Cosmological phase transitions with hidden scale invariance

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S. Arunasalam, A. Kobakhidze, CL, S. Liang, A. Zhou, PLB 776 (2018) 48-53
S. Arunasalam, A. Kobakhidze, CL, in preparation

22nd Conference on Particle Physics and Cosmology - Daejeon
August 28, 2018

Motivation and overview

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This talk: a study of the electroweak and chiral phase transitions in an extension of the Standard Model with classical scale invariance.

Standard Model with hidden scale invariance

- Scale invariant models are attractive to address the hierarchy problem

  e.g.: [K. Meissner, H. Nicolai, PLB 648 (2007) 312] [R. Foot et al., PRD 77 (2008) 035006] [S. Iso et al., PLB 676 (2009) 81]

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- Focus on low-energy effective field theory:
  - Standard Model Higgs potential at UV scale Λ

$$V(\phi^\dagger\phi) = V_0(\Lambda) + \lambda(\Lambda)\left[\phi^\dagger\phi - v_{ew}^2(\Lambda)\right]^2 + \ldots$$

  - spontaneously broken scale invariance manifests through dilaton field χ

$$\Lambda \to \Lambda\frac{\chi}{f_\chi} \equiv \alpha\chi$$

$$v_{ew}^2(\Lambda) \to \frac{v_{ew}^2(\alpha\chi)}{f_\chi^2}\chi^2 \equiv \frac{\xi(\alpha\chi)}{2}\chi^2$$

$$V_0(\Lambda) \to \frac{V_0(\alpha\chi)}{f_\chi^4}\chi^4 \equiv \frac{\rho(\alpha\chi)}{4}\chi^4$$

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\end{aligned}$$

We get an effective scale invariant potential:

$$V(\phi^\dagger\phi,\chi) = \lambda(\alpha\chi)\left[\phi^\dagger\phi - \frac{\xi(\alpha\chi)}{2}\chi^2\right]^2 + \frac{\rho(\alpha\chi)}{4}\chi^4$$

Hierarchy and light dilaton

- Scale invariance is broken by quantum effects:

$$\lambda^{(i)}(\alpha\chi) = \lambda^{(i)}(\mu) + \beta_{\lambda^{(i)}}(\mu)\ln(\alpha\chi/\mu) + \beta'_{\lambda^{(i)}}(\mu)\ln^2(\alpha\chi/\mu) + \ldots$$

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$$\left.\frac{\partial V}{\partial \chi}\right|_{\phi=v_{ew},\chi=v_\chi} = 0, \quad \left.\frac{\partial V}{\partial \phi}\right|_{\phi=v_{ew},\chi=v_\chi} = 0, \quad V(v_{ew}, v_\chi) = 0$$

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- We obtain dimensional transmutation and hierarchy of VEVs ($\Lambda \sim v_\chi$):

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- Approximate dilaton mass: for $v_\chi \sim M_P$, $m_\chi(v_{ew}) \sim 10^{-8}$ eV

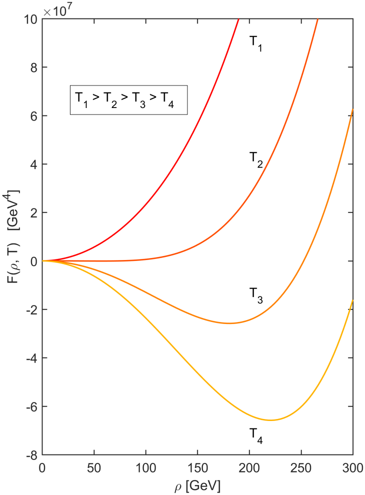
Early universe phase transitions

Hot Big Bang scenario:

- early Universe $\sim$ hot plasma (high T)
- scalar field(s) behaviour dictated by their free energy density $\mathcal{F}(\rho, T)$
- dynamics depend on the underlying particle physics model

2nd-order transition / crossover:

- smooth dynamics
- no particular signatures



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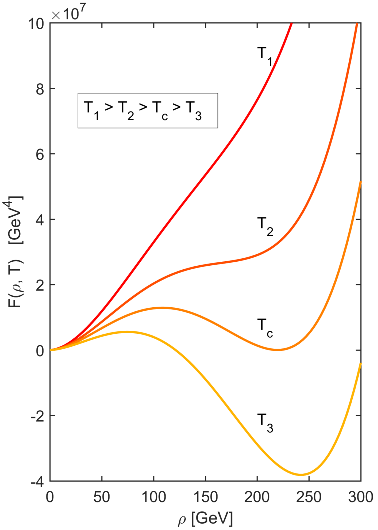

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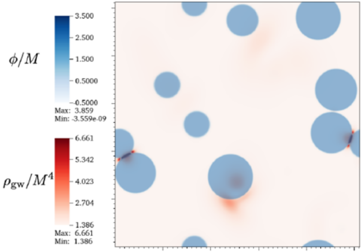
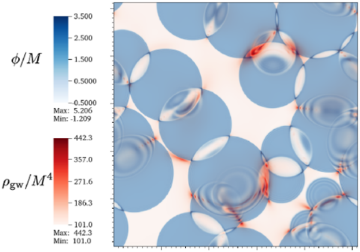


(a) $t/R_* = 0.35$



(b) $t/R_* = 0.66$

[D. Cutting, M. Hindmarsh, D. Weir, arXiv:1802.05712]

Electroweak and QCD phase transitions

In the Standard Model, both electroweak and QCD PTs are crossover

[K. Kajantie et al., Phys. Rev. Lett. 77 (1996) 2887] [Y. Aoki et al, Nature 443 (2006) 675]

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- vacua are degenerate $\Rightarrow$ no EWPT until $T \ll T_{EW}$

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QCD-induced electroweak phase transition:

- supercooling until $T \sim T_{QCD}$
- at T_{QCD}: chiral phase transition with 6 massless quarks
- quark condensates reduce the barrier in the Higgs potential $\Rightarrow$ EWPT

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See also: [W. Buchmuller, D. Wyler, PLB 249 (1990) 281] [S. Iso et al., PRL 119 (2017) 141301] [B. von Harling, G. Servant, JHEP 1801 (2018) 159]

Thermal Higgs-dilaton potential + quark condensates

- Thermal contributions to the Higgs-dilaton potential ⇒ barrier along the flat direction:

$$V_T(h,\chi(h)) \approx AT^4 + \frac{1}{48}\left[4\lambda(\Lambda) + 6y_t^2(\Lambda) + \frac{9}{2}g^2(\Lambda) + \frac{3}{2}g'^2(\Lambda)\right] h^2T^2 + \dots$$

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- Quark-antiquark condensate with N massless quarks [J. Gasser, H. Leutwyler, PLB 184 (1987) 83] :

$$\langle\bar{q}q\rangle_T = \langle\bar{q}q\rangle\left[1 - (N^2-1)\frac{T^2}{12Nf_\pi^2} - \frac{1}{2}(N^2-1)\left(\frac{T^2}{12Nf_\pi^2}\right)^2 + \dots\right]$$

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- This linear term dominates over the barrier for small enough T

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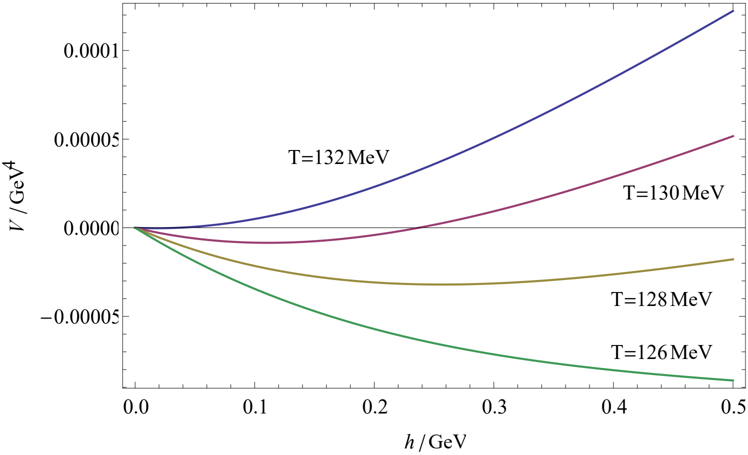

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$$f_0 = f_p \frac{a(t_c)}{a(t_0)} \approx 1.65 \cdot 10^{-8} \frac{v}{R_c\, H_c} \frac{T_c}{100 \text{ MeV}} \text{ Hz} \approx 10^{-7} - 10^{-4} \text{ Hz}$$

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- A refined analysis currently under investigation from linear sigma model

- $U(N_f) \times U(N_f)$ linear sigma model for the pions: $\Phi = T_a(\sigma_a + i\pi_a)$:

$$\mathcal{L}_{\text{pions}} = \text{Tr}\left(\partial_\mu \Phi^\dagger \partial^\mu \Phi\right) - m^2 \Phi^\dagger \Phi - \lambda_1 \left[\text{Tr}\left(\Phi^\dagger \Phi\right)\right]^2 - \lambda_2 \text{Tr}\left(\Phi^\dagger \Phi\right)^2$$

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- Confirm 1st-order PT for $N_f \geq 3$ and massless quarks

Linear sigma model with Higgs and dilaton (in progress)

- Modify the previous model to incorporate scale invariance explicitly:

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$$V = \frac{1}{4}(\lambda_\varphi + \delta\lambda_\varphi) + A \, \varphi^4 + B \, \varphi^4 \ln \frac{\varphi}{\mu}$$

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$$V = \frac{1}{4}(\lambda_\varphi + \delta\lambda_\varphi) + A\,\varphi^4 + B\,\varphi^4 \ln\frac{\varphi}{\mu}$$

- Next steps: compute thermal corrections, dynamics of the PT and GW spectrum

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  - small dilaton mass: $m_\chi \approx 10^{-8}$ eV
  - low temperature QCD-induced electroweak transition
  - GW production from 1st order phase transition
- Under investigation:
  - joint dynamics of the Higgs, dilaton and pions
  - precise computation of the GW frequency and amplitude
  - production of primordial black holes

Backup slides

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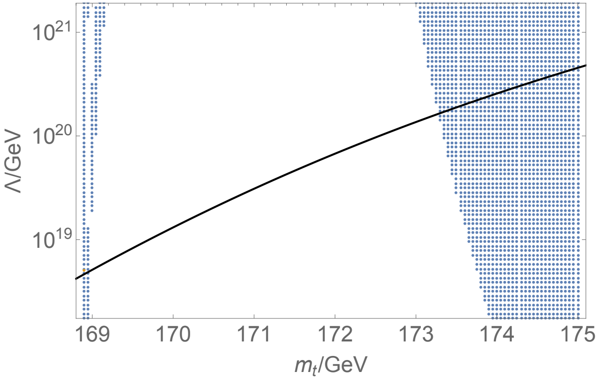

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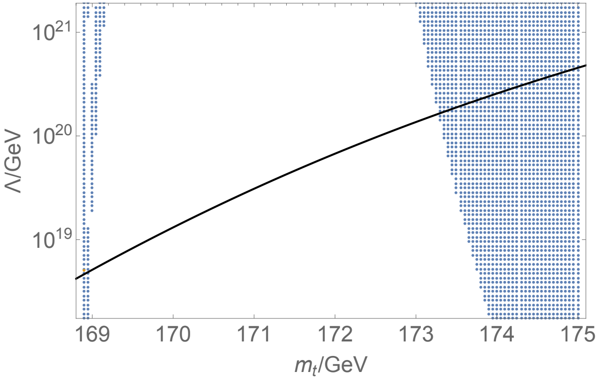

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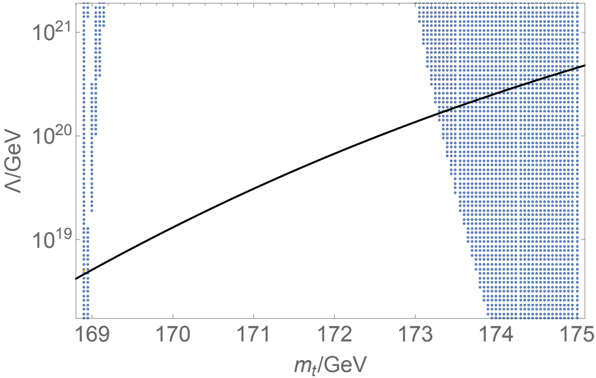
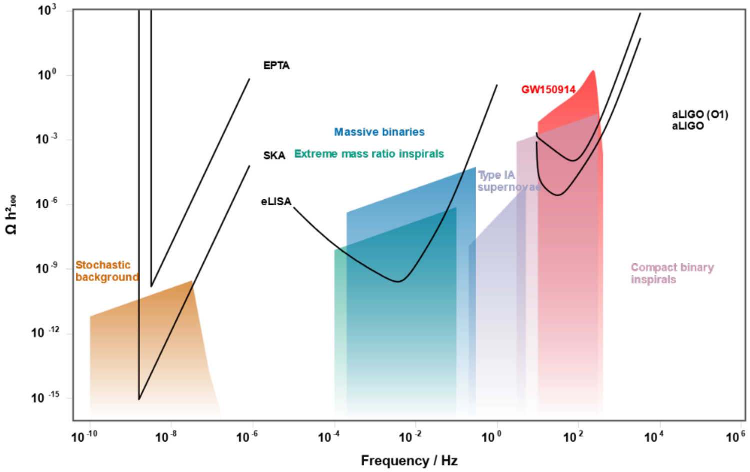


- Dilaton mass at $v_\chi \sim \Lambda \sim M_P$: $m_\chi \sim 10^{-8}$ eV
- Indicative only and requires higher-loop corrections

Gravitational Waves



[From rhcole.com/apps/GWplotter/]