Somme aspects of Dark Energy models

Cosmo-18
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- Matter perturbations can be characterized by the "growth function" $f = \frac{d \ln \delta}{d \ln a} \equiv \frac{d \ln \delta}{dN}$

$$\frac{df}{dN} + f^2 + \frac{1}{2}(1 - 3\, w_{\rm eff})\, f = \frac{3}{2}\, g\, \Omega_m$$

$$\frac{G_{\rm eff}}{G} \equiv g$$

- A convenient "parameterization" $f = \Omega_m^{\gamma}$.
  Actually

$$\delta_m(z, k) \Leftrightarrow \gamma = \gamma(z, k)$$

- In ΛCDM: $\gamma \simeq 0.55$
  Quasi-constant
  It can be very different in modified gravity models!

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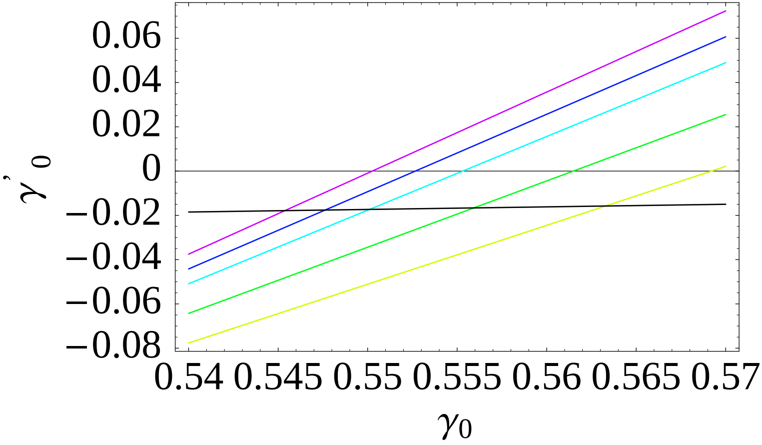

$\Omega_{m,0} = 0.3$ and from top to bottom:
$w_{DE,0} = -1.4,\ -1.3,\ -1.2,\ -1,\ -0.8.$
The black line gives the **true** value of γ_0 realized:
same non vanishing $\gamma'_0 \approx -0.02$.

Exact results in GR (g=1) when γ is constant

All background quantities given in parametric form:

$$a(\Omega_m), t(\Omega_m), H(\Omega_m), z(\Omega_m), \ldots$$

- In particular:

$$w_{DE} = -\frac{1}{3(2\gamma - 1)} \frac{1 + 2\Omega_m^{\gamma} - 3g\Omega_m^{1-\gamma}}{1 - \Omega_m}$$

- Asymptotic future ($\Omega_m \to 0$)

$$w_{\infty} = -\frac{1}{3(2\gamma - 1)} \quad \Rightarrow \quad 0.5 < \gamma < 1$$

Asymptotic past ($\Omega_m \to 1$)

$$w_{-\infty} = \frac{5\gamma - 3}{3(2\gamma - 1)} \quad \Rightarrow \quad 0.5 < \gamma < 0.6$$

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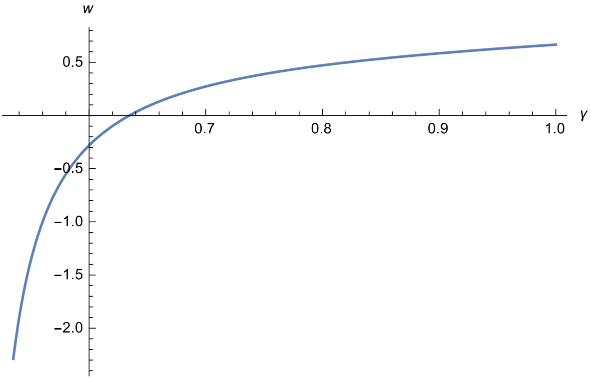

$$0.554 \leq \gamma \leq 0.568 \quad \Leftrightarrow \quad -1.2 \leq w_0 \leq -0.8$$

- Further consequences:

$$0.5 < \gamma < \frac{6}{11} \quad \Leftrightarrow \quad w_{-\infty} < -1$$

$$0.5 < \gamma < \frac{2}{3} \quad \Leftrightarrow \quad w_{\infty} < -1$$

Deep in the DE dominated era:
phantom regime unavoidable!

- $\gamma = 0.6 : w_{-\infty} = 0$

  $\gamma = \frac{6}{11} = 0.545454 : w_{-\infty} = -1$

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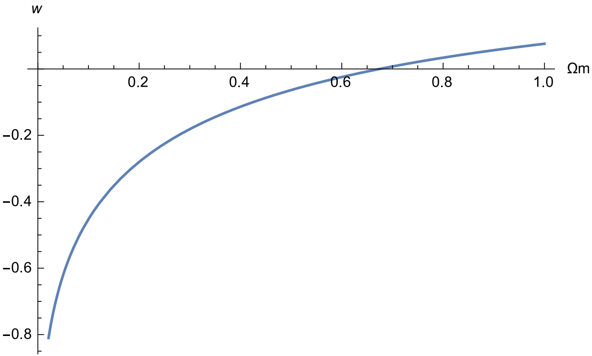
$\Rightarrow$ Cannot be realized by quintessence!

....Unless tracking DE ($\Omega_{-\infty} \equiv \Omega_m(N)|_{N\to-\infty} \neq 1$)

Tracking models correspond to the roots of $F(\Omega_m;\gamma)$

$$F(\Omega_m;\gamma) \equiv \frac{1 + 2\Omega_m^{\gamma} - 3\Omega_m^{1-\gamma}}{1-\Omega_m}$$



Infinite number of roots $\Omega_{-\infty} < 1$ for $0.6 < \gamma < 1$

Particular cases:

$$\gamma = 0.6 + \frac{3}{125}\,\epsilon\,, \qquad \epsilon \equiv 1 - \Omega_{-\infty} \ll 1\,,$$

From observations (CMB) $\epsilon < 1\%$, anyway $w_{\infty} < -1$

The following cases avoid $w_{\infty} < -1$ but $\Omega_{-\infty}$ too small

$$\gamma = \frac{2}{3}\,, \qquad \Omega_{-\infty} = \frac{1}{8}$$

$$\gamma = 1 - \frac{\ln 3}{\ln \Omega_{-\infty}^{-1}}\,, \qquad \Omega_{-\infty} \to 0$$

Constant γ is ruled out for quintessence and for constant w_{DE}

A constant w_{DE} possible beyond GR

- w_{DE} constant during all the expansion

$$\frac{G_{\rm eff}}{G} \equiv g = \frac{1}{3}\left(\Omega_m^{\gamma} + 2\Omega_m^{2\gamma-1}\right)$$

- weaker: w_{DE} constant during part of the expansion

$$g_0 = \frac{1}{3}\left[(1-\beta)\,\Omega_{m,0}^{\gamma-1} + \beta\,\Omega_{m,0}^{\gamma} + 2\,\Omega_{m,0}^{2\gamma-1}\right]$$

$$\beta \equiv -3w_0(2\gamma - 1)$$

Strongly violated in many modified gravity DE models

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- (massless) ST model

$$L = \frac{1}{2}\left(F(\Phi)\,R - Z(\Phi)\,g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi\right) - U(\Phi) + L_m(g_{\mu\nu})$$

$$G_{\rm eff} = \frac{1}{8\pi F}\left(1 + \frac{1}{2\omega_{BD}+3}\right)$$

$$G = G_{\rm eff,0} \approx \frac{1}{8\pi F_0} \quad \Rightarrow g_0 = 1$$

- $f(R)$ model

$$g(z,k) = \left(\frac{df}{dR}\right)^{-1}\left[1 + \frac{1}{3}\,\frac{\left(\frac{\lambda_c}{\lambda}\right)^2}{1+\left(\frac{\lambda_c}{\lambda}\right)^2}\right]$$

$$\frac{df}{dR}(R \gg R_0) \simeq 1\,, \quad \frac{d^2f}{dR^2} > 0 \quad \Rightarrow g \geq 1 \;\; \forall z,k$$

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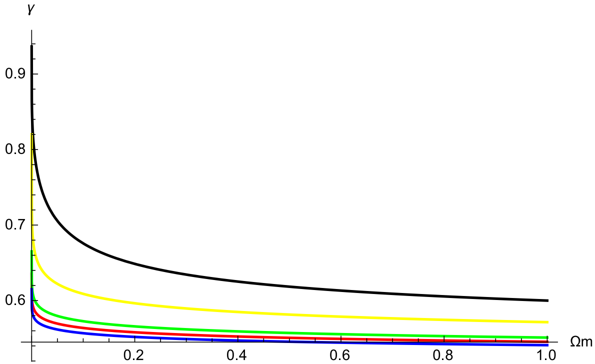
True evolution inside GR

$$6w_{DE}\ \Omega_m\ \ln\Omega_m\ \frac{d\gamma}{d\Omega_m} + 3w_{DE}(2\gamma - 1) + F(\Omega_m;\gamma) = 0$$

Simple examples: w_{DE} =constant



$$\gamma_{-\infty} = \frac{3(1 - w_{DE})}{5 - 6\ w_{DE}}$$

$$\gamma(\Omega_m) = \gamma_{-\infty} + \frac{(3-\alpha)(2-\alpha)}{2(2\alpha-5)^2(5-4\alpha)}(1-\Omega_m) + \frac{(3-\alpha)(2-\alpha)(36\alpha^2-140\alpha+97)}{12(5-2\alpha)^3(5-4\alpha)(5-6\alpha)}(1-\Omega_m)^2 + \mathcal{O}\left((1-\Omega_m)^3\right) \qquad \alpha = 3w_{DE}$$

Remarkably accurate at 1st order for $\gamma_0 = \gamma(\Omega_{m,0})$
For assessment of DE models: $\gamma(z)$ around γ_0

$$\gamma = \gamma_0 + \gamma_0' \frac{z}{1+z} + \mathcal{O}\left[\left(\frac{z}{1+z}\right)^2\right]$$

$$\gamma_0' = \frac{1-\Omega_{m,0}}{2\ln\Omega_{m,0}}\left[3w_0(2\gamma_0-1) + F(\Omega_{m,0};\gamma_0)\right]$$

γ_0 is calculated numerically

Accuracy below the percent level up to $z \sim 3$

Interesting problem: asymptotic future

$$\gamma \sim \frac{3w_\infty - 1}{6w_\infty} + \frac{C}{\ln \Omega_m} \quad \to \gamma_\infty = \frac{3w_\infty - 1}{6w_\infty} \qquad\qquad \Omega_m \to 0$$

In particular: $\gamma_\infty(w) = \gamma(w_\infty = w)$

$$\text{However :} \quad \frac{d\gamma}{d\Omega_m} \quad \sim \quad -C\left([\ln \Omega_m]^2\ \Omega_m\right)^{-1} \qquad \text{diverges!}$$

$$\frac{d\gamma}{dz} \quad \sim \quad 3Cw_{DE}\ [\ln \Omega_m]^{-2} \qquad \text{diverges too!}$$

But: $\frac{d\gamma}{d\ln a} \to 0 \qquad (w_\infty \text{ is finite})$

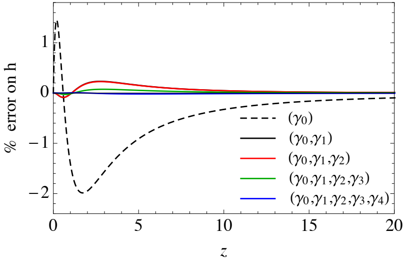
Asymptotic past: $\gamma \sim \gamma_{-\infty} + \frac{D}{1-\Omega_m}$

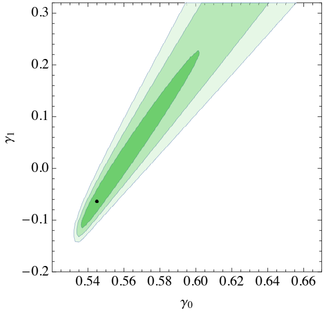
$$2 \ln \Omega_m \frac{d\gamma}{dN} + (2\gamma - 1) \frac{d \ln \Omega_m}{dN} + (1 - \Omega_m) F(\Omega_m, \gamma, g)$$

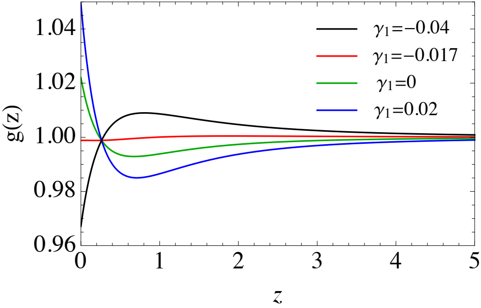
Assume the behaviour: $\gamma = \gamma_0 + \gamma_1 (1 - x) \qquad x \equiv \frac{a}{a_0}$

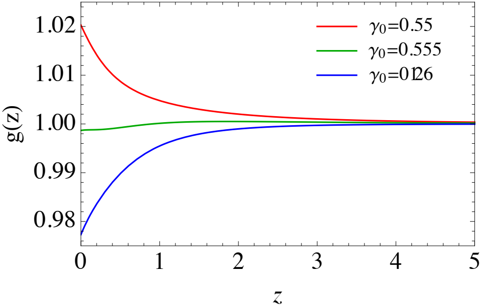
We can reconstruct the background evolution in function of the parameters $\gamma_0,\ \gamma_1$

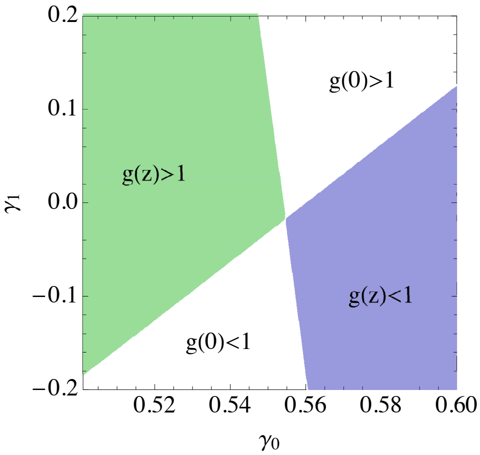
From low z SNIa data: $\gamma_0 \gtrsim 0.53$ and $\gamma_1 \gtrsim -0.15$






γ_1=−0.04
γ_1=−0.017
γ_1=0
γ_1=0.02
g(z)
z
1.04
1.02
1.00
0.98
0.96
0
1
2
3
4
5


γ_0=0.55
γ_0=0.555
γ_0=0126
g(z)
z
1.02
1.01
1.00
0.99
0.98
0
1
2
3
4
5


0.2
0.1
0.0
−0.1
−0.2
γ_1
0.52
0.54
0.56
0.58
0.60
γ_0
g(z)>1
g(0)>1
g(z)<1
g(0)<1

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- On low redshifts:

$$G_{\rm eff} \simeq G[1 + A(w_{DE,0}, \Omega_{m,0}, \Omega_{DE,0})\, z^2]$$

- For ΛCDM, we obtain (generic case):

$$G_{\rm eff} \simeq G[1 + \Delta^2\, z^2]$$

Always increasing for $w_{DE,0} < -1$

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Conclusion

Constant γ in GR:
Constant w_{DE} not possible during the entire evolution
$w_\infty < -1$ for non-tracking DE
For tracking DE, either $w_\infty < -1$ or $\Omega_{m,-\infty}$ too small

True evolution in GR:
γ is quasi-constant for wCDM in the past
Constant γ is strongly violated for $\Omega_m \to 0$ with $\frac{d\gamma}{dz} \to -\infty$

Constant γ beyond GR: constant w_{DE} requires $G_{\rm eff} < G$, not possible in $f(R)$, (massless) ST
Tracking DE possible with $G_{\rm eff} > G$ or $G_{\rm eff} < G$

Background reconstruction: $\gamma_0 \gtrsim 0.53$, $\gamma_1 \gtrsim -0.15$ in GR using SNIa data on $z \leq 0.3$