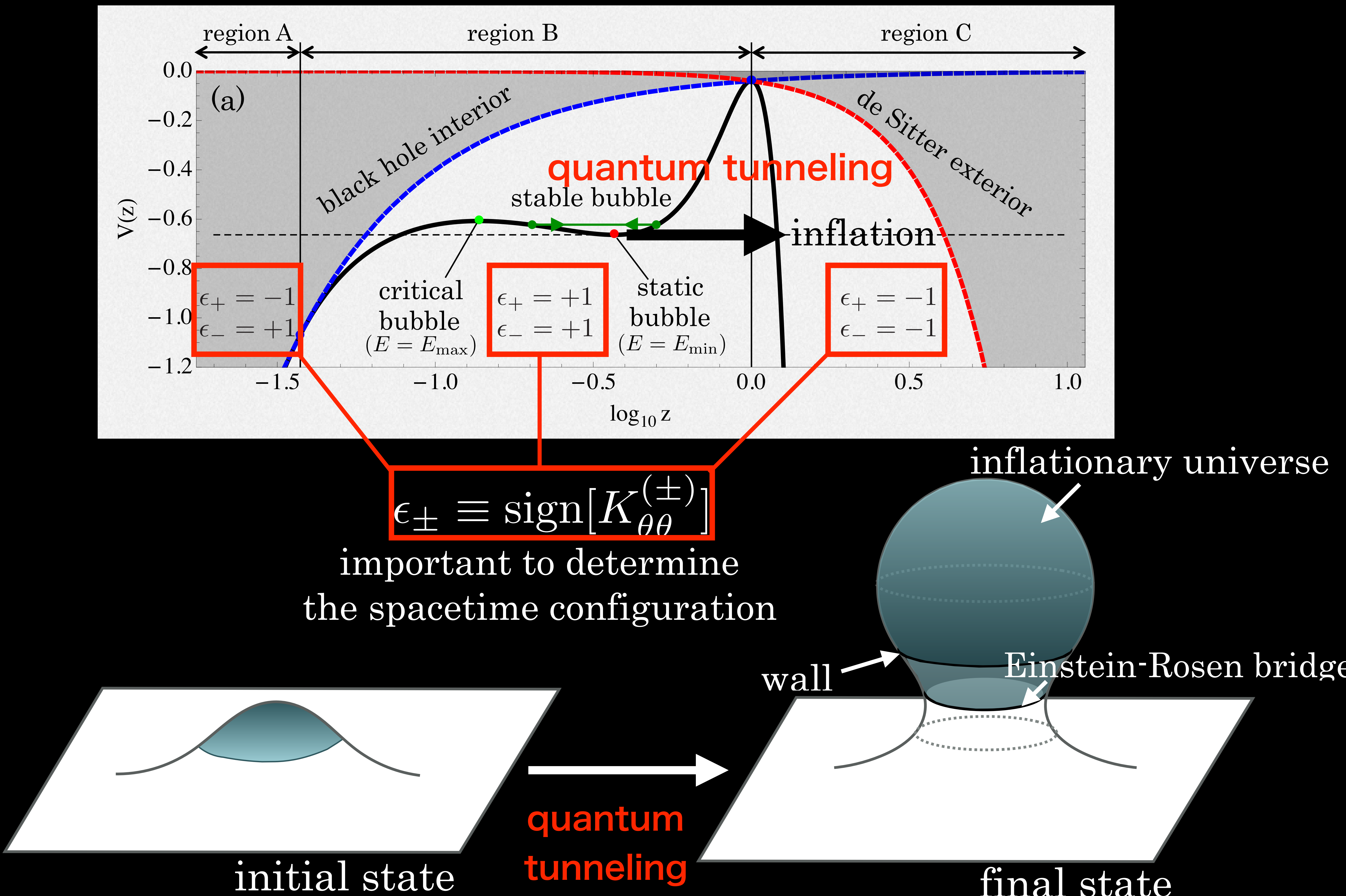
There is NO quasi-homogeneous mode.

→ Density fluctuation never be red-shifted.

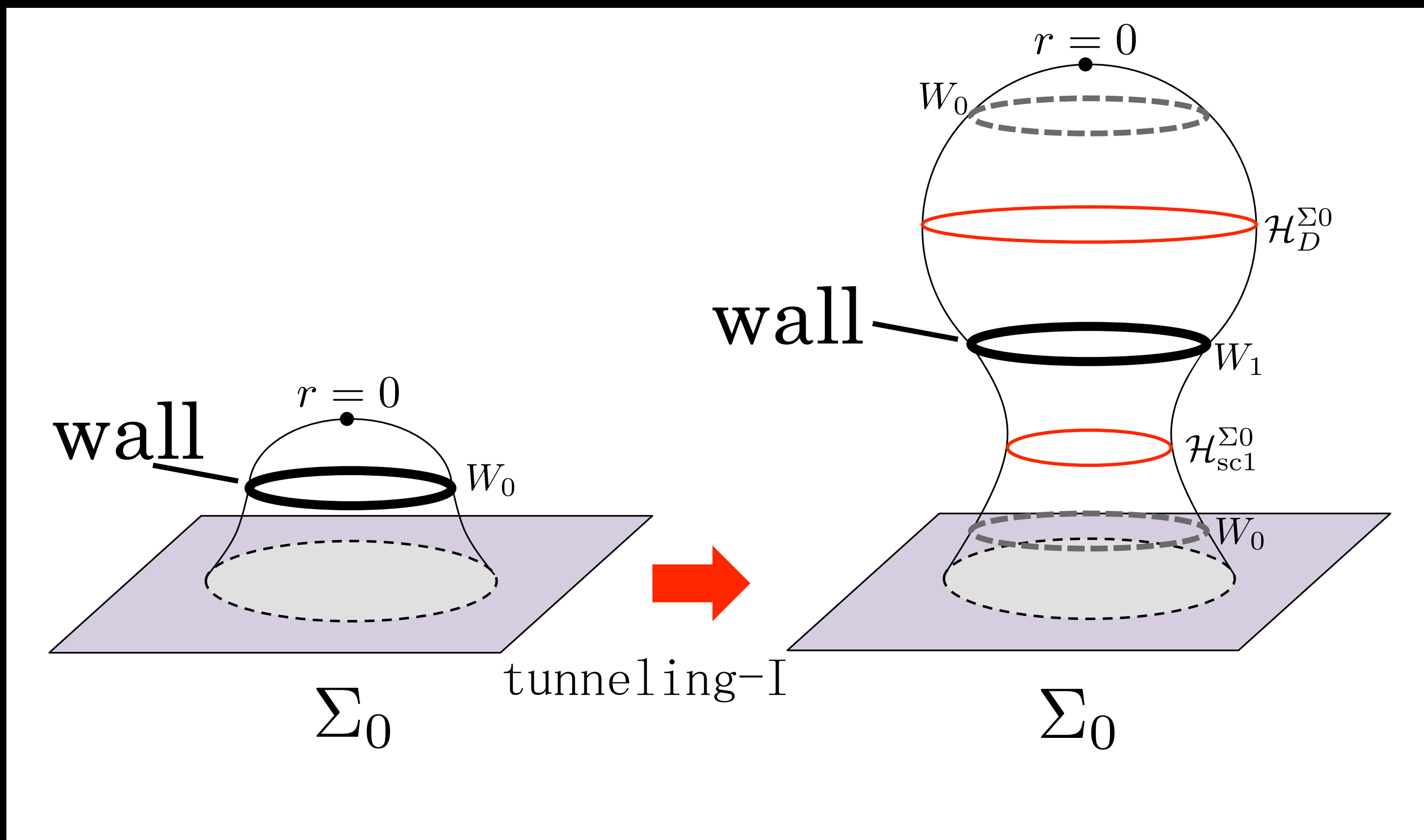


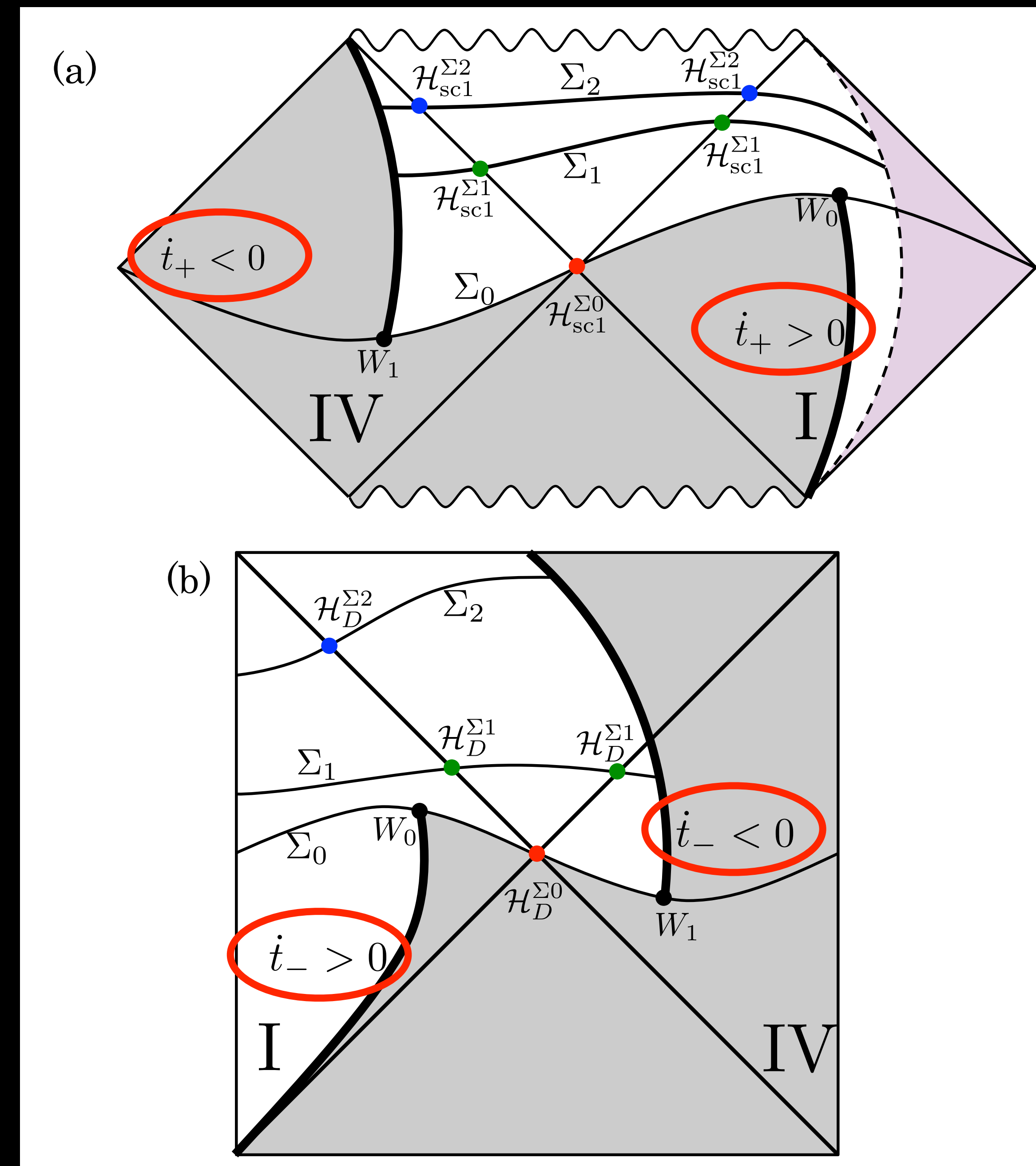


Quantum effect on inhomogeneous space



Configuration of the final state





$$K_{11}^{(\pm)} = K_{22}^{(\pm)} = \frac{\dot{t}_\pm f_\pm}{R}$$

$$\epsilon_\pm \equiv \mathrm{sign}[K_{\theta\theta}^{(\pm)}] = \mathrm{sign}[\dot{t}_\pm] \quad \text{for} \quad f_\pm > 0$$

How to calculate the tunneling rate

Gregory-Moss-Withers formula (2014)

$$\Gamma \simeq \tau_d^{-1} e^{-B+\Delta S}$$

$-B$: the Euclidean dynamics of wall

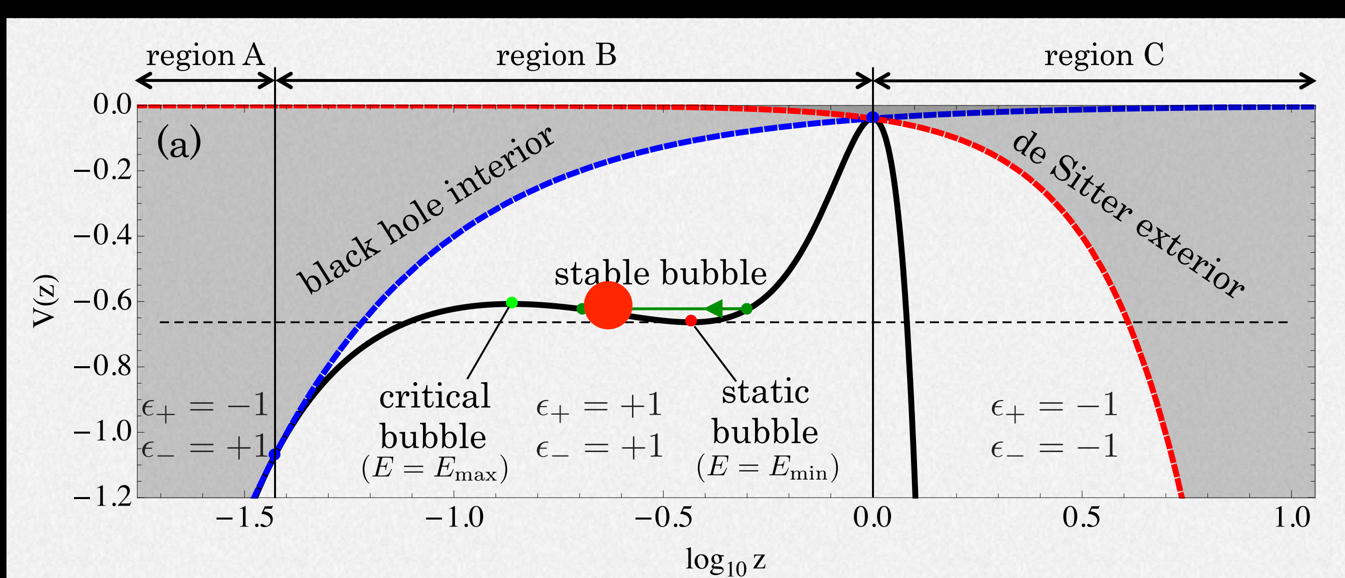
ΔS: boundary of the E-H action

the total Euclidean action

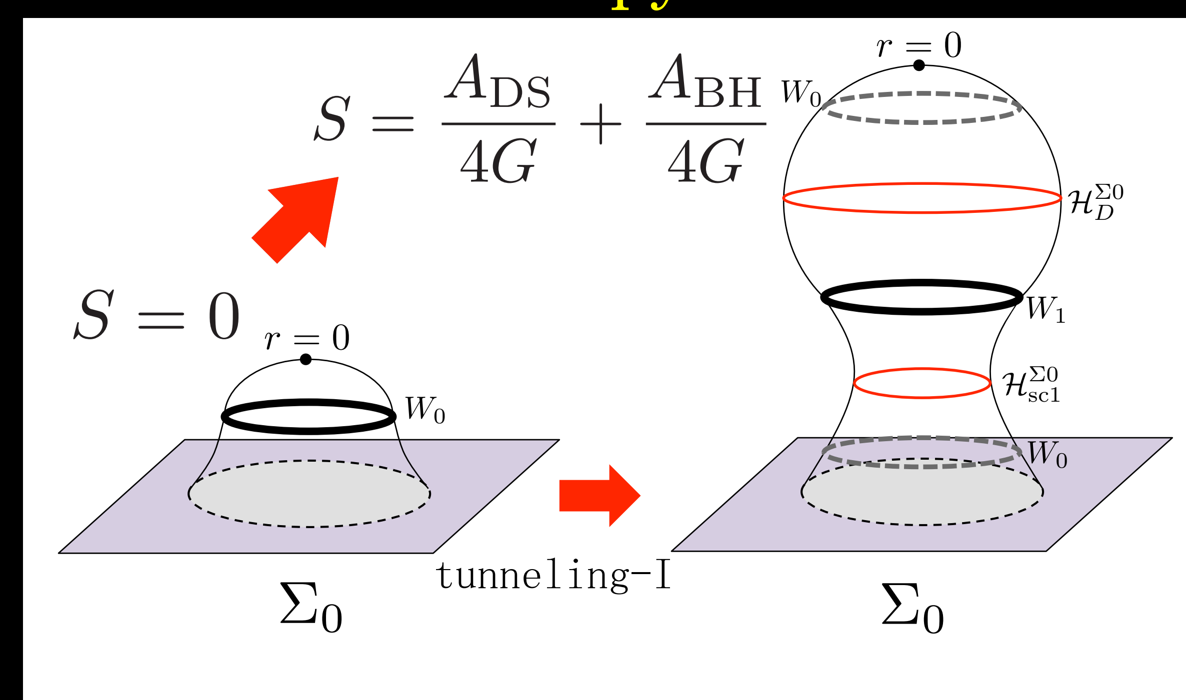
$$I \equiv B - \Delta S$$

$$\Delta S = \frac{A_{\rm DS}}{4G} + \frac{A_{\rm BH}}{4G}$$

increment of entropy



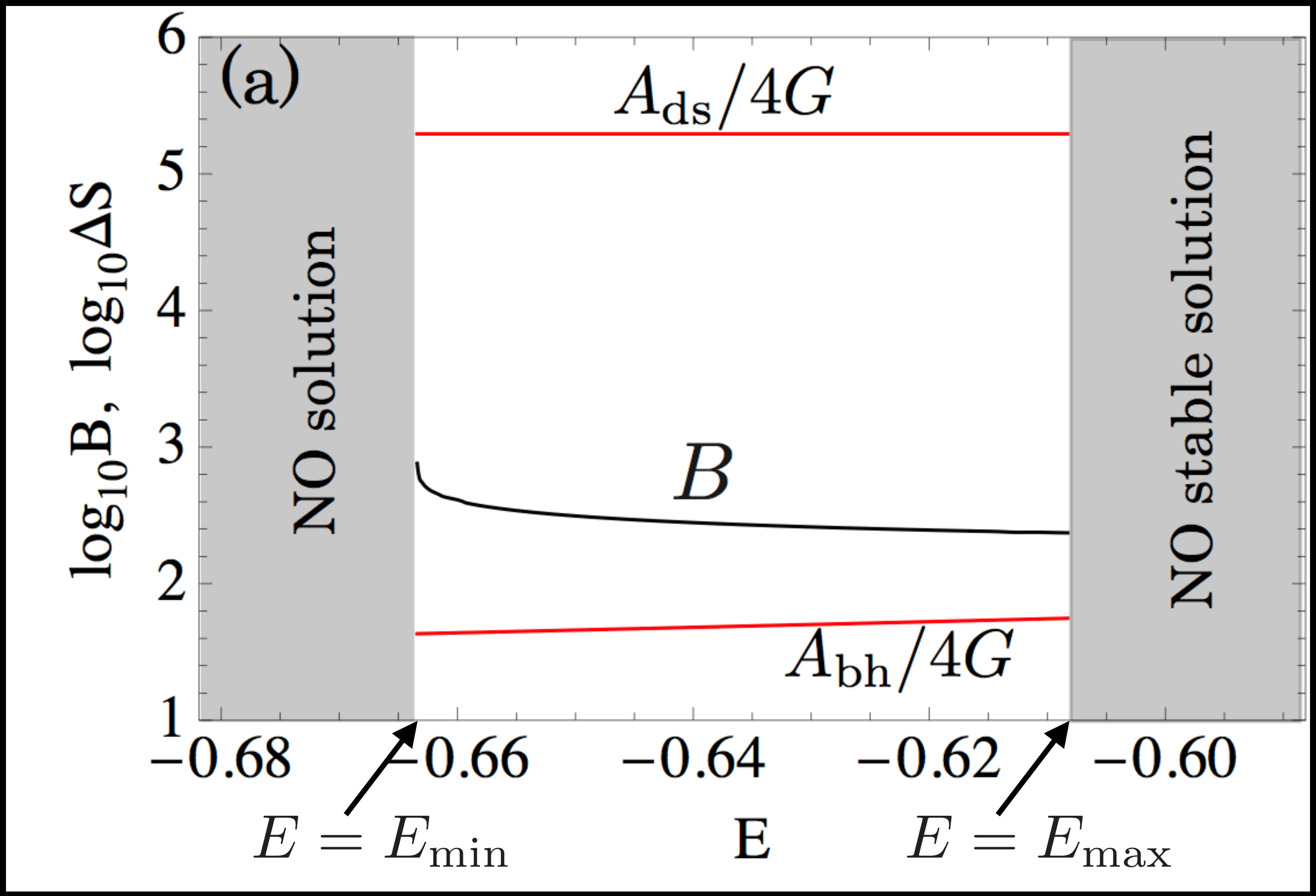
the Euclidean dynamics of wall



B v.s. ΔS

$$\Gamma \simeq \tau_d^{-1} e^{-B+\Delta S}$$

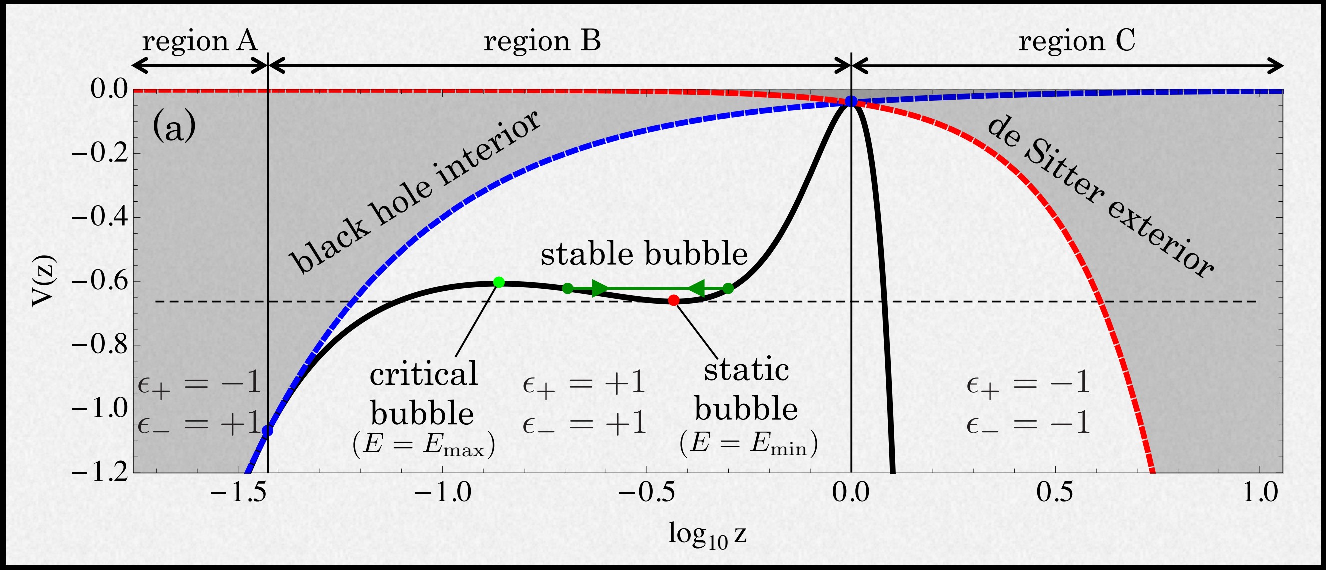
$$\Delta S > B$$

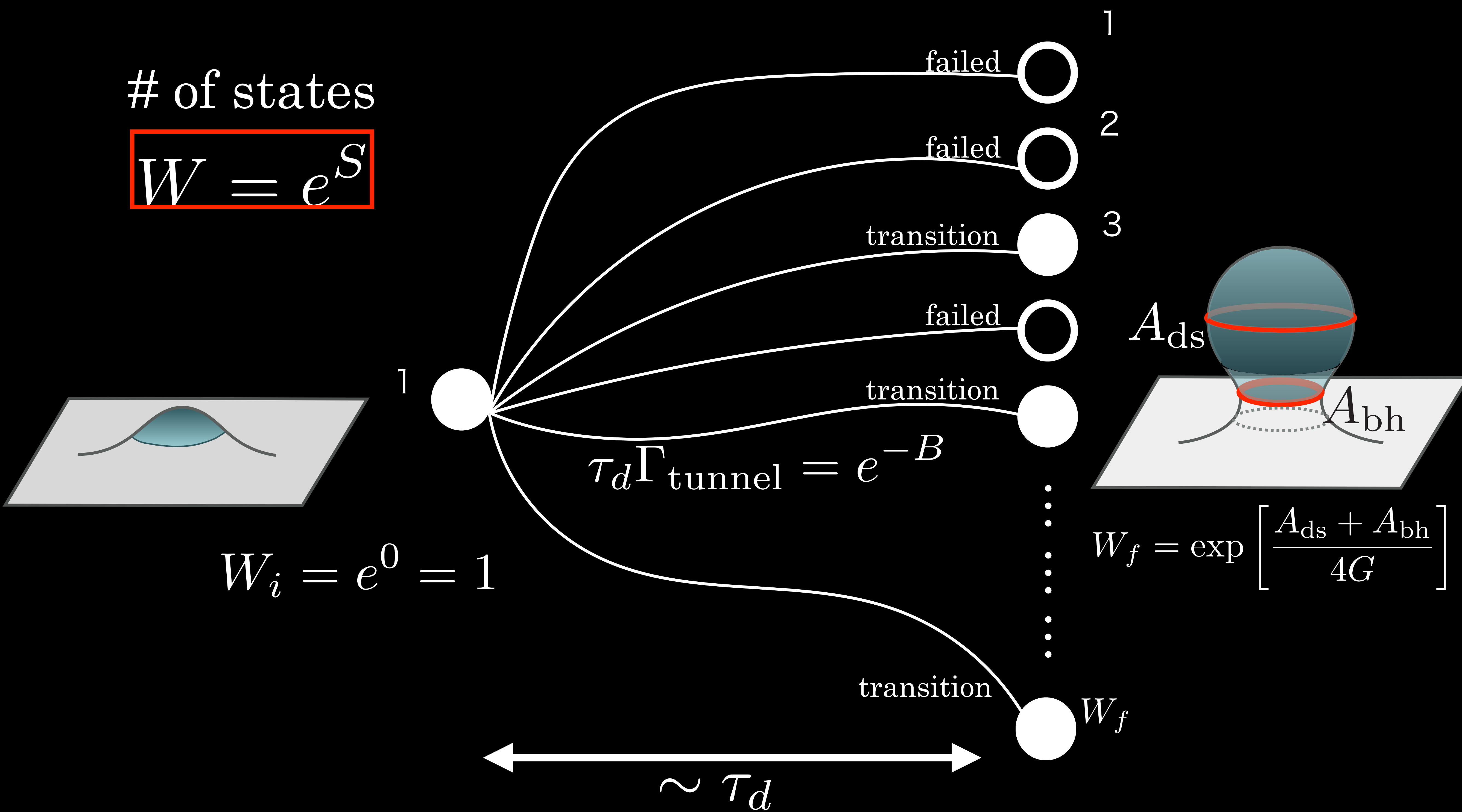


$$e^{-B+\Delta S} \gg 1$$

exponential enhancement

How should we interpret this?





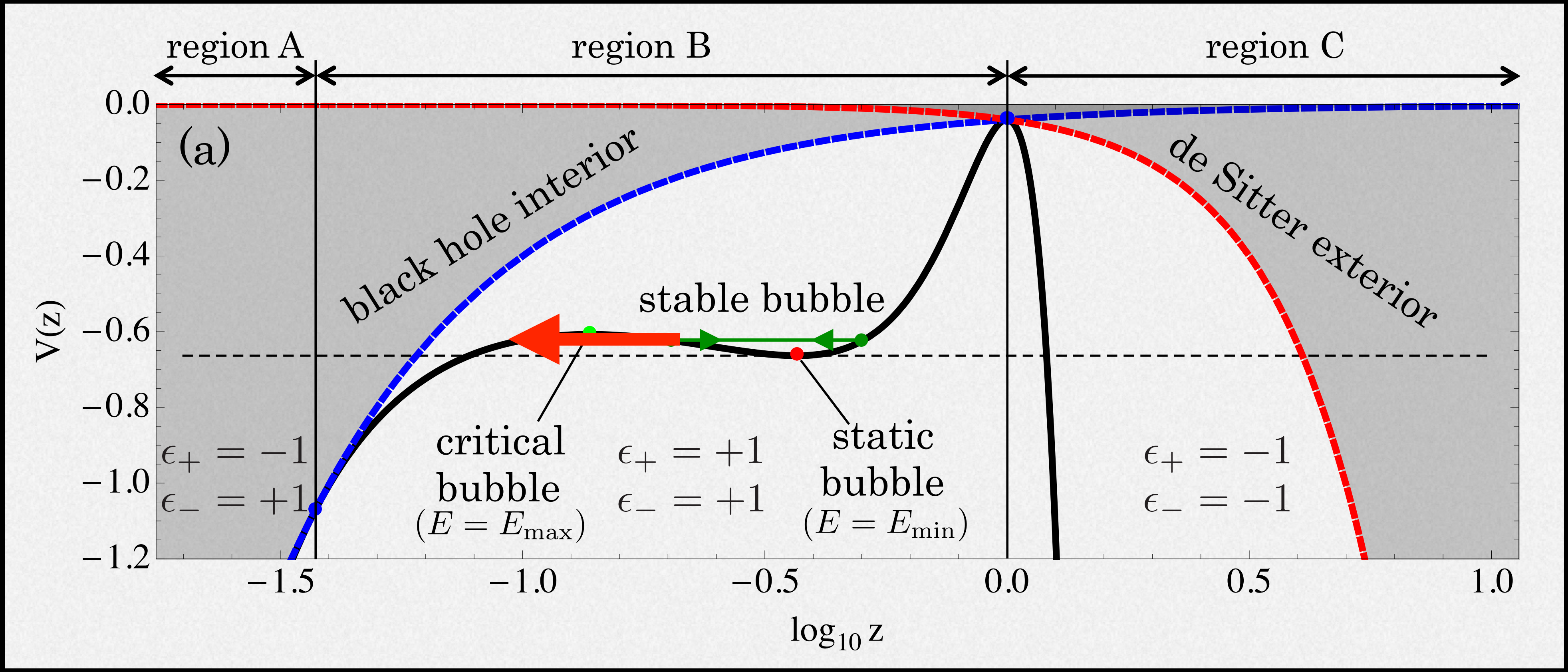
$$\left[\text{expectation number of } \bullet\right] \equiv \tau_d \Gamma_{\rm tunnel} \times W_f = \underline{e^{-B+\Delta S}}$$

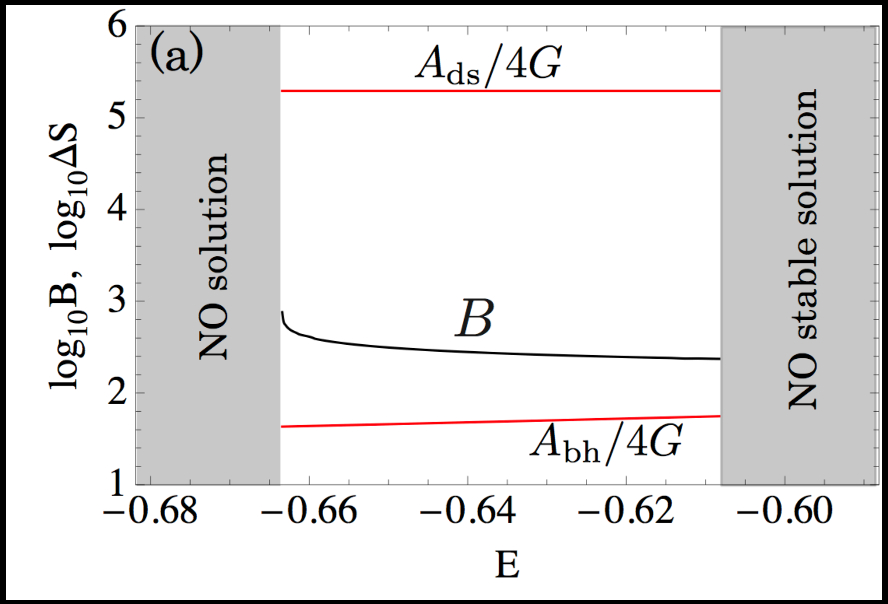
Conclusions

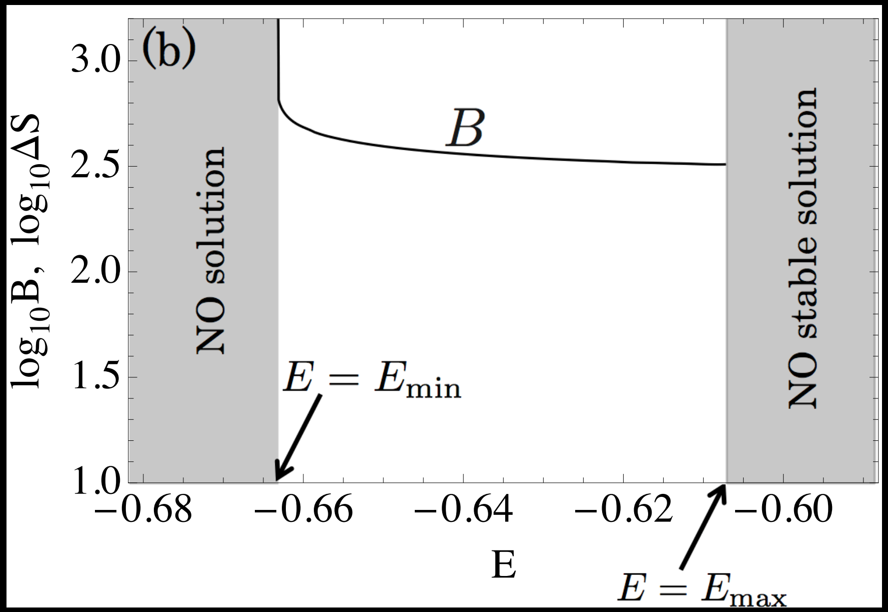
- We modeled a part of inhomogeneous space by a FVB.
- Assuming the thin wall approximation, we can solve simplified equations (Israel junction condition).
- We considered the most conservative situation, i.e., there is no quasi-homogeneous mode in the system.
- Once the bubble tunnels to a larger one, it would start inflation beyond the Einstein-Rosen bridge.
- The quantum tunneling may be promoted by the increment of the Bekenstein entropy.
- Here we assume that gravitational interaction among vacuum energy bumps does not change our main conclusions. Confirming this assumption will be the next work.
- Investigating our proposal in the case of thick walls is ongoing.

BONUS SLIDES

Collapse of bubble







$$e^{-B+\Delta S} \gg 1$$

$$e^{-B+\Delta S} \ll 1$$