

# Cosmology of String Moduli

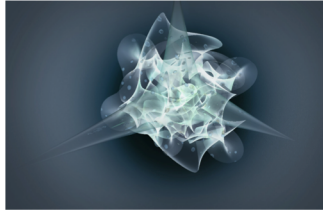
Fernando Quevedo  
ICTP + Cambridge  
COSMO 2018  
Daejon, South Korea, 2018

# Based on Articles

- S. Antusch, F. Cefala, S. Krippendorff, F. Muia, S. Orani and FQ:  
*“Oscillons from String Moduli,”*  
JHEP {1801} (2018) 083, [arXiv:1708.08922].
- S. Krippendorff, F. Muia and FQ,  
*“Moduli Stars,”*  
JHEP {1808} (2018) 070, [arXiv:1806.04690].
- M Cicoli, S. de Alwis, A. Maharana, F. Muia and FQ  
*“de Sitter vs Quintessence in String Theory,”*  
[arXiv:1808.xxxx]..

# Strings and Moduli

- String theory predicts (6 or 7) extra dimensions
- Major problem: Fixing size and shape of extra dimensions (moduli)



- Progress to fix all moduli: only this century (GKP, KKLT, LVS,...)
- In some cases the 4D space = de Sitter space ( $\Lambda > 0$ )

# Physics of Moduli

- Moduli: scalar particles in 4D: candidates for inflatons
- Gravitational strength couplings
- Mass of moduli  $\sim$  gravitino mass
- Each modulus equivalent to saxion+axion
- Number of moduli order 100-1000

# Moduli Stabilisation in IIB

- Moduli  $S, T_i, U_a$

$$V_F = e^K \left( K_{MN}^{-1} D_M W \bar{D}_{\bar{M}} \bar{W} - 3|W|^2 \right)$$

$$W_{\text{tree}} = W_{\text{flux}}(U, S) \quad K_{i\bar{j}}^{-1} K_i K_{\bar{j}} = 3 \quad \text{No-scale}$$

$$V_F = e^K \left( K_{a\bar{b}}^{-1} D_a W D_{\bar{b}} \bar{W} \right) \geq 0$$

Fix  $S, U$  but  $T$  arbitrary

- Quantum corrections

$$\delta V \propto W_0^2 \delta K + W_0 \delta W$$

- Three options:  $W_0 \gg \delta W \quad \delta K \gg \delta W$  Runaway: Dine-Seiberg problem

$$\begin{aligned} W_0 &\sim \delta W = W_{\text{np}} \\ W_0 &\ll 1 \end{aligned}$$

Fix T-modulus: KKLT

$$\begin{aligned} \delta K &\sim W_0 \delta W \\ \delta K &\sim 1/\mathcal{V} \text{ and } \delta W \sim e^{-a\tau} \end{aligned}$$

Fix T-moduli: LVS

# Inflation and Strings

- Some inflationary EFTs describe CMB + other data very well.
- Inflation needs an UV completion.
- Some EFTs of string compactification can describe inflation
- Challenges: Moduli stabilisation and

$$M_{\text{planck}} > M_{\text{string}} > M_{\text{kk}} > M_{\text{inf}} .$$

# Inflation and Strings 2

- Epochs: Pre-inflation, inflation, post-inflation (pre-BBN)
- Chiral spectrum implies  $N=0,1$  in 4D (work with  $N=1$ )
- Strings relevant in postinflation? (yes: moduli).

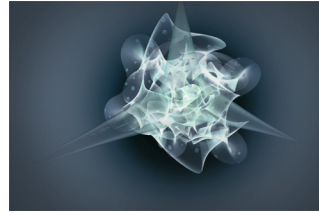
“Generically”: If eft is suspersymmetric then the moduli survive at low energies until susy breaks:

$$\text{mass}_{\text{moduli}} \approx m_{\text{gravitino}}.$$

(but interesting exceptions!)

# Kahler moduli

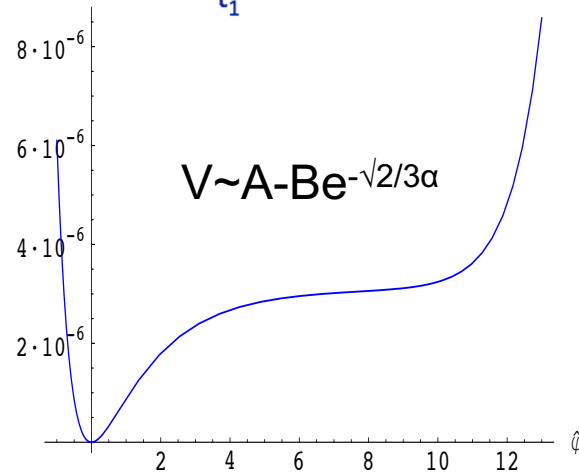
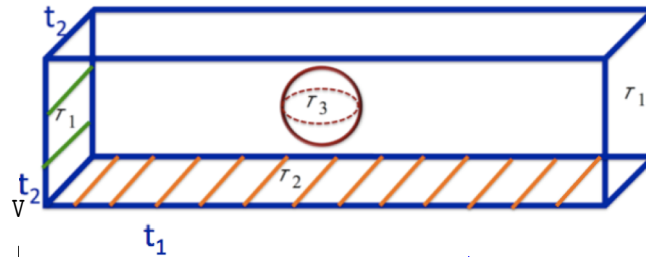
- Overall volume



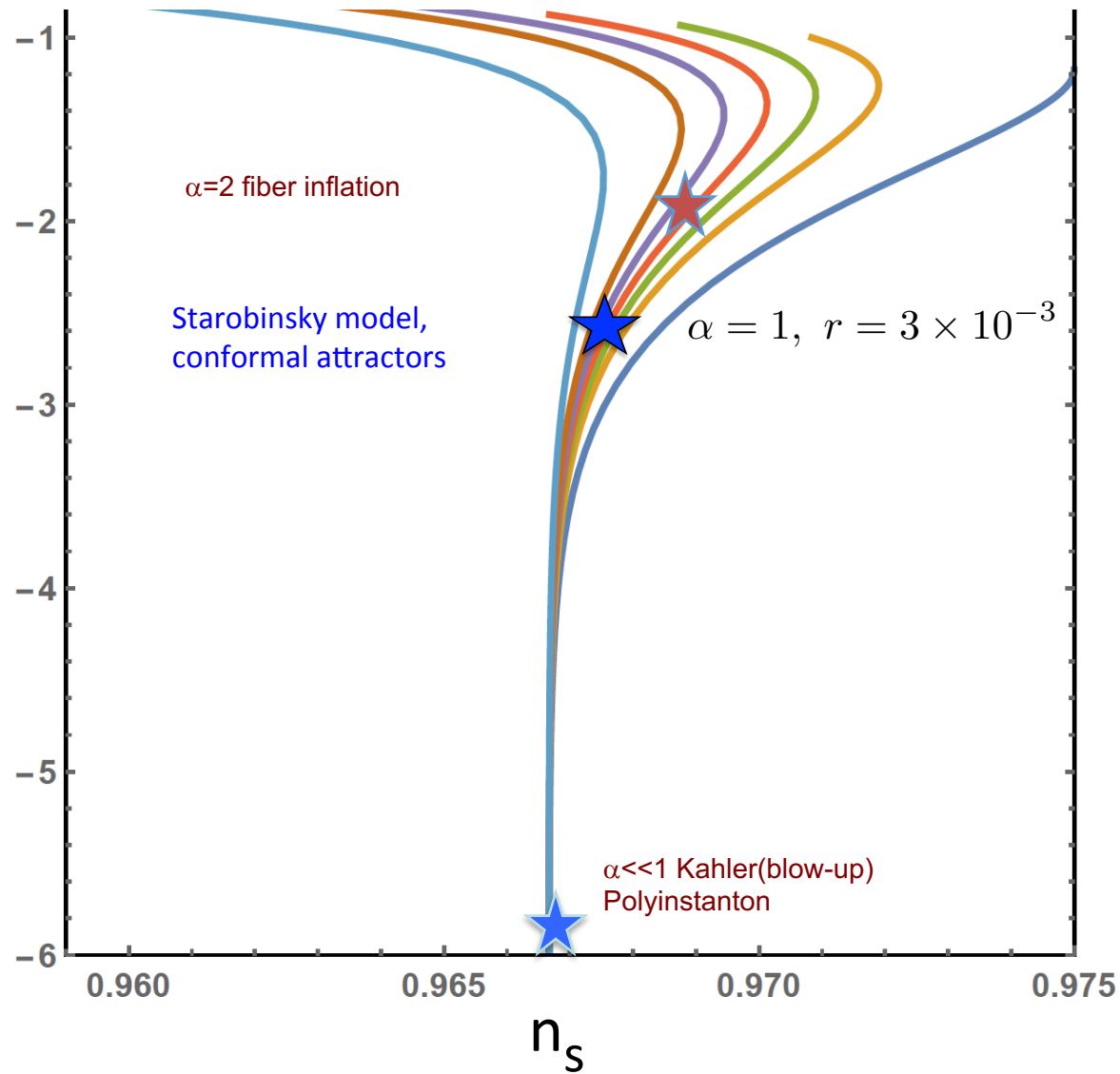
- Blow-up



- Fibre moduli



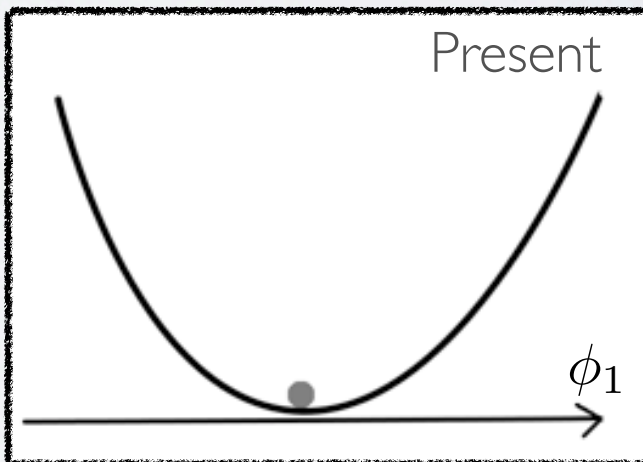
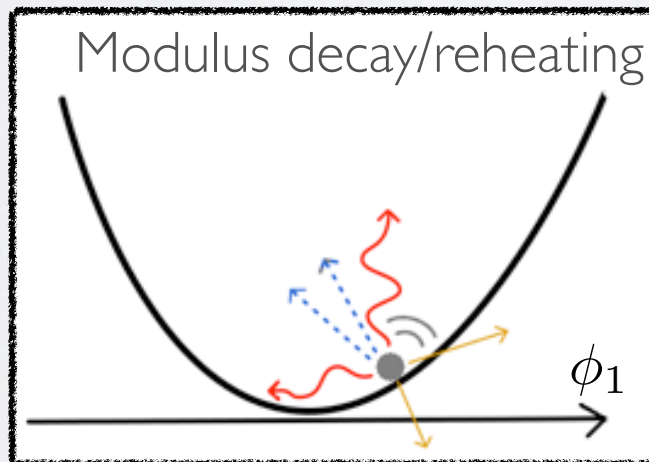
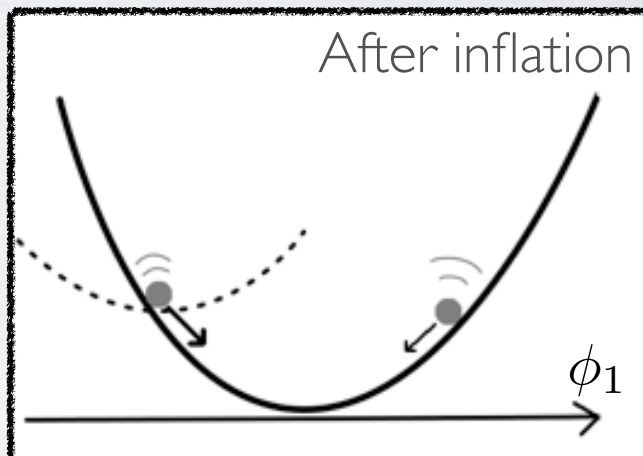
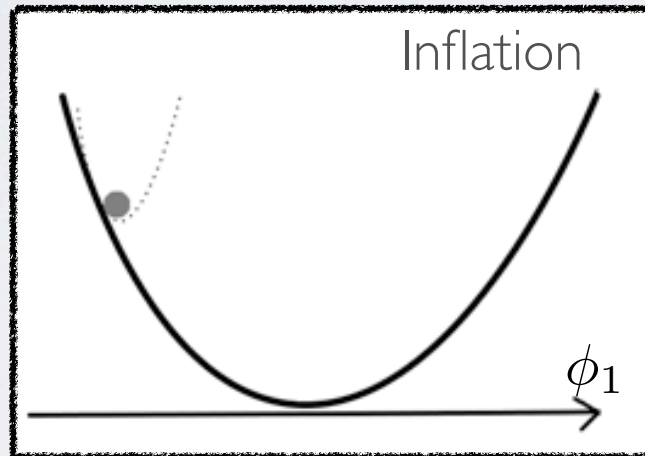
# Inflation: Fibre+Blow-up



Adapted from  
Kallosh, Linde  
 $\alpha$ -attractors

# Post Inflation

# Moduli Domination



$$\Gamma_\phi \sim \frac{1}{8\pi} \frac{m_\phi^3}{M_{\text{Pl}}^2}$$

$$T > O(1 \text{ MeV}), \text{ so } m_\phi \gtrsim 3 \cdot 10^4 \text{ GeV}$$

Coughlan et al 1983, Banks et al, de Carlos et al 1993

# Oscillons\* from String Moduli

Antusch, Cefalá, Krippendorf, Muia, Orani, FQ

[arXiv:1708.08922](#)

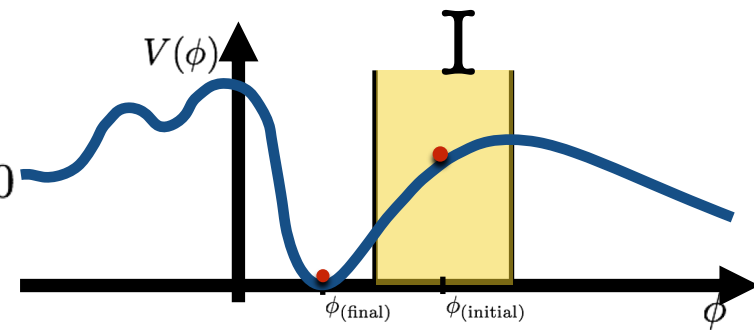
\*localised, long-lived, non-linear excitations of the scalar fields.

# Generalities

- Exponentially growing solutions:

$$\ddot{\phi}(t) + 3H\dot{\phi}(t) + V'(\phi(t)) = 0$$

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \left( \frac{k^2}{a^2(t)} + V''(\phi(t)) \right) \delta\phi_k = 0$$

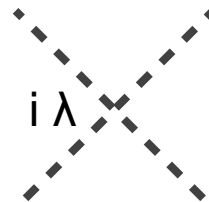


- Conditions for unstable solutions:
  - parametric resonance
  - tachyonic preheating (modulus displaced in I)
 
$$k^2/a^2 + \partial^2 V / \partial \phi^2 < 0$$
  - tachyonic oscillations (oscillations reach I)
 
$$k_p \sim \sqrt{\partial^2 V / \partial \phi^2}|_{\text{min}} \equiv m$$

# Necessary Conditions for Oscillons production

- Quantum fluctuations of the field grow as it oscillates around the minimum.
- The growth of fluctuations is sufficiently strong for non-linear interactions to become important.
- The potential is shallower than quadratic away from the minimum in some field space region relevant for the dynamics of the field.

$$V = \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4 + \dots$$



Attractive 'force'  
for  $\lambda > 0$

# Lattice simulations\*

- LatticeEasy: to analyse strong growth of perturbations.

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\nabla^2\phi + \frac{\partial V}{\partial\phi} = 0 \quad H^2 = \frac{1}{3M_{\text{Pl}}^2} \left( V + \frac{1}{2}\dot{\phi}^2 + \frac{1}{2a^2}|\nabla\phi|^2 \right)$$

- Modified version to calculate also metric perturbations:

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$$

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{1}{a^2}\nabla^2 h_{ij} = \frac{2}{M_{\text{Pl}}^2}\Pi_{ij}^{\text{TT}} \quad \Pi_{ij}^{\text{TT}} = \frac{1}{a^2} [\partial_i\phi\partial_j\phi]^{\text{TT}}$$

$$\Omega_{\text{GW}}(k) = \frac{1}{\rho_{\text{c}}} k \frac{d\rho_{\text{GW}}}{dk} \quad \rho_{\text{GW}}(t) = \frac{M_{\text{Pl}}^2}{4} \left\langle \dot{h}_{ij}(\mathbf{x}, t) \dot{h}_{ij}(\mathbf{x}, t) \right\rangle_{\text{V}}$$

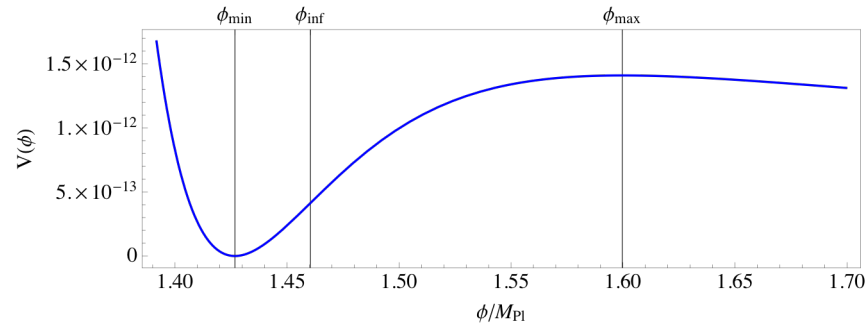
\*Plus Floquet analysis

# KKLT Oscillons

$$V/M_{\text{Pl}}^4 = \frac{e^{K_{\text{cs}}}}{6\tau^2} \left( aA^2(3 + a\tau)e^{-2a\tau} - 3aAe^{-a\tau}W_0 \right) .$$

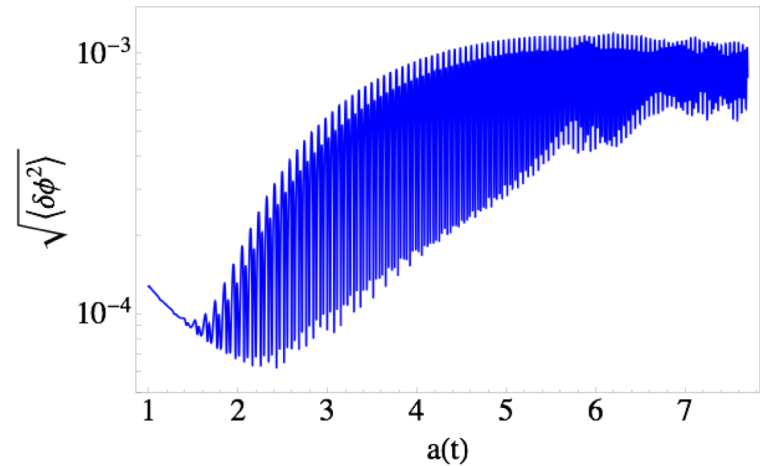
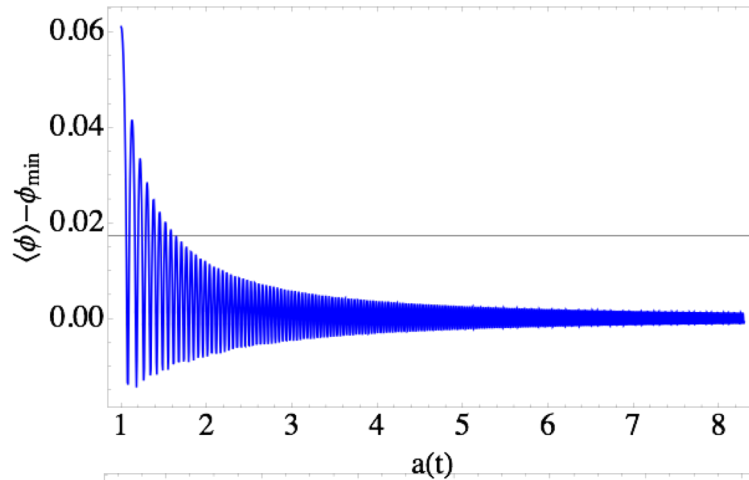
$$\phi/M_{\text{Pl}} = \frac{\sqrt{3}}{2} \log (T + \bar{T}) .$$

$$10^{-12} \leq W_0 \leq 10^{-5}, \quad 1 \leq A \leq 10, \quad 1 \leq a \leq 2\pi .$$

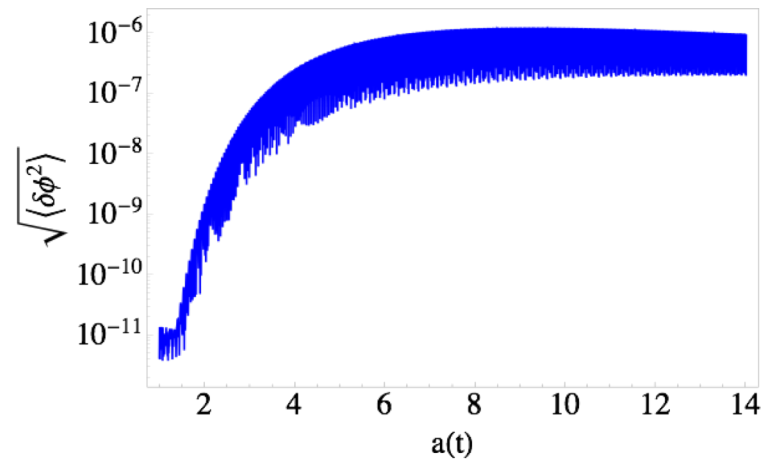
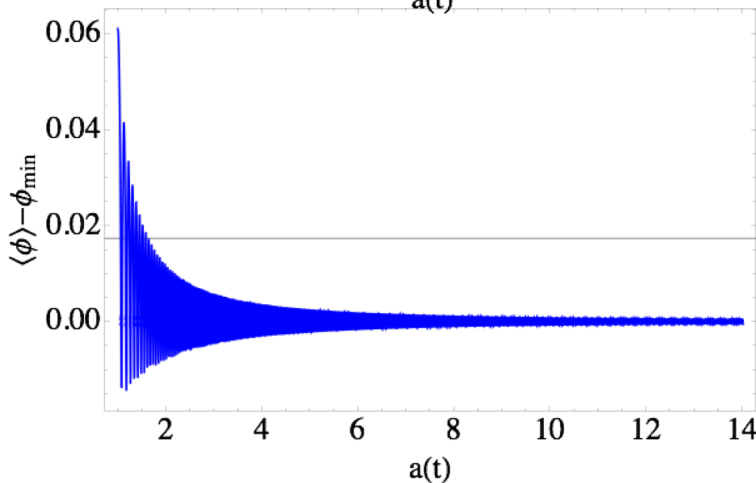


# KKLT results

$W_0=10^{-12}$   
(no oscillons)



$W_0=10^{-5}$   
oscillons  
generated

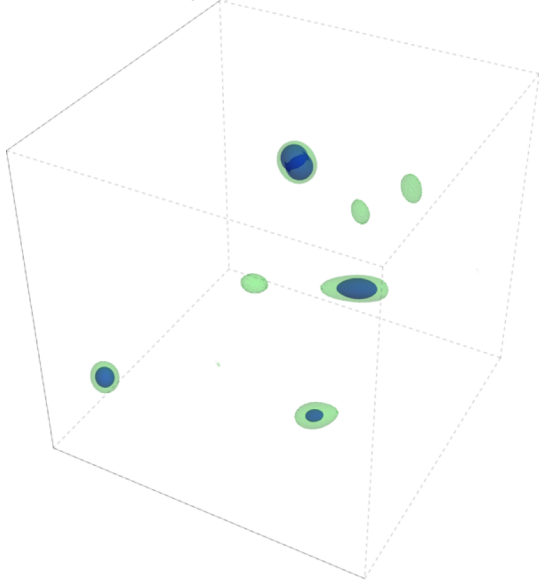


$512^3$  points

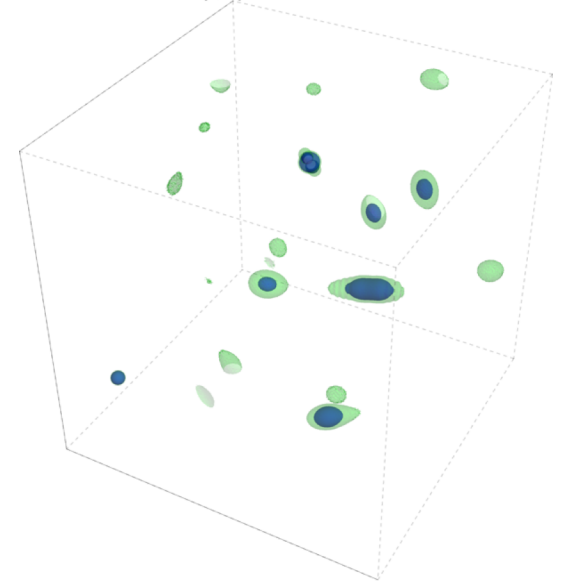
$$L^3 \simeq (0.7/H_{\text{initial}})^3$$

# Snapshots

$\rho/\langle\rho\rangle$  at  $a = 6.41$

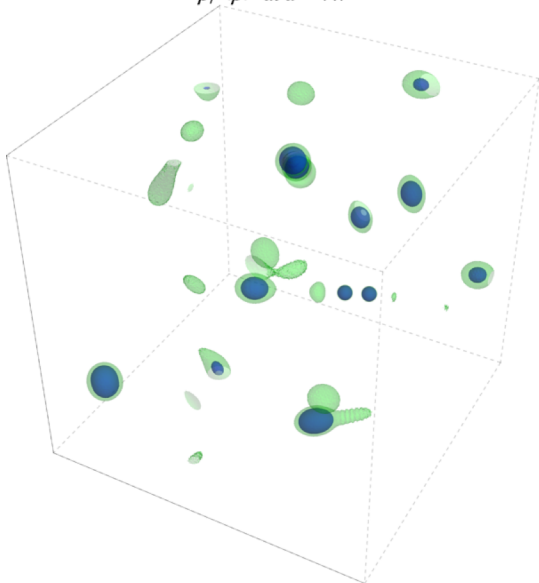


$\rho/\langle\rho\rangle$  at  $a = 7.07$

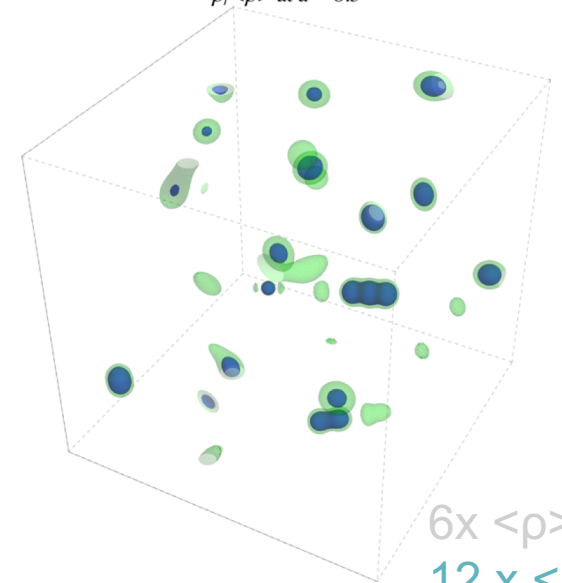


$W_0=10^{-5}$   
 $A=10$   
 $a=2\pi$

$\rho/\langle\rho\rangle$  at  $a = 7.7$

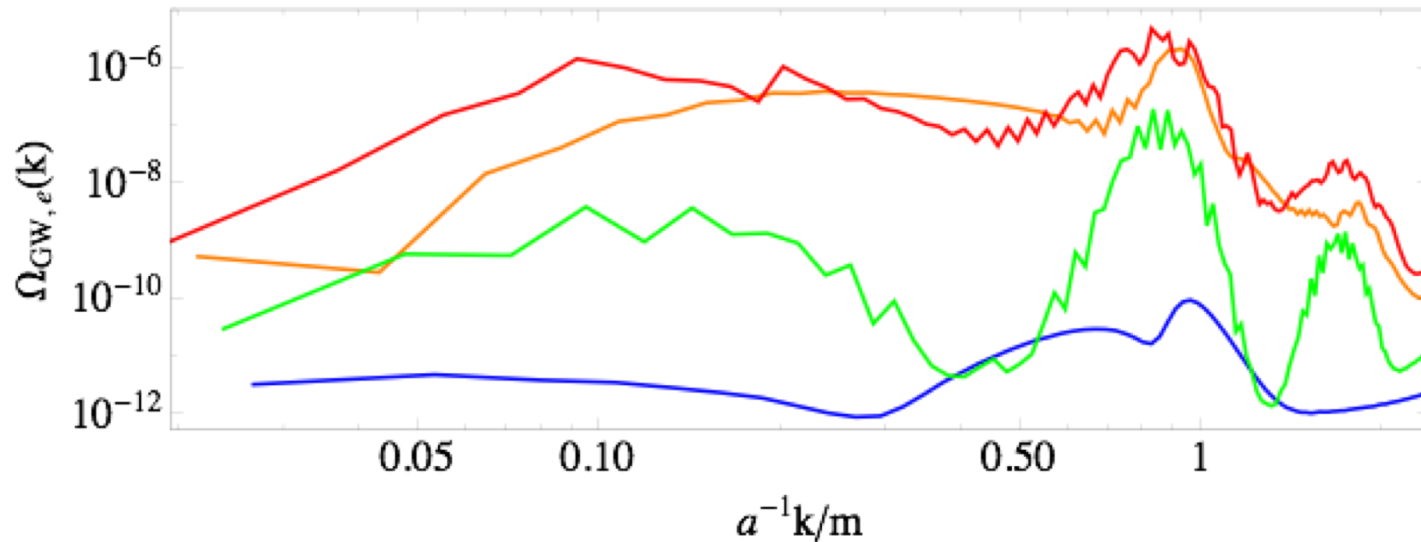


$\rho/\langle\rho\rangle$  at  $a = 8.3$



6x  $\langle\rho\rangle$   
12x  $\langle\rho\rangle$

# GW spectrum: KKL T



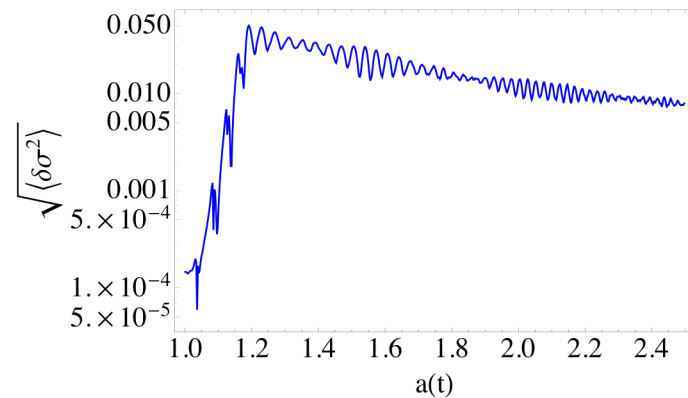
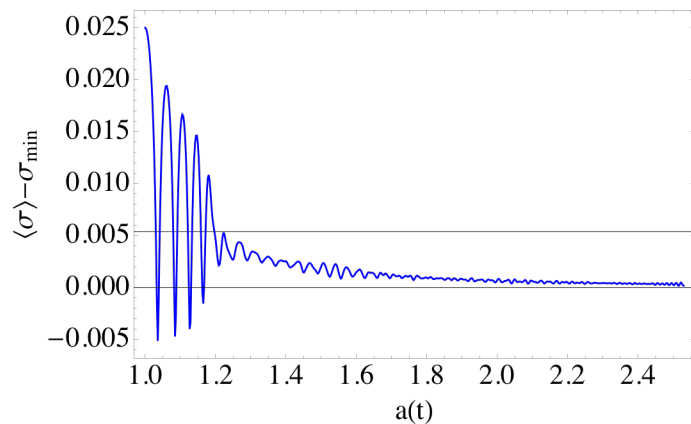
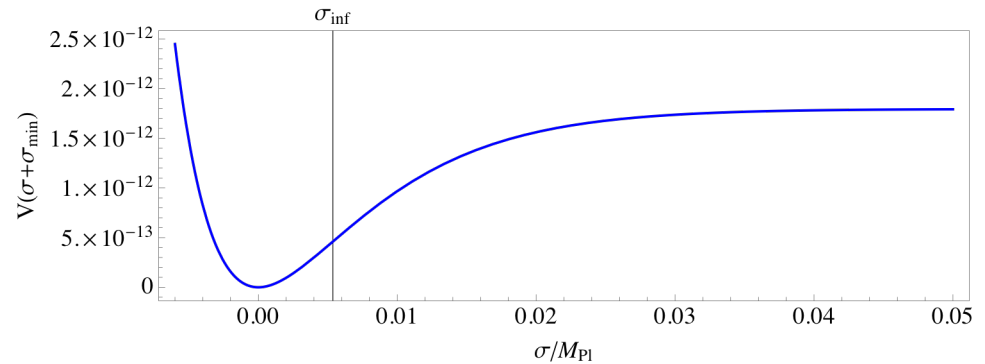
$$f_{0,\text{peak}} \sim 10^9 \text{ Hz}$$

$$\Omega_{\text{GW},0}(f_{0,\text{peak}}) \sim 3 \times 10^{-11}$$

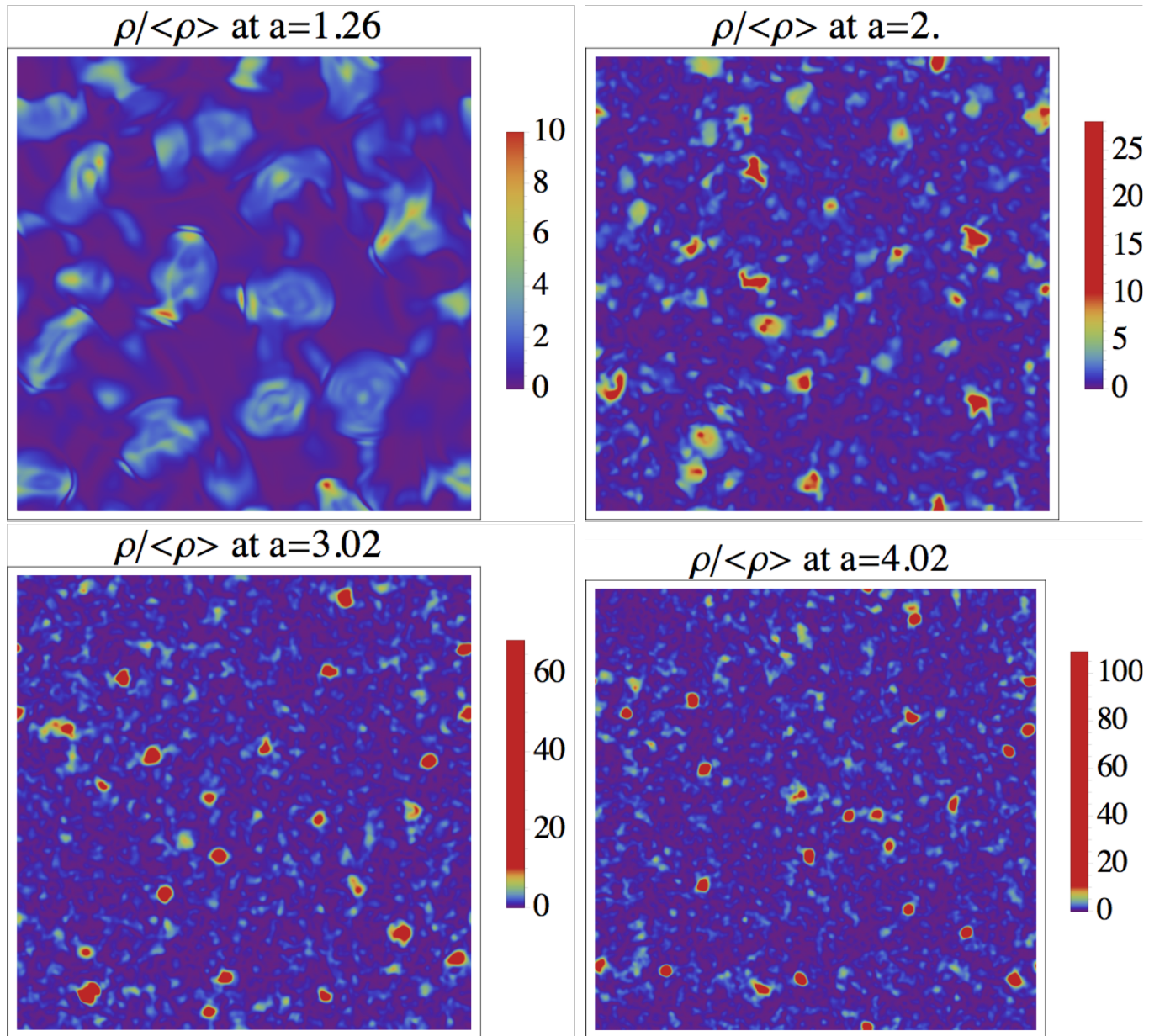
\*Overall scaling can lower frequency but also lower the amplitude

# Blow-up Oscillations

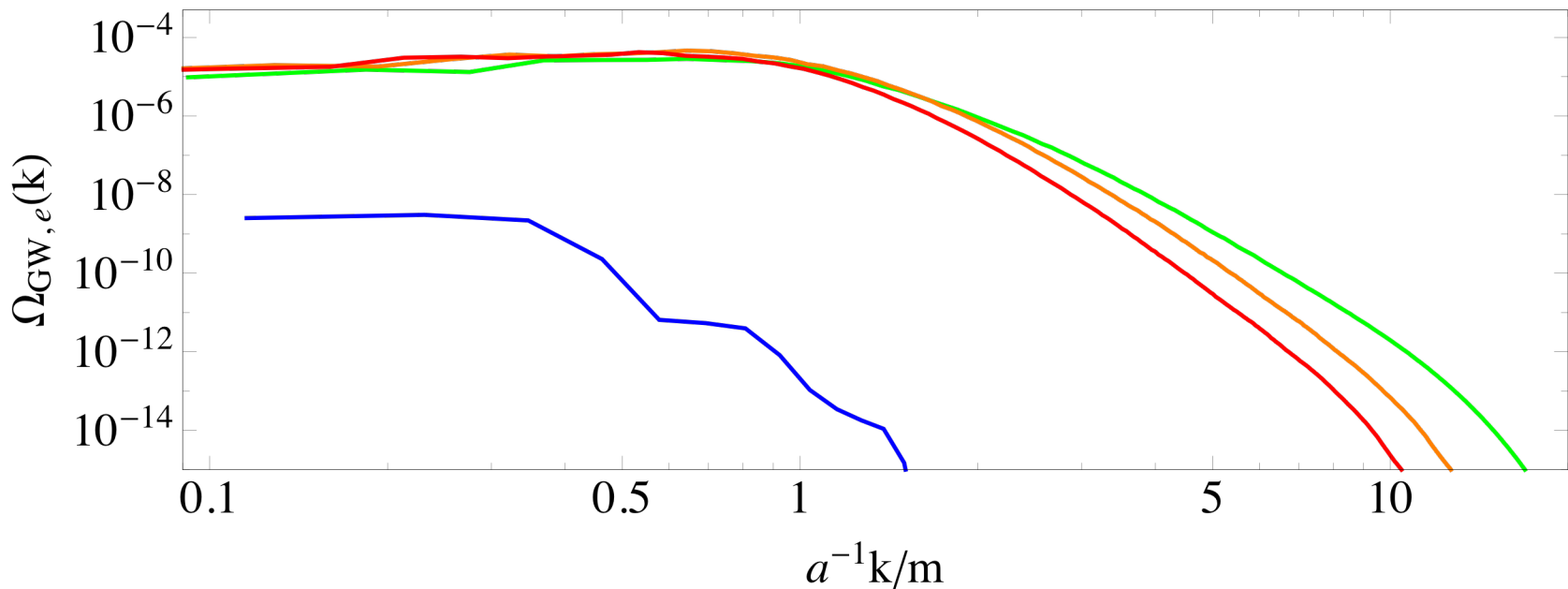
$$V \sim V_0 \left( 1 - \kappa(\sigma) e^{-\alpha \sigma^{4/3}} \right)^2 ,$$



# Oscillons from Blow-up mode



# Gravitational Waves



$$f_0 \sim 10^8 \text{ Hz} - 10^9 \text{ Hz}, \quad \text{with} \quad \Omega_{\text{GW},0} \sim 10^{-10} - 5 \times 10^{-10}.$$

\*No oscillons for volume nor fibre moduli but also no overshooting!

# Moduli Stars

Krippendorf, Muia + FQ  
JHEP {1808} (2018) 070, [[arXiv:1806.04690](#)].

# Boson and Fermion Stars

- Fermion stars: Gravity vs fermion pressure

$$GM^2/R \sim N^{4/3}/R, \quad N = M/m$$

$$M_{\max} \sim \frac{M_{\text{P}}^3}{m_f^2} \quad R_{\min} \sim \frac{M_{\text{P}}}{m_f^2}.$$

(e.g.  $M \sim M_{\odot}$  for  $m \sim 1 \text{ GeV}$  neutron star)

- Boson stars: Gravitational BEC

Heisenberg  $R > 1/m$   
Schwarschild  $R \sim 2GM$

$$R_{\min} \sim \frac{1}{m} \quad M_{\max} \sim \frac{M_{\text{P}}^2}{m}.$$

But adding interactions

$$M_c \sim M_p^3/m^2$$

# Bosonic Compact Objects

- Q-balls
  - Oscillons
- } Repulsive pressure vs attractive interaction

## Gravity vs Repulsive pressure

- Boson stars
- Mini-boson stars
- Oscillatons (e.g. axion stars)  
(+ axion miniclusters)

# Are there stringy boson/fermion stars?

Candidates:

Long-lived (stable) gravitationally coupled fields:

- hidden sector fermions/bosons,
- moduli,
- modulini,
- gravitini

# Stringy Fermion Stars

Gravitino and modulini:

$$M_{\max} \sim \frac{M_{\text{P}}^3}{m_f^2} \qquad m_f = m_{3/2} = \frac{W_0}{\mathcal{V}}$$

Validity of EFT and Cosmological moduli problem:  $10^3 \leq \mathcal{V} \leq 10^9$

$$1 \text{ g} \lesssim M \lesssim 10^{15} \text{ g}, \qquad 10^{-27} \text{ cm} \lesssim R \lesssim 10^{-15} \text{ cm}$$

**Recall:**  $M_{\odot} \simeq 2 \times 10^{33} \text{ g} \simeq 10^{57} \text{ GeV}$ .  $1 \text{ GeV} \simeq 1.8 \times 10^{-24} \text{ g}$

# Volume modulus stars

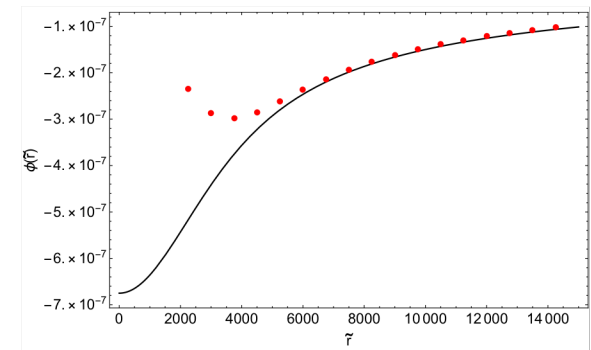
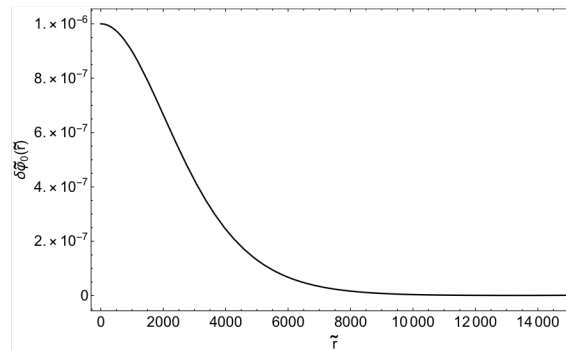
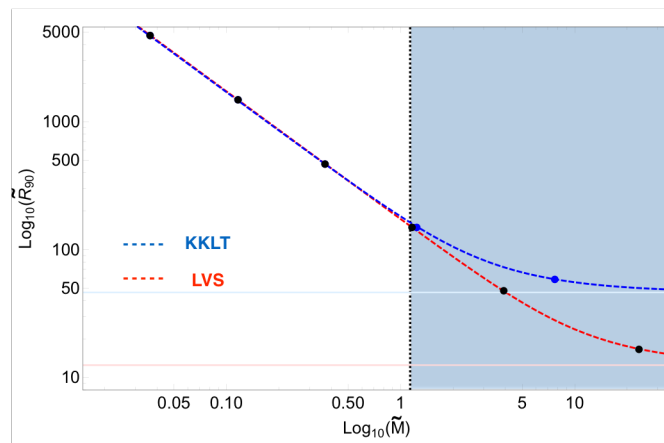
$$S = \int d^4x \sqrt{-g} \left[ -\frac{g^{\mu\nu}}{2} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right]$$

$$\varphi(r, t) = \varphi_0(r) \cos(\omega t) , \quad ds^2 = -(1 + 2\phi) dt^2 + (1 - 2\phi) dr^2 + r^2 d\Omega^2 ,$$

$$\tilde{\varphi}_0''(\tilde{r}) + \frac{2}{\tilde{r}} \tilde{\varphi}_0'(\tilde{r}) = 2(\phi(\tilde{r}) - \epsilon) \tilde{\varphi}_0(\tilde{r}) ,$$

$$\phi''(\tilde{r}) + \frac{2}{\tilde{r}} \phi'(\tilde{r}) = \frac{\tilde{\varphi}_0^2(\tilde{r})}{4} ,$$

$$M(r) = \left( \frac{\Lambda^2}{m} \right) \tilde{M}(\tilde{r}) , \quad \tilde{M}(\tilde{r}) = 4\pi \int_0^{\tilde{r}} d\tilde{r}' \tilde{r}'^2 \tilde{\rho}(\tilde{r}') .$$



# Q-balls in string theory?

Global symmetries?

1. From (non) anomalous U(1)
2. From Peccei-Quinn symmetries

**\*Open strings:**

$$U_D = g^2 \left( \xi - \sum_i q_i |\Phi_i|^2 \right)^2$$
$$U_{\text{soft}} = \sum_i m_i^2 |\Phi_i|^2 + \left( \sum_{ijk} A_{ijk} \Phi_i \Phi_j \Phi_k + \sum_{ij} B_{ij} \Phi_i \Phi_j + h.c. \right)$$
$$E^2 = \frac{2U}{\sum_i q_i |\Phi_i|^2} = \frac{2(U_D + U_{\text{soft}})}{\sum_i q_i |\Phi_i|^2}$$

Minimum for nonvanishing:  $\rho^2 = \sum_i q_i \rho_i^2 = \sum_i q_i |\Phi_i|^2$

e.g. Kusenko (1997) for  
MSSM

# Closed string sector

- Massive moduli + axion  
(generalised axion stars,  $m > 1 \text{ TeV}$ )
- Axion much lighter  
(Ultra-light axion)  $V_\psi = \frac{g_s}{2\pi} a_b A_b \frac{e^{-a_b \tau_b}}{\tau_b^2} [1 + \cos(a_b \psi_b)]$  ,
- PQ symmetry almost exact (PQ-balls?)

# Q-Balls

...Coleman (1985)...

Complex scalar, U(1) global symmetry

$$\mathcal{L} = \int d^3x \left( \frac{1}{2} \partial^\mu \Phi \partial_\mu \Phi^* - U(|\Phi|) \right)$$

U minimum at  $\Phi=0$

Noether current and conserved charge

$$J_\mu = \frac{1}{2i} (\Phi^* \partial_\mu \Phi - \Phi \partial_\mu \Phi^*); \quad Q = \int d^3x J^0 = \frac{1}{2i} \int d^3x (\Phi^* \dot{\Phi} - h.c.)$$

Extrema of energy

$$\begin{aligned} E_\omega &= \int d^3x \left( \frac{1}{2} |\dot{\Phi}|^2 + \frac{1}{2} |\nabla \Phi|^2 + U(|\Phi|) \right) + \omega \left( Q - \frac{1}{2i} \int d^3x (\Phi^* \dot{\Phi} - h.c.) \right) \\ &= \int d^3x \left( \frac{1}{2} |\dot{\Phi} - i\omega \Phi|^2 + \frac{1}{2} |\nabla \Phi|^2 + \hat{U}(|\Phi|) \right) + \omega Q \end{aligned}$$

$$\hat{U}_\omega(|\Phi|) = U(|\Phi|) - \frac{1}{2} \omega^2 |\Phi|^2.$$

$$\Phi(x, t) = \varphi(x) e^{i\omega t}$$

Thin wall approximation (large Q)

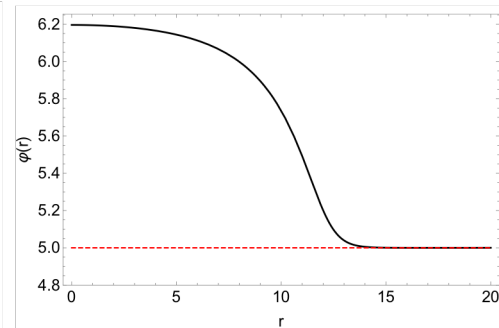
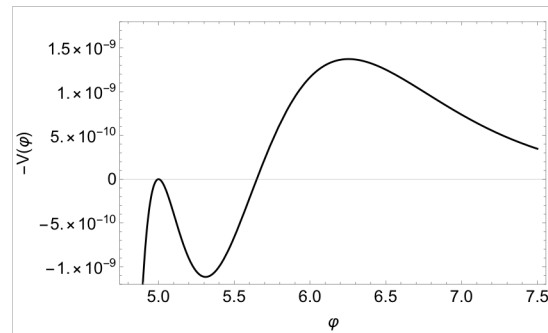
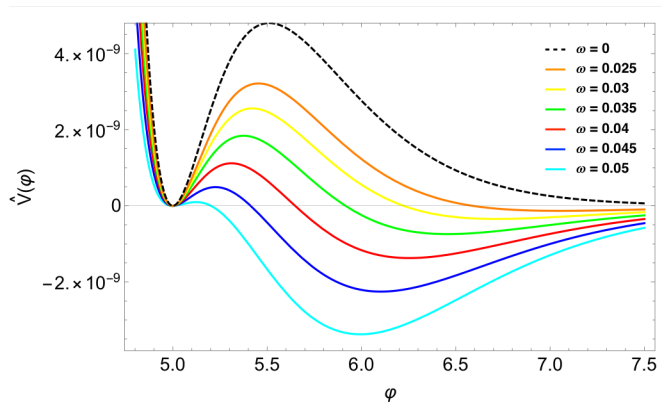
$$E = Q \sqrt{\frac{2U(\varphi_0)}{\varphi_0^2}}$$

# PQ Balls?

$$S = \int d^4x \mathcal{L} = \int d^4x [-f(\tau) [\partial_\mu \tau \partial^\mu \tau + \partial_\mu \theta \partial^\mu \theta] - V(\tau)]$$

$$\dot{\theta} = \omega, \quad \nabla \theta = 0. \quad \hat{V}(\tau) = V(\tau) - \omega^2 f(\tau)$$

$$Q = \omega \int d^3x f(\tau) \propto \int 4\pi r^2 dr \frac{\omega}{r^2} \rightarrow \infty.$$



# Spinning Axions?

$$\mathcal{S} = \int d^4x \sqrt{-g} [-g^{\mu\nu} (\partial_\mu \varphi \partial_\nu \varphi + f(\varphi) \partial_\mu \theta \partial_\nu \theta) - V]$$

$$f = \alpha/\tau^2 = \alpha e^{-\sqrt{2/\alpha}\varphi}.$$

$$\ddot{\varphi} + 3H\dot{\varphi} + \partial_\varphi V = \frac{q^2 \partial_\varphi f}{4a^6 f^2}.$$

$$V = V_0 e^{-\kappa_1 \varphi}, \quad f = \alpha e^{-\kappa_2 \varphi},$$

$$\varphi(t) = B \ln t - C, \quad a(t) = t^{\frac{\kappa_1 + \kappa_2}{3\kappa_1}} \quad w(\varphi) = \frac{\kappa_1 - \kappa_2}{\kappa_1 + \kappa_2}.$$

Similar to  
spintessence

# Formation Mechanisms?

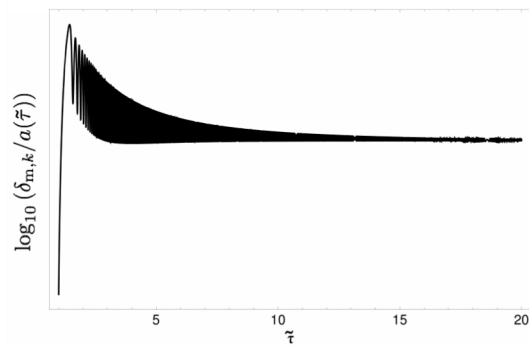
I) There is some initial localized overdensity;

II) The initial overdensity collapses due to the effect of attractive interactions.

$$\delta_{m,k} \equiv \frac{\delta\rho_{m,k}}{\langle\rho\rangle} \propto a(t) \sim t^{2/3}, \quad k \gg aH.$$

$$\Psi = \frac{\delta_{m,k}(t_{\text{dec}})}{\delta_{m,k}(t_{\text{mat}})} \approx \left(\frac{t_{\text{dec}}}{t_{\text{mat}}}\right)^{2/3} \approx \left(\frac{H_{\text{mat}}}{H_{\text{dec}}}\right)^{2/3} \approx \left(\frac{m}{\Gamma}\right)^{2/3} \approx \left(\frac{M_{\text{P}}}{m}\right)^{4/3},$$

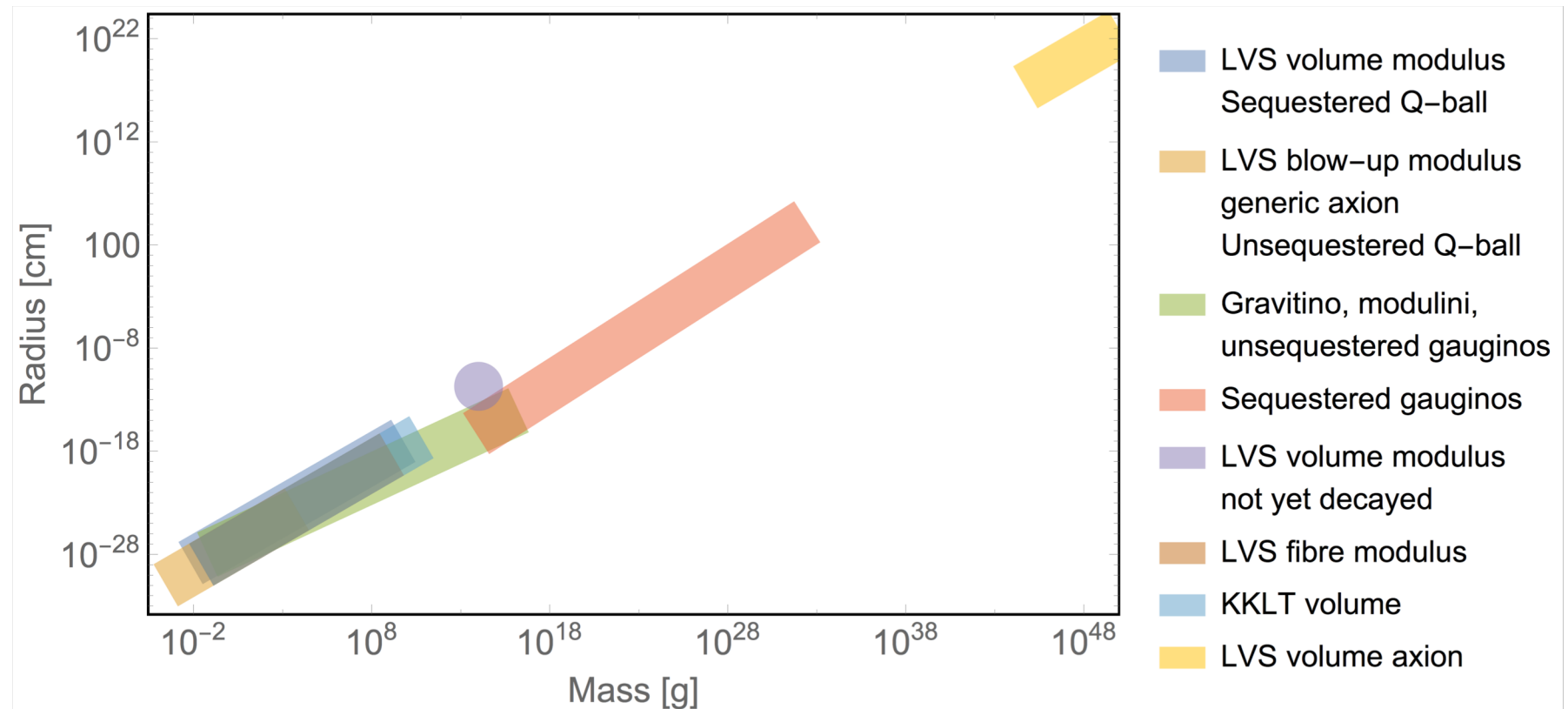
$$\Psi = \left. \frac{\delta_{m,k}(\tau_{\text{dec}})}{\delta_{m,k}(\tau_{\text{mat}})} \right|_{\nu} \approx \left(\frac{M_{\text{P}}}{M_{\text{P}}/\nu^{3/2}}\right)^{4/3} = \nu^2, \quad \text{Enhancement factor!}$$



# Properties of Moduli Stars

Particle	State mass	Star mass	Star radius	Enhancement
LVS volume modulus	$M_{\text{P}}/\mathcal{V}^{3/2}$	$M_{\text{P}}\mathcal{V}^{3/2}$	$l_{\text{P}}\mathcal{V}^{3/2}$	$\mathcal{V}^2$
LVS blow-up modulus Generic axion	$M_{\text{P}}/\mathcal{V}$	$M_{\text{P}}\mathcal{V}$	$l_{\text{P}}\mathcal{V}^{5/3}$	$\mathcal{V}^{4/3}$
LVS fibre moduli	$M_{\text{P}}/\mathcal{V}^{5/3}$	$M_{\text{P}}\mathcal{V}^{5/3}$	$l_{\text{P}}\mathcal{V}^{5/3}$	$\mathcal{V}^{20/9}$
LVS volume axion	$M_{\text{P}}e^{-\alpha\mathcal{V}^{2/3}}$	$M_{\text{P}}e^{\alpha\mathcal{V}^{2/3}}$	$l_{\text{P}}e^{\alpha\mathcal{V}^{2/3}}$	$e^{4/3\alpha\mathcal{V}^{2/3}}$
KKLT volume modulus	$M_{\text{P}} W_0 /\mathcal{V}$	$M_{\text{P}} W_0 ^{-1}\mathcal{V}$	$l_{\text{P}} W_0 ^{-1}\mathcal{V}$	$( W_0 ^{-1}\mathcal{V})^{4/3}$
Gravitino, modulini, unsequestered gauginos	$M_{\text{P}} W_0 /\mathcal{V}$	$M_{\text{P}}\mathcal{V}^2/ W_0 ^2$	$l_{\text{P}}\mathcal{V}^2/ W_0 ^2$	$\mathcal{V}^{4/3}/ W_0 ^{4/3}$
Sequestered gauginos	$M_{\text{P}}/\mathcal{V}^2$	$M_{\text{P}}\mathcal{V}^4$	$l_{\text{P}}\mathcal{V}^4$	$\mathcal{V}^{8/3}$
Unsequestered Q-balls	$M_{\text{P}}/\mathcal{V}$	$M_{\text{P}}\mathcal{V}$	$l_{\text{P}}\mathcal{V}$	$\mathcal{V}^{4/3}$
Sequestered Q-balls	$M_{\text{P}}/\mathcal{V}^{3/2}$	$M_{\text{P}}\mathcal{V}^{3/2}$	$l_{\text{P}}\mathcal{V}^{3/2}$	$\mathcal{V}^2$

# Size and Mass of Moduli Stars



**de Sitter vs Quintessence**

# de Sitter Challenges

- Define S-matrix (resonance?)
- Classical no-go theorems (atoms are unstable classically!)
- No dS solution of string theory under full calculational control (KKLT, LVS,...?)

# Challenges to KKLT, LVS,...

- Fluxes under control only in SUSY 10D      Sethi
- All SUSY breaking part is 4D EFT (with string inputs).  
Trust EFT?
- Tuning  $W_0 \ll 1$ ? in KKLT
- Higher correction in LVS?
- Antibranes (by hand, non susy, singularity?)
- T-branes in a control region?      Bena et al.
- Antibranes and non-perturbative effects?      Moritz et al.

# Swampland conjectures

- Swampland: Quantum gravity vs EFT ! Vafa et al.
- Weak gravity conjecture
- Distance conjecture See M. Reece talk.
- New conjecture:  $M_p \frac{|\nabla V|}{V} \gtrsim c,$  Obied et al

(It would imply quintessence and no de Sitter  
and hard to have inflation!).

# de Sitter Achievements

- Remarkable: well defined prescription exists that includes all stringy ingredients: branes, orientifolds, warping, anti (T)-branes, perturbative, non-perturbative effects, etc.
- IIB with fluxes~ Calabi-Yau (moduli space understood).
- $W_0 \ll 1$  is plausible (not achieved yet) due to the large number of fluxes.
- Perturbative effects in LVS in better control as the volume is exponentially large. All computed so far harmless.
- Antibrane: nonlinearly realised SUSY (nilpotent superfield)
- Hierarchies:  $E \ll M_{\text{KK}} = \frac{M_s}{\mathcal{V}^{1/6}} \ll M_s \equiv \frac{1}{\ell_s} \equiv \frac{1}{2\pi\sqrt{\alpha'}} = g_s^{1/4} \frac{M_p}{\sqrt{4\pi\mathcal{V}}}.$

# Challenges for the new conjecture

- Higgs potential with quintessence field? (at the  $\langle H \rangle = 0$  point. Denef et al.
- If  $V$  asymptotes to infinite from above even supersymmetric AdS forbidden. Conlon
- Both addressed if modify conjecture (allow saddle points for  $V > 0$ ).

see e.g. Andriot

# Quintessence from Strings?

- Need stabilise all moduli except for quintessence field:  
as difficult as getting de Sitter
- Or have many fields rolling but slower than  
quintessence. Difficult.
- Fifth force and varying couplings constraints (e.g.  
volume modulus or dilaton problematic)

# Quintessence Candidates

- Modulus (fibre, blow-up) that does not couple directly to SM. It also would require a very small string scale (e.g.  $M_s \sim \text{TeV}$ )

Cicoli, et al

- Axions

$$\mathcal{L} = -\frac{1}{2}\partial^\mu\theta\partial_\mu\theta - \mu^4 \left(1 - \cos\left(\frac{\theta}{f}\right)\right),$$

K. Choi  
Panda et al  
Kaloper et al.

# Axion Quintessence 1

$$m_a \simeq \sqrt{\frac{g_s}{8\pi}} \frac{M_p}{\mathcal{V}^{2/3}} e^{-\frac{\pi}{N} \mathcal{V}^{2/3}} M_p, \quad \text{Naturally very small!}$$

$$V = \Lambda^4 - \sum_{i=1}^{N_{\text{ULA}}} \Lambda_i^4 \cos\left(\frac{a_i}{f_i}\right) + \dots, \quad \text{Minimum not necessarily at zero}$$

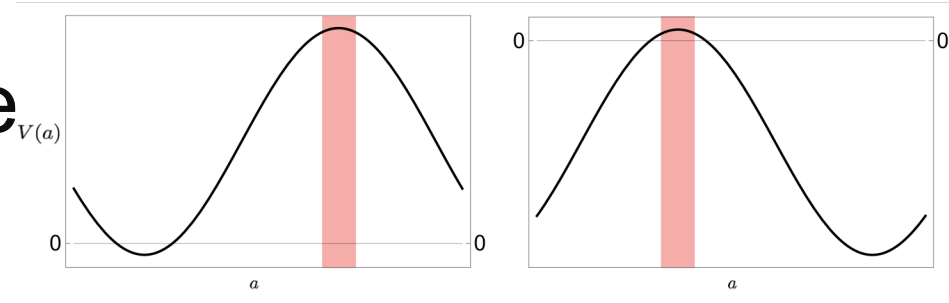
$$\epsilon = \frac{1}{2} \left[ \left( \frac{\Lambda_\ell}{\Lambda} \right)^4 \frac{M_p}{f_\ell} \right]^2 \frac{\sin^2(a_\ell/f_\ell)}{\left( 1 - (\Lambda_\ell/\Lambda)^4 \cos(a_\ell/f_\ell) \right)^2} < 1. \quad \text{Slow-roll}$$

$$f_\ell \gtrsim M_p. \quad \text{Not necessarily}$$

# Axion Quintessence 2

- Hilltop Quintessence

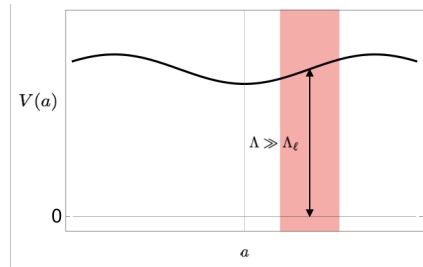
$$\Lambda^4 + \Lambda_i^4 > 0$$



- Quasi-natural quintessence

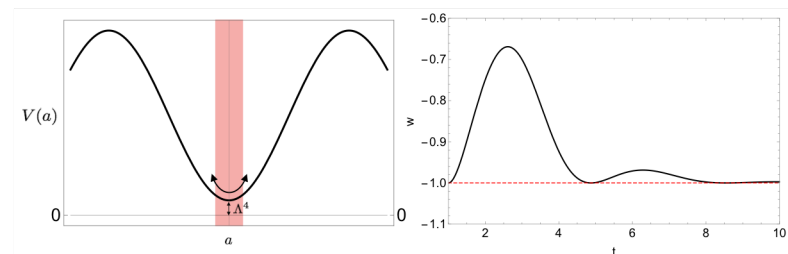
$$f_\ell \gtrsim \left(\frac{\Lambda_\ell}{\Lambda}\right)^4 M_p$$

$$f_\ell < M_p \quad \Lambda \gg \Lambda_\ell.$$



$$w = \frac{p}{\rho} = \frac{\frac{\dot{a}^2}{2} - V}{\frac{\dot{a}^2}{2} + V} \sim -\frac{1 - \frac{1}{3}\epsilon}{1 + \frac{1}{3}\epsilon} \sim -1 + \frac{2}{3}\epsilon.$$

- Oscillating quintessence



# Conclusions 1

- Concrete string models of inflation  $r \leq 10^{-2} - 10^{-3}$
- Strings: Inflation only one component, postinflation is very important
- Strings: Post inflation (dark energy, matter, radiation, different thermal history (moduli domination), baryogenesis, cdm,...)
- Correlations (large  $r$  no TeV SUSY, etc.)

## Conclusions 2

- Rich spectrum of compact objects (oscillons, gravitino, modulini, moduli, axion stars)  
Gravitational waves spectrum ('hear the shape of the extra dimensions?')
- de Sitter vs Quintessence: Many achievements, challenges, open questions

*The report of my death was an exaggeration.*  
*Mark Twain*