

Relativistic stars in degenerate higher-order scalar-tensor theories after GW170817

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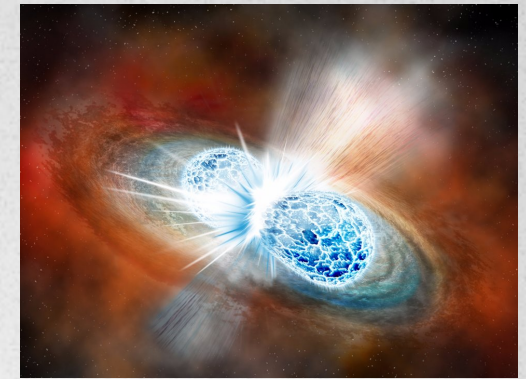


立教大学



GW170817

Abbott *et al.* (2017)



- Nearly simultaneous detection of GWs and gamma-ray burst from binary neutron star merger

$$\longrightarrow |c_{\text{GW}}^2 - 1| < \mathcal{O}(10^{-15})$$

- Tight constraint on modified gravity (**scalar-tensor**, vector-tensor, Horava gravity, ...)

Creminelli & Vernizzi; Sakstein & Jain; Ezquiaga & Zumalacarregui; Baker *et al.*; Gumrukcuoglu *et al.*, Gong *et al.*; Oost *et al.*; Kase & Tsujikawa; ... (2017, 2018)

- *Caveat: This talk concerns modified gravity in the low-redshift Universe ($z \lesssim 0.01$); modification in the early Universe is free from this constraint*

Healthy scalar–tensor theories

before 2015

Horndeski (2+1 dofs)

2nd-order EOMs

GR (2 dofs) $\longrightarrow \bullet$

$$\begin{aligned}\mathcal{L} = & G_2(\phi, X) - G_3(\phi, X)\Box\phi \\ & + G_4(\phi, X)R + G_{4X} [(\Box\phi)^2 - \dots] \\ & + G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi + \dots\end{aligned}$$

The most general scalar-tensor theory with **2nd-order** EOMs
(= trivially Ostrogradsky stable)

Horndeski (1974)

Deffayet *et al.* (2011)

TK, Yamaguchi, Yokoayama (2011)

Healthy scalar–tensor theories

after 2015

Degenerate **H**igher-**O**rder **S**calar-**T**ensor theories = **DHOST**

Horndeski (2+1 dofs)

2nd-order EOMs

GR (2 dofs) $\longrightarrow$ •



Higher-order EOMs, but **degenerate**

$\longrightarrow$ 2+1 dofs, Ostrogradsky stable

Langlois & Noui (2015); Crisostomi *et al.* (2016); Ben Achour *et al.* (2016)

Higher-order EOMs, non-degenerate $\longrightarrow$ More than 3 dofs, Ostrogradsky unstable

Healthy scalar–tensor theories

after GW170817

Horndeski (2+1 dofs)

2nd-order EOMs

GR (2 dofs) → •

$$c_{\text{GW}}^2 = 1$$

Viabke Horndeski after GW170817

Horndeski (2+1 dofs)

2nd-order EOMs

GR (2 dofs) → •

$$\mathcal{L} = G_4(\phi)R + G_2(\phi, X) - G_3(\phi, X)\square\phi$$

$$X := \nabla_\mu\phi\nabla^\mu\phi$$

$$c_{\text{GW}}^2 = 1$$

Viabale DHOST after GW170817

Horndeski (2+1 dofs)

2nd-order EOMs

GR (2 dofs) → •

This talk:
Quadratic DHOST

$$c_{\text{GW}}^2 = 1$$

Quadratic DHOST

Langlois & Noui (2015);
Crisostomi *et al.* (2016)

- Generalization of “G₄” (G₂, G₃ + 6 functions)

$$\mathcal{L} = G_2(\phi, X) - G_3(\phi, X)\Box\phi + \mathcal{L}_{\text{qDHOST}}$$

$$\mathcal{L}_{\text{qDHOST}} = f(\phi, X)R + \sum_{I=1}^5 \mathcal{L}_I$$

$$\sim (\partial\partial\phi)^2 \quad \phi_\mu := \nabla_\mu\phi$$

- Degeneracy conditions
(complicated...)

→ **3** conditions on f, A_I

- $c_{\text{GW}}^2 = 1$ constraint

→ $6 - 3 - 1 = \underline{\mathbf{2}} \text{ free functions}$ (+ G₂, G₃)
(f, A_3)

$$\begin{aligned} \mathcal{L}_1 &:= A_1(\phi, X)\phi_{\mu\nu}\phi^{\mu\nu} \\ \mathcal{L}_2 &:= A_2(\phi, X)(\Box\phi)^2 \\ \mathcal{L}_3 &:= A_3(\phi, X)\Box\phi\phi^\mu\phi_{\mu\nu}\phi^\nu \\ \mathcal{L}_4 &:= A_4(\phi, X)\phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu \\ \mathcal{L}_5 &:= A_5(\phi, X)(\phi^\mu\phi_{\mu\nu}\phi^\nu)^2 \end{aligned}$$

Creminelli & Vernizzi;
Sakstein & Jain;
Ezquiaga & Zumalacarregui;
Baker *et al.* (2017)

Weak gravity in DHOST

$$ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Psi)(dr^2 + r^2 d\Omega^2)$$

$$\frac{d\Phi}{dr} = \frac{G_N M}{r^2} + \Upsilon_1 \frac{G_N M''}{4}$$

← Newtonian

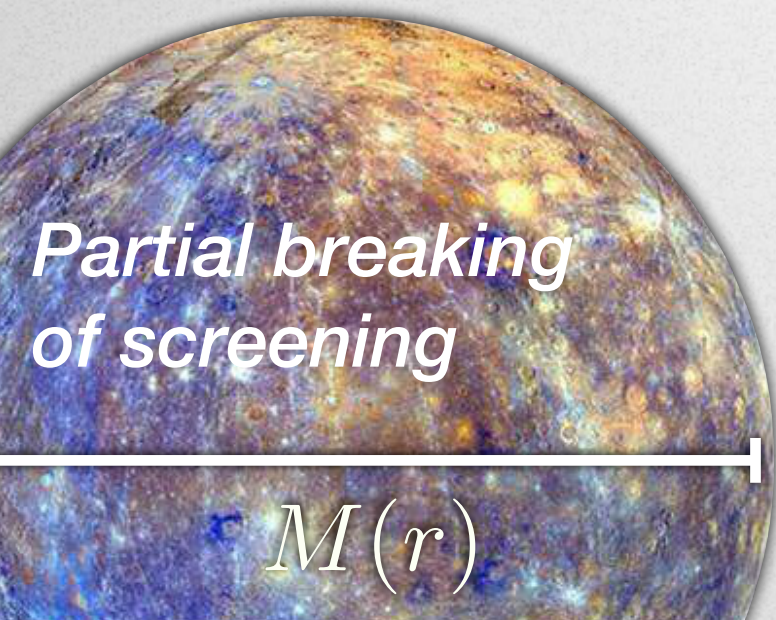
$$\frac{d\Psi}{dr} = \frac{G_N M}{r^2} - \frac{5\Upsilon_2}{4} \frac{G_N M'}{r} + \Upsilon_3 G_N M''$$

← Post-Newtonian,
relativistic

$$M(r) : \text{ enclosed mass} \quad (8\pi G_N)^{-1} = 2f - 2X f_X - \frac{3}{2} X^2 A_3, \quad \Upsilon_i = \Upsilon_i[f, A_3]$$

- Gravity is perfectly GR near and outside the source; modified only inside (**partial breaking of Vanishtein screening**)

TK, Watanabe, Yamauchi (2015);
Crisostomi & Koyama (2017);
Langlois et al. (2017);
Dima & Vernizzi (2017)



*Partial breaking
of screening*

Successful screening

$M(r)$

$M = \text{const}$

Testing Vainshtein–breaking theories

- Constraints on Υ_i : not so strong

Koyama & Sakstein (2015); Jain *et al.* (2015);
Sakstein *et al.* (2016, 2017); Salzano *et al.* (2017);
Saltas *et al.* (2018)

- Newtonian stellar structure

Gravity must be attractive
near the center

$$-2/3 < \Upsilon_1 < 1.6$$

Minimum mass for hydrogen burning
< Mass of observed red dwarfs

Sakstein (2015)

Saito *et al.* (2015)

- improved using WDs: $\Upsilon_1 \leq 0.14$

Saltas *et al.* (2018)

- X-ray + lensing profiles of galaxy clusters ($\Upsilon_3 = 0$ theories)

$$\Upsilon_1 = -0.11^{+0.93}_{-0.67}, \quad \Upsilon_2 = -0.22^{+1.22}_{-1.19}$$

Sakstein *et al.* (2016)

This work:

Babichev *et al.* (2016);
Sakstein *et al.* (2016)

Go to strong gravity regime! — **Relativistic stars**

TOV system in qDHOST

■ Lagrangian:

Cf. Chagoya & Tasinato (2018); De Felice *et al.* (2015);
Kase *et al.* (2016) > angular deficit

$$\mathcal{L} = \cancel{G_2 - G_3 \Box \phi} + \mathcal{L}_{\text{qDHOST}} + \mathcal{L}_{\text{matter}}$$

$$\mathcal{L}_{\text{qDHOST}} = f(X)R + A_3(X)\Box\phi\phi^\mu\phi_{\mu\nu}\phi^\nu + \dots$$

Shift-symmetric qDHOST

$$X := \nabla_\mu \phi \nabla^\mu \phi$$

■ Ansatz:

$$ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 d\Omega^2$$

$$\phi = vt + \psi(r)$$

$$T_\mu^\nu = \text{diag}(-\rho, P, P, P)$$

TOV system in qDHOST

- Field equations: $\mathcal{E}_\mu^\nu = T_\mu^\nu$, $\mathcal{E}_\phi = 0$, $\nabla_\nu T_\mu^\nu = 0$

$$\mathcal{E}_t^t = b_1 \nu'' + b_2 X'' + \dots = -\rho$$

$$\mathcal{E}_r^r = c_1 \nu'' + c_2 X'' + \dots = P$$

$$\mathcal{E}_{tr} = d_1 \nu'' + d_2 X'' + \dots = 0 \quad (\longrightarrow \mathcal{E}_\phi = 0 \text{ is automatic})$$

$$P' = -\frac{\nu'}{2}(\rho + P)$$

$$X = -e^{-\nu} v^2 + e^{-\lambda} (\psi')^2$$

- X is more convenient than ψ'
- Higher derivatives can be eliminated in the end by combining the equations because the system is *degenerate*

TOV system in qDHOST

- Final form of the field equations:

$$e^\lambda = \mathcal{F}_\lambda(\nu, \nu', X, X', P)$$

$$X' = \mathcal{F}_1(\nu, X, \rho, P)\nu' + \frac{\mathcal{F}_2(\nu, X, \rho, P)}{r}$$

$$\nu' = \mathcal{F}_3(\nu, X, \rho, \rho', P)$$

$$P' = -\frac{\nu'}{2}(\rho + P) \quad + \text{EOS}$$

- Explicit expressions of the right-hand sides are messy

Here we present the explicit expression for $\mathcal{F}_1$, $\mathcal{F}_2$, and $\mathcal{F}_3$ that appear in Eqs. (30) and (31):

$$\mathcal{F}_1 = \frac{U_1}{V}, \quad \mathcal{F}_2 = \frac{U_2}{V}, \quad \mathcal{F}_3 = \frac{U_3}{W}, \quad (\text{A1})$$

where

$$\frac{U_1}{2fe^\nu X^2} = 2r^2 v^2 (P + \rho) (B_1 f - X f_X) + e^\nu X \{ B_1 f [r^2 (\rho - 5P) - 8f] + 2X f_X (4f + Pr^2) \}, \quad (\text{A2})$$

$$\frac{U_2}{2fe^\nu X^2} = -2v^2 (B_1 f - X f_X) [4f + r^2 (P - \rho)] + e^\nu X \{ B_1 f [r^2 (\rho - 5P) - 8f] + 2X f_X (4f + Pr^2) \}, \quad (\text{A3})$$

$$V = -8r^2 v^4 (P + \rho) (B_1 f - X f_X)^2 + 2e^\nu v^2 X (X f_X - B_1 f) \{ B_1 f [r^2 (5\rho - 9P) - 20f] + 4X f_X (4f + Pr^2) \} \\ + 3B_1 f e^{2\nu} X^2 \{ B_1 f [8f + r^2 (5P - \rho)] - 2X f_X (4f + Pr^2) \}, \quad (\text{A4})$$

$$U_3 = -16e^{3\nu} f r^4 f_X^2 [B_1 (X f_X - 2f) + X (f B_{1X} + f_X - X f_{XX})] (\rho - 3P)^2 X^5 \\ + 8e^{2\nu} r^2 (f B_1 - X f_X)^2 \{ 16[e^\nu f_X + (v^2 + e^\nu X) f_{XX}] (3P - \rho) f^2 \\ + 2[f_{XX} (\rho - 3P) ((2v^2 + e^\nu X) \rho - (2v^2 + 5e^\nu X) P) r^2 + e^\nu f_X (\rho^2 - 8P\rho + 15P^2) r^2 \\ + 4f_X^2 (6(v^2 + 2e^\nu X) \rho - 18(v^2 + 2e^\nu X) P + r(v^2 + 3e^\nu X) \rho')] f \\ + r^2 f_X^2 [(-408e^\nu X P^2 + (8(21v^2 + 29e^\nu X) \rho + r(28v^2 + 33e^\nu X) \rho') P \\ - \rho(8(7v^2 + 4e^\nu X) \rho + r(16v^2 + 9e^\nu X) \rho'))] \} X^4 \\ - 2e^{2\nu} r^2 f_X (4f B_1 - 4X f_X) (3P - \rho) \{ -e^\nu r^2 f_X^2 (10\rho - 30P + 3r\rho') X^3 \\ + 16f^2 (v^2 + e^\nu X) [B_1 (X f_X - 2f) + X (f B_{1X} + f_X - X f_{XX})] \\ + 2f r^2 [\rho (e^\nu X^3 f_{XX} - (2v^2 + e^\nu X) (B_1 (X f_X - 2f) + X (f B_{1X} + f_X - X f_{XX}))) \\ - (3e^\nu X^3 f_{XX} - (2v^2 + 5e^\nu X) (B_1 (X f_X - 2f) + X (f B_{1X} + f_X - X f_{XX}))) P] \} X^3 \\ + 8e^\nu (f B_1 - X f_X)^3 \{ f_X [2(68v^4 - 24e^\nu X v^2 - 305e^{2\nu} X^2) P^2 + (4(8v^4 + 122e^\nu X v^2 + 73e^{2\nu} X^2) \rho \\ + r(12v^4 + 76e^\nu X v^2 + 39e^{2\nu} X^2) \rho') P - (2v^2 + e^\nu X) \rho (52\rho v^2 + 34e^\nu X \rho + r(14v^2 + 9e^\nu X) \rho')] r^4 \\ + 2f [e^\nu (\rho - 5P) ((2v^2 + e^\nu X) \rho - (2v^2 + 5e^\nu X) P) r^2 + 8f_X (3(2v^4 + 9e^\nu X v^2 + 5e^{2\nu} X^2) \rho \\ + 3(2v^4 - 9e^\nu X v^2 - 19e^{2\nu} X^2) P + r(v^4 + 6e^\nu X v^2 + 3e^{2\nu} X^2) \rho')] r^2 + 128e^\nu f^3 (v^2 + e^\nu X) \\ + 16e^\nu f^2 [-(3v^2 + 2e^\nu X) \rho r^2 + (7v^2 + 10e^\nu X) P r^2 - 8X (v^2 + e^\nu X) f_X] \} X^2 \\ - 8(f B_1 - X f_X)^4 \{ 4(16v^6 - 60e^\nu X v^4 + 20e^{2\nu} X^2 v^2 + 75e^{3\nu} X^3) P^2 r^4 \\ + (2v^2 + e^\nu X)^2 (4v^2 + 3e^\nu X) \rho (4\rho + r\rho') r^4 - 8e^\nu f_X (2v^2 + e^\nu X) [6(4v^2 + 3e^\nu X) \rho + r(5v^2 + 3e^\nu X) \rho'] r^2 \\ - P [-(2v^2 + e^\nu X) (8(8v^4 - 12e^\nu X v^2 - 15e^{2\nu} X^2) \rho + r(8v^4 - 18e^\nu X v^2 - 15e^{2\nu} X^2) \rho') r^2 \\ - 48e^\nu f_X (-8v^4 + 10e^\nu X v^2 + 15e^{2\nu} X^2)] r^2 + 384e^{2\nu} f^2 X^2 (v^2 + e^\nu X) \}, \quad (\text{A5})$$

$$\frac{W}{r} = 16e^{3\nu} f r^4 f_X^2 [B_1 (X f_X - 2f) + X (f B_{1X} + f_X - X f_{XX})] (\rho - 3P)^2 X^5 \\ - 4e^{2\nu} r^2 (f B_1 - X f_X)^2 \{ [3r^2 (16v^2 - 77e^\nu X) P^2 + 4((3v^2 + 71e^\nu X) \rho r^2 + 6f (v^2 - 9e^\nu X)) P \\ + \rho (24f (v^2 + 7e^\nu X) - r^2 (36v^2 + 61e^\nu X) \rho)] f_X^2 - 4e^\nu f (\rho - 3P) (-\rho r^2 + 5P r^2 + 8f) f_X \\ - 4f f_{XX} (\rho - 3P) [-2v^2 \rho r^2 - (2v^2 - 5e^\nu X) P r^2 + e^\nu X (8f - r^2 \rho)] \} X^4 \\ + e^{2\nu} r^2 f_X (4f B_1 - 4X f_X) (3P - \rho) \{ -e^\nu r^2 f_X^2 (17\rho - 15P) X^3 + 32e^\nu f^2 [B_1 (X f_X - 2f) + X (f B_{1X} + f_X - X f_{XX})] X \\ + 4f r^2 [\rho (e^\nu X^3 f_{XX} - (2v^2 + e^\nu X) (B_1 (X f_X - 2f) + X (f B_{1X} + f_X - X f_{XX}))) \\ - (3e^\nu f_{XX} X^3 + (2v^2 - 5e^\nu X) (B_1 (X f_X - 2f) + X (f B_{1X} + f_X - X f_{XX}))) P] \} X^3 \\ - 4e^\nu (f B_1 - X f_X)^3 \{ 4e^\nu f (-\rho r^2 + 5P r^2 + 8f) [-2v^2 \rho r^2 - (2v^2 - 5e^\nu X) P r^2 + e^\nu X (8f - r^2 \rho)] \\ - f_X [5(20v^4 - 40e^\nu X v^2 + 79e^{2\nu} X^2) P^2 r^4 + 4v^4 \rho (12f - 5r^2 \rho) r^2 + 36e^\nu v^2 X \rho (3r^2 \rho - 4f) r^2 \\ + 2((40v^4 - 46e^\nu X v^2 - 199e^{2\nu} X^2) \rho r^2 + 24f (v^4 - 3e^\nu X v^2 + 18e^{2\nu} X^2)) P r^2 + e^{2\nu} X^2 (71\rho^2 r^4 - 480f \rho r^2 + 256f^2)] \} X^2 \\ + 4(f B_1 - X f_X)^4 \{ -48r^4 \rho^2 v^6 + 12e^\nu r^2 X \rho (\rho r^2 + 20f) v^4 + 8e^{2\nu} r^2 X^2 \rho (9r^2 \rho - 47f) v^2 \\ - r^4 (48v^6 - 180e^\nu X v^4 + 200e^{2\nu} X^2 v^2 - 225e^{3\nu} X^3) P^2 + 3e^{3\nu} X^3 (9\rho^2 r^4 - 104f \rho r^2 + 256f^2) \\ - 4r^2 [24r^2 \rho v^6 - 12e^\nu X (4\rho r^2 + 5f) v^4 + 2e^{2\nu} X^2 (16\rho r^2 + 47f) v^2 - 15e^{3\nu} X^3 (14f - 3r^2 \rho)] P \}. \quad (\text{A6})$$

Boundary conditions

- Exact exterior solution: Schwarzschild

$$e^\nu = e^{-\lambda} = 1 - \frac{2G_N \mu}{r}, \quad \mu = \text{const}$$
$$X(r) = -v^2$$

- Center

$$\nu = \nu_c, \quad X = -e^{-\nu_c} v^2 \quad (\leftarrow \psi'(0) = 0), \quad \rho = \rho_c$$

 shooting parameter

- Matching at the surface $r = R$

$$X(R) = -v^2, \quad e^{\nu(R)} = 1 - \frac{2G_N \mu}{R}$$

 integration const
determined

Concrete model

- Model: $f = \frac{M_{\text{Pl}}^2}{2} + \alpha X^2$, $A_3 = -8\alpha - \beta$ ($\beta = 0 \rightarrow \text{GLPV}$)
- Viable self-accelerating cosmology Crisostomi & Koyama (2017)
- Dimensionless parameters

$$\bar{\alpha} = \frac{\alpha v^4}{M_{\text{Pl}}^2}, \quad \bar{\beta} = \frac{\beta v^4}{M_{\text{Pl}}^2}$$

Vainshtein-breaking in weak gravity regime

$$\Upsilon_i \sim \bar{\alpha}, \bar{\beta} \lesssim \mathcal{O}(0.1)$$

Hulse-Taylor pulsar constraint

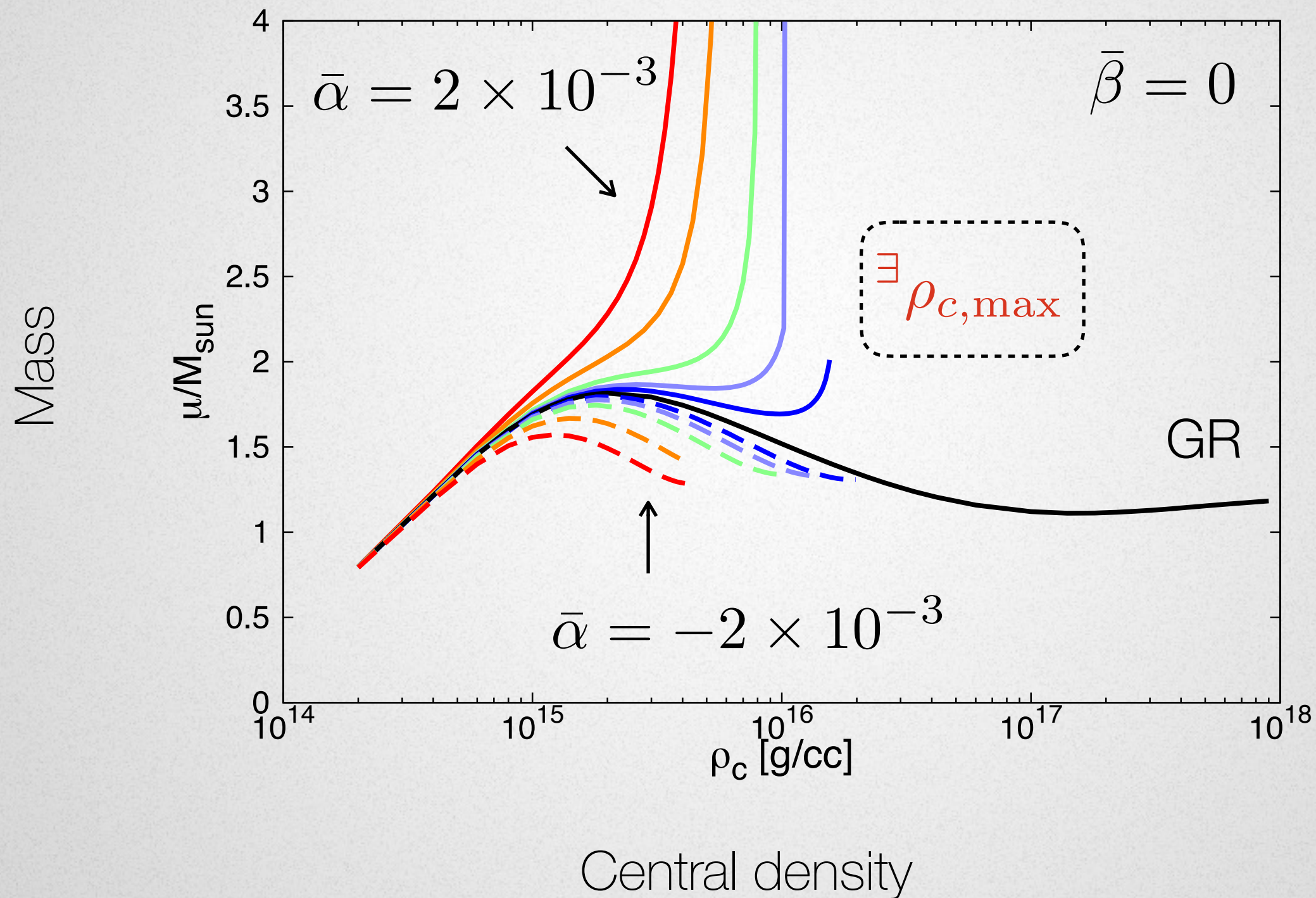
Beltran Jimenez *et al.* (2015);
Dima & Vernizzi (2017)

“G” for GWs 

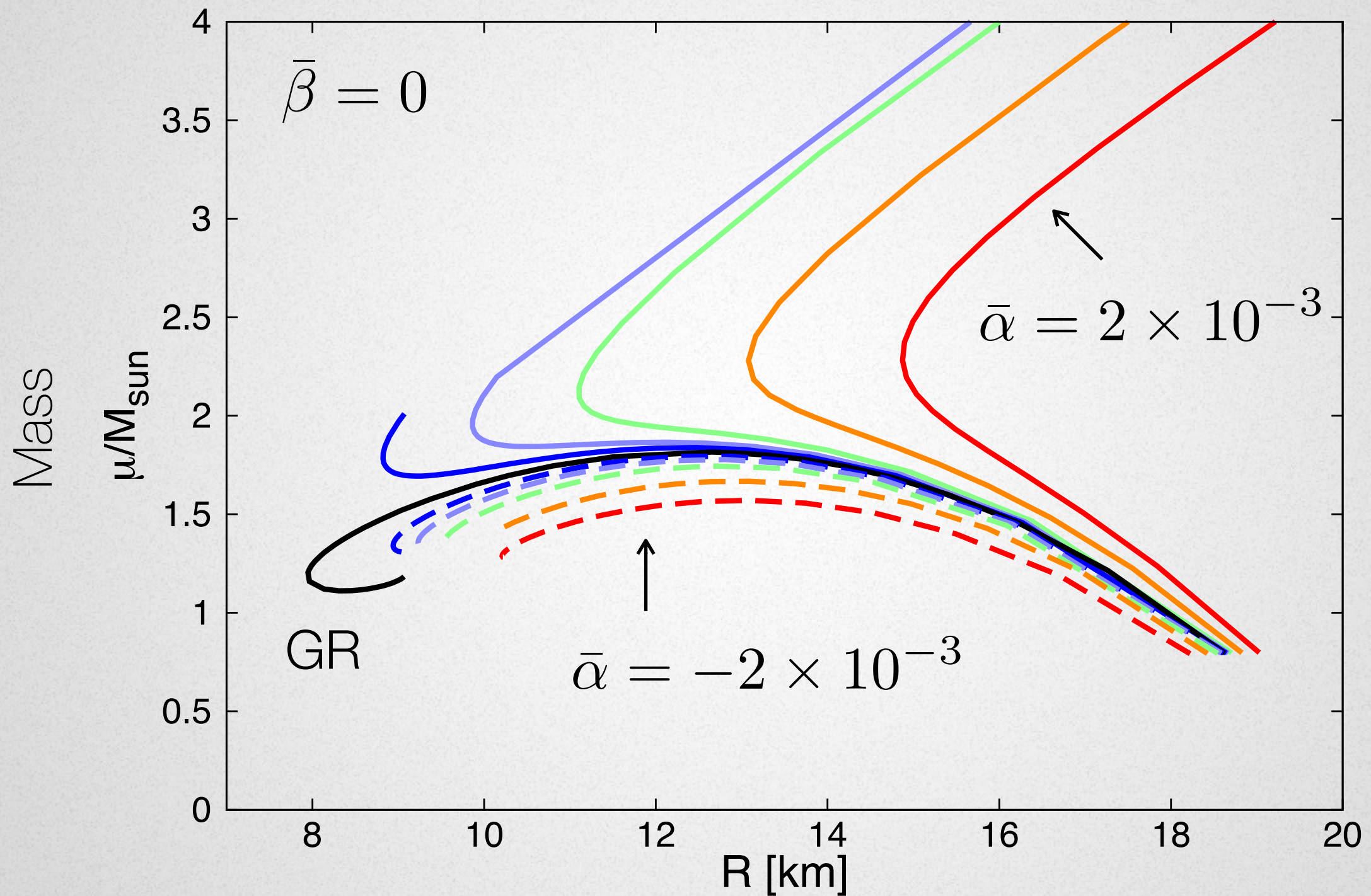
$$G_{\text{GW}}/G_N - 1 \sim \bar{\alpha}, \bar{\beta} < \mathcal{O}(10^{-3})$$

Results

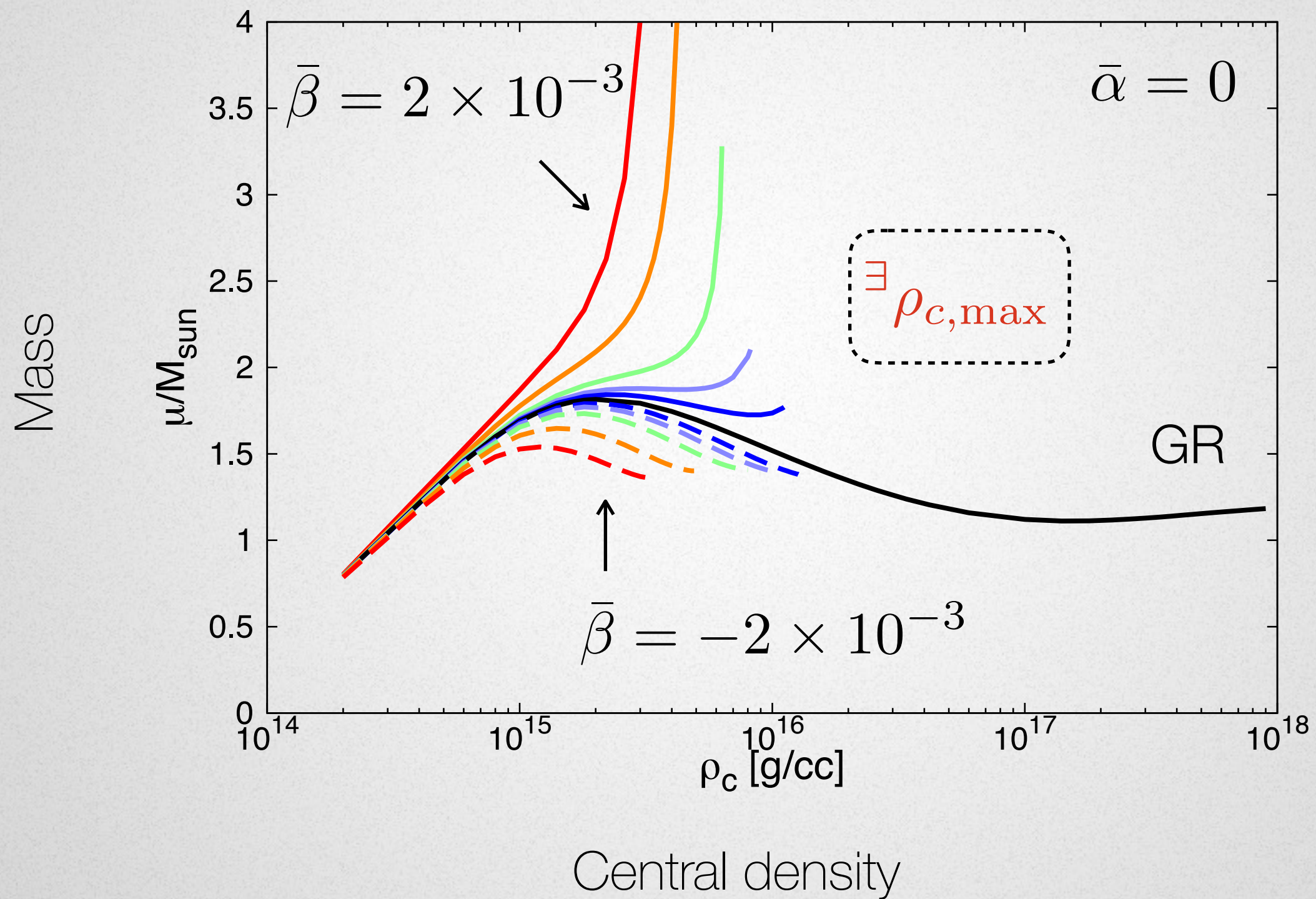
$$\rho = \left(\frac{P}{K} \right)^{1/2} + P, \quad K = 123 M_{\odot}^2$$



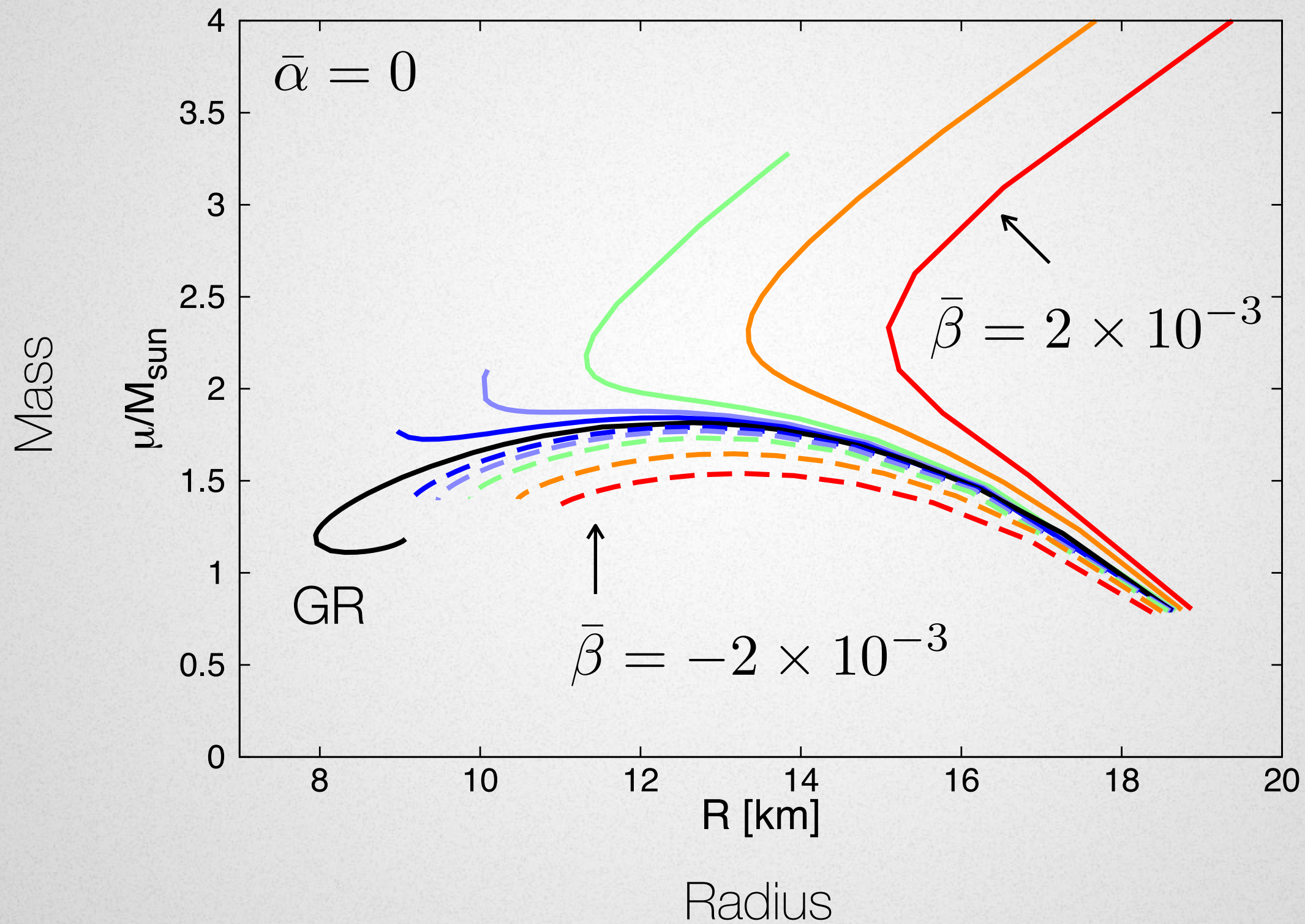
Results



Results



Results



Summary

- qDHOST survived after GW170817
 - Partial breaking of Vainshtein screening inside matter
 - Constraints in weak gravity regime still not so strong
- Toward strong gravity tests: Relativistic stars in qDHOST
 - Large modification even if parameters are small and only tiny corrections are expected in weak gravity regime
 - To what extent model-dependent results?