A Real-Time Semiclassical Picture of Vacuum Decay

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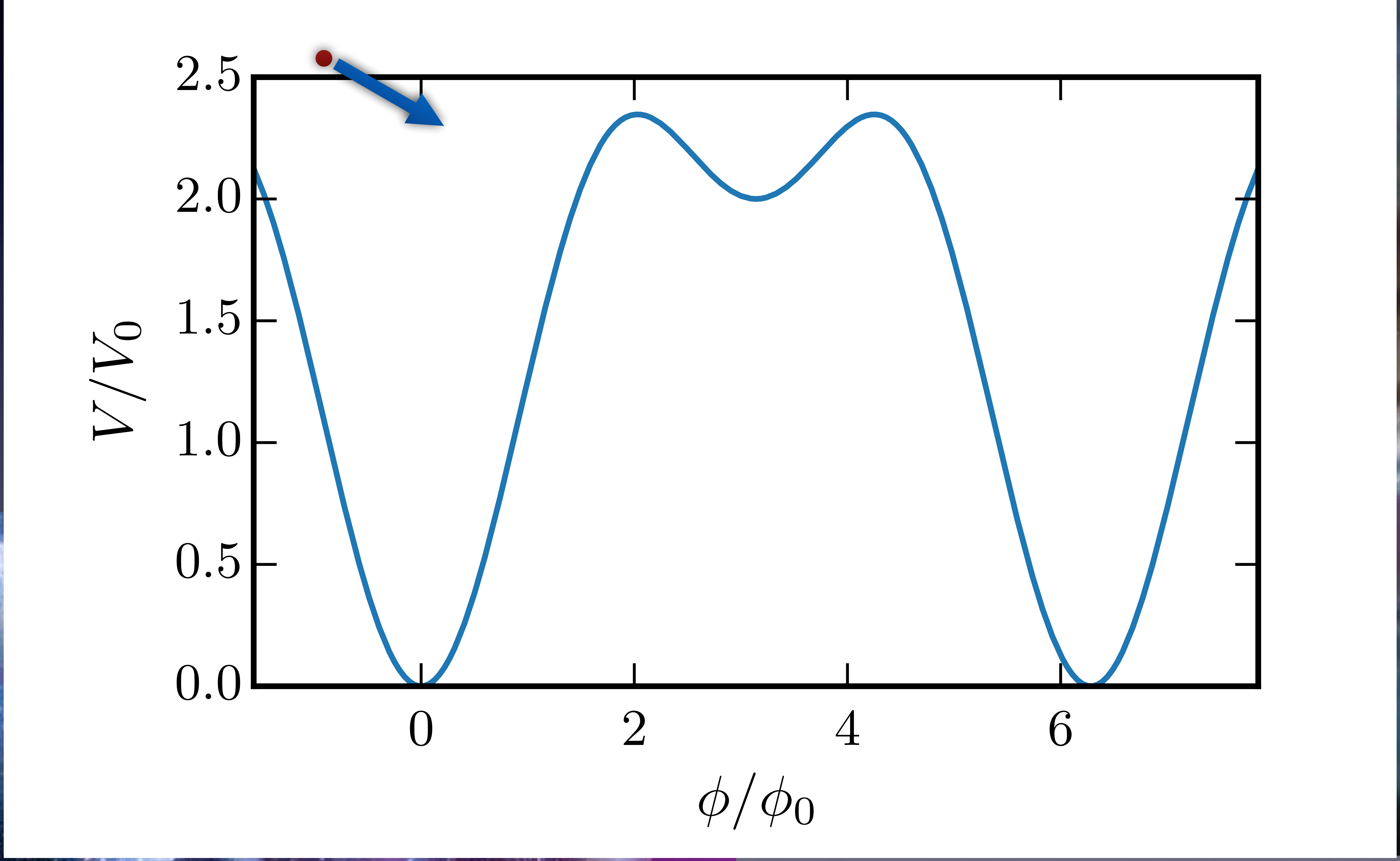
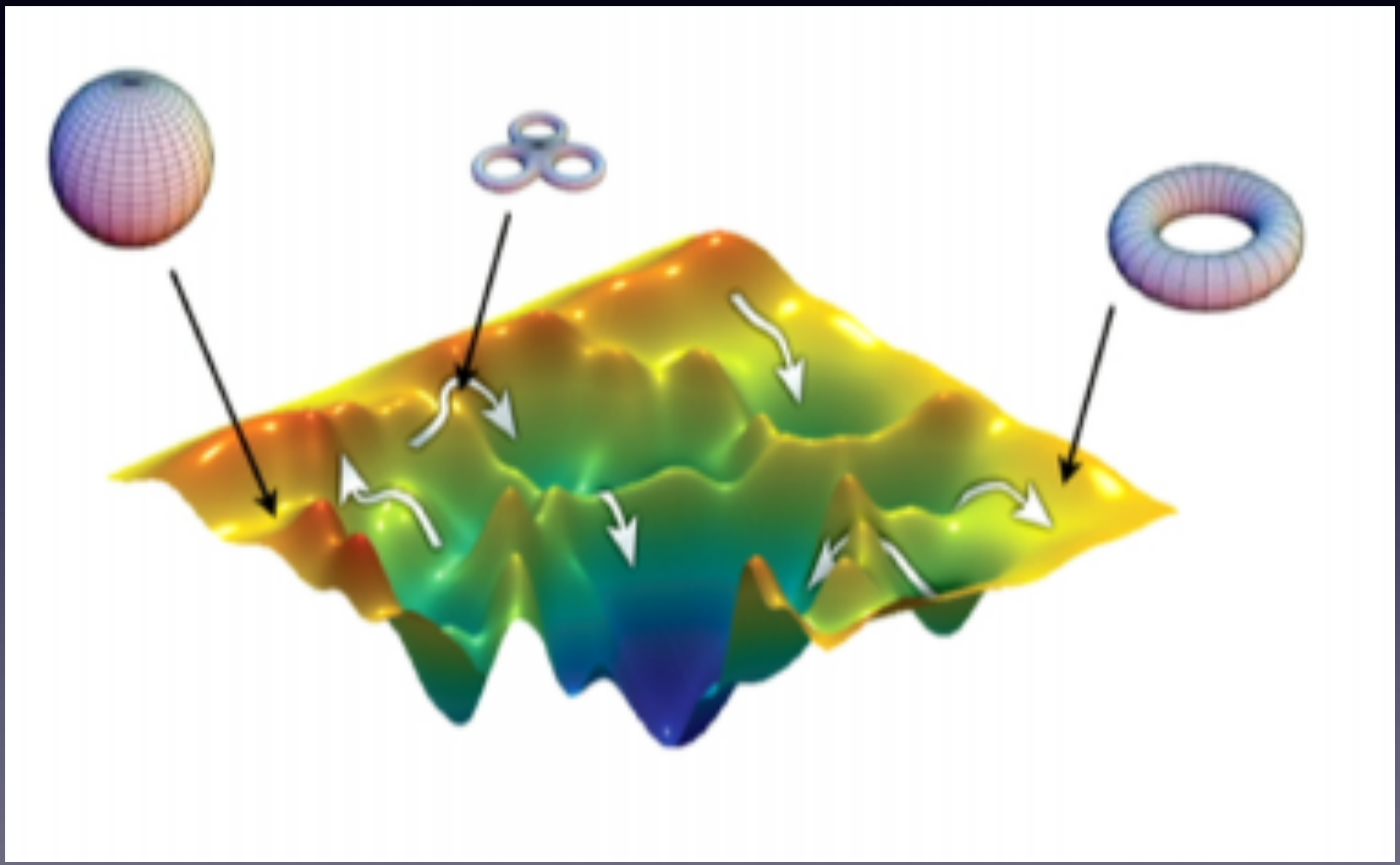
w/ Matt Johnson, Hiranya Peiris, Andrew Pontzen, and Silke Weinfurtner
1712.02356 (accepted to JHEP), 1806.06069, and in progress

First Order Phase Transitions

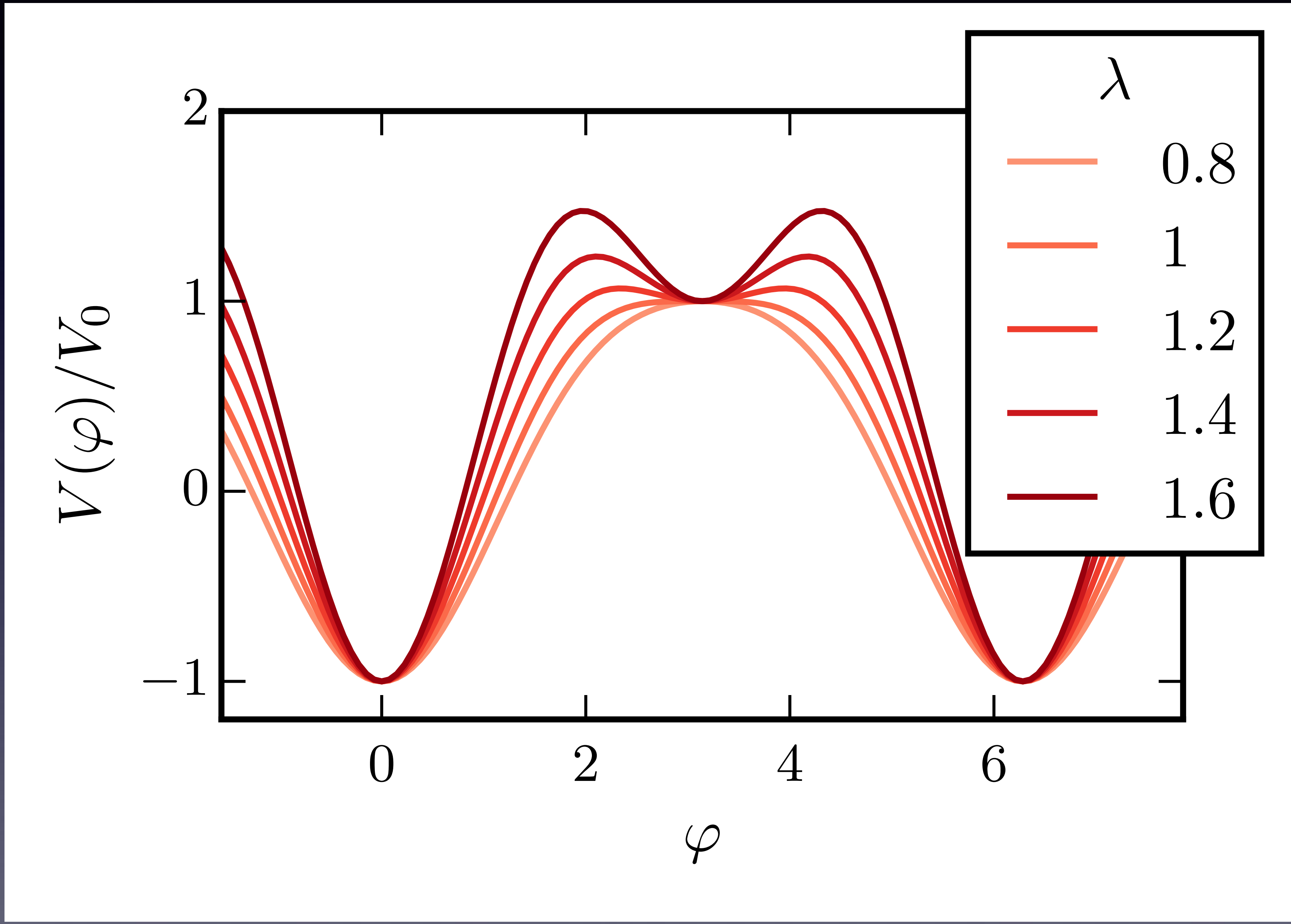


First Order Phase Transitions




INSTA

Model

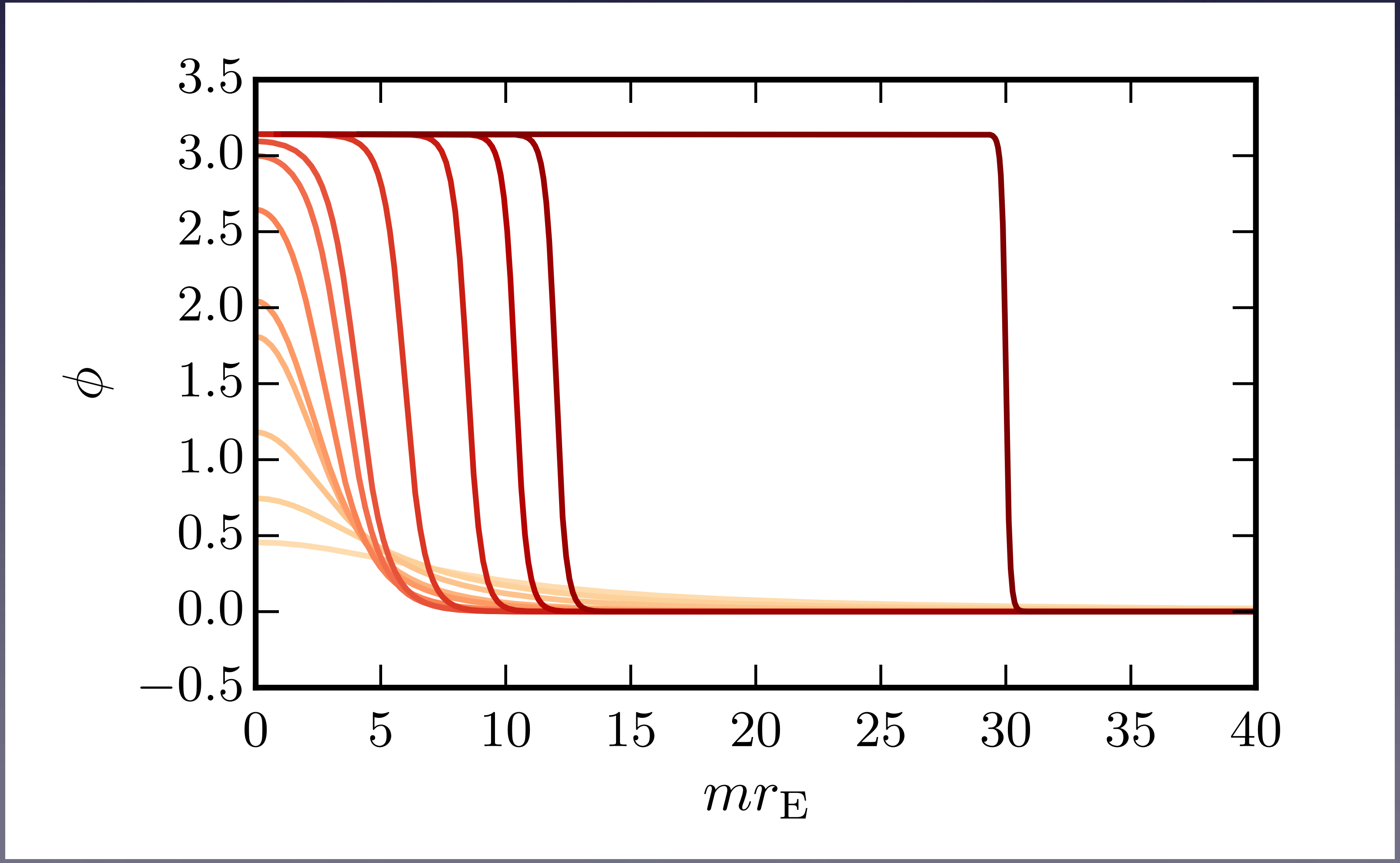


$$V(\phi) = V_0 \left(-\cos\left(\frac{\phi}{\phi_0}\right) + \frac{\lambda^2}{2}\sin^2\left(\frac{\phi}{\phi_0}\right) + 1 \right)$$

Standard Description

$$r_{\mathrm{E}}^2 = \tau^2 + \mathbf{r}^2 \qquad \tau = it$$

$$\frac{\partial^2 \phi_{\mathrm{I}}}{\partial r_{\mathrm{E}}^2} + \frac{d}{r_{\mathrm{E}}} \frac{\partial \phi_{\mathrm{I}}}{\partial r_{\mathrm{E}}} - \frac{\partial V}{\partial \phi} = 0 \qquad \frac{\partial \phi_{\mathrm{I}}}{\partial r_{\mathrm{E}}}(0) = 0 \qquad \phi(\infty) = \phi_{\mathrm{fv}}$$



Decay Rates

$$S_{\mathrm{E}} = A_{\mathrm{d+1}} \int dr_{\mathrm{E}} r_{\mathrm{E}}^{d} \left(\frac{\phi'^2}{2} + V(\phi) \right)$$

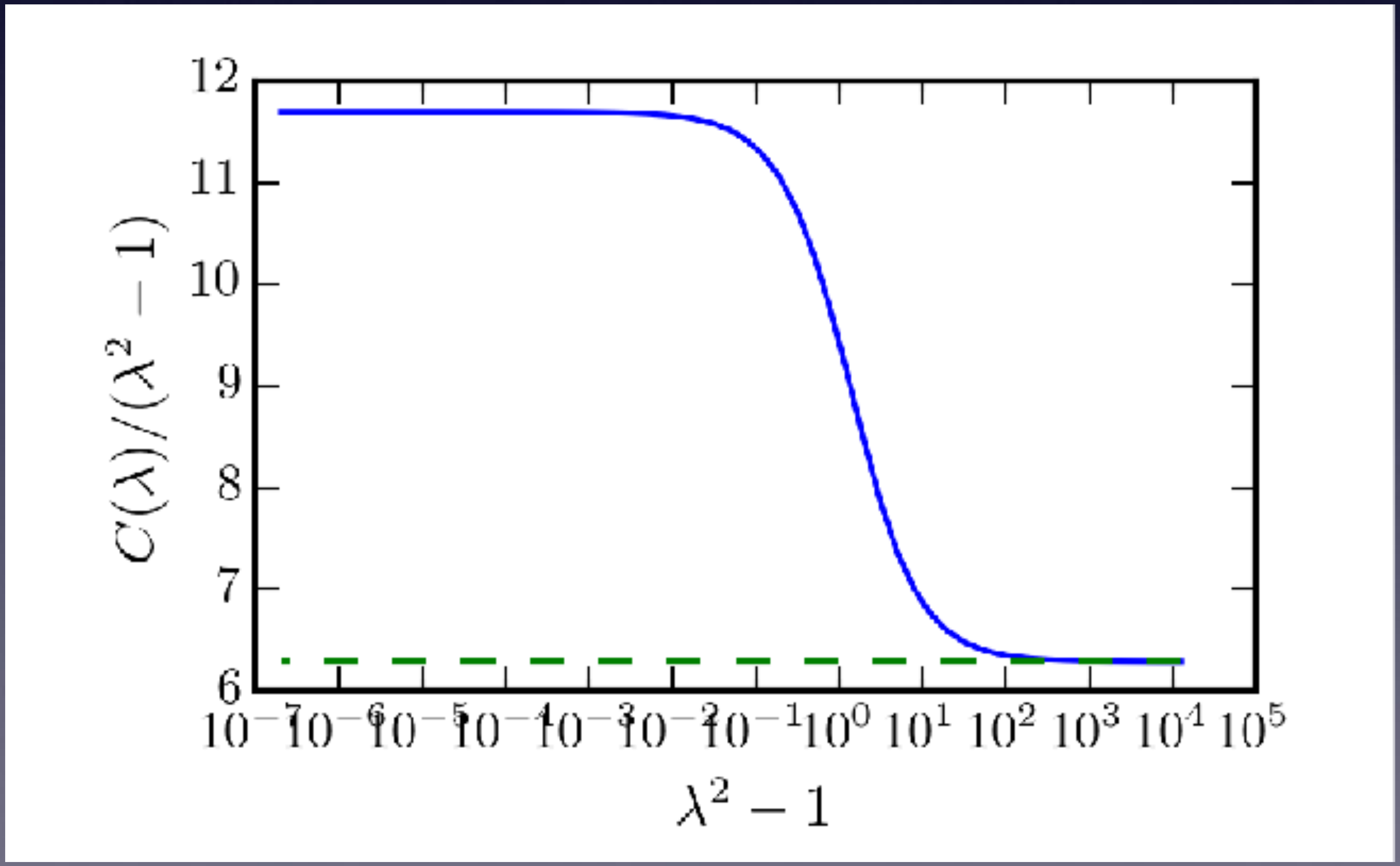
$$S_{\mathrm{I}} = S_{\mathrm{E}}[\phi_{\mathrm{B}}] - S_{\mathrm{E}}[\phi_{\mathrm{fv}}]$$

- Single negative eigenmode

$$\frac{\Gamma}{V} = \left(\frac{S_{\mathrm{I}}}{2\pi} \right)^{D/2} \sqrt{\frac{\det \delta^2 S_{\mathrm{E}}[\phi_{\mathrm{fv}}]}{\det' \delta^2 S_{\mathrm{E}}[\phi_{\mathrm{B}}]}} e^{-S_{\mathrm{I}}} \left(1 + \mathcal{O}(\hbar)\right)$$

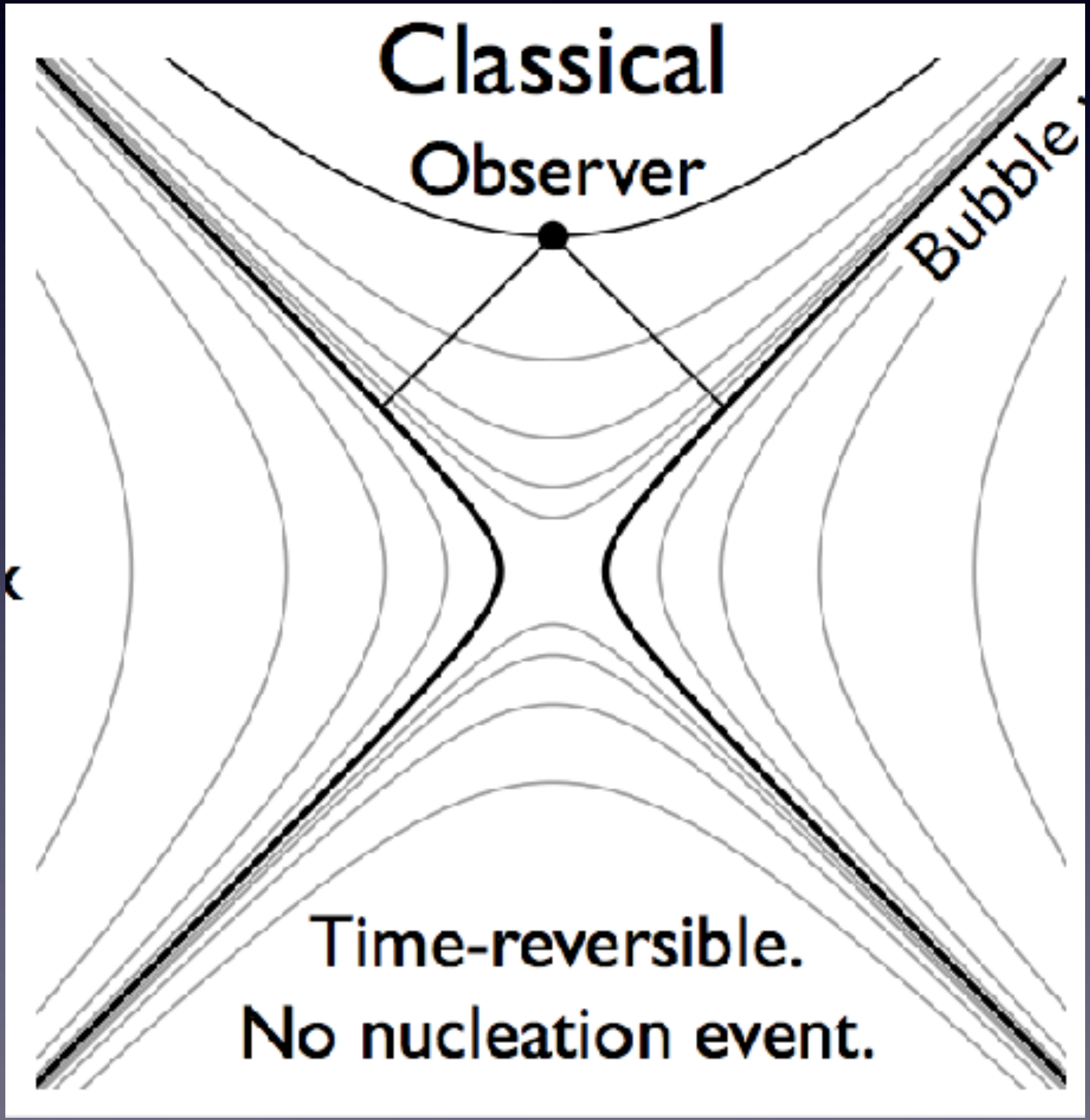
Nucleation Rates

$$\frac{\Gamma}{V} \approx g(\lambda)\left[m_{\mathrm{eff}}^2\right]^{\frac{D}{2}}\left(\frac{S_{\mathrm{I}}}{2\pi}\right)^{\frac{D}{2}} e^{-S_{\mathrm{I}}}$$



Real-Time Interpretation

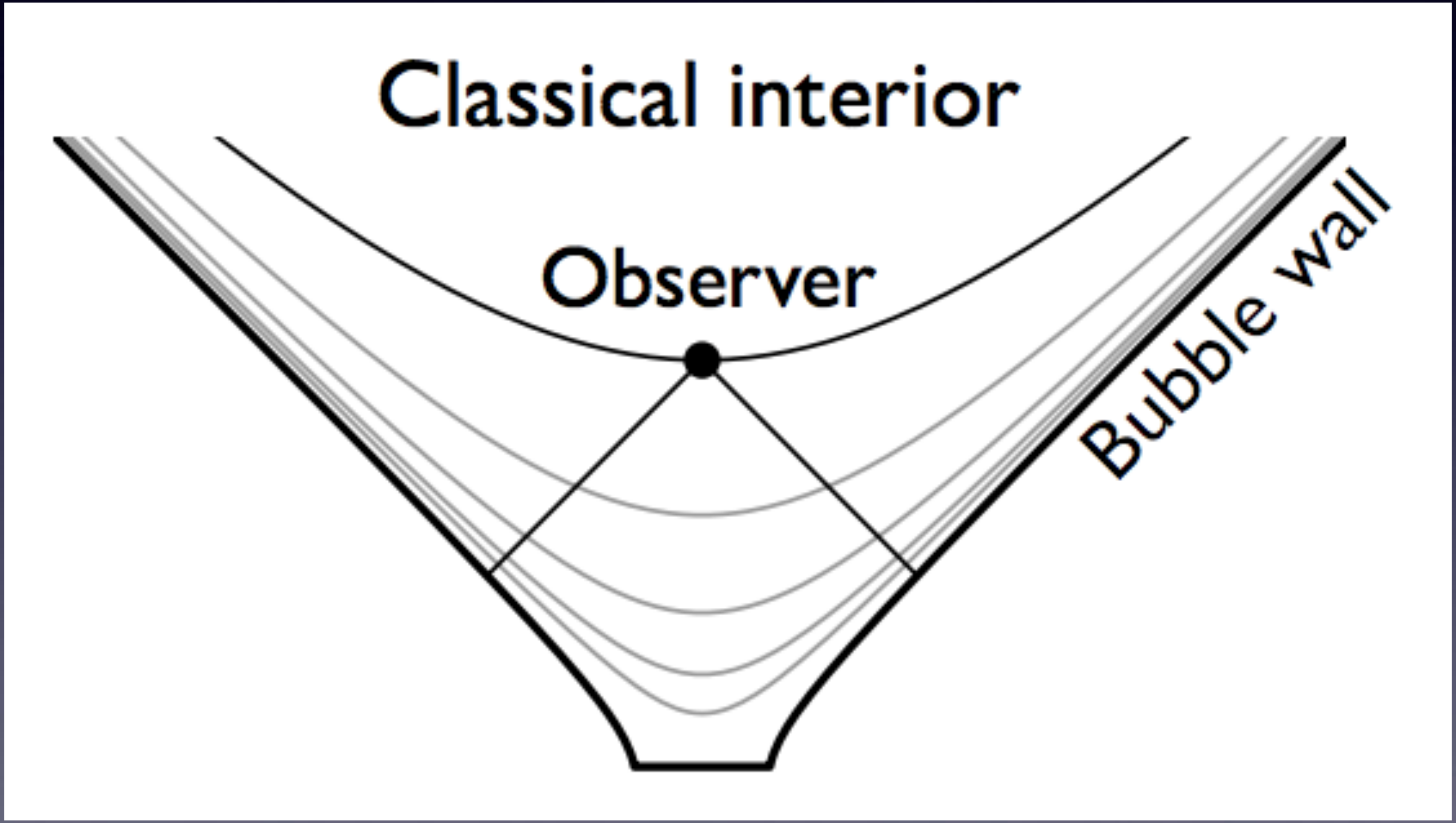
$$\phi(x,t) = \phi_{\mathrm{I}}(\sqrt{x^2 - t^2})$$



[Figure courtesy of Andrew Pontzen]

Ad-Hoc Nucleation

$$\phi(\mathbf{x}, t = 0) = \phi_{\mathrm{I}}(|\mathbf{x}|)$$



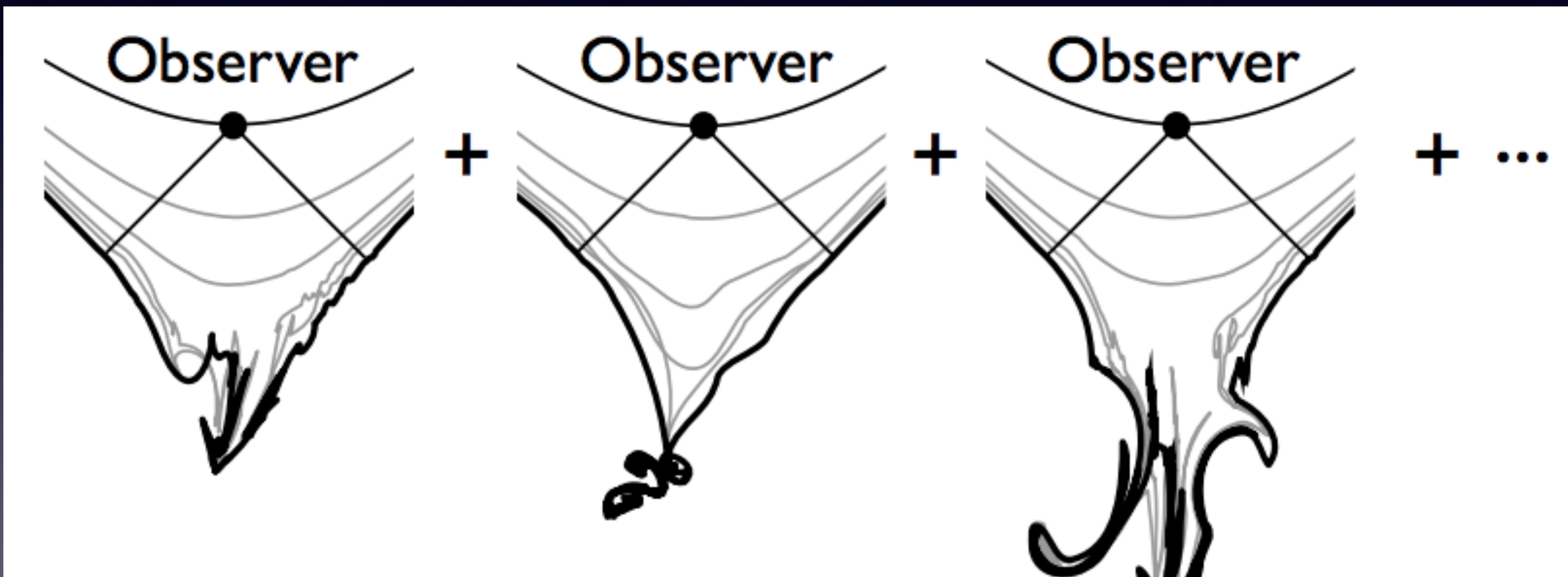
No real-time classical description

[Figure courtesy of Andrew Pontzen]

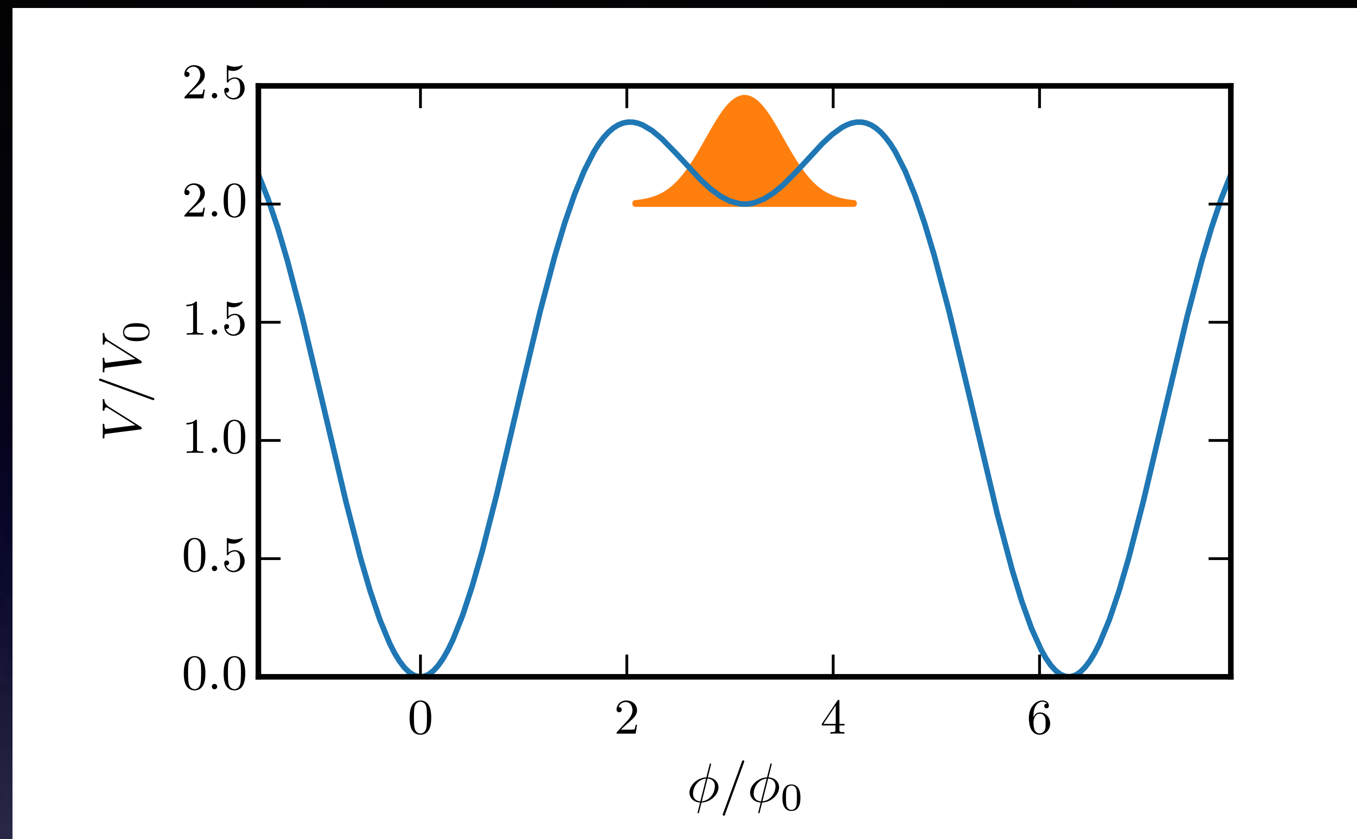
Some Questions

- QFT exponentially complex. Approximations needed.
- Time-dependent description of nucleation
    - Bubble precursor? Init. cond. at nucleation
- Bubble-bubble correlations
- Fast decay/large fluctuation limit?
- Time evolving background/potential
- Nonvacuum state

Full Evolution?



[Figure courtesy of Andrew Pontzen]



$$\boldsymbol{\phi} = \phi_{\mathrm{fv}} + \delta\hat{\phi}(\mathbf{x}, t)$$

$$\mathbf{\Pi} = \mathbf{0} + \delta\hat{\Pi}(\mathbf{x}, t)$$

$$\langle \delta\tilde{\phi}_k \delta\tilde{\phi}_p^* \rangle = \frac{1}{2\omega_k}\delta(k-p) \qquad \langle \delta\tilde{\Pi}_k \delta\tilde{\Pi}_p^* \rangle = \frac{\omega_k}{2}\delta(k-p)$$

Quantum Commutators

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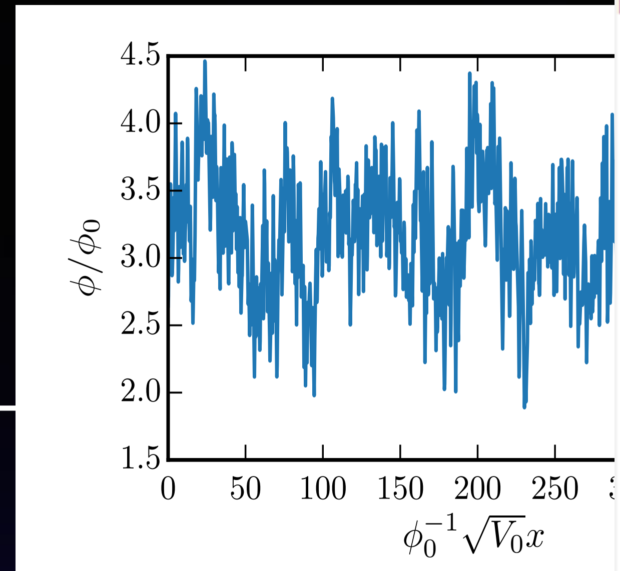
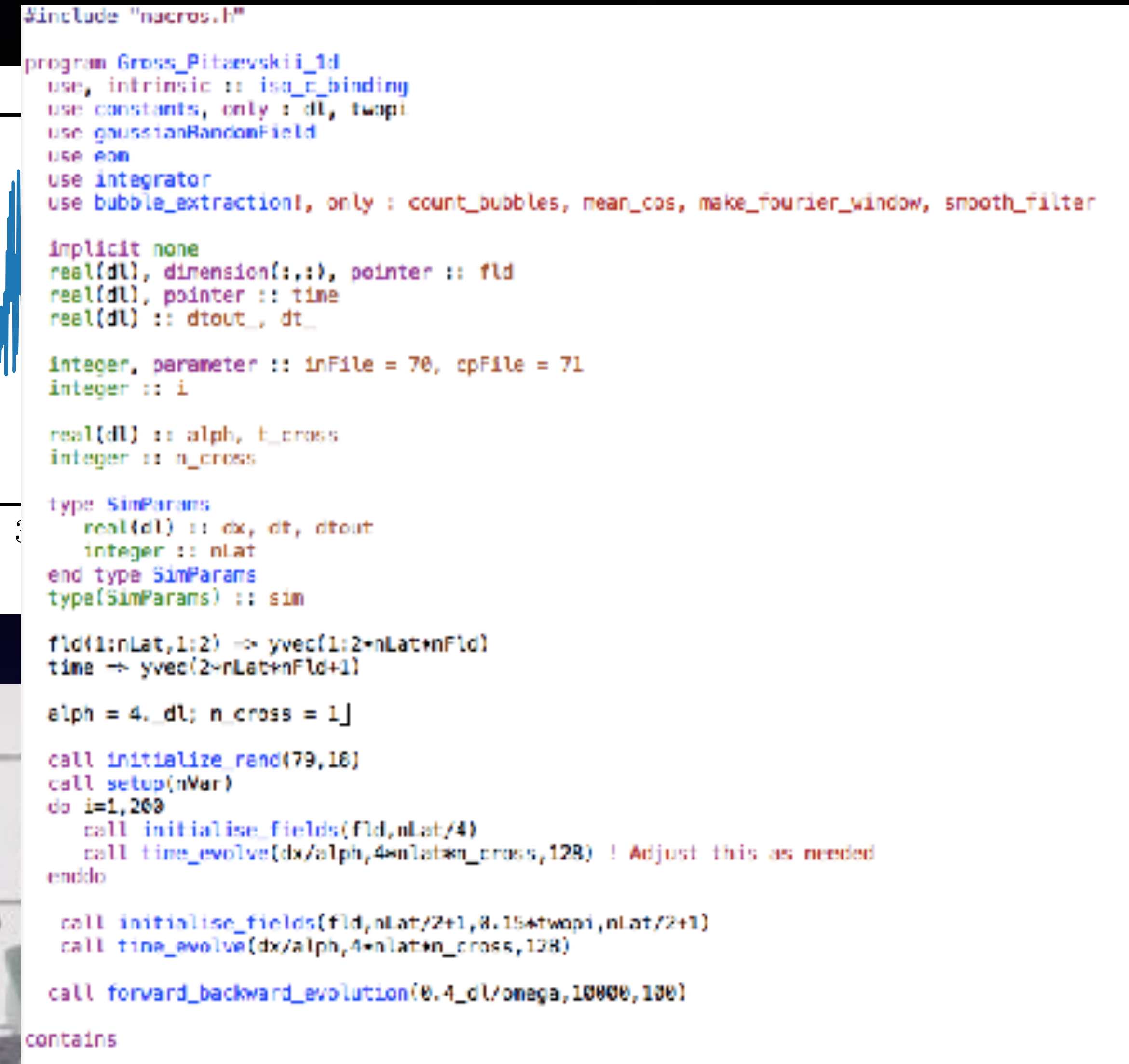
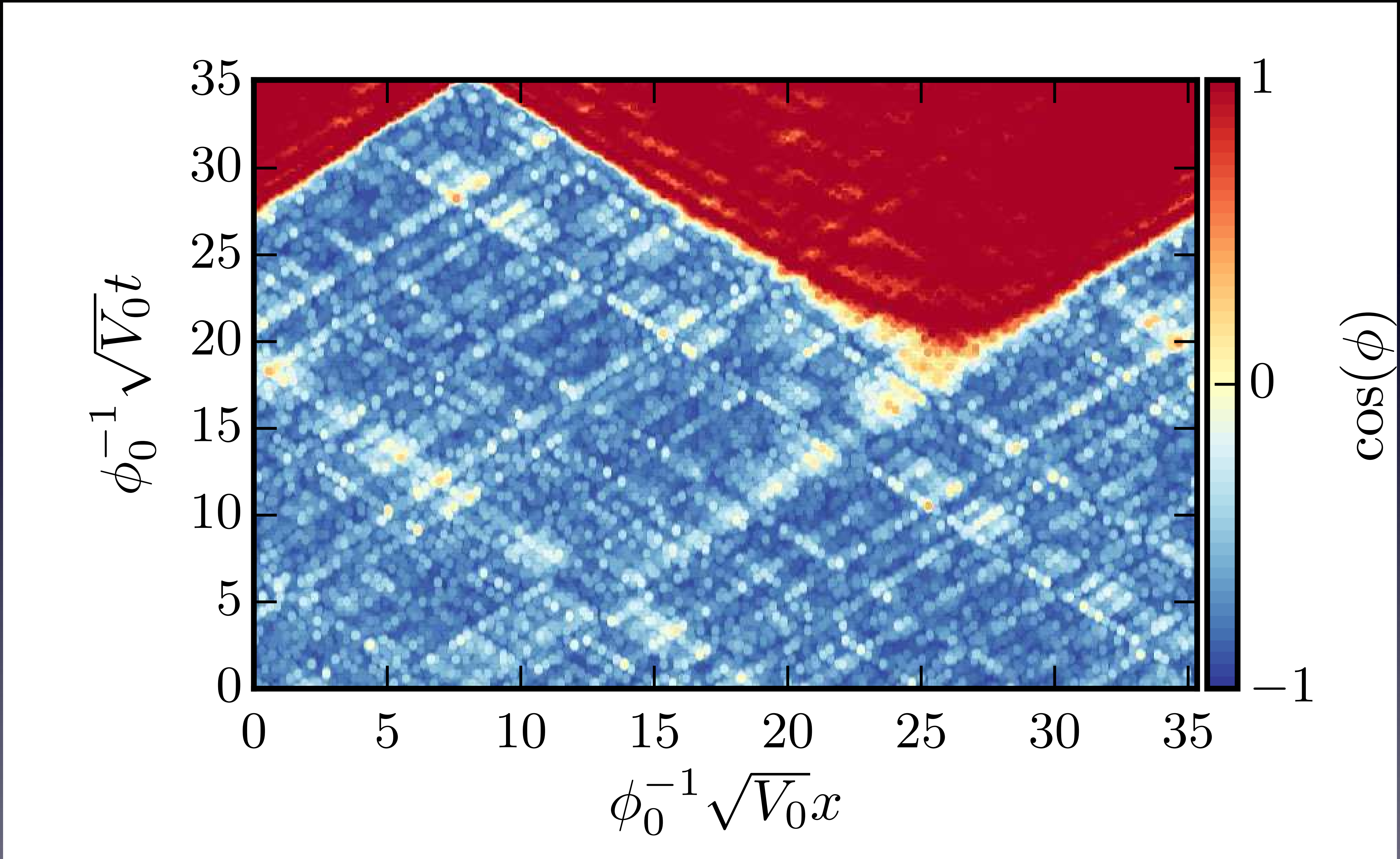


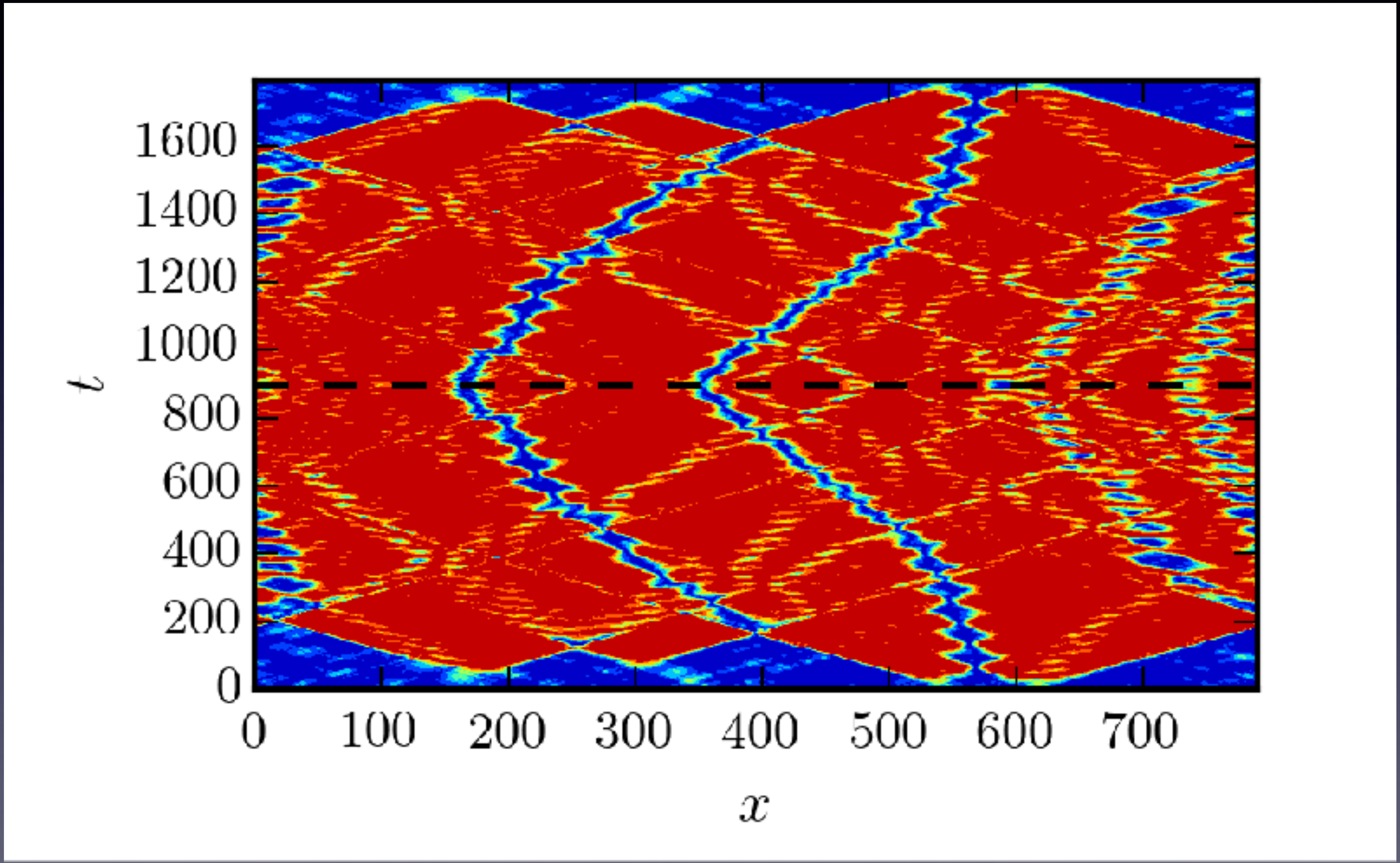


```
#include "macros.h"

program Gross_Pitaevskii_1d
  use, intrinsic :: iso_c_binding
  use constants, only : dl, twopi
  use gaussianRandomField
  use eom
  use integrator
  use bubble_extractionI, only : count_bubbles, mean_cos, make_fourier_window, smooth_filter

  implicit none
  real(dl), dimension(:,:), pointer :: fld
  real(dl), pointer :: time
  real(dl) :: dtout_, dt_

  integer, parameter :: inFile = 70, cpFile = 71
  integer :: i

  real(dl) :: alph, t_cross
  integer :: n_cross

  type SimParams
     real(dl) :: dx, dt, dtout
     integer :: nLat
  end type SimParams
  type(SimParams) :: sim

  fld(1:nLat,1:2) => yvec(1:2*nLat*nFld)
  time => yvec(2*nLat*nFld+1)

  alph = 4._dl; n_cross = 1

  call initialize_rand(79,18)
  call setup(nVar)
  do i=1,200
     call initialise_fields(fld,nLat/4)
     call time_evolve(dx/alph,4*nlat*n_cross,128) ! Adjust this as needed
  enddo

   call initialise_fields(fld,nLat/2+1,0.15*twopi,nLat/2+1)
   call time_evolve(dx/alph,4*nlat*n_cross,128)

  call forward_backward_evolution(0.4_dl/omega,10000,100)

contains
```

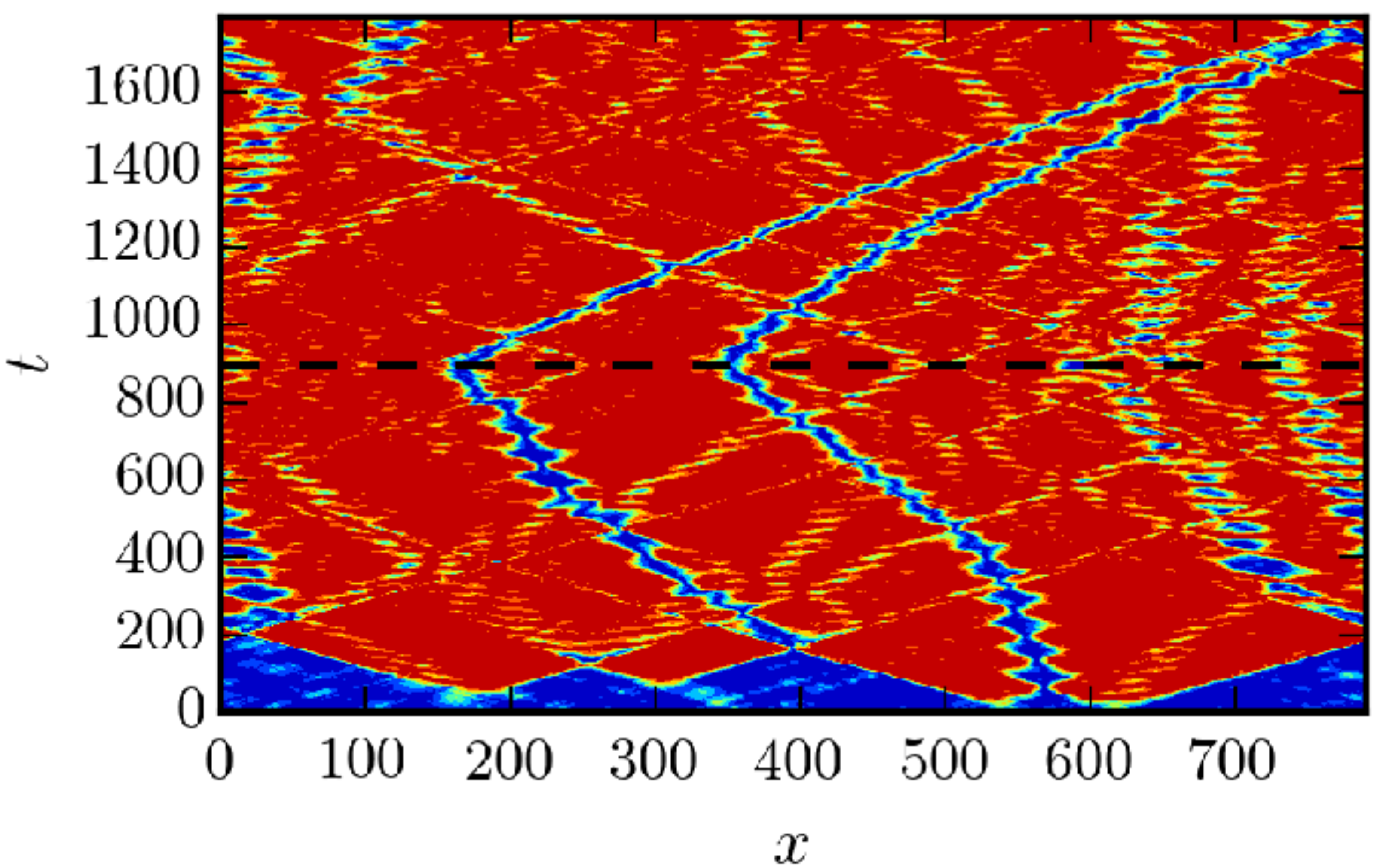


Classically-Allowed Vacuum Decay

Numerical Reversibility



Destroyed by Addition of Noise



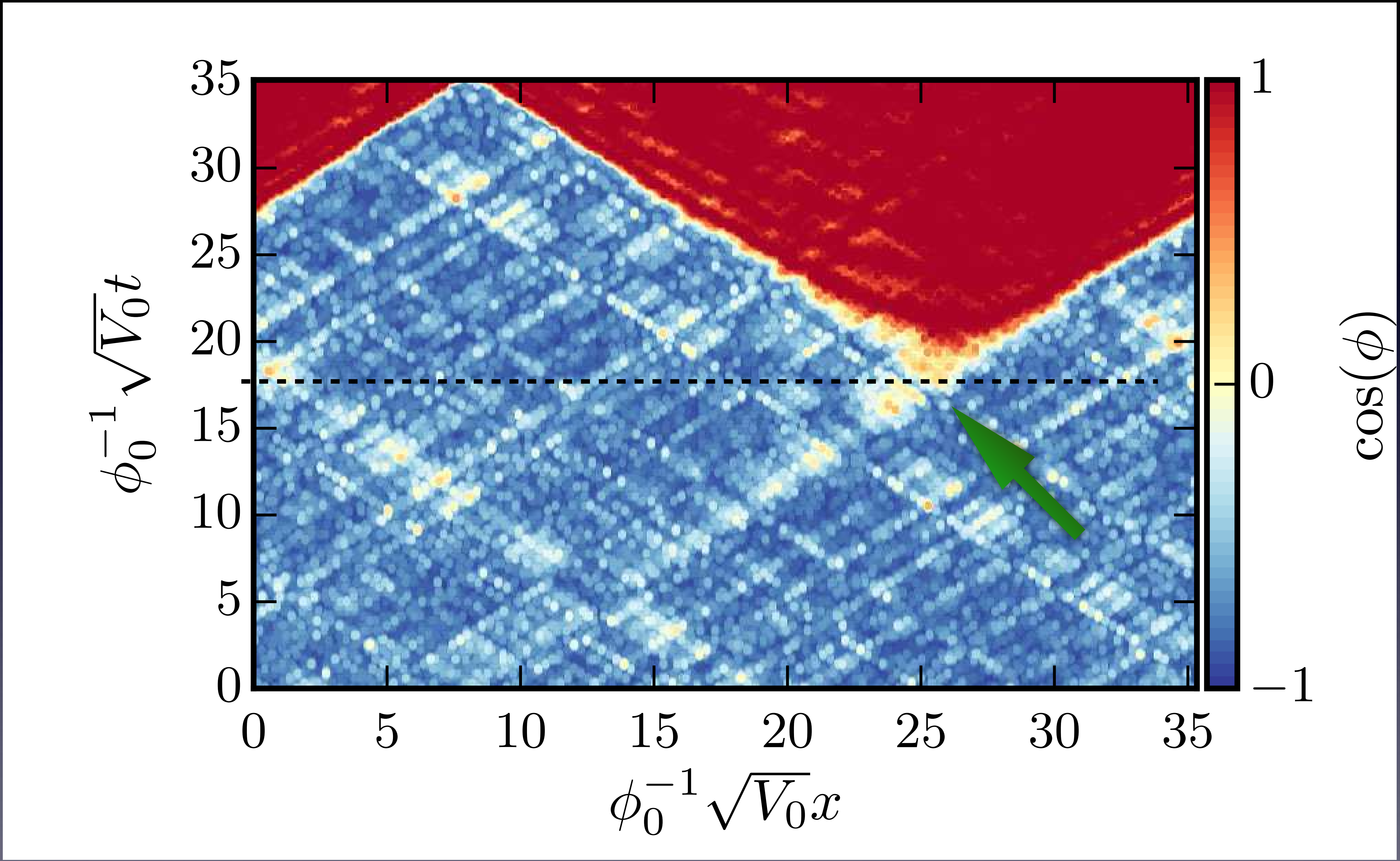
Decay Rates?

Prediction

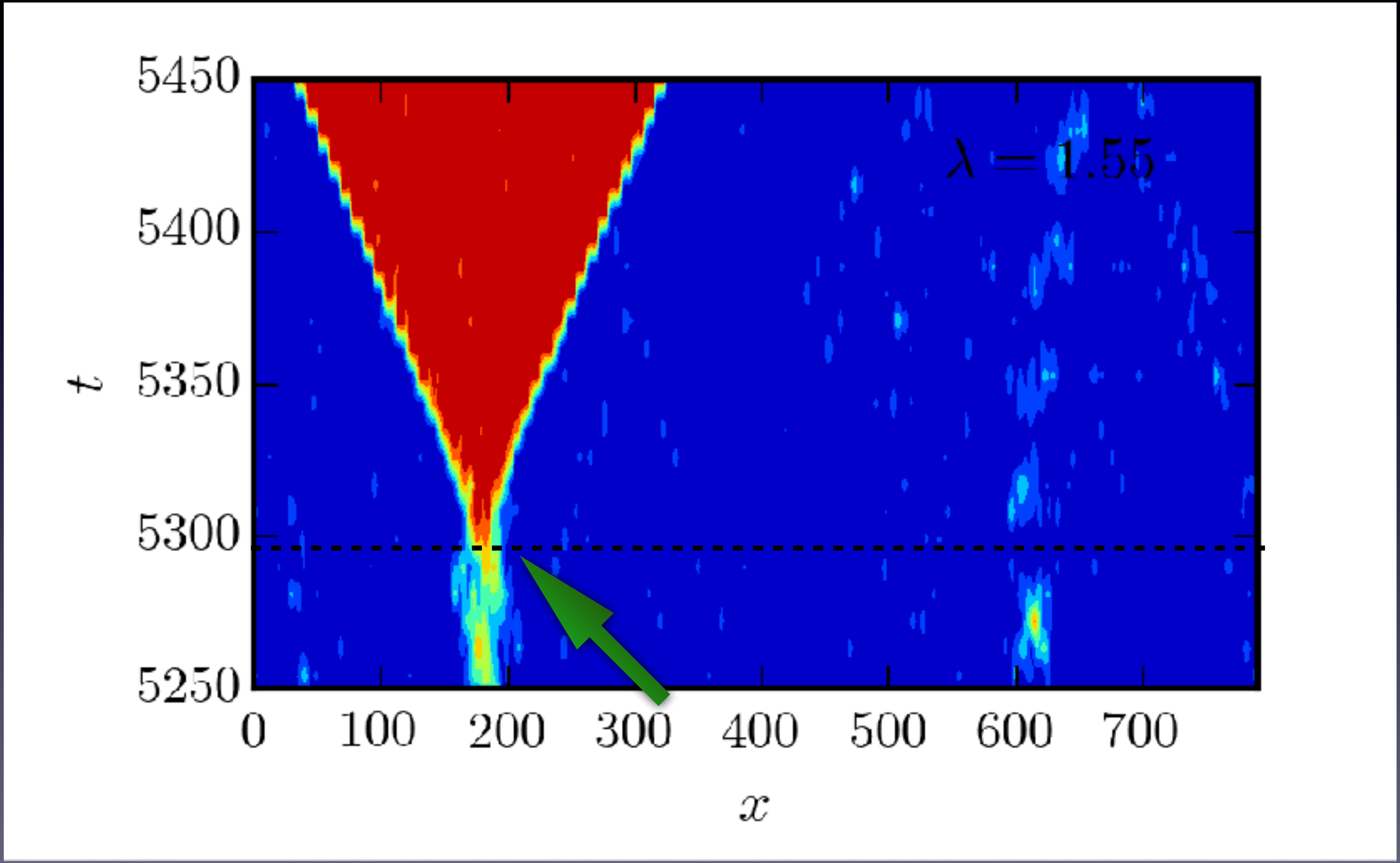
$$\frac{\Gamma_{\mathrm{I}}^{(1+1)}}{L} \approx g(\lambda, V_0, \phi_0) m_{\mathrm{eff}}^2 \phi_0^2 C(\lambda) e^{-2\pi\phi_0^2 C(\lambda)}$$

$\mathcal{O}(1)$ $\quad\sim V''(\phi_{\mathrm{fv}})$ Instanton

$$V(\phi) = V_0 \left(-\cos\left(\frac{\phi}{\phi_0}\right) + \frac{\lambda^2}{2}\sin^2\left(\frac{\phi}{\phi_0}\right) + 1 \right)$$

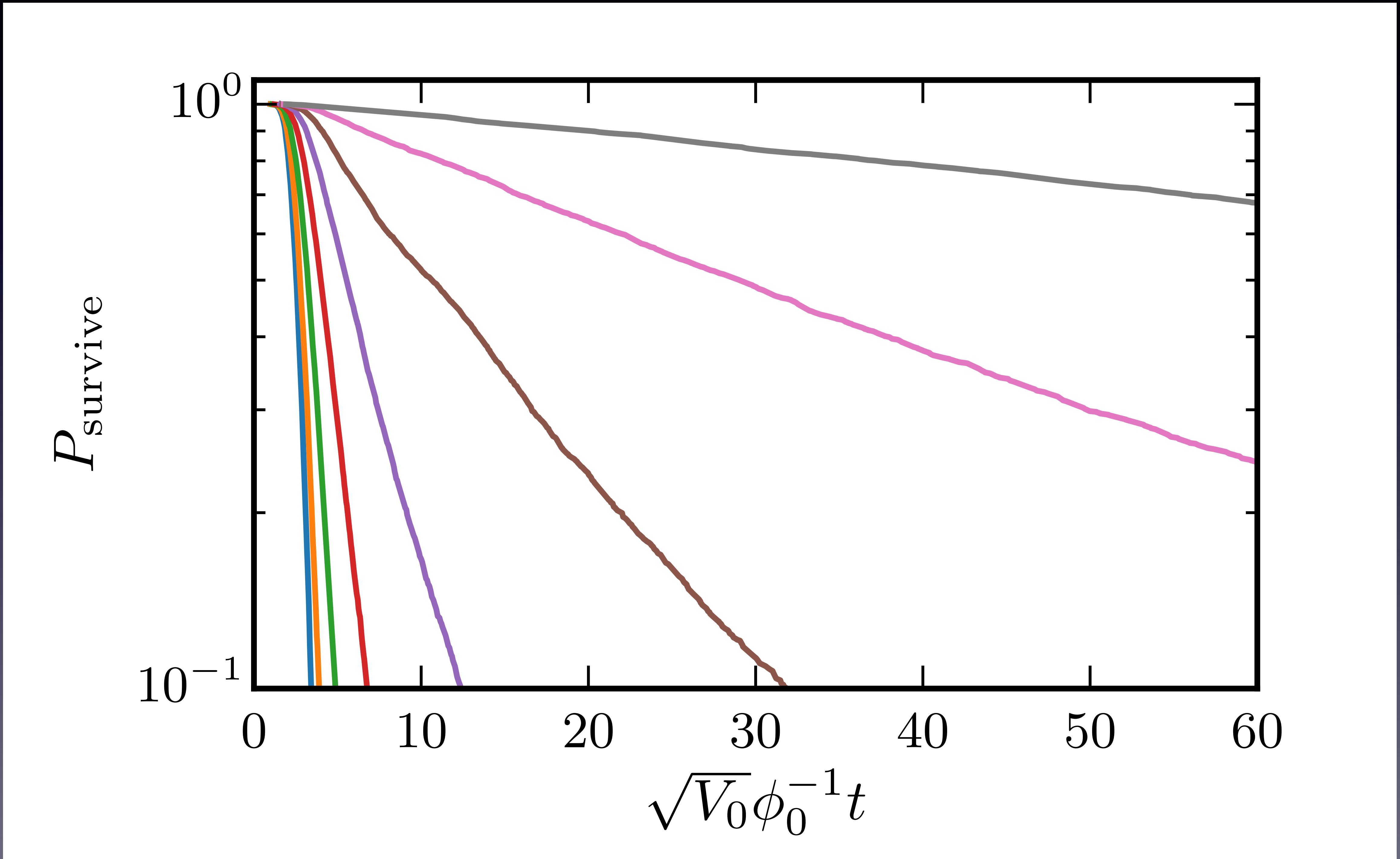


$t^{(i)}_{\mathrm{decay}}$

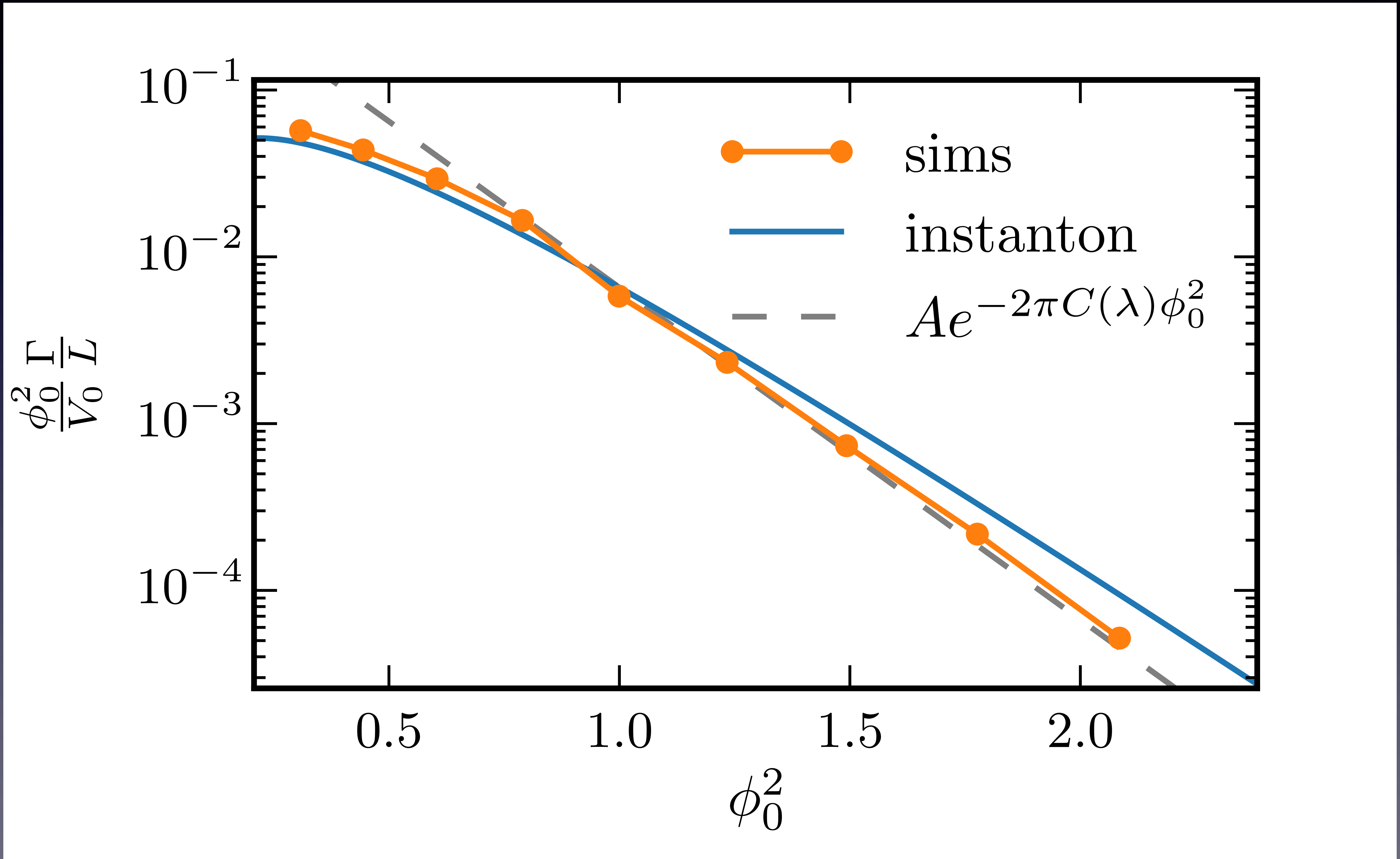


$$t^{(i)}_{\mathrm{decay}}$$

$$P_{\text{survive}} \sim e^{-\Gamma(t-t_0)}$$



$$\frac{\Gamma_{\mathrm{I}}^{(1+1)}}{L} = g(\lambda, \phi_0) m_{\mathrm{eff}}^2 \phi_0^2 e^{-2\pi\phi_0^2 C(\lambda)}$$



[JB, Johnson, Peiris, Pontzen, Weinfurtner, 1806.06069]

Conclusions

Physical Process: False Vacuum Decay

- False Vacuum decay **can** occur via classical time-evolution (quantum is in initial state)
- Decay rates ~ Euclidean Calculations
  - Alternative description of instanton (no tunnelling)
  - Complimentary to instanton (Euclidean rate wrong)
  - (Magic cancellation of amplitudes)

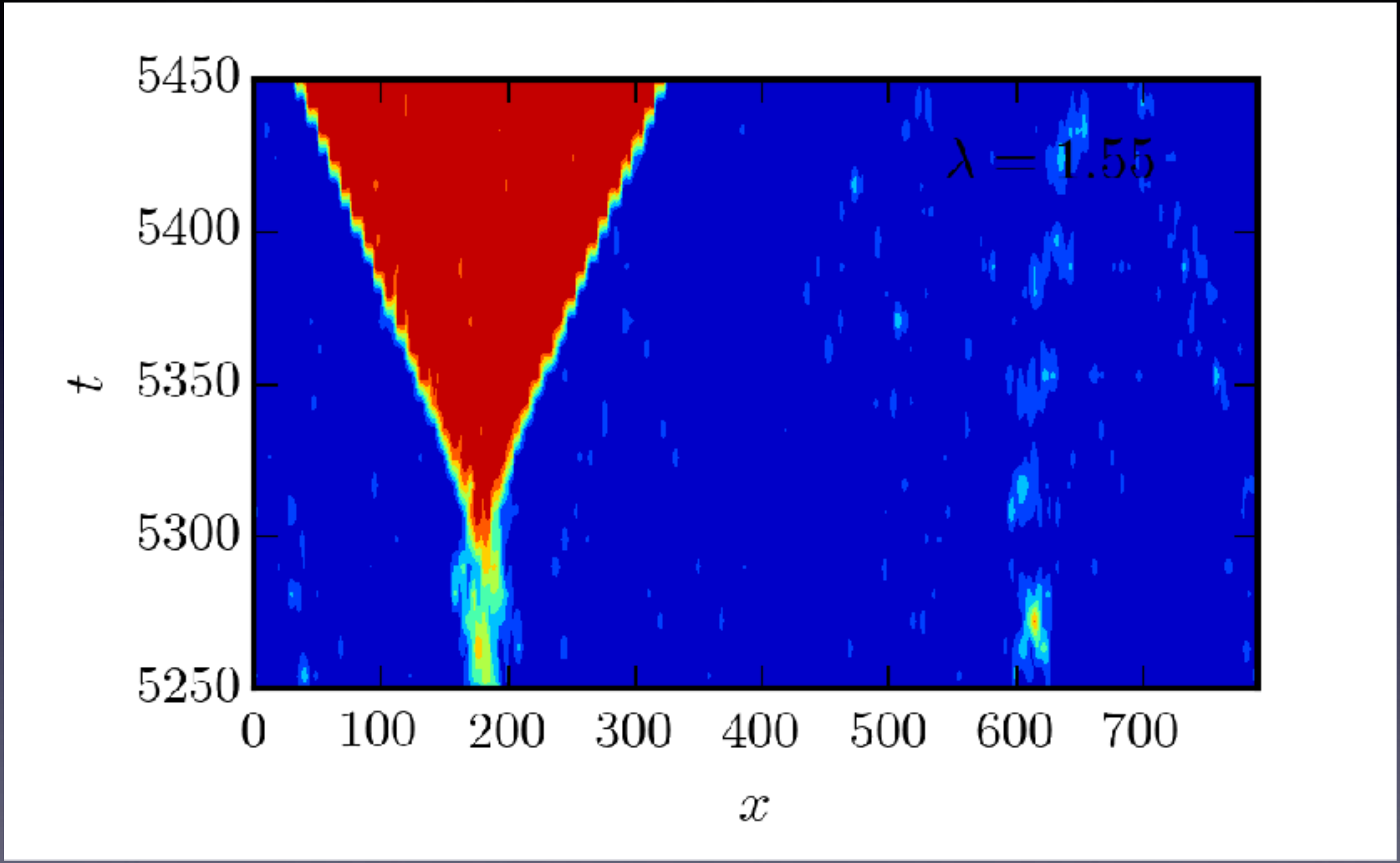
Conclusions

Physical Process: False Vacuum Decay

- False Vacuum decay **can** occur via classical time-evolution (quantum is in initial state)
- Decay rates ~ Euclidean Calculations
  - **Alternative description of instanton (no tunnelling)**
  - Complimentary to instanton (Euclidean rate wrong)
  - (Magic cancellation of amplitudes)

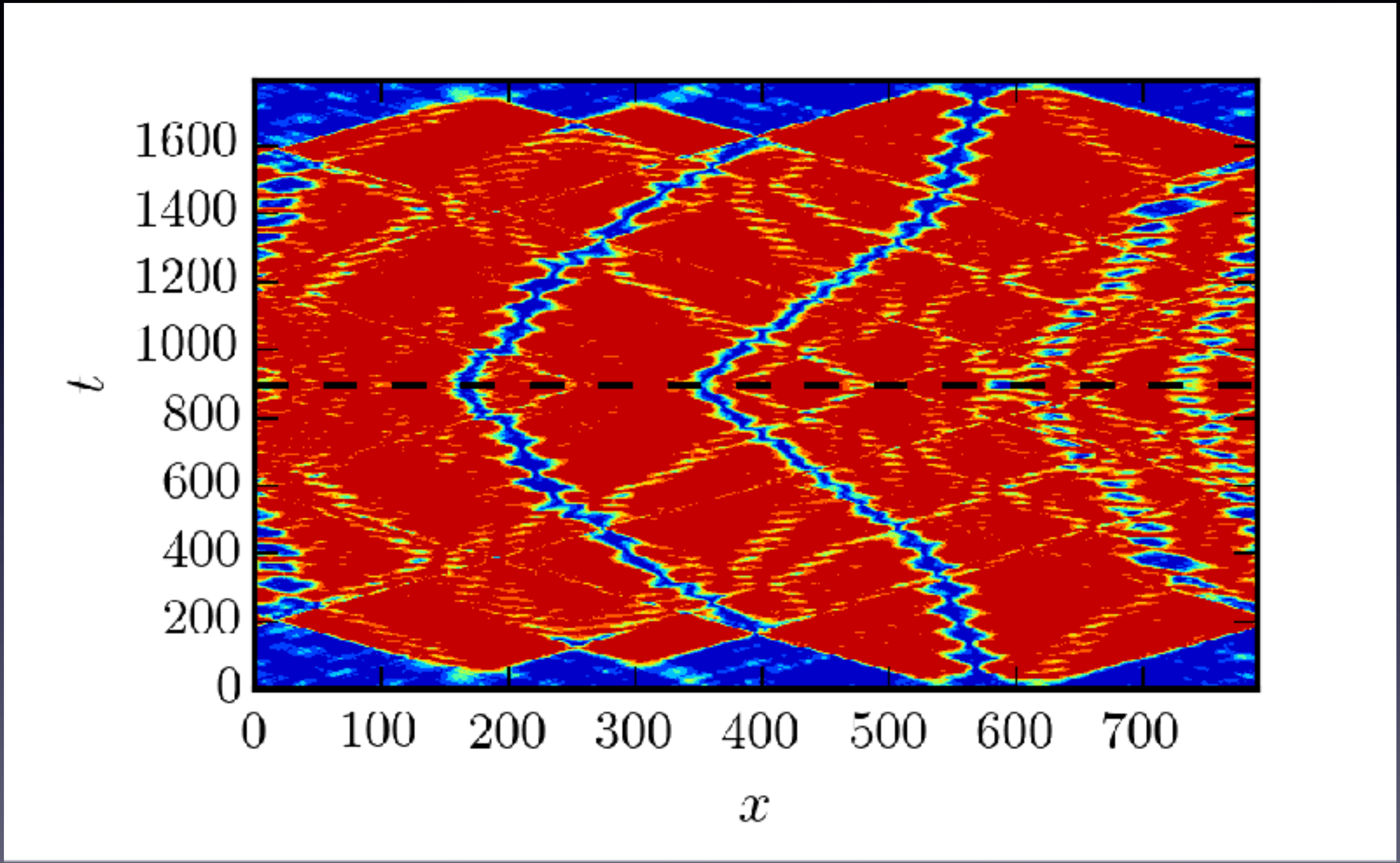
Current/Future Work

- Real-time $\longleftrightarrow$ Instanton
    - Renormalisation, Fluc. Determinant, Wigner
- Mean bubble profile = instanton?
- Bubble-bubble correlations?
- Time-dependent background or potential
- Non-vacuum initial states (pure or mixed)



5450
5400
5350
5300
5250
t
$\lambda = 1.55$
0
100
200
300
400
500
600
700
x

Numerical Reversibility



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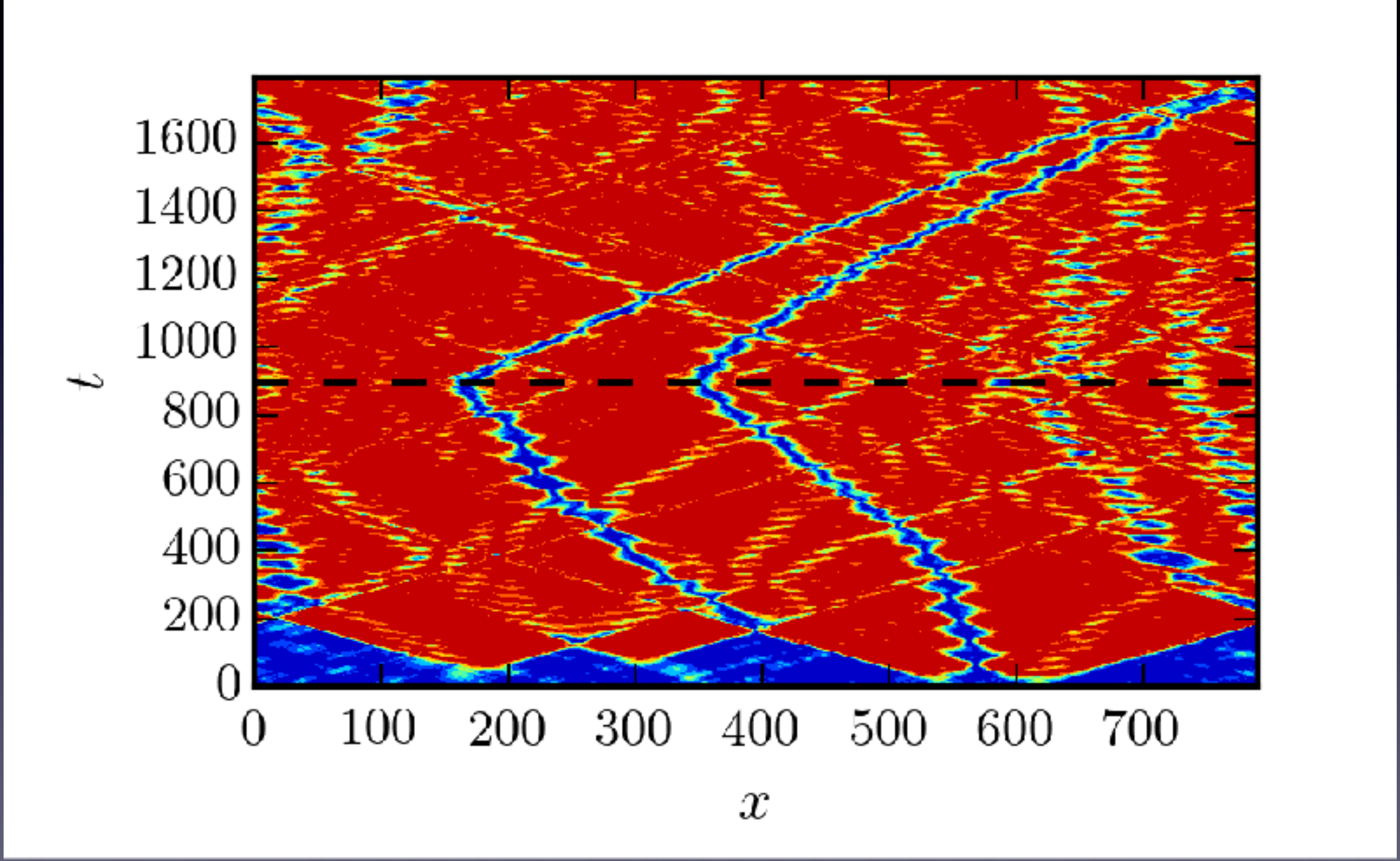

Defining Decays