Inflaton Fragmentation in E-models of cosmological α-attractors

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BASED ON 1710.07487 WITH FUMINORI HASEGAWA

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APR.11, 2018

Introduction

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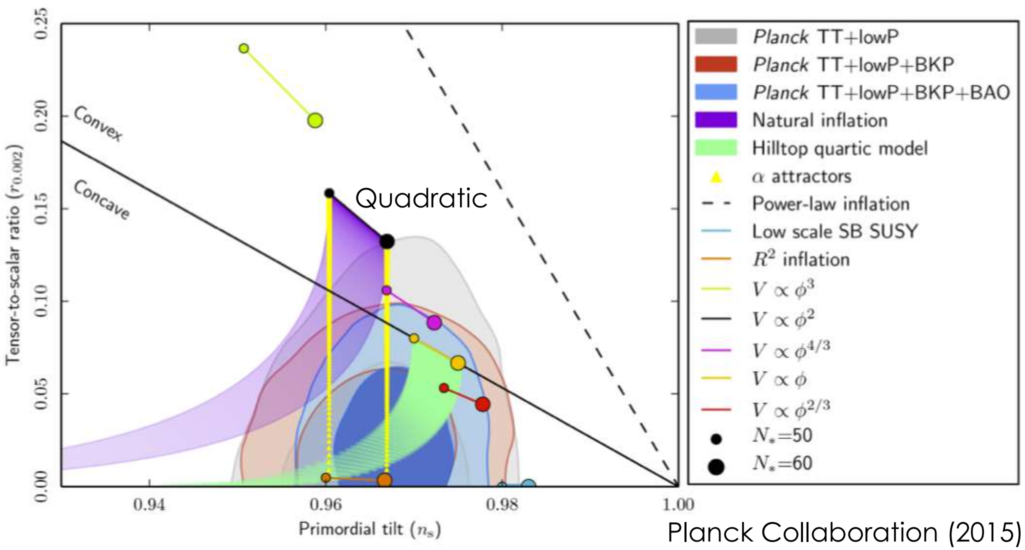

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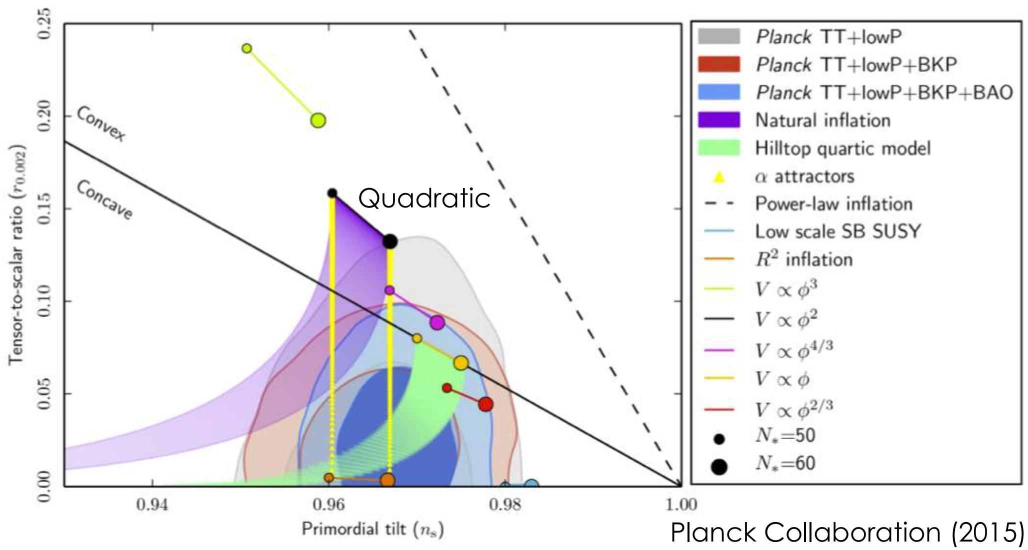

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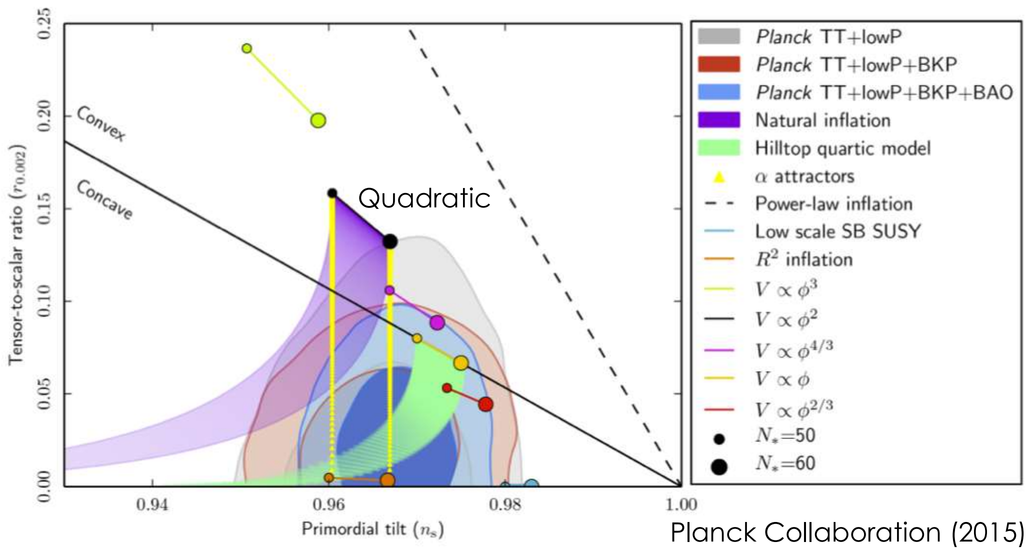

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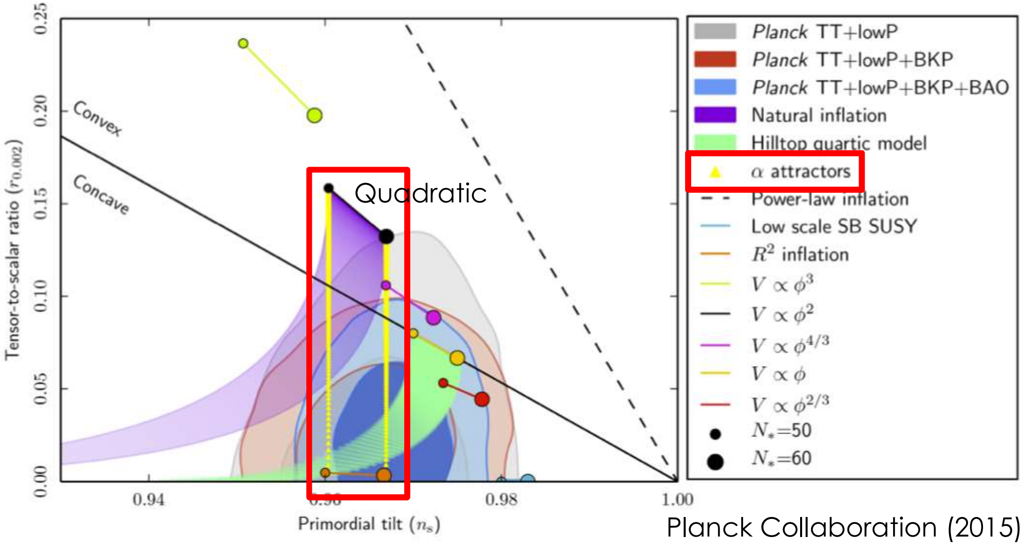

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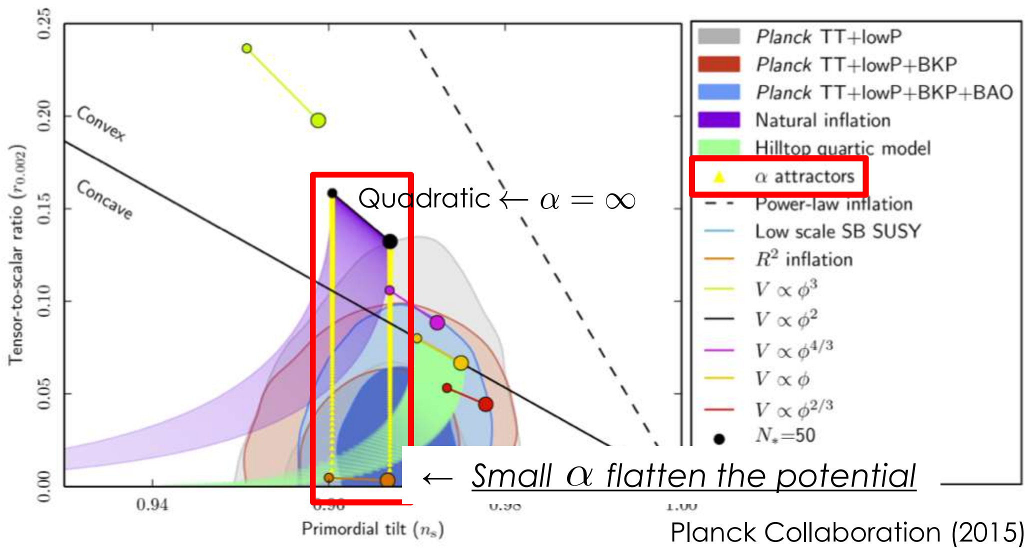

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→ *May resolve the degeneracy*

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- *We estimate the threshold in E-models*

  : also from theoretical curiosity on the model itself

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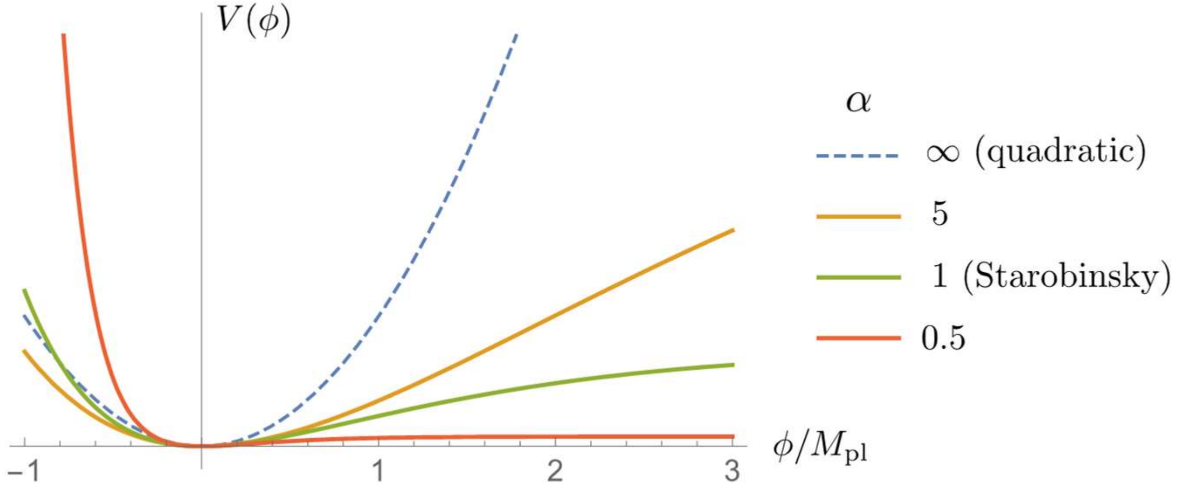

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$$\mathcal{P}_\zeta \equiv \frac{V}{24\pi^2 \epsilon M_{\rm pl}^4} \simeq \frac{N^2 m^2}{24\pi^2 M_{\rm pl}^2},$$

$$n_s \equiv 1 - 6\epsilon + 2\eta \simeq 1 - \frac{2}{N},$$

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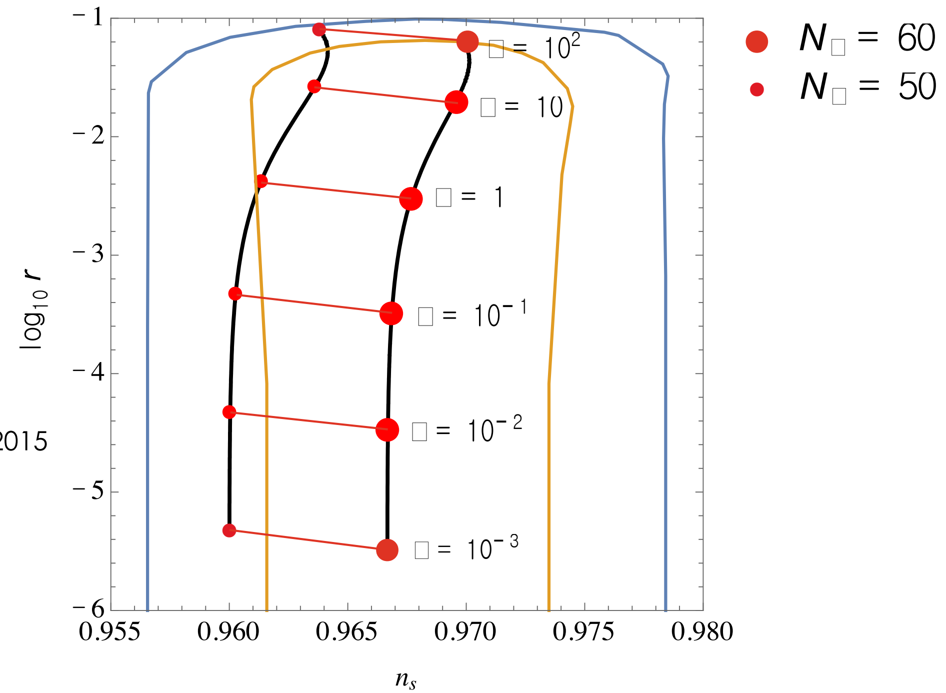
$$\mathcal{P}_\zeta(k_*) \simeq 2.4 \times 10^{-9}.$$

$$m \simeq 1.4 \times 10^{-5} M_{\rm pl} \left(\frac{N_*}{55}\right)^{-1}$$

Planck 2015



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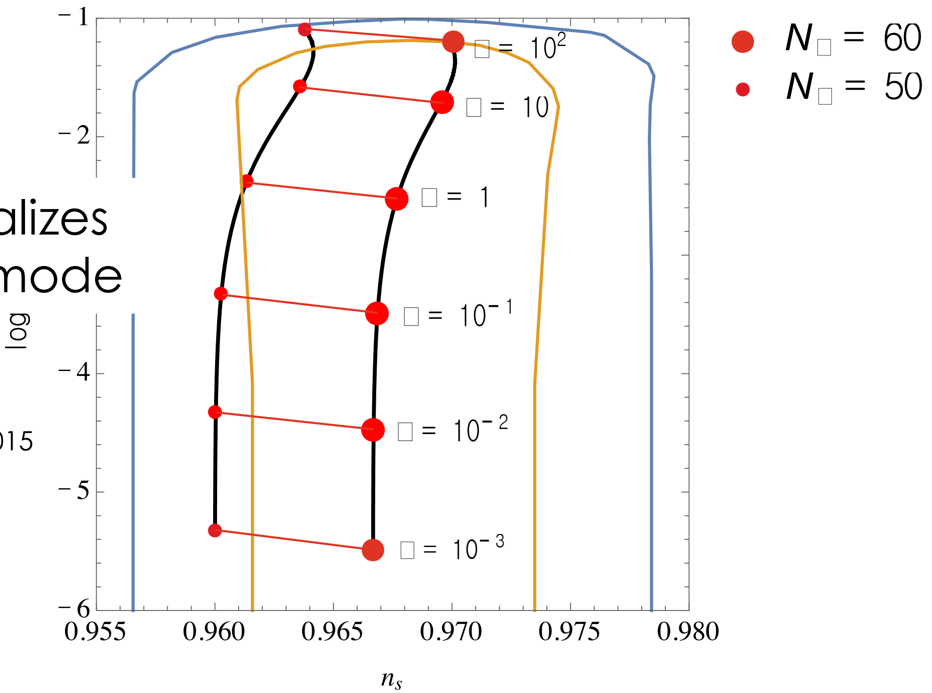

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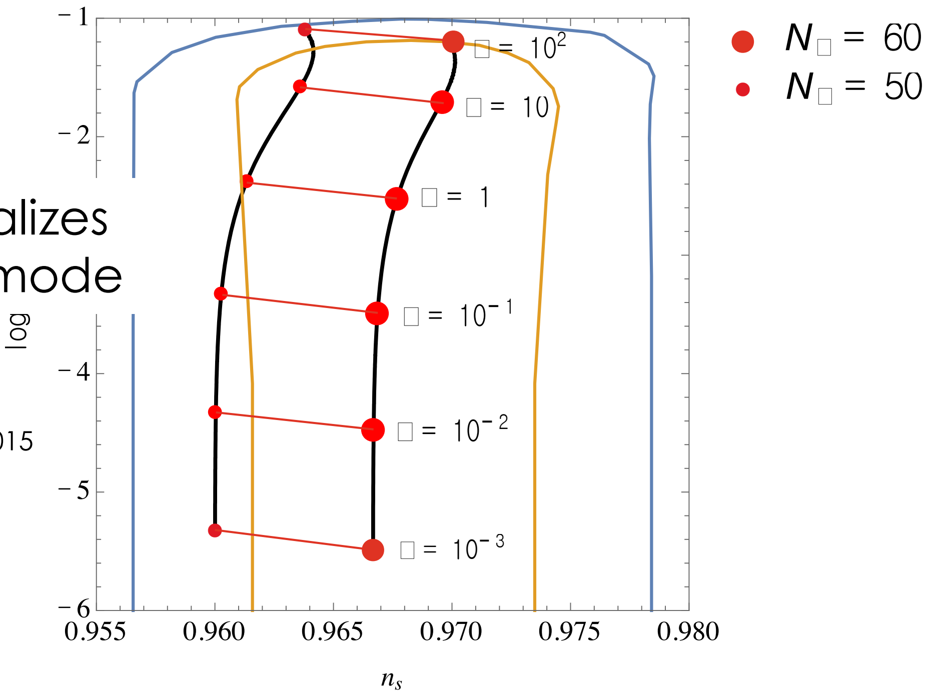

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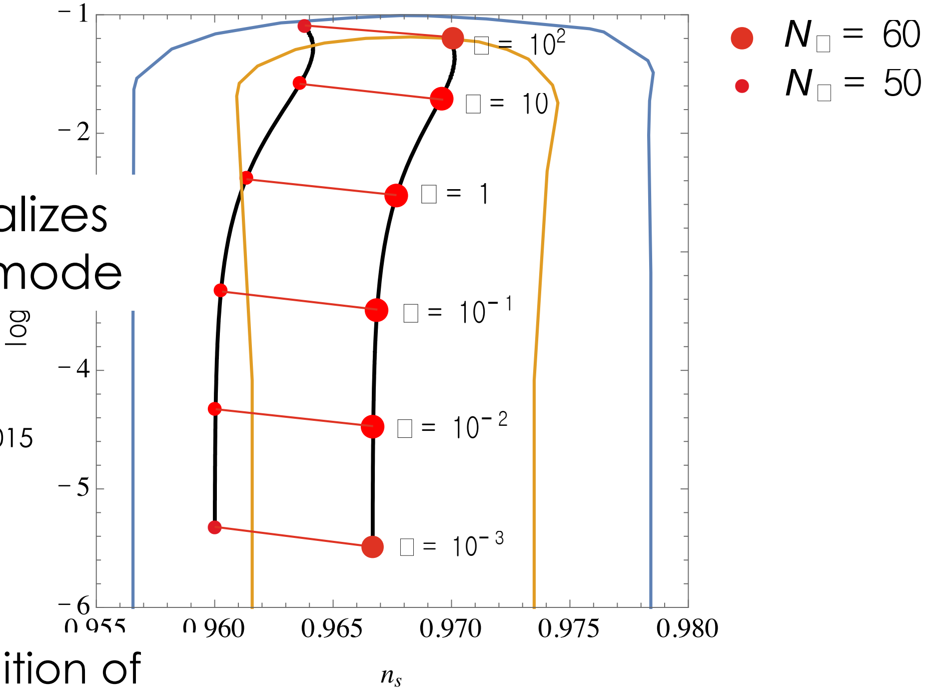

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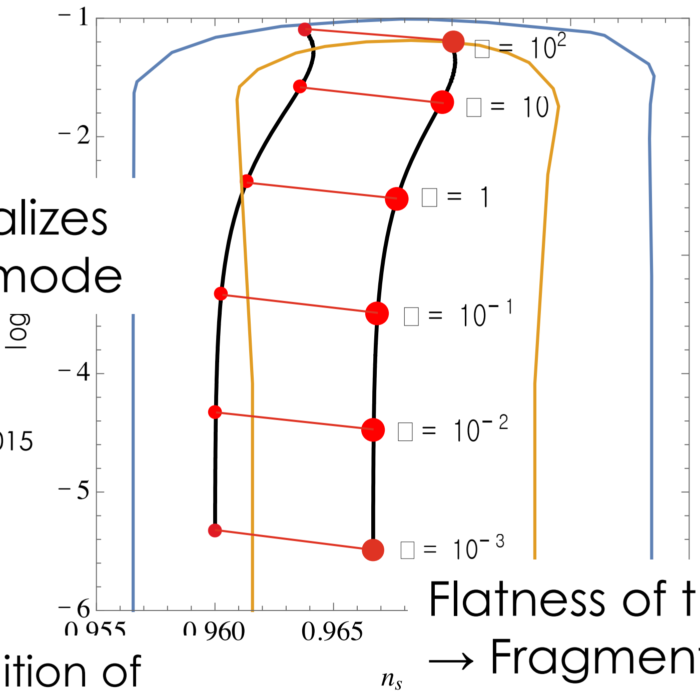

Flatness of the potential
→ Fragmentation into
I-balls (Oscillons)

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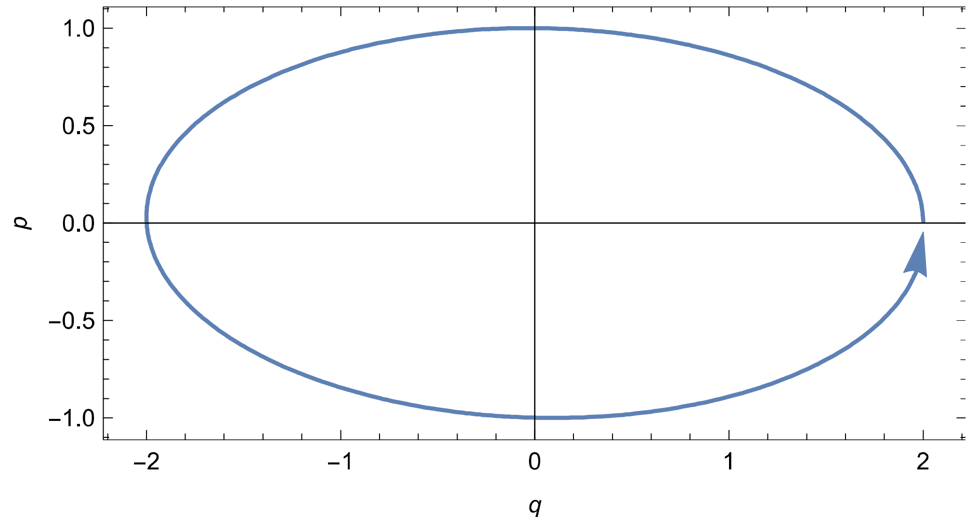

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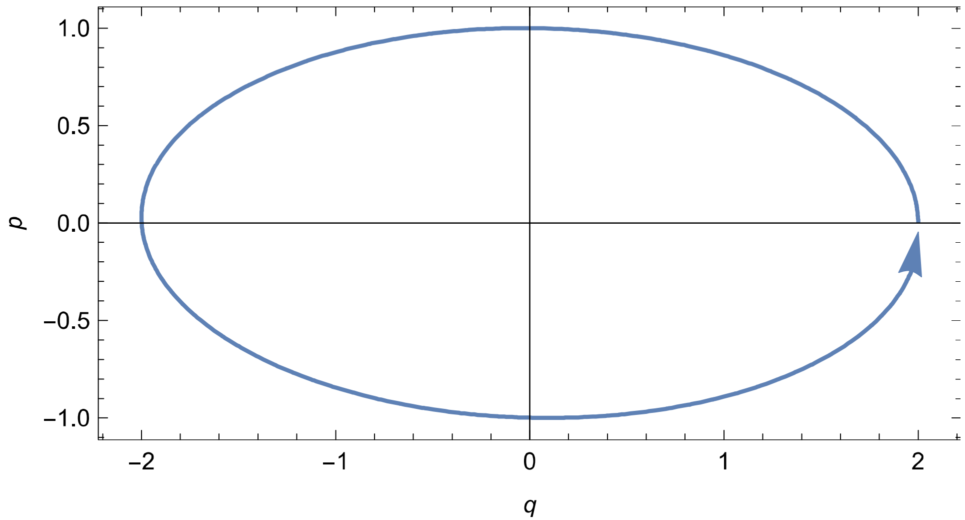

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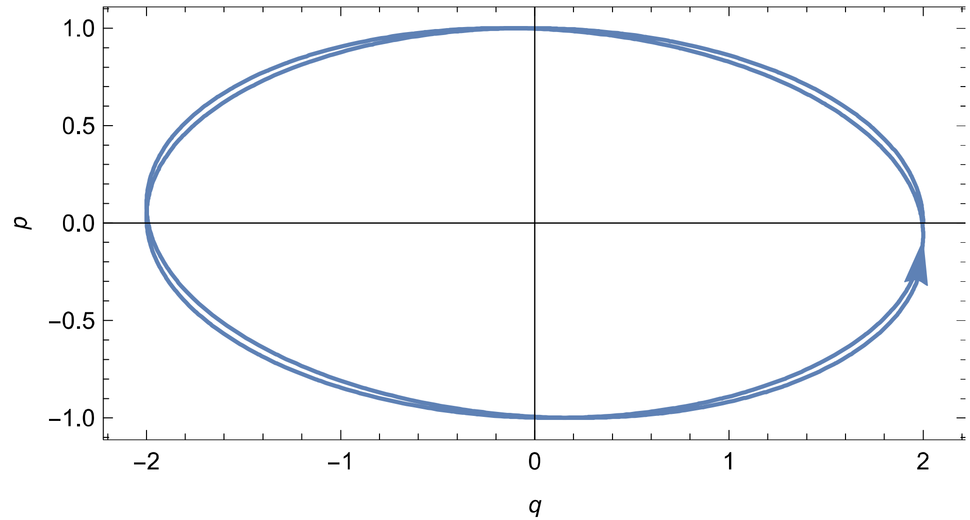

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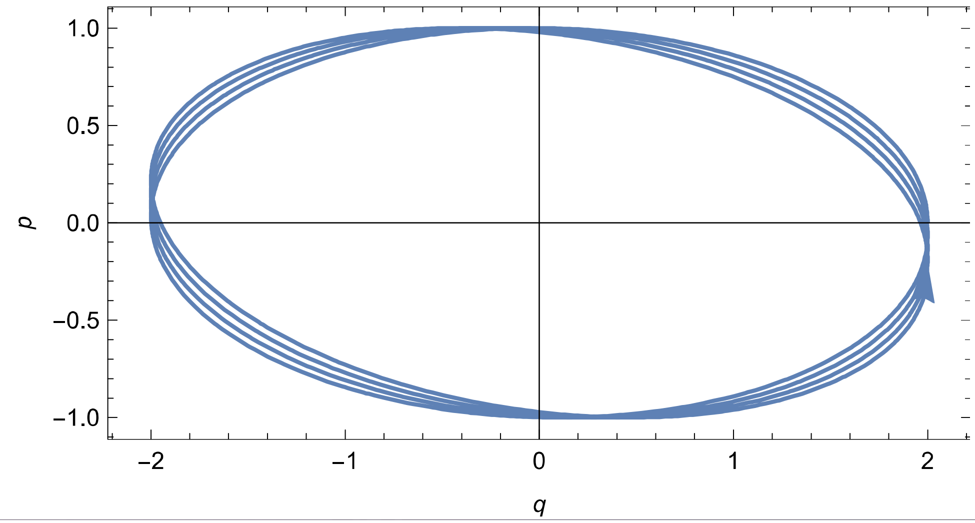

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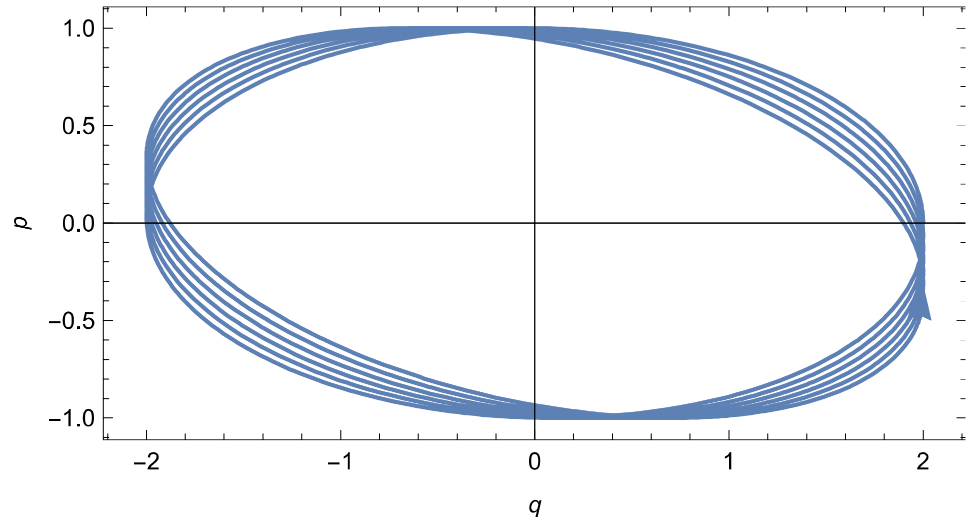

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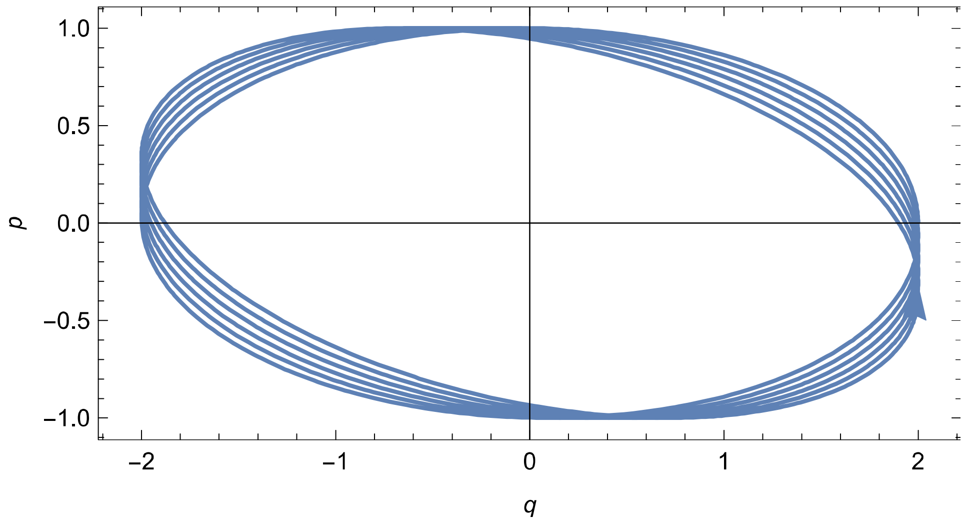
    : *Adiabatic invariant*



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$r \rightarrow t$: 1D dynamics of point mass

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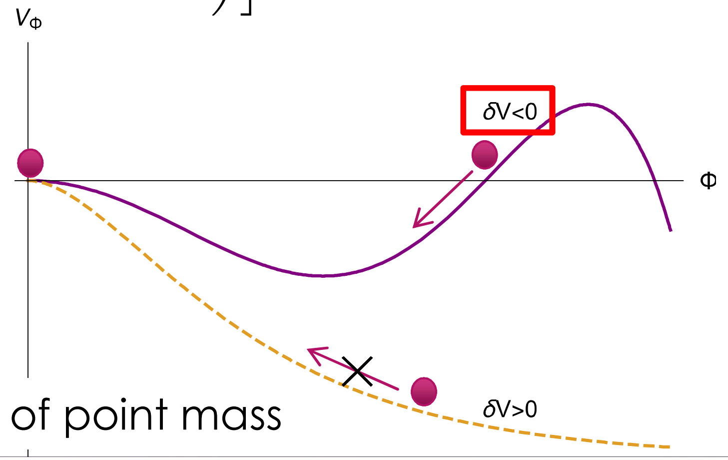


I-ball

S. Kasuya, M. Kawasaki, and F. Takahashi (2002)

- I-ball is lowest energy configuration with a fixed I
  - Consider nearly harmonic oscillation: $V = \frac{1}{2}m^2\phi^2 + \delta V, \quad \phi(\mathbf{x},t) \simeq \Phi(\mathbf{x})\cos mt,$

$$\Rightarrow \overline{E_{\tilde{\omega}}} \equiv \overline{E} + \tilde{\omega}\left[I - \frac{1}{m}\int d^3x\overline{\dot{\phi}^2}\right]$$

$$= \int d^3x \left[\frac{1}{2}\left(1 - 2\frac{\tilde{\omega}}{m}\right)\overline{\dot{\phi}} + \frac{1}{2}\overline{(\nabla\phi)^2} + \overline{V}\right] + \tilde{\omega}I$$

$$\simeq \frac{1}{4}\int d^3x \left[(\nabla\Phi)^2 + m^2\left(\left(1 - 2\frac{\tilde{\omega}}{m}\right)\Phi^2 + 4\overline{V}(\Phi)\right)\right] + \tilde{\omega}I,$$

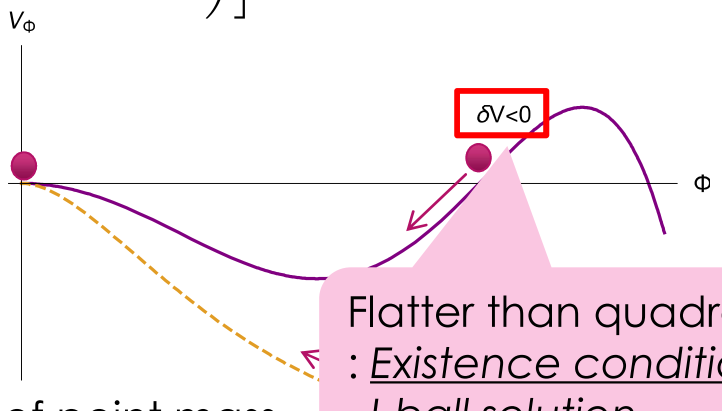
: Spatial gradient of the solution coming from the higher-order terms must be small since it violates the adiabaticity (in order to fix I)

$$\Rightarrow \quad \frac{d^2}{dr^2}\Phi + \frac{2}{r}\frac{d}{dr}\Phi + \frac{dV_\Phi}{d\Phi} = 0,$$

$$V_\Phi = -m^2\left(1 - \frac{\tilde{\omega}}{m}\right)\Phi^2 - 2\overline{\delta V}(\Phi),$$

$$\frac{d\Phi}{dr}(0) = 0, \quad \Phi(\infty) = 0$$

$r \to t$: 1D dynamics of point mass



Flatter than quadratic
: *Existence condition of I-ball solution*

I-ball profiles in E-models

F. Hasegawa and J. P. Hong (2017)

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I-balls are formed when $\epsilon \lesssim 1$

$\to \quad R \sim m^{-1} \sim 10^5 M_{\rm pl}^{-1}$

$M \sim m^2 \Phi(0)^2 R^3 \sim \alpha m^{-1} M_{\rm pl}^2 \sim 10^5 \alpha M_{\rm pl}$

Linear instability analysis

F. Hasegawa and J. P. Hong (2017)

- Equation of motion for linear fluctuation:

$$\frac{d^2}{dT^2}\delta\phi_k + \frac{1}{1-(2/3)(\Phi_0/\sqrt{\alpha}M_{\rm pl})^2}\left[4\left(\frac{k}{ma}\right)^2 + 4 - \frac{4}{3}\left(\frac{\Phi_0}{\sqrt{\alpha}M_{\rm pl}}\right)^2 - 4\sqrt{6}\left(\frac{\Phi_0}{\sqrt{\alpha}M_{\rm pl}}\right)\cos(2T)\right]\delta\phi_k \simeq 0, \qquad T \equiv \tau/2$$

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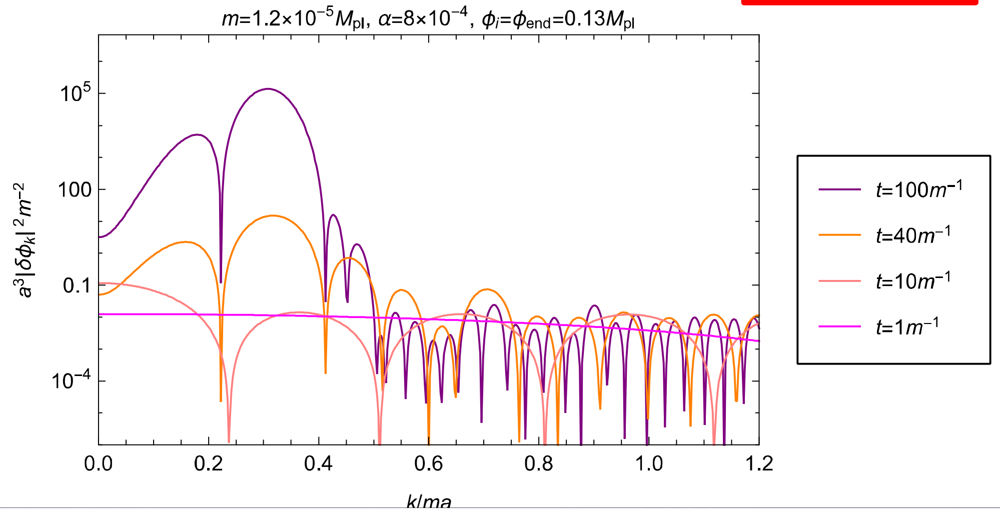

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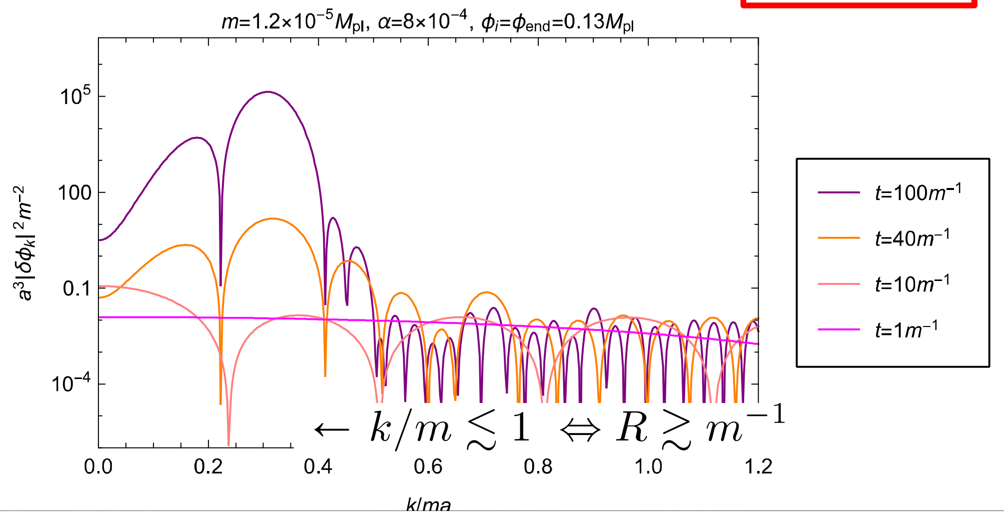

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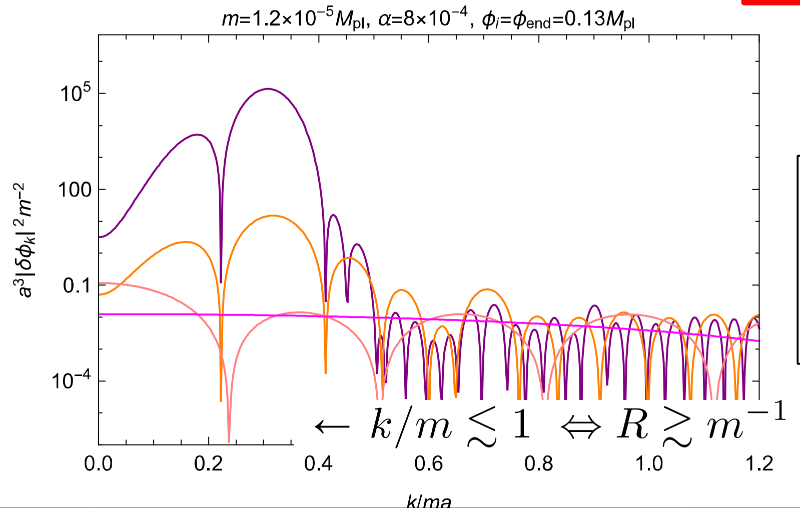

: Consistent with the analytical estimation

Lattice simulations

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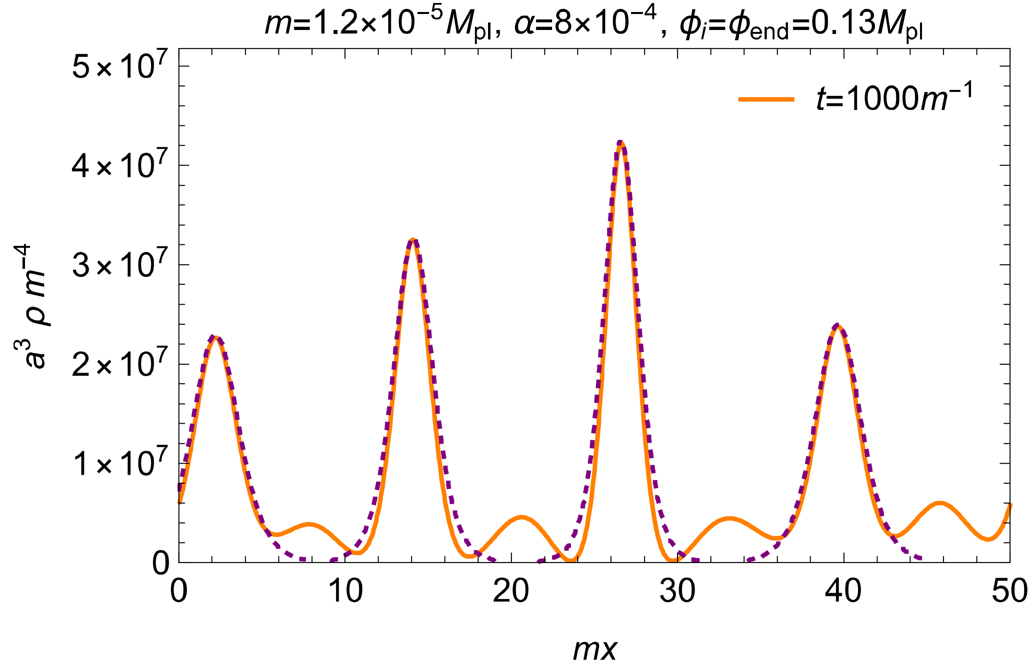

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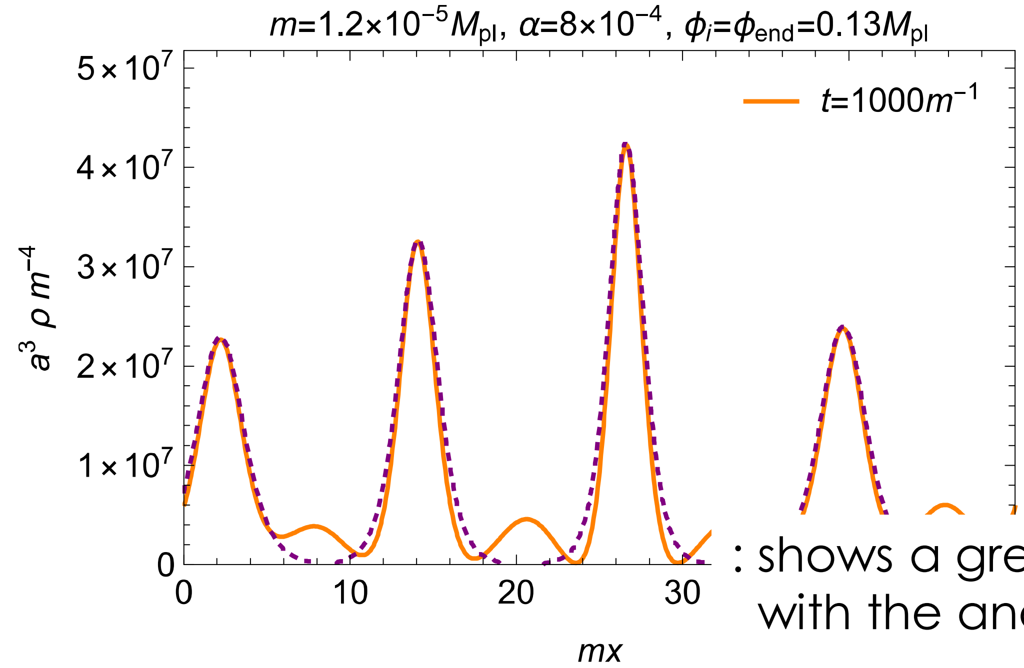
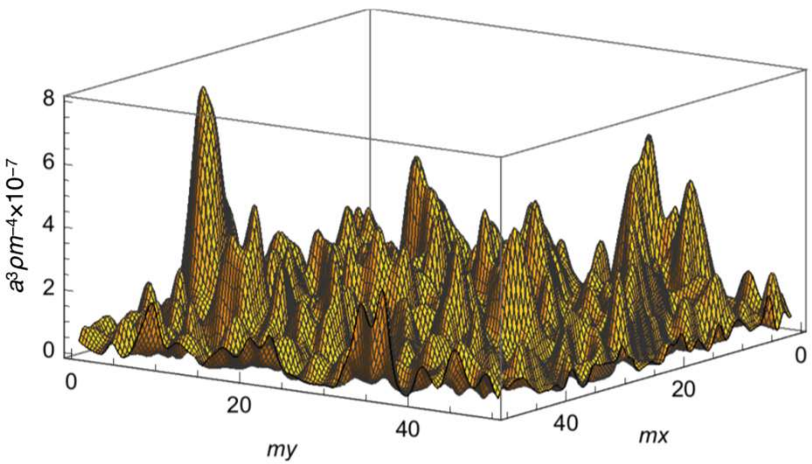


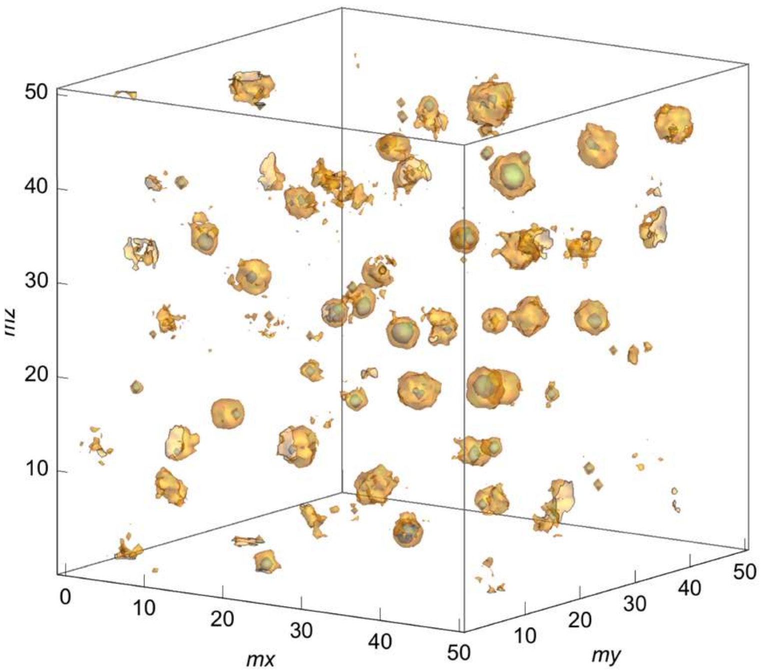
: shows a great agreement with the analytical profiles

Lattice simulations

F. Hasegawa and J. P. Hong (2017)

- I-ball formation for $\alpha \lesssim 10^{-3}$ is confirmed in 2D, 3D simulations as well:





Implications

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- However, the frequency is *not in currently observable range*:

$$f \sim H_{\rm end} \sim m\sqrt{\alpha} \lesssim 10^{36}~{\rm Hz}, \quad m \sim 10^{13}~{\rm GeV} \quad \alpha \lesssim 10^{-3}$$

,

$$\Rightarrow \quad f_0 = \frac{a_{\rm end}}{a_0} f = \frac{a_{\rm end}}{a_R}\frac{a_R}{a_0} f$$

$$\sim 5.5 \times 10^{-32} \left(\frac{M_{\rm pl}}{\Gamma}\right)^{1/2} \left(\frac{H_{\rm end}}{\Gamma}\right)^{-2/3} H_{\rm end},$$

$$\lesssim 10^{8}~{\rm Hz} \times \left(\frac{\Gamma}{10^{-7} M_{\rm pl}}\right)^{1/6},$$

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  - Locally generated radiation must diffuse throughout the space

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$$\begin{aligned} l_{\rm d} &\sim l_{\rm mf}\sqrt{N_t} \leftarrow \text{Number of collision during } t \\ &\sim \sqrt{l_{\rm mf} t} \\ &\sim \frac{1}{\sqrt{n\sigma H}} \\ &\sim \frac{1}{\sqrt{\alpha_{\rm s}^2 TH}} \quad (\sigma \sim \alpha_{\rm s}^2/T^2) \end{aligned}$$

I-ball number per horizon ($N_{\rm I}$)

$$l_{\text{I-I}} \sim N_{\rm I}^{-1/3} H^{-1} \sim N_{\rm I,form}^{-1/3} \left(\frac{H}{H_{\rm form}}\right)^{1/3} H^{-1},$$

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$$l_{\rm d} \gtrsim l_{\text{I-I}}$$
$$\Rightarrow T_{\rm RH} \lesssim 10^8\ {\rm GeV}\,\alpha_{\rm s}^{-6} N_{\rm I,form}^2 \alpha \left(\frac{m}{10^{13}\ {\rm GeV}}\right)^2$$

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  - Observational degeneracy of T- and E-models may be resolved for $10^{-4} \lesssim \alpha \lesssim 10^{-3}$
  - Even for $\alpha \lesssim 10^{-4}$ the difference in oscillon properties including size, number density may lead to different phenomena

    $\leftarrow$ $\mathcal{O}(1)$ change in radius may lead to sizable change in number density, since (Volume)$\propto$(Radius)^3

Summary

- We studied Inflaton fragmentation in E-models of α-attractors
- Instability overcomes cosmic expansion and fragments into I-balls for $\alpha \lesssim 10^{-3}$
- GWs are expected to be produced but the frequency is out of observable range
- Spatially localized reheating from I-balls may lead to possible delay of usual homogeneous radiation era
- Observational degeneracy with T-models may be resolved