Exploring multifield inflation with numerical methods (PyTransport)

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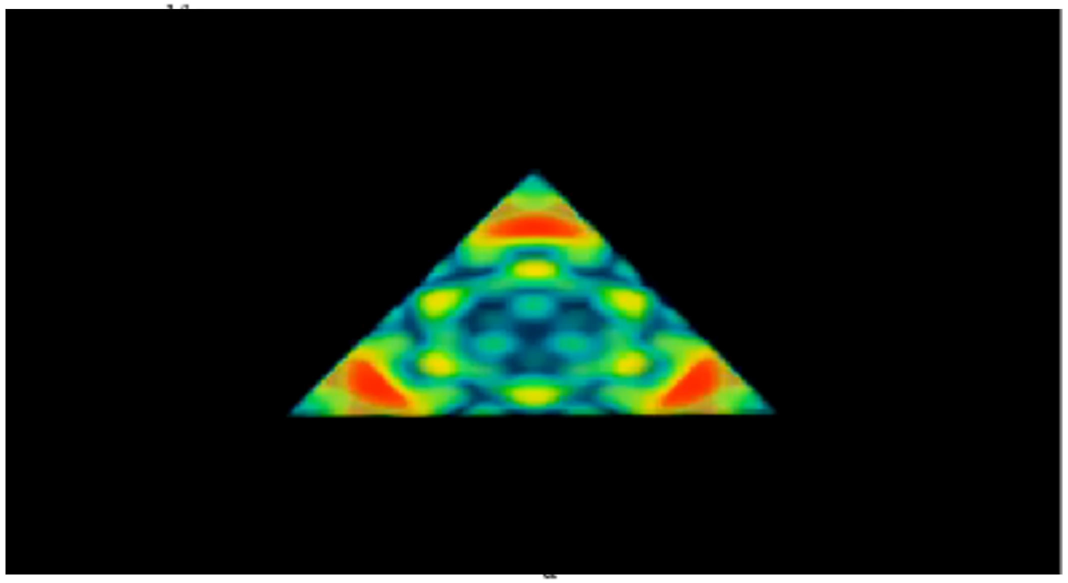
arXiv:1609.00379; arXiv:1708.07130 + forthcoming

Motivation

- Inflation with more than one field may be generic
- New effects in (higher order) correlation functions could potentially allow us to detect new fields
- In many systems the large N limit has interesting properties. To probe this limit for inflation, however, numerics are essential
- Without numerics, theory error even for simple models can be greater than observational uncertainty
- At very least we should be able to take any model of inflation and confront with (improving) observations

PyTransport

- PyTransport and sibling code CppTransport (developed by David Seery) solves transport equations for inflationary perturbations to produce full power spectrum and bispectrum
- Deals with models with arbitrary numbers of scalar fields, with a curved field space metric — soon perturbative reheating
- Includes all tree-level effects on sub and super-horizon scales
- Publicly available and automated in sense user need only provide potential (and field space metric)



Observational quantities

- Statistical quantities we want to evaluate

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2)P(k)$$

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3)$$

$$f_{\rm NL} = \frac{5}{6}\frac{B(k_1,k_2,k_3)}{P(k_1)P(k_2)+P(k_1)P(k_3)+P(k_2)P(k_3)}$$

- Basic predictions

$$P(k) \sim Ak^{-3}$$

$$f_{\rm NL} \sim \text{slow roll (for canonical single field)}$$

Calculating statistics

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_p^2 R + \mathcal{L}_m\right]$$

$$\mathrm{d}s^2 = -(1+2\Phi)\mathrm{d}t^2 + a^2(\delta_{ij} + h_{ij})\mathrm{d}x^i\mathrm{d}x^j$$

$$\mathcal{L}_m = -G_{IJ}g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - V$$
$$\phi^I + \delta\phi^I$$

action expanded order by order in fluctuations Q^I
and gravitational waves (tensor) h_{ij}

Calculating statistics

$$S = S_{(2)} + S_{(3)}$$

$$\mathcal{O}(2) \text{ in } Q^I \qquad \mathcal{O}(3) \text{ in } Q^I$$

Maldacena 2003; Seery and Lidsey 2006; Chen *et al*. 2007; Elliston *et al*. 2012; many others

Lagranian or Hamiltonian equations of motion for Q^I

$$\langle Q^I(k_1) Q^J(k_2) \rangle = (2\pi)^3 \delta(\mathbf{k_1} + \mathbf{k_2}) \boxed{\Sigma^{IJ}(k_1)}$$

$$\langle Q^I(k_1) Q^J(k_2) Q^K(k_3) \rangle = (2\pi)^3 \delta(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3}) \boxed{B^{IJK}(k_1, k_2, k_3)}$$

$$Q^I \to \zeta$$

Calculating the statistics — transport method

- Our approach (schematically)

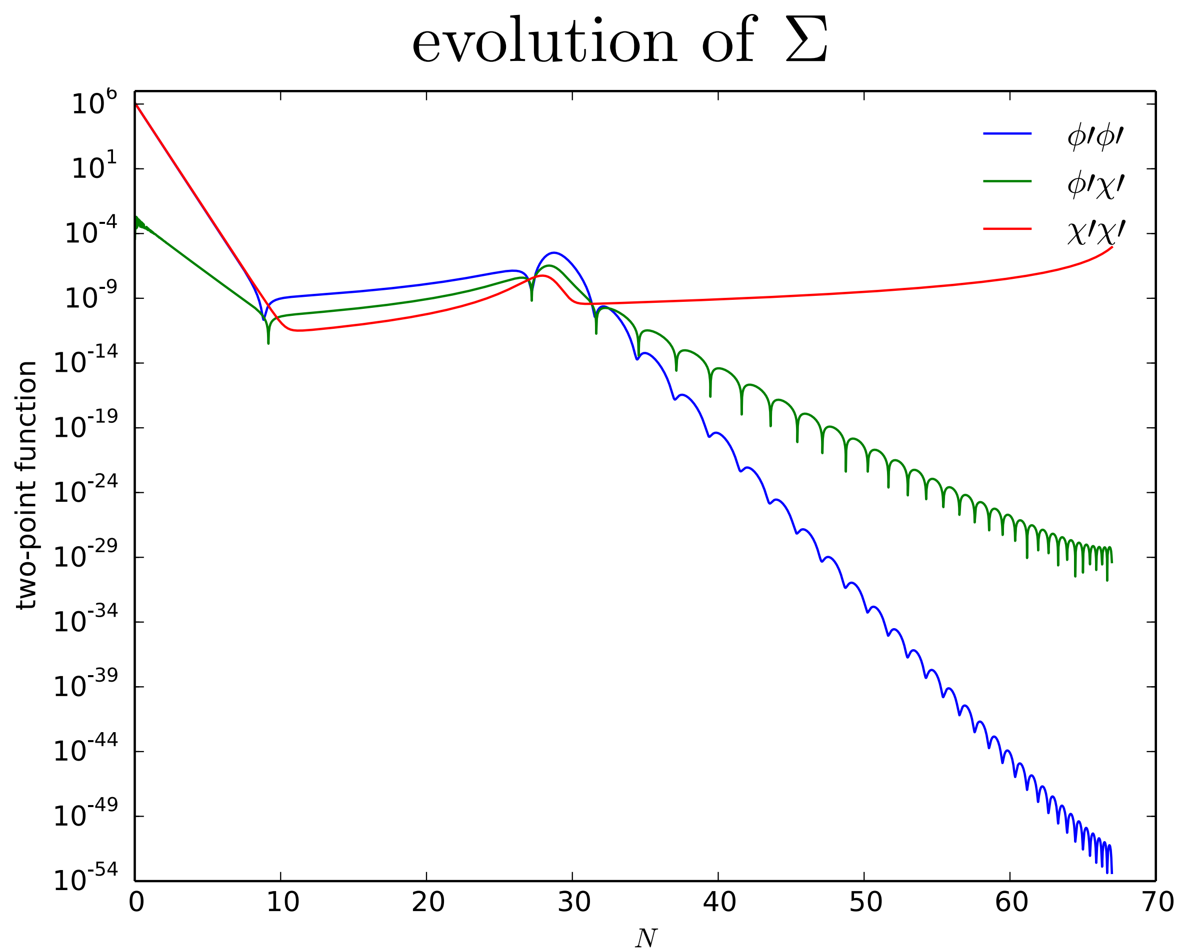
$$\frac{\mathrm{d}Q^I}{\mathrm{d}t} = u^I{}_J Q^J + \frac{1}{2} u^I{}_{JK} Q^J Q^k$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\Sigma^{IJ} = \boxed{u^I{}_K}\Sigma^{KJ} + \boxed{u^J{}_K}\Sigma^{IK}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}B^{IJK} = \boxed{u^I{}_L}B^{LJK} + \boxed{u^I{}_{LM}}\Sigma^{JL}\Sigma^{KM} + \text{cyclic perms}$$

Background and k dependent quantities

Ideal for a numerical implementation — solve from Bunch Davis vacuum


evolution of Σ
two-point function
N
$\phi\prime\phi\prime$
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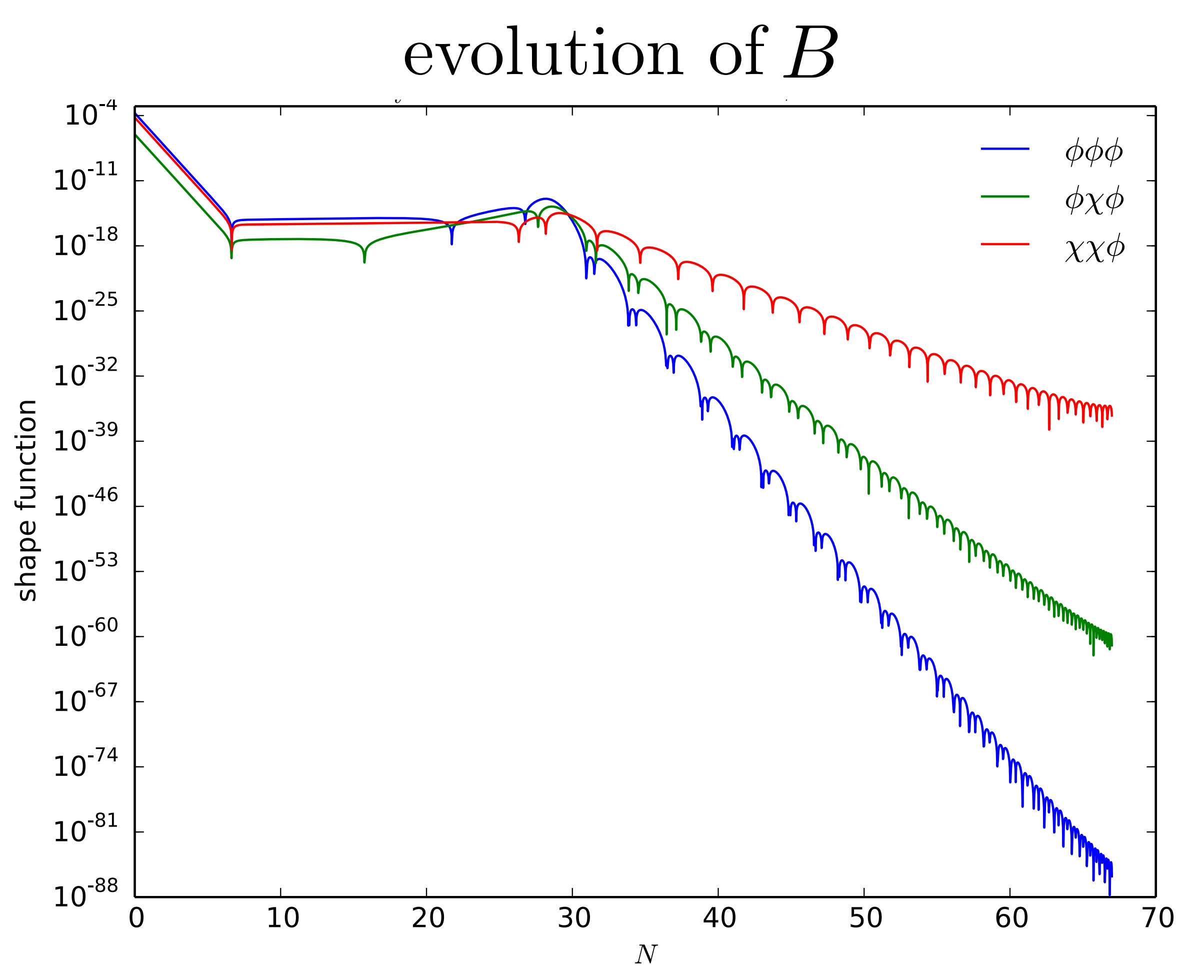
evolution of B
shape function
N
$\phi\phi\phi$
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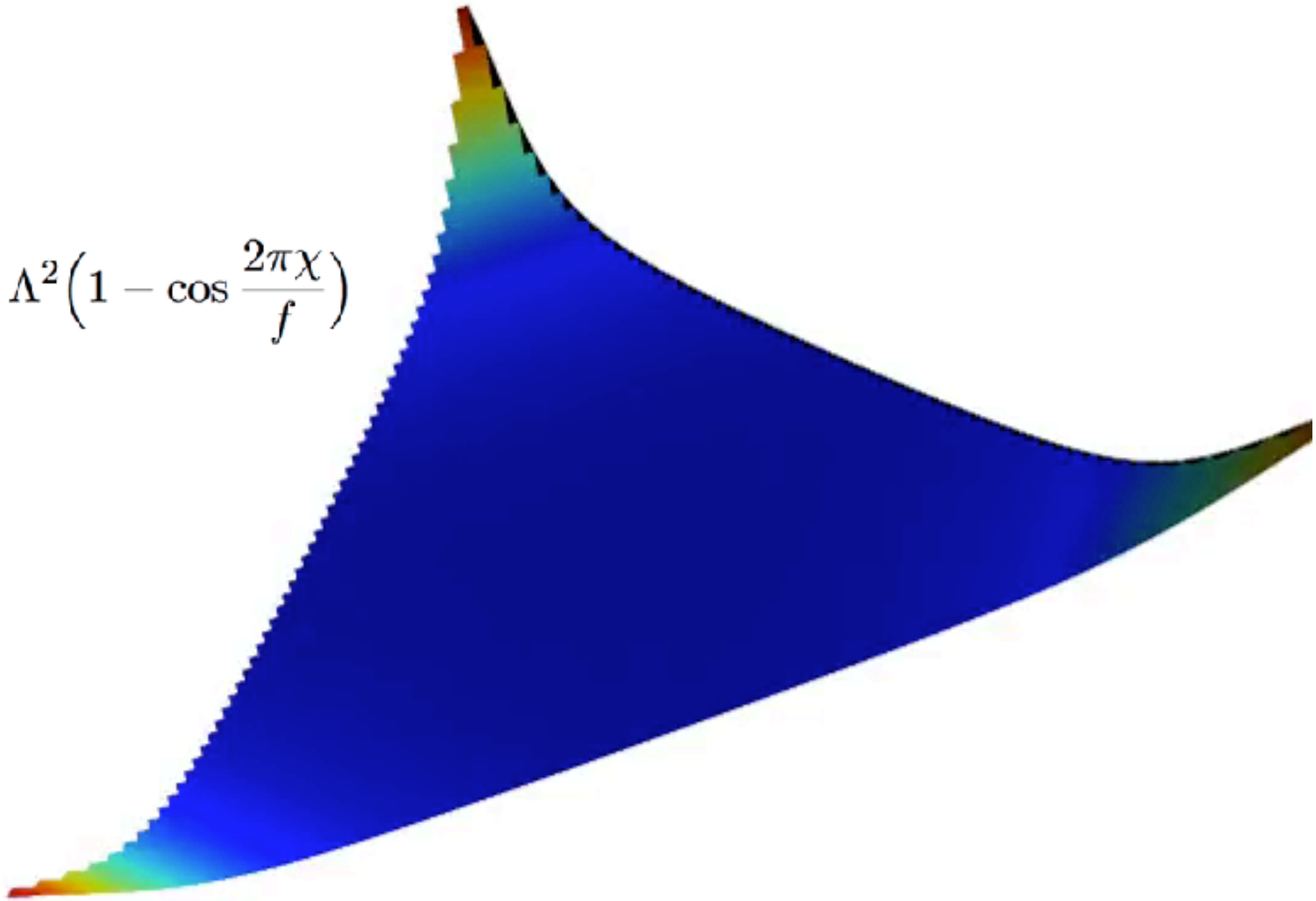
$$V = \frac{1}{4}g\phi^4 + \Lambda^2\left(1 - \cos\frac{2\pi\chi}{f}\right)$$



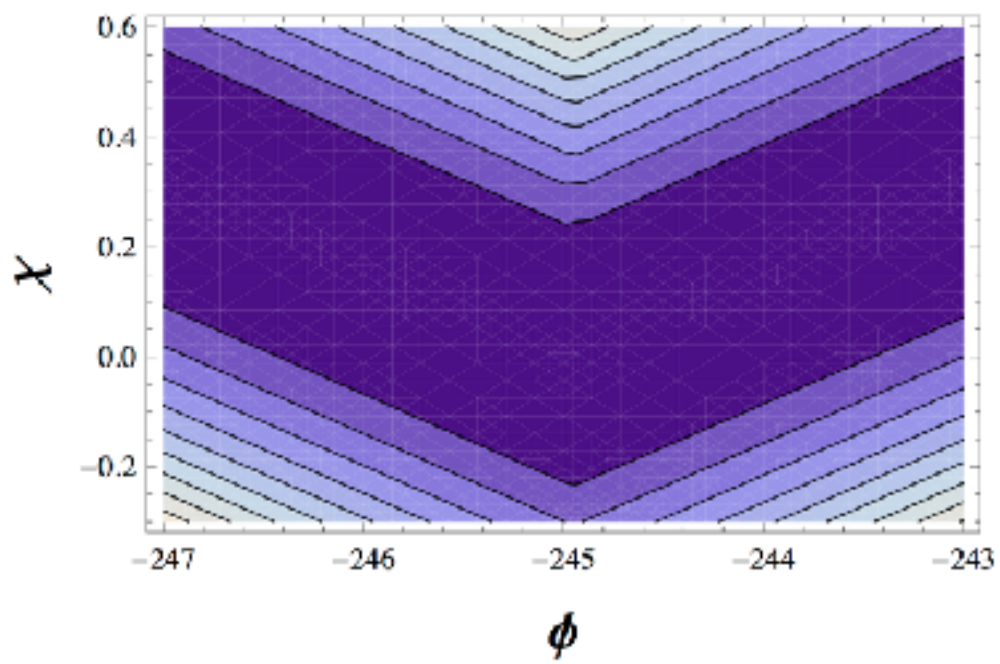
Slice through reduced bispectrum with $k_1 + k_2 + k_3$ fixed

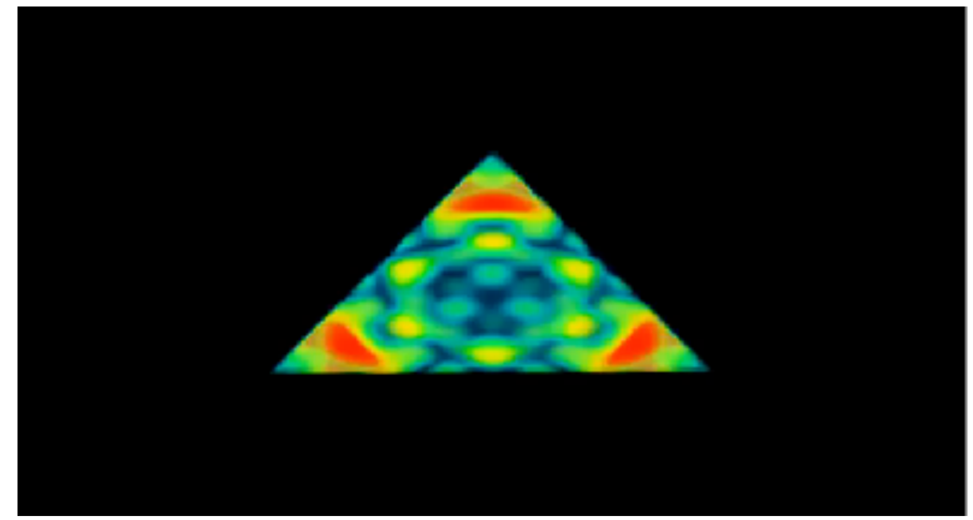
Multifield models

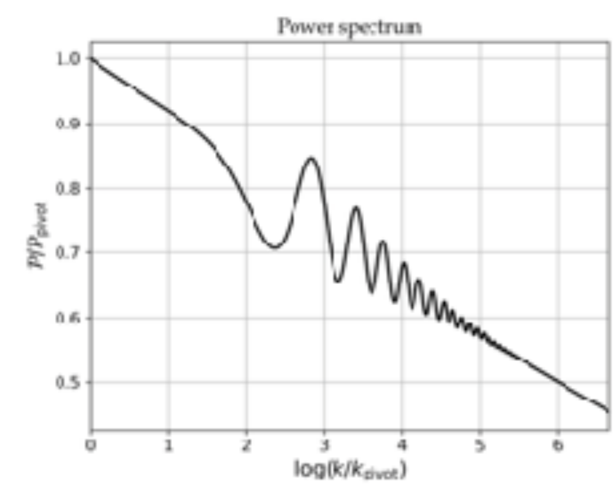
- **Model driven** - string theory, supergravity, MSSM, Standard Model. At a minimum we should be able to test all models
    - Either concrete models, or random potentials e.g. Dias, Frazer and Marsh (2017), Bjorkmo and Marsh (2017)
- **Phenomenological** - how do multi-field dynamics differ from single field dynamics? - the great hope is that we could detect new fields!
- **New effects** - extra light/heavy fields, curved field space metric -> curved trajectories, isocurvature modes -> Non-Gaussianity Byrnes et al. 2008; Hall and Choi Chen & Wang 2009; Tolley and M. Wyman 2010; Achúcarro et al. 2011
- Effects can be **during inflation** or **during reheating** (curvaton, modulated reheating etc) Enqvist and sloth e.g. Enqvist and Sloth 2001; Lyth and Wands 2001; Zaldarriaga 2003
- **Probabilistic** interpretation may be needed e.g. Frazer 2014

Heavy field with turn

Goa and Langlois (2014)



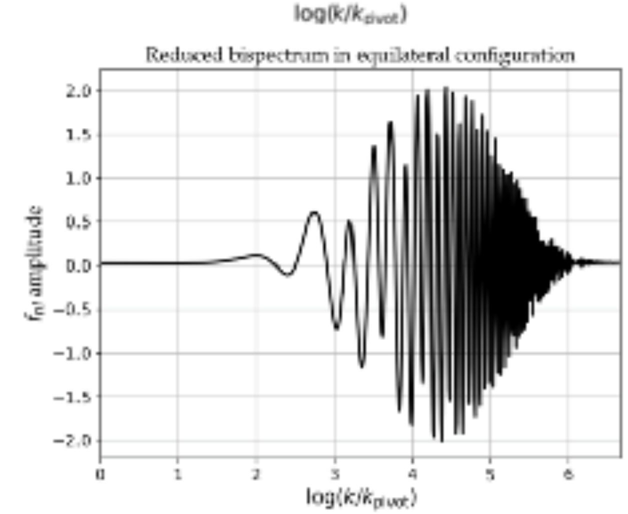




Achucarro, Hardeman, Palma, Patil (2010)

$$\Gamma(\phi_1) = \frac{\Gamma_0}{\cosh^2\left(2\left(\frac{\phi_1 - \phi_{1(0)}}{\Delta\phi_1}\right)\right)}$$

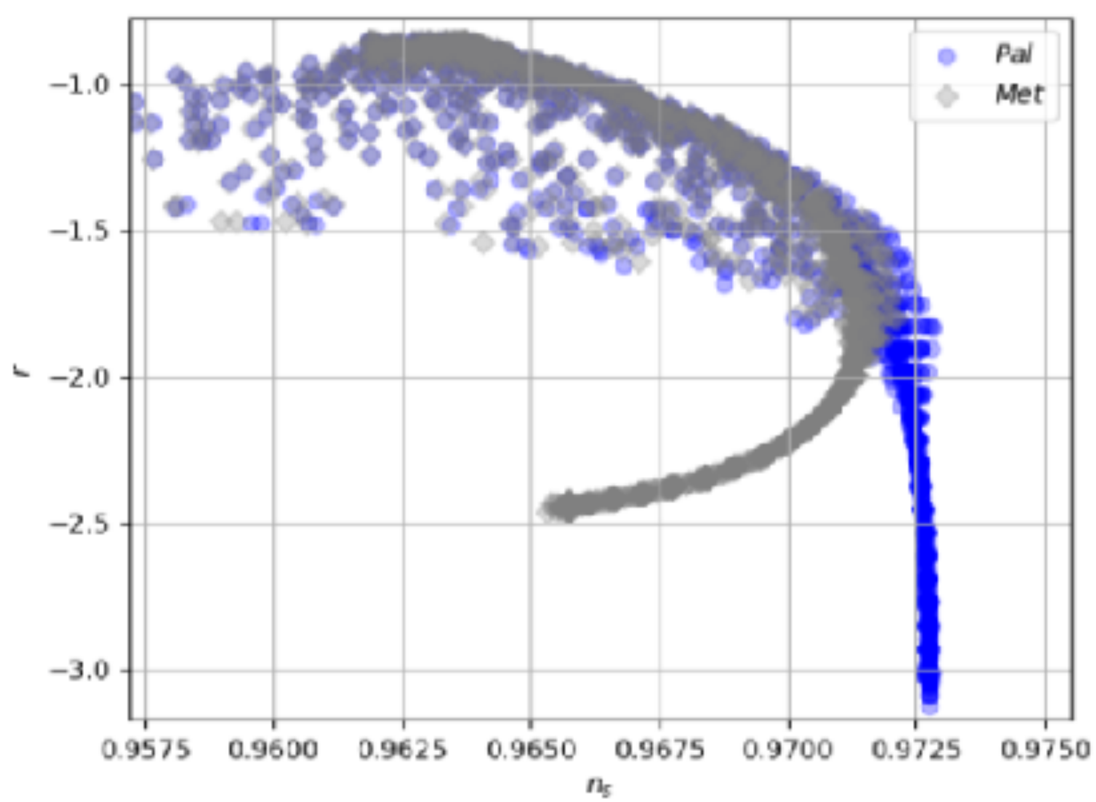
$$G_{IJ} = \begin{pmatrix} 1 & \Gamma(\phi_1) & 0 \\ \Gamma(\phi_1) & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

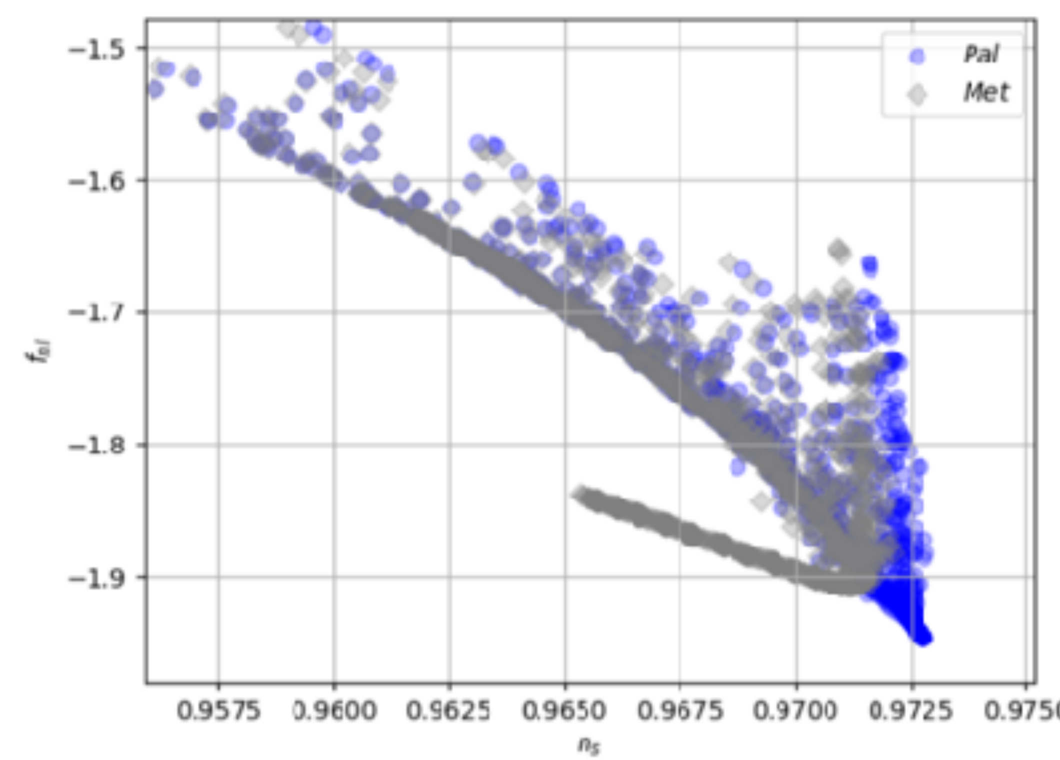


Multifield alpha attractors (Palatini and Metric formulations)

Ronayne, Carrilho, DJM and Tenkanen (2018)

$$S_J = \int d^4x\sqrt{-g}\left(\frac{1}{2}\delta_{IJ}g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - \frac{M_{\rm P}^2}{2}\left(1+f(\phi^I)\right)g^{\mu\nu}R_{\mu\nu}(\Gamma) - V(\phi^I)\right)$$

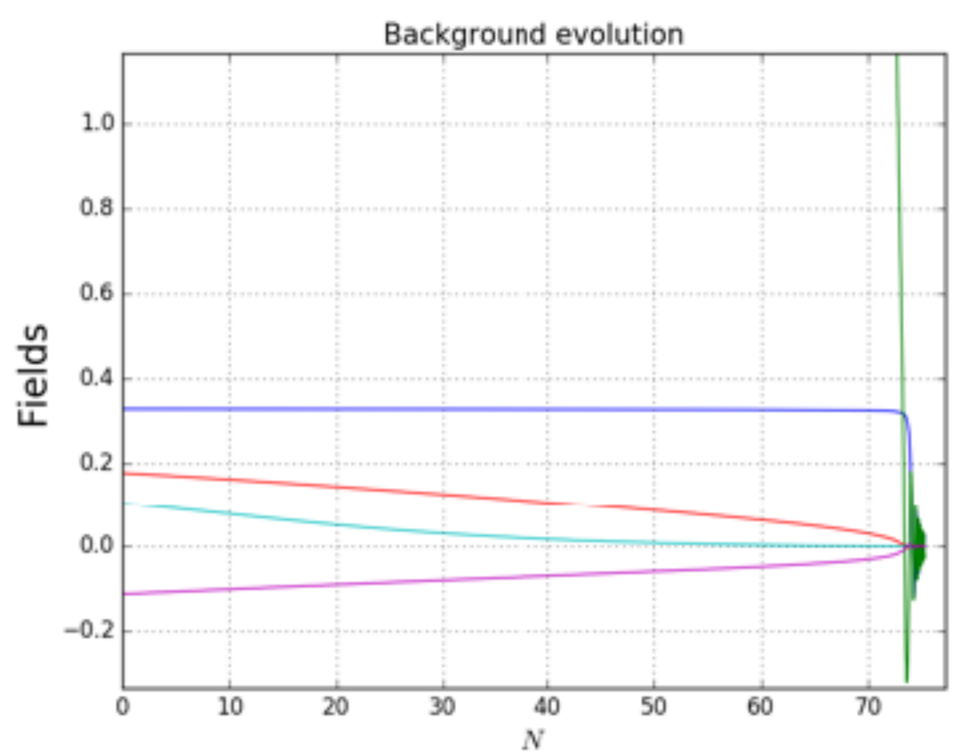


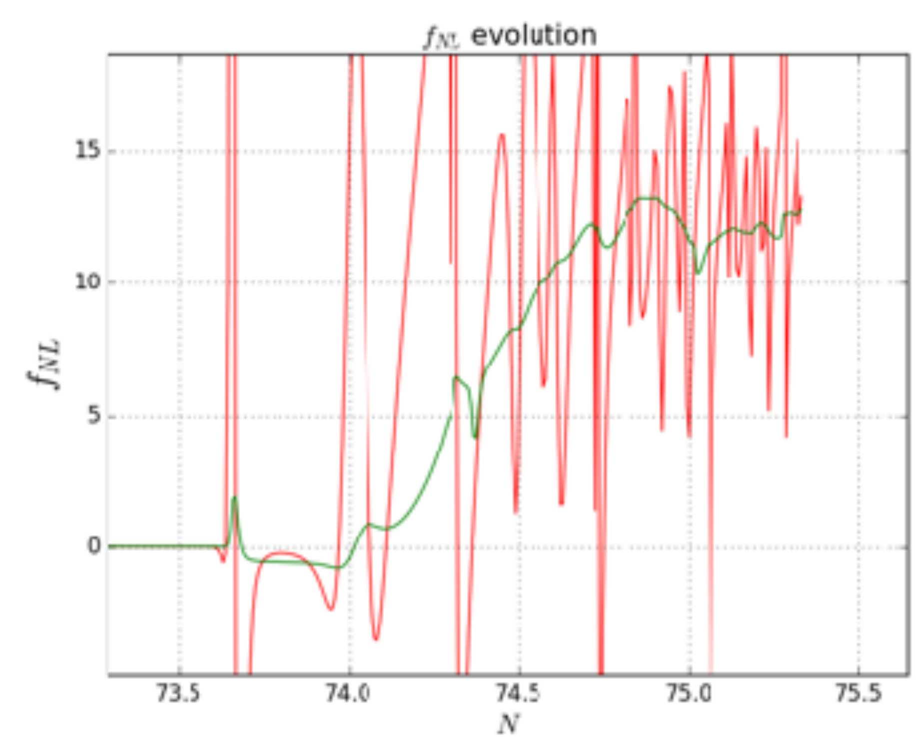


Many field models (in progress)

- Dyson Brownian motion Marsh, Dias, Frazer and Marsh (2017), Gaussian Random potentials e.g. Marsh, McAllister, Pajer, Wrase (2013); Dias, Frazer and Marsh (2017), Bjorkmo and Marsh (2017)
- E.G. N-axion (c.f. Kim, Liddle, Seery (2009), uncoupled case leads to observable non-Gaussianity)

$$\mathcal{L} = \frac{1}{2} G_{IJ} \partial \phi^I \partial \phi^J + \sum_K \Lambda_K^4 \left(1 - \cos\left(\phi^K\right)\right)$$





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