Thermodynamics of Dark Energy Models in Loop Quantum Cosmology

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Layout

- Cosmological Parameters
- Thermodynamics Analysis

The Friedmann equations for loop quantum cosmology (LQC) are given by

$$H^2 = \frac{\rho}{3}\left(1 - \frac{\rho}{\rho_1}\right), \tag{1}$$

$$\dot{H} = -\frac{\rho + p}{3}\left(1 - 2\frac{\rho}{\rho_1}\right). \tag{2}$$

We consider the universe to be comprised of two components such as scalar field dark energy (DE) model and dark matter (DM). For this scenario, the Friedmann equations can be written in effective form as

$$3H^2 = \rho_{eff}\left(1 - \frac{\rho_{eff}}{\rho_1}\right), \tag{3}$$

$$\dot{H} = -\frac{1}{2}(\rho_{eff} + p_{eff})\left(1 - \frac{2\rho_{eff}}{\rho_1}\right). \tag{4}$$

where $\rho_{eff} = \rho_\phi + \rho_m$.

It is assumed that the scalar field and DM components do not conserve separately but interact with each other through a source term Q (Ferreira et al. PRD 2017).

The conservation equations can be expressed as

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = -Q, \quad \dot{\rho}_m + 3H(\rho_m + p_m) = Q. \tag{5}$$

Following (Das et al. APSS 2015), Q can be expressed as

$$Q = \zeta H(\dot{\phi})^4, \tag{6}$$

where ζ appears as a constant. We consider cold DM which is pressureless ($p_m = 0$) and leads to $p_{eff} = p_\phi$.

However, the energy density and pressure of non-canonical scalar field can be defined as (Das and Mamon, APSS 2015)

$$\rho_\phi = \frac{3}{4}(\dot{\phi})^4 + V(\phi), \quad (7)$$

$$p_\phi = \frac{1}{4}(\dot{\phi})^4 - V(\phi). \quad (8)$$

In the following, we consider two different forms of equation of state (EoS) parameter ω_ϕ for elaborating the cosmological parameters such as

- Constant EoS parameter $(\omega_\phi = \omega)$
- Variable EoS parameter $(\omega_\phi = \omega(z))$

Constant EoS Parameter:

In this case, we consider EoS parameter ω_ϕ as constant (ω) which can be written as

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} = \frac{X^2 - V}{X^2 + V} = \omega. \tag{9}$$

With the help of above assumption, the various components of non-canonical scalar field model can be expressed as (Das and Mamon, APSS 2015)

$$V(z) \;=\; v_0(1+z)^\epsilon, \quad \rho_\phi = \frac{4}{1-3\omega}V(z), \quad p_\phi = \frac{4\omega}{1-3\omega}V(z). \tag{10}$$

Using Eq.(5), we evaluate ρ_m as

$$\rho_m = \frac{4v_0\zeta(1+\omega)}{(1-3\omega)(3-\epsilon)}(1+z)^\epsilon + (1+z)^3. \tag{11}$$

Here $\epsilon = (1+\omega)(3+\zeta)$ and v_0 is a positive constant.

To investigate the stability of this particular interacting non-canonical scalar field model, we calculate square speed of sound which is defined as

$$C_s^2 = \frac{\partial p_{eff}}{\partial \rho_{eff}} = \frac{\frac{\partial p_{eff}}{\partial z}}{\frac{\partial \rho_{eff}}{\partial z}}. \tag{12}$$

For this EoS parameter, the above expression for square speed of sound turns out to be

$$C_s^2 = \frac{\frac{4\omega v_0 \epsilon (1+z)^{\epsilon-1}}{1-3\omega}}{\frac{4v_0\epsilon(1+z)^{\epsilon-1}}{1-3\omega} - \frac{4v_0\zeta(1+\omega)\epsilon(1+z)^{\epsilon-1}}{(1-3\omega)(3-\epsilon)} + 3(1+z)^2}. \tag{13}$$

It can be observed that square speed of sound remains positive for all three selected values of ω. This exhibits the stability of under consideration interacting non-canonical scalar field model.

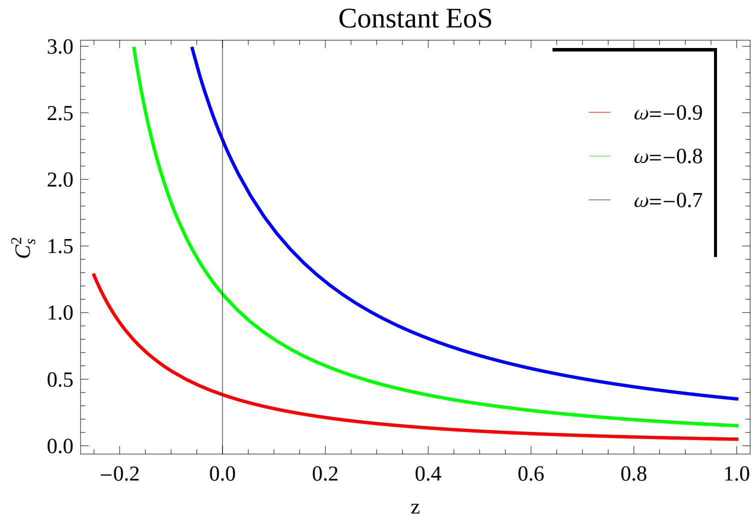

Figure 1: Plot of C_s^2 versus z. The values of other constant parameters are $v_0 = 2$ and $\zeta = 1$

Using Eqs.(10) and (11), we can get

$$\rho_{eff} + p_{eff} = \frac{4(1+\omega)}{1-3\omega}V(z) + \frac{4v_0\zeta(1+\omega)}{(1-3\omega)(3-\epsilon)}(1+z)^{\epsilon} + (1+z)^3. \tag{14}$$

The plot of $\rho_{eff} + p_{eff}$ versus redshift parameter is shown in Figure **2** which exhibits the quintessence as well as vacuum behavior of the universe ($\because 0 \leq \rho_{eff} + p_{eff} \leq 0.7$ which leads to $-1 \leq \omega_{eff} \leq -\frac{1}{3}$).

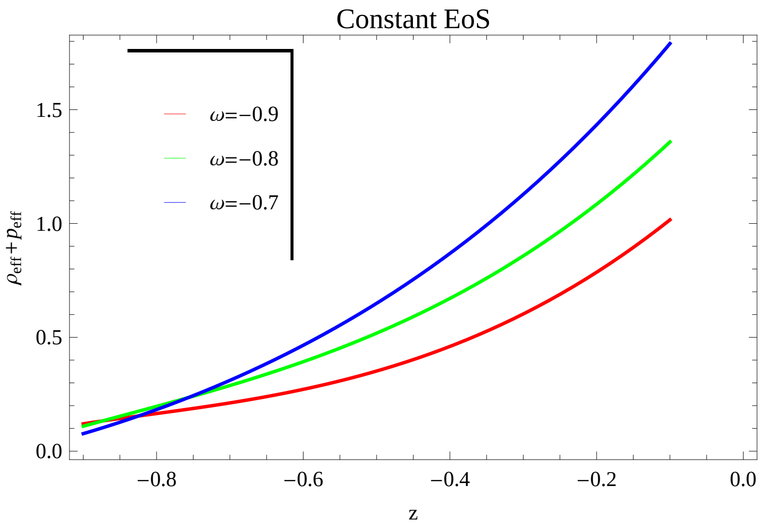

Figure 2: Plot of $\rho_{eff} + p_{eff}$ versus z.

The expression for deceleration parameter q is given by

$$q = -1 - \frac{\dot{H}}{H^2}. \tag{15}$$

By inserting the corresponding values in above equations, we obtain

$$\begin{aligned} q \;=\; & -1 + \frac{3}{2}\Bigg(\Bigg(\frac{4(1+\omega)}{1-3\omega}V(z) + \frac{4v_0\zeta(1+\omega)}{(1-3\omega)(3-\epsilon)}(1+z)^\epsilon + (1+z)^3\Bigg) \\ & \Bigg(1 - \frac{2}{\rho_1}\Bigg(\frac{4}{1-3\omega}v_0(1+z)^\epsilon + \frac{4v_0\zeta(1+\omega)}{(1-3\omega)(3-\epsilon)}(1+z)^\epsilon + (1+z)^3\Bigg)\Bigg)\Bigg) \\ & \Bigg(\Bigg(\frac{4}{1-3\omega}v_0(1+z)^\epsilon + \frac{4v_0\zeta(1+\omega)}{(1-3\omega)(3-\epsilon)}(1+z)^\epsilon + (1+z)^3\Bigg) \\ & \Bigg(1 - \frac{1}{\rho_1}\Bigg(\frac{4}{1-3\omega}v_0(1+z)^\epsilon + \frac{4v_0\zeta(1+\omega)}{(1-3\omega)(3-\epsilon)}(1+z)^\epsilon + (1+z)^3\Bigg)\Bigg)\Bigg)^{-1}. \end{aligned} \tag{16}$$

The plot of deceleration parameter with respect to redshift variable z is given by Figure **3**. Recent observations (Riess et al. AJ 1998, Linder PRL 2003) verified that the present universe is experiencing accelerated phase of expansion and the value lies in between $-1 \leq q \leq 0$.

In our case, q remains negative for all values of ω i.e. $\omega = -0.9,\ \omega = -0.8,\ z = -0.7$.
This means q represents accelerated expansion of universe for this choice of EoS parameter.

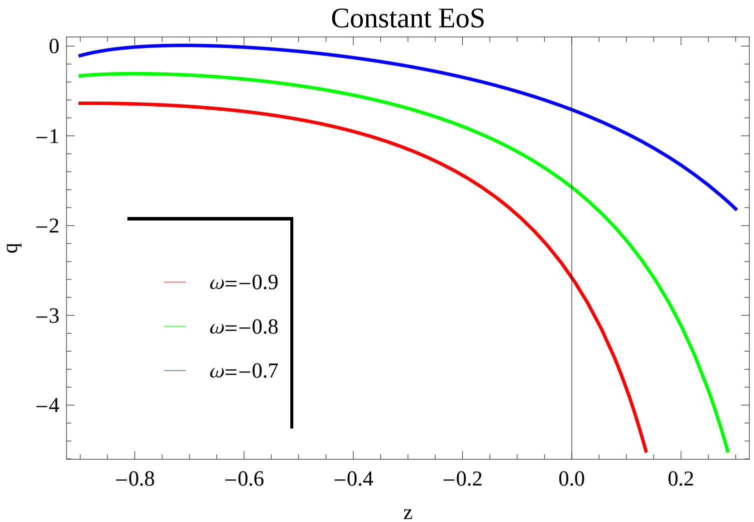

Figure 3: Plot of deceleration parameter q versus z.

Variable EoS Parameter:

We consider Chevallier-Polarski-Linder (CPL) parametrization of EoS parameter which is defined as (Chevallier and Polarski 2001)

$$\omega(z) = \omega_0 + \omega_a \frac{z}{1+z}. \tag{17}$$

Using Eq.(5), we can get the value of $V(z)$ as follow

$$V(z) = v_0 e^{\omega_a (3+\zeta)(1+z)^{-1}} (1+z)^{\alpha}, \tag{18}$$

here $\alpha = (3+\zeta)(1+\omega_0+\omega_a)$.

The energy density and pressure components take the following form

$$\rho_\phi = \frac{4v_0}{1-3(\omega_0+\omega_a \frac{z}{1+z})} e^{\omega_a (3+\zeta)(1+z)^{-1}} (1+z)^{\alpha}, \tag{19}$$

$$p_\phi = \frac{4(\omega_0+\omega_a \frac{z}{1+z})v_0}{1-3(\omega_0+\omega_a \frac{z}{1+z})} e^{\omega_a (3+\zeta)(1+z)^{-1}} (1+z)^{\alpha}. \tag{20}$$

The value of ρ_m can be calculated by making use of Eq.(5) as given below

$$
\begin{aligned}
\rho_m &= 4v_0\zeta(1+z)^3\Bigg(\frac{(1+\omega_0+\omega_a)(1+z)^{-3+\alpha}}{(3-\alpha)(1-3(\omega_0+\omega_a))}{}_2F_1\big(3-\alpha,1,4-\alpha,\frac{-3\omega_a(1+z)^{-1}}{1-3(\omega_0+\omega_a)}\big) \\
&+ \frac{\omega_a(3+\zeta)(1+\omega_0+\omega_a)(1+z)^{-4+\alpha}}{(4-\alpha)(1-3(\omega_0+\omega_a))}{}_2F_1\big(4-\alpha,1;5-\alpha;\frac{-3\omega_a(1+z)^{-1}}{1-3(\omega_0+\omega_a)}\big) \\
&- \frac{\omega_a(1+z)^{-4+\alpha}}{(4-\alpha)(1-3(\omega_0+\omega_a))}{}_2F_1\big(4-\alpha,1;5-\alpha;\frac{-3\omega_a(1+z)^{-1}}{1-3(\omega_0+\omega_a)}\big) \\
&- \frac{(\omega_a)^2(3+\zeta)(1+z)^{-5+\alpha}}{(5-\alpha)(1-3(\omega_0+\omega_a))}{}_2F_1\big(5-\alpha,1;6-\alpha;\frac{-3\omega_a(1+z)^{-1}}{1-3(\omega_0+\omega_a)}\big)\Bigg) + (1+z)^3.
\end{aligned}
\tag{21}
$$

By inserting the corresponding values of this EoS parameter case in Eq.(12), we can get the squared speed of sound for this case. The C_s^2 remains positive which exhibits the stability of this model.

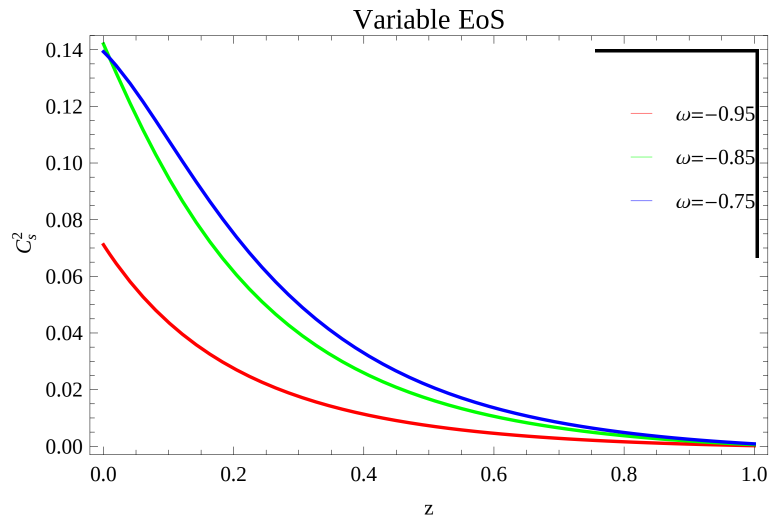

Figure 4: Plot of C_s^2 for variable EoS parameter.In this plot, we have chosen the values of ω_a and ω_0 as suggested in the literature, i.e., $\omega_a = -1.58(1+\omega_0)$ (Bennet et al. AJS 2003) and $\omega_0 = -0.95, -0.85, -0.75$ with $-1 < \omega_0 < -0.7$ (Linder PRD 2015).

$\rho_{eff} + p_{eff}$:

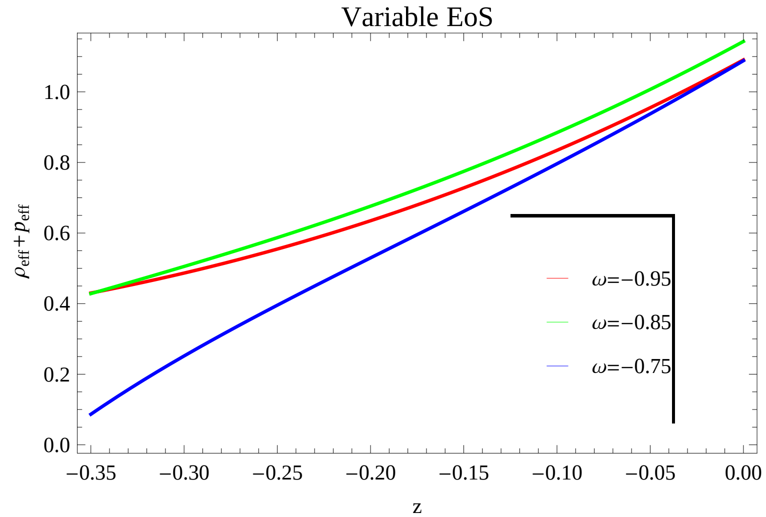

Figure 5: Plot of $\rho_{eff} + p_{eff}$ for variable EoS parameter.

In this case, the behavior of $\rho_{eff} + p_{eff}$ shown in Figure **5** which exhibits the quintessence as well as vacuum-like behavior of the universe
($\because 0 \leq \rho_{eff} + p_{eff} \leq 0.7$ which leads to $-1 \leq \omega_{eff} \leq -\frac{1}{3}$).

Deceleration Parameter:

From Figure **6**, it can be observed that plot of deceleration parameter remains positive in case of $\omega_0 = -0.75$. This exhibits the decelerated $(q > 0)$ phase of the universe in this case. For $\omega_0 = -0.85$ and $\omega_0 = -0.95$, we can see transition from decelerated phase towards accelerated phase of expansion of universe.

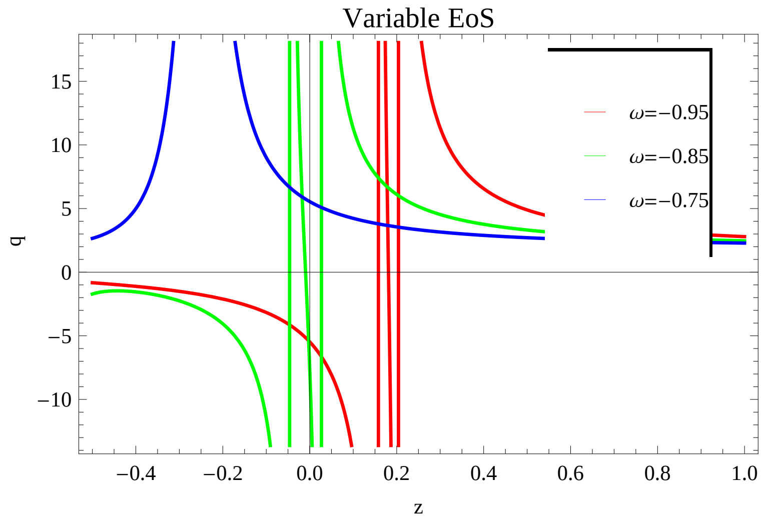

Figure 6: Plot of deceleration parameter q for variable EoS parameter.

Inspired by BH thermodynamics (Hawking CTP 1975), it was realized that there should be a connection between gravity and thermodynamics. For this purpose, Jacobson (PRL 1996) derived a relation between thermodynamics and the Einstein field equations on the basis of entropy-horizon area proportionality relation along with first law of thermodynamics $dQ = TdS$. Here $dQ,\ T$ and dS indicate the exchange in energy, temperature and entropy change for a given system.

It was found that the field equations can be expressed as follows

$$TdS = dE + pdV, \tag{22}$$

Here, $E = E_A = \rho_{eff} V_A,\ p = p_{eff}$ and $V = V_A = \frac{4}{3}\pi R_A^3$ represent the internal energy, pressure and volume of the spherical system for any spherically symmetric spacetime in any horizon.

We discuss the thermodynamics at apparent horizon which is defined as (Cai and Wang JCAP 2005)

$$R_A = \frac{1}{H}, \qquad \dot{R}_A = \frac{\dot{H}}{H^2}. \tag{23}$$

Also, the temperature can be defined on apparent horizon as (Cai and Wang JCAP 2005)

$$T_A = \frac{1}{2\pi R_A}\left(1 - \frac{\dot{R}_A}{2HR_A}\right) = \frac{R_A}{4\pi}\left(\dot{H} + 2H^2\right). \tag{24}$$

The rate of change of internal entropy function can be obtained as

$$T_A \dot{S}_I = 4\pi R_A^2 (\dot{R}_A - HR_A)(\rho_{eff} + p_{eff}). \tag{25}$$

Now we discuss the GSLT for a cosmological system which is the generalization of GSLT for a system containing BH as proposed by Bakenstein (PRD 1973). Bakenstein argued that the common entropy in the BH exterior plus the BH entropy never decreases. This statement is based on the proportionality relation between entropy of BH horizon and horizon area.

Thus GSLT can be described as follows

$$\frac{dS_{tot}}{dt} \geq 0. \qquad (26)$$

Here $S_{tot} = S + S_{BH}$, S represents the entropy of matter (body) outside a BH and S_{BH} is the entropy of BH. This proposal has been generalized towards the cosmological system where it can be defined as the sum of all entropies of the constituents (mainly DM and DE) and entropy of boundary (either it is Hubble or apparent or event horizon) of the universe can never decrease.

In the present scenario, we check the validity of GSLT taking into account apparent horizon as the boundary of universe

It is suggested that the horizon entropy is directly proportional to surface area of its horizon which is defined as

$$S_A = \frac{A}{4G},$$

where $A = 4\pi R_A^2$ is the surface area and R_A is the radius of apparent horizon.

The rate of change of entropy of horizon on the apparent horizon leads to (Cai and Wang JCAP 2005; Davis et al. AJ 2007; Bhattacharya et al. CJP 2011)

$$T_A \dot{S}_A \quad = \quad 4\pi R_A^3 H(p_{eff} + \rho_{eff}), \tag{27}$$

Using Eqs.(25) and (27), the total time rate of change of entropy function becomes

$$T_A \dot{S}_{tot} = 4\pi R_A^2 H(p_{eff} + \rho_{eff})\dot{R}_A. \tag{28}$$

Inserting all the corresponding values in Eq.(28), we get the GSLT as

$$\frac{dS_{tot}}{dz} = \frac{16\pi^2}{H^4(1+z)(\dot{H}+2H^2)}\bigg(\dot{H}(\dot{H}+2H^2) + (\rho_{eff}+p_{eff})(\dot{H}+H^2)\bigg). \tag{29}$$

In order to examine the thermodynamic equilibrium, we have to require the following expression

$$\begin{aligned}\frac{d^2S_{tot}}{dz^2} &= 16\pi^2\Bigg(\Bigg(-\frac{4}{H^5(1+z)}\frac{dH}{dz}-\frac{1}{H^4(1+z)^2}\Bigg)\Bigg(\dot{H}+\frac{(\rho_{eff}+p_{eff})(\dot{H}+H^2)}{\dot{H}+2H^2}\Bigg)\\ &+ \frac{1}{H^4(1+z)}\Bigg(\frac{d\dot{H}}{dz}-\frac{(\rho_{eff}+p_{eff})(\dot{H}+H^2)}{(\dot{H}+2H^2)^2}(\frac{d\dot{H}}{dz}+4H\frac{dH}{dz})+\frac{(\dot{H}+H^2)}{(\dot{H}+2H^2)^2}\\ &\Bigg(\frac{d\rho_{eff}}{dz}+\frac{p_{eff}}{dz}\Bigg)+\frac{(\rho_{eff}+p_{eff})}{(\dot{H}+2H^2)^2}(\frac{d\dot{H}}{dz}+2H\frac{dH}{dz})\Bigg)\Bigg).\end{aligned} \quad (30)$$

Constant EoS Parameter:

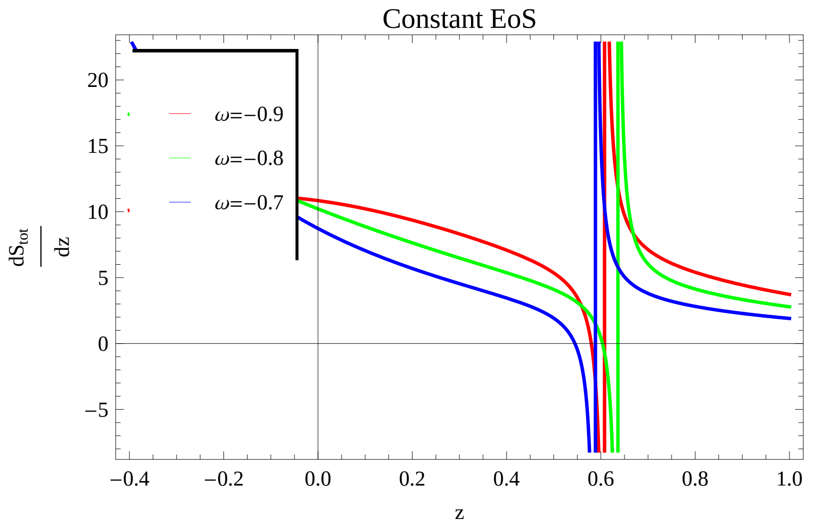

Figure 7: Plot of $\frac{dS_{tot}}{dz}$.

Figure **7** shows the behavior of $\frac{dS_{tot}}{dz}$ versus redshift z. For $\omega = -0.9$, $\frac{dS_{tot}}{dz}$ remains positive before and after $z = 0.6$. This means that GSLT holds everywhere excluding neighborhood of $z = 0.6$. For $\omega = -0.8$, $\frac{dS_{tot}}{dz}$ remains positive except for $0.6 \leq z \leq 0.65$ and hence GSLT holds excluding described values of z. The trajectory of $\frac{dS_{tot}}{dz}$ exhibits positive behavior except for $0.55 \leq z \leq 0.6$ in case of $\omega = -0.7$ and hence GSLT holds everywhere excluding the range $0.55 \leq z \leq 0.6$.

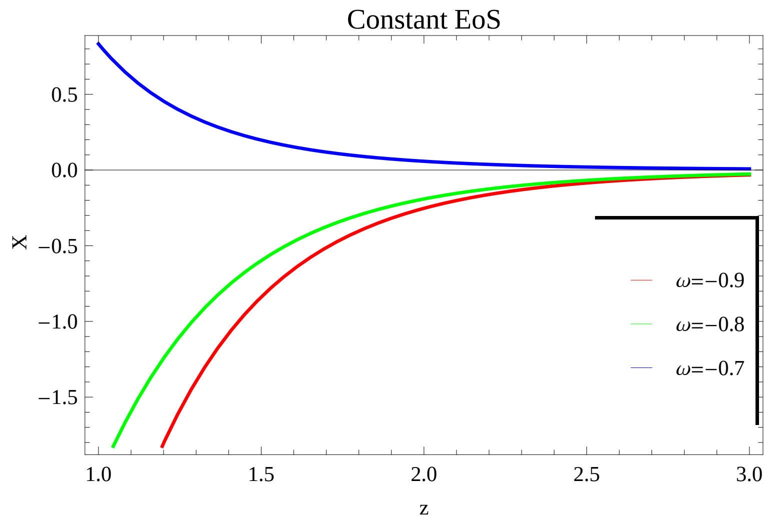

Figure 8: Plot of $X = \frac{d^2 S_{tot}}{dz^2}$.

It is quite obvious from Figure **8** that $X = \frac{d^2 S_{tot}}{dz^2}$ remains negative when plotted versus red shift parameter z in the range $1 \leq z \leq 3$ for $\omega = -0.9$ and $\omega = -0.8$. This shows the fulfillment of thermodynamic equilibrium condition.

Variable EoS Parameter:

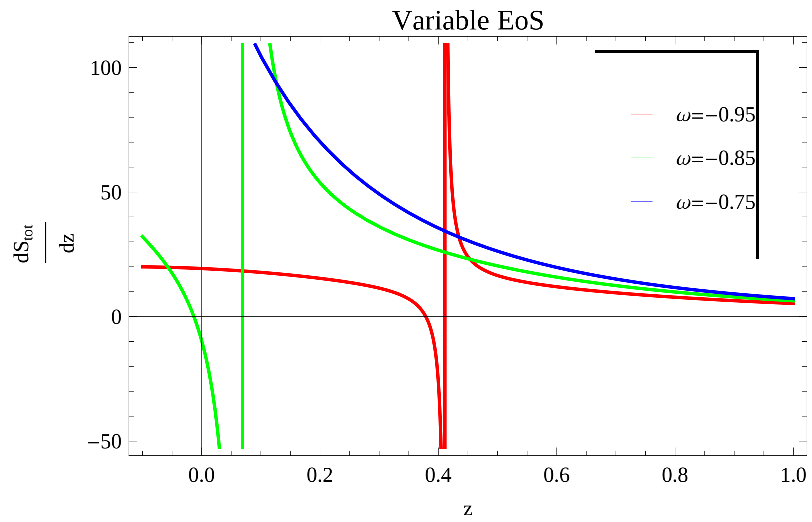

Figure 9: Plot of $\frac{dS_{tot}}{dz}$.

It can be observed from Figure 9 that for $\omega_0 = -0.95$, $\frac{dS_{tot}}{dz} > 0$ excluding neighborhood of $z = 0.4$. In case of $\omega_0 = -0.85$, $\frac{dS_{tot}}{dz} < 0$ only when $-0.01 < z < 1$ otherwise it remains positive. When we take $\omega_0 = -0.75$, $\frac{dS_{tot}}{dz}$ remains positive everywhere. Hence GSLT is satisfied everywhere excluding some points described in case of $\omega_0 = -0.95$ and $\omega_0 = -0.85$.

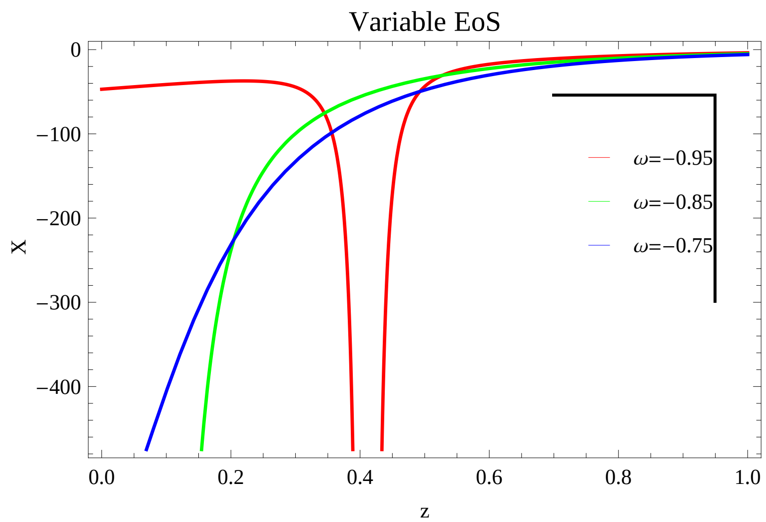

Figure 10: Plot of $\frac{dS^2_{tot}}{dz^2}$.

As far as thermodynamic equilibrium is concerned, it is verified from Figure **10** that model under consideration is in thermodynamic equilibrium for variable EoS in case of Bekenstein entropy acting as entropy of horizon.

Concluding Remarks

- The squared speed of sound exhibits the stability of current model in both cases of EoS parameter.
- The deceleration parameter also favor the current cosmic acceleration.
- GSLT also remains valid.
- Thermal equilibrium condition is also preserved.

THANKS