The no-boundary proposal in
loop quantum cosmology

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Why quantum cosmology?

“... it appears that quantum theory would have to modify not only Maxwellian electrodynamics, but also the new theory of gravitation.” – Albert Einstein, 1916.

→ Need to look for indirect probes for quantum gravity in early universe cosmology when very high energy scales were naturally reached ⇒ Quantum cosmology.
Ingredients for quantum cosmology

→ Quantum cosmology entails treating the universe as a quantum system.

→ Two parts of the final theory:

  • The Hamiltonian (or action) determines the dynamics ⇒ Corrections from quantum gravity?
  • The quantum state of the universe ⇒ Initial conditions Set by some ‘topological’ principle?
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The no-boundary proposal

→ **Restrict to minisuperspace, spatially closed cosmologies with a cosmological constant or a single scalar field.**

→ Wavefunction specified by the value of the 3–metric and spatial field configuration on a final spacelike surface \( \Sigma \Rightarrow \Psi = \Psi[h_{ab}, \chi] \)

→ **Saddle-point approximation** [J. Hartle & S. Hawking, 1983]

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\Psi[h_{ab}, \chi] := \int (h, \chi) \mathcal{D}[g] \mathcal{D}[\varphi] e^{-S[g, \varphi]/\hbar} \approx e^{-S_{\text{ext}}[h_{ab}, \chi]/\hbar}
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→ **No-boundary saddle-points**: Extrema of the action (generally complex but Euclidean for the simplest cases), with \((h_{ab}, \chi)\) on the boundary at late times and are regular everywhere else.

→ **Quantum completion for inflation ⇒ Principle for setting initial conditions for cosmological perturbations.**

→ For minisuperspace models, this implies the boundary conditions \(a(0) = 0, \ \varphi(0) = 0\). (Regularity at the South Pole)
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Basic continuum quantities of spatial geometry, such as areas and volumes, are represented by operators with discrete spectra. An infinitesimal change of these quantities in time — or, more geometrically, the extrinsic curvature of space — no longer has a linear and local expression in space but is instead exponentiated and extended one-dimensionally, along an eponymous loop. [A. Ashtekar, M. Bojowald, T. Thiemann ...]

For a cosmological model, they imply two main corrections:

- **Holonomy modifications**: No operator for extrinsic curvature $\dot{a}$ or the Hubble parameter $\dot{a}/a$ ⇒ Well-defined operators only for $SU(2)$ holonomy matrix elements, which are periodic functions such as $\dot{a} \to \sin(\ell(a)\dot{a})/\ell(a)$ with $\ell(a) \sim l_P/a$.

- **Inverse-volume corrections**: Using $\hat{h}^{-1}[\hat{h}, \sqrt{\hat{a}}] = -\frac{1}{2} \hbar \ell \hat{a}^{-1/2}$ (where $\hat{h} = \exp(i\ell p_a)$) to get $a^{-1} = f(a)/a$ with $f(a)$ some quantum correction function which goes to 1 for large $a$. The small-$a$ behaviour eliminates the divergence of a direct inverse at $a = 0$. 
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Loop quantum gravity corrections

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In the path integral form for the no-boundary proposal, this implies replacing the Einstein-Hilbert action by an effective LQC action, which includes the said corrections.

In the canonical picture, instead of solving the standard WDW operator, one solves a “difference” equation in LQC → Quantum geometry corrections imply a modified Hamiltonian constraint in $\hat{H}_{\text{LQC}} \psi = 0$. Still need boundary conditions for specific solutions. Naturally, the Friedmann equation is also modified in LQC as a result.

The role played by modified constraints crucial in LQG ⇒ They result in deformed gauge transformations. Since background is modified, covariant perturbations imply an effective line-element $ds_β^2 = -βN^2dt^2 + a(t)^2dΩ_k$ where $β(a, \dot{a})$ changes sign at large curvature resulting in dynamical signature change.

South-Pole regularity conditions modified for LQC –

EH: $a(0) = 0, \dot{a}(0) = 1$ ⇔ LQC: $a(0) = 0, \dot{a}(0) = 0$
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In the canonical picture, instead of solving the standard WDW operator, one solves a “difference” equation in LQC \( \Rightarrow \) Quantum geometry corrections imply a modified Hamiltonian constraint in \( \hat{H}_{LQC} \psi = 0 \). Still need boundary conditions for specific solutions. Naturally, the Friedmann equation is also modified in LQC as a result.

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South-Pole regularity conditions modified for LQC –
\( \text{EH: } a(0) = 0, \ \dot{a}(0) = 1 \ \Leftrightarrow \text{LQC: } a(0) = 0, \ \dot{a}(0) = 0 \)
Hartle-Hawking proposal

→ For minisuperspace cosmologies, in the saddle-point approximation, the no-boundary wavefunction simplifies

\[ \Psi_{HH}[\tilde{a}, \chi] \approx e^{-S_{EH}[\tilde{a}, \chi]/\hbar} \]

→ For simplest models, say with only a cosmological constant, our (Lorentzian) universe tunnels from nothing via an Euclidean region.

→ Friedmann equation: \[ \dot{a}^2 = -V(a) \] and on-shell action

\[ S_{EH} = -\frac{3\pi}{2} \int_0^{\tilde{a}} a \sqrt{|V(a)|}. \]

The nucleation probability of a universe \( \mathcal{P} \approx e^{-2S_{LQC}} \).
Pure de Sitter

[S.B. & D.-h. Yeom, 2018]

\[-\dot{a}^2 = \mathcal{V} := \frac{8\pi a^2}{3} f^2(a) \left[ \frac{\rho}{f'(a)} - \rho_1 \right] \left[ \frac{\rho_2 - \frac{\rho}{f(a)}}{\rho_c} \right] \]

\[\rightarrow\] A typical solution $a(\tau)$ for some numerical values of $\Lambda$ & $l_{Pl}$.

\[\rightarrow -2S_E^{LQC} \simeq \frac{A}{4} + c + d \log A, \quad d > 0 \text{ where } A = 4\pi \dot{a}^2\]

\[\rightarrow\] LQC correction rather small $\Rightarrow$ There is a potential barrier for both EH ($-\dot{a}^2 \sim -1 + \Lambda a^2$) and LQC scenarios.
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\rightarrow \text{LQC correction rather small } \Rightarrow \text{There is a potential barrier for both EH} \quad (-\ddot{a}^2 \sim -1 + \Lambda a^2) \text{ and LQC scenarios.}
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Massless scalar field

→ Usual KG equation $\ddot{\varphi} + 3H\dot{\varphi} = 0 \Rightarrow \dot{\varphi} = 0$ and non-dynamical solution. In EH theory, no way to get interesting solutions.

→ Modified equations of motion [S.B. & D.-h. Yeom, 2018]

$$V = \frac{8\pi G}{3} a^2 f^2(a) \left[ \frac{a^6}{4\sqrt{3}\gamma^3 l_P^6} \left( \frac{\rho}{\rho_c} \right) \left( \frac{g(a)}{f(a)} \right) - \rho_1 \right] \left[ \frac{1}{\rho_c} \left( \rho^2 - \frac{a^6}{4\sqrt{3}\gamma^3 l_P^6} \left( \frac{\rho}{\rho_c} \right) \left( \frac{g(a)}{f(a)} \right) \right) \right]$$

$$\ddot{\varphi} - \left( \frac{\dot{B}(a)}{B(a)} \right) \dot{\varphi} = 0 \text{ Classically, } B(a) \sim a^{-3} \text{ & } B(a) \sim a^{12} \text{ in QG regime}$$

→ New instantonic solutions for NBWF $\leftrightarrow$ New physical interpretations for LQC
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→ New instantonic solutions for NBWF ⇔ New physical interpretations for LQC
Loops rescue the no-boundary proposal

→ Euclidean $\int^h D[g] e^{-S_E/h}$ (compact Euclidean 4-geometries bounded by $h$) vs. Lorentzian $\int_0^h D[g] e^{iS/h}$ (Lorentzian 4-geometries interpolating between a vanishing initial 3-geometries and $h$). [HH, 1983; A. Vilenkin, 1982]

→ Euclidean path integral diverges for $\Lambda > 0$ for all contours of the lapse ⇒ Lorentzian path integral can be made well-defined by applying Piecard-Lefshetz theory to yield a convergent integral by deforming the lapse contour.

→ Unsuppressed runaway perturbations on the final 3-geometry due to an inverse Gaussian weighting for perturbations ⇒ Old problem of the scale factor having wrong-sign kinetic term. [J. Feldbrugge, J.-l. Lehners & N. Turok, 2017]

→ Dynamical signature change in makes these inverted Gaussians have the correct sign for having a Bunch-Davies state at the onset of inflation. [M. Bojowald & S.B., 2018]
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Timeless stability of perturbations

→ The mode equation
\[ \ddot{\nu} \approx \frac{1}{4} \left( (n - 2\epsilon)(n + 2) + \epsilon(\epsilon + 2) - \beta \frac{N^2 \ell(\ell+2)}{c^2} \right) \frac{\nu}{t^2}, \]
and its solution is \( \nu_+ = \nu_1 t^{\frac{1}{2}(1+\gamma)} \) where

\[ \gamma = \sqrt{1 + n(n + 2) - \beta \frac{\ell(\ell + 2)N^2}{c^2}} \]

→ For EH, \( \beta = 1, n = 0 \), \( \gamma \) and the solutions \( \nu_{\pm} \) have branch cuts on the real \( N \)-axis ⇒ The action evaluated on the regular solution \( \nu_+ \) is equal to \( S_+(\nu_1) = \frac{1}{4} N^{-1}(\gamma - 1)\nu_1^2 \) and has a negative imaginary part above the branch cut. This result leads to a Gaussian with positive exponent in the path integral of perturbations.

→ With dynamical signature change, that is \( \beta < 0 \), \( \gamma \) is always real for real \( N \). Its branch cuts in the complex plane are now on the imaginary \( N \)-axis where they do not affect the Lorentzian path integral ⇒ The action \( S_+ \) is always real and finite and does not lead to unbounded contributions to the path integral.
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Summary

→ Conclusions: Fruitful confluence between different approaches to quantum cosmology.

- Loops provide necessary quantum geometry corrections which expands the solution space for the no-boundary proposal.
- The no-boundary wave function is necessary to discover new physical phenomenon in loops which cannot be probed otherwise.
- Remarkable similarity in dynamical signature-change coming loops and the Euclidean (generally, complex) phase in the Hartle-Hawking proposal.

→ Looking ahead:

- Implications for LQC corrections in other type of models of the NB proposal – Hawking-Turok instanton? Perhaps some of the divergences of the instantonic solutions ameliorated by loops?
- No-boundary state made compatible with dynamical signature-change ⇒ New route towards dS/CFT? [Hartle, Hertog, Hawking ... ]
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