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# Gravitational waves

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Ryusuke Jinno (IBS-CTPU)



Jul. 2018 @ IBS summer school



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# SELF INTRODUCTION

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- Ryusuke (류스케 / 隆介) Jinno (진노 / 神野)

- 2016/3 : Ph.D @ Univ. of Tokyo (particle physics group, supervised by Takeo Moroi)

- 2016/4-8 : JSPS fellow (PD) @ KEK, Japan

- 2016/9- : Research Fellow @ IBS-CTPU, Korea



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# SELF INTRODUCTION

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- Research interests & recent works

- Machine learning : Application to QFT tunneling problem
- Gravitational waves : Analytic approach to GW production in phase transitions
- (P)reheating : Preheating in Higgs inflation (newly-found main channel: "spike preheating")
- Inflation : Hillclimbing inflation (an inflationary attractor)

Hillclimbing Higgs inflation (new realization of Higgs inflation with hillclimbing)



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Hillclimbing Higgs inflation (new realization of Higgs inflation with hillclimbing)





## Introduction

- What's gravitational waves?
- Some history

## astrophysical GWs

- New probe to the Universe  
(focussing on recent aLIGO)

## cosmological GWs

- What's interesting?
- "Standard" cosmic history
- Various sources

## Summary

**My own  
work**





## Introduction

- What's gravitational waves?
- Some history

## astrophysical GWs

**Produced in present Universe**  
**Point source (e.g. stars)**

## cosmological GWs

**Produced in early Universe**  
**Stochastic**

**Summary**

**My own  
work**

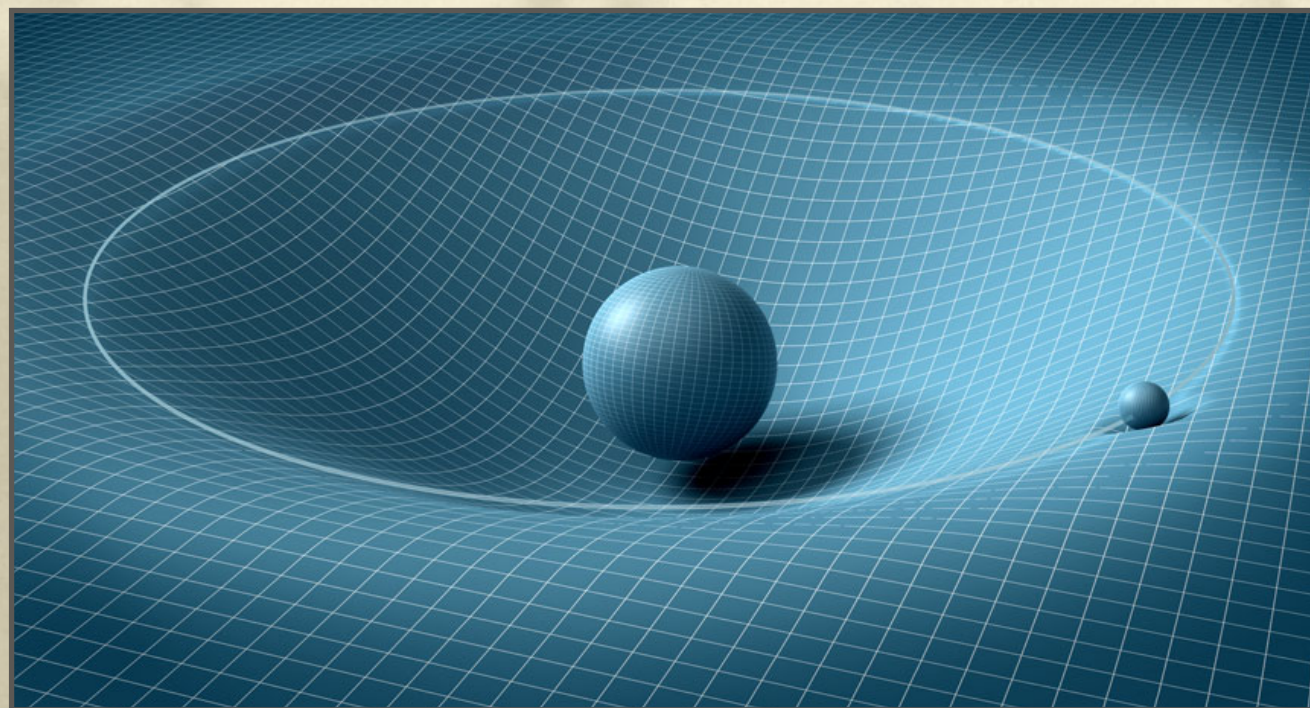


# WHAT'S GWS?

- General relativity [A.Einstein, 1916]

- Theory of gravity
- Spacetime tells matter how to move, and matter tells spacetime how to curve

[ J.A.Wheeler ]





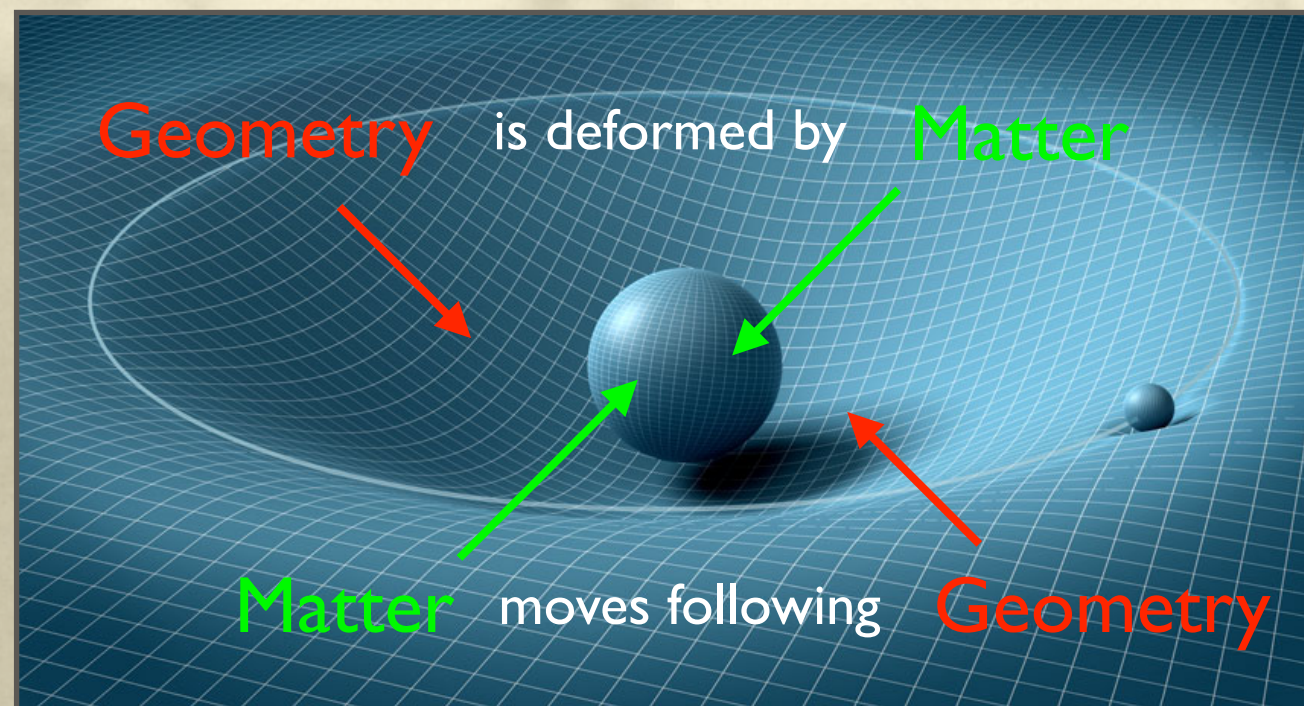
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[ J.A.Wheeler ]

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi GT_{\mu\nu}$$

(or simply  $G_{\mu\nu}$ )



# WHAT'S GWS?

## ■ General relativity [A.Einstein, 1916]

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[ J.A.Wheeler ]

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \overbrace{8\pi G}^{\text{constant}} T_{\mu\nu}$$

made from metric  $g_{\mu\nu}$

(which determines geometry of spacetime)

energy & momentum

carried by matter



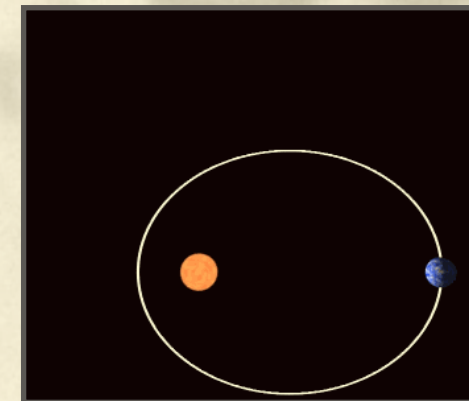
# WHAT'S GWS?

- Important predictions of general relativity

- Perihelion precession of Mercurie

575" (Mercurie's perihelion precession for 100 yrs)

= 532" (perturbations from other planets) + 43" (General relativity)



[ Wikipedia ]

- Black holes

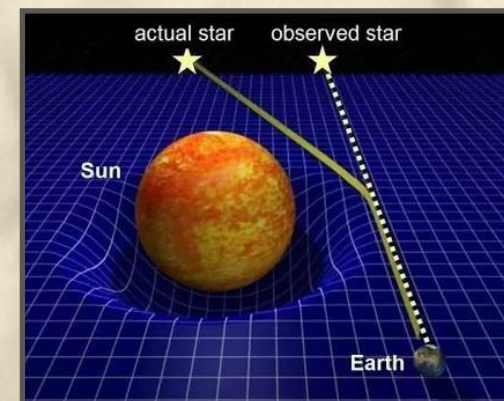


[ NASA/JPL-Caltech ]

- Gravitational lensing (especially the Sun)

General relativity's prediction =  $2 \times$  classical physics prediction

- Gravitational waves



[ <https://www.quora.com/How-different-is-gravitational-lensing-from-optical-lensing> ]



# WHAT'S GWS?

## ■ Gravitational waves

- Let's assume (almost) **homogeneous isotropic Universe** & **small amount of GWs**.

Then we have metric  $ds^2 = -dt^2 + \underbrace{a^2(t)}_{\substack{\text{scale factor} \\ = \text{"size of the Universe"}}} (\underbrace{\delta_{ij} + h_{ij}}_{\substack{\text{GWs} \\ \text{We assume } \ll 1}}) dx^i dx^j$

- Note : in some cases other perturbations are important (e.g. around BHs)
- GWs  $h_{ij}(t, \vec{x})$  satisfy traceless cond.  $h_{ii} = 0$  & transverse cond.  $\frac{\partial}{\partial x^i} h_{ij} = 0$

and obey the equation of motion

$$\square h_{ij}(t, \vec{x}) = -16\pi G P_{ij,kl} T_{kl}(t, \vec{x})$$



# WHAT'S GWS?

$$\square h_{ij}(t, \vec{x}) = -16\pi G P_{ij,kl} T_{kl}(t, \vec{x})$$

## ■ Rough picture

- In the absence of matter (i.e.  $T_{ij} = 0$ ), GWs are just waves:  $\square h_{ij} = 0$

- Energy-momentum tensor  $T_{ij}$  acts as a source,  
and projection  $P_{ij,kl}$  picks up some component of  $T_{ij}$

- Newton constant part:  $8\pi G \sim \frac{1}{(10^{18} \text{ GeV})^2} \sim \frac{1}{(\text{Huge energy scale})^2}$



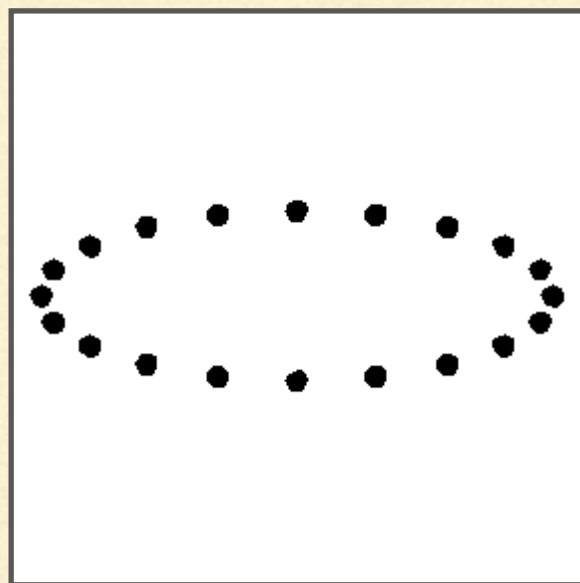
# WHAT'S GWS?

$$\square h_{ij}(t, \vec{x}) \sim T_{ij}(t, \vec{x})$$

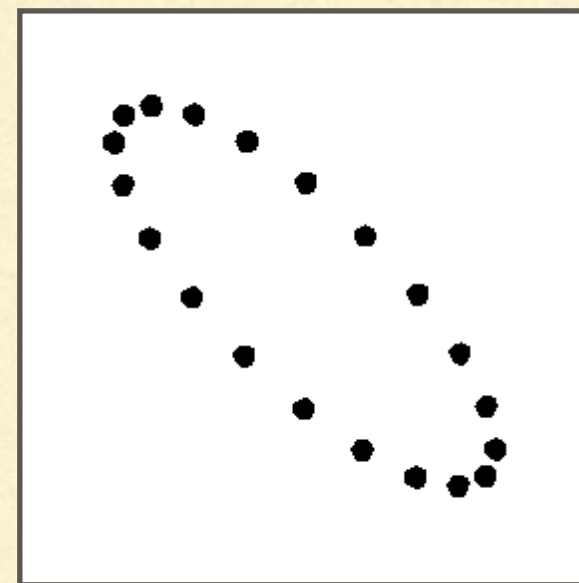
## ■ 2 modes of GWs

- 2 dof: 6 ( $3 \times 3$  symmetric matrix) - 1 (traceless  $h_{ii} = 0$ ) - 3 (transverse  $\partial_i h_{ij} = 0$ )

Mode +



Mode ×





# WHAT'S GWS?

$$\square h_{ij}(t, \vec{x}) \sim T_{ij}(t, \vec{x})$$

## ■ 2 modes of GWs

- 2 dof: 6 ( $3 \times 3$  symmetric matrix) - 1 (traceless  $h_{ii} = 0$ ) - 3 (transverse  $\partial_i h_{ij} = 0$ )

- Decomposition into various  $\vec{k}$  makes it easier:  $h_{ij}(t, \vec{x}) \sim \int d^3k e^{i\vec{k} \cdot \vec{x}} h_{ij}(t, \vec{k})$

- Traceless ( $h_{ii}(t, \vec{k}) = 0$ ) & transverse ( $k_i h_{ij}(t, \vec{k}) = 0$ )

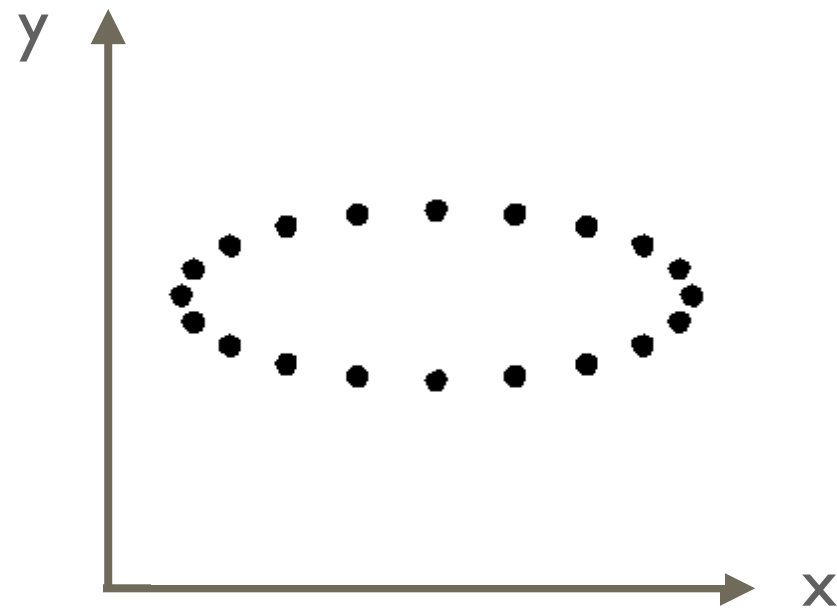
waves ( $\sim \sin(kt)$ , in the absence of matter) in  $\vec{k} \propto \hat{z}$  direction has the form:

$$h_{ij}(t, \vec{k}) = h_+ \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \sin(kt + \delta_+) + h_\times \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \sin(kt + \delta_\times)$$



# WHAT'S C/A/C?

Spatial metric  $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} h_+ & 0 & 0 \\ 0 & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix} \sin(kt + \delta_+)$

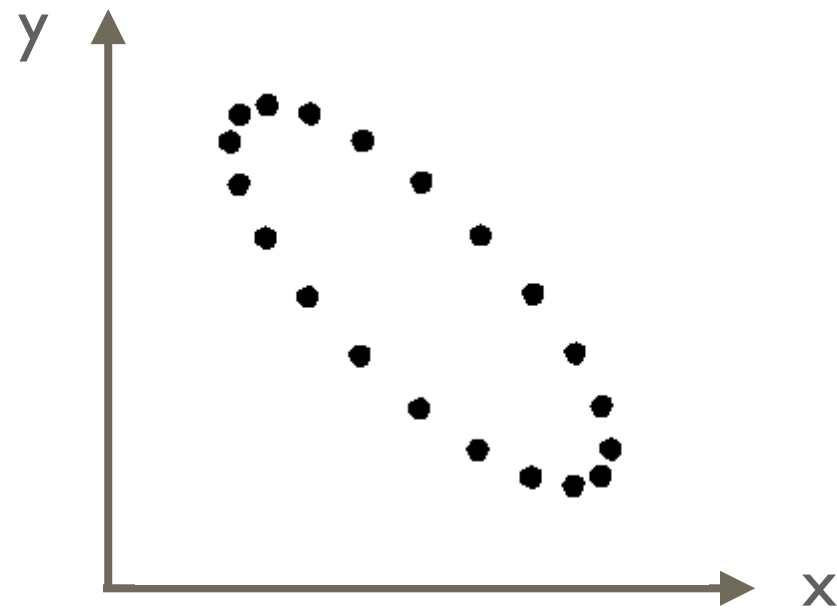


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# WHAT'S CVM/C?

Spatial metric  $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & h_{\times} & 0 \\ h_{\times} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \sin(kt + \delta_{\times})$



wave

$j = 0)$

$(t, \vec{k})$

n:

$$h_{ij}(t, \vec{k}) = h_{+} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \sin(kt + \delta_{+}) + h_{\times} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \sin(kt + \delta_{\times})$$



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# BRIEF HISTORY OF GWS

[ Wikipedia ]

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- 1893, Heaviside: Analogy of electromagnetism applied to gravity  
[ Heaviside, *Electromagnetic Theory*, 1893, p.281-282,359 ]
- 1905, Poincare: Proposal of gravitational waves emitted from accelerating body  
[ Poincare, Membres de l'Académie des sciences depuis sa création, 1905 ]
- 1916, Einstein: General theory of relativity → prediction of GWs  
(note : at that time, 3 types are proposed: LL, LT, TT)
- 1922, Doubt on the existence of GWs  
(e.g. Eddington: they "propagate at the speed of thought")
- 1936, Einstein & Rosen wrote a paper saying that GWs do not exist



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Il importait d'examiner cette hypothèse de plus près et en particulier de rechercher quelles modifications elle nous obligerait à apporter aux lois de la gravitation. C'est ce que j'ai cherché à déterminer; j'ai été d'abord conduit à supposer que la propagation de la gravitation n'est pas instantanée, mais se fait avec la vitesse de la lumière. Cela semble en contradiction avec un résultat obtenu par Laplace qui annonce que cette propagation est, sinon instantanée, du moins beaucoup plus rapide que celle de la lumière. Mais, en réalité, la question que s'était posée Laplace diffère considérablement de celle dont nous nous occupons ici. Pour Laplace, l'introduction d'une vitesse finie de propagation était la *seule* modification qu'il apportait à la loi de Newton. Ici, au contraire, cette modification est accompagnée de plusieurs autres; il est donc possible, et il arrive en effet, qu'il se produise entre elles une compensation partielle.

Quand nous parlerons donc de la position ou de la vitesse du corps attirant, il s'agira de cette position ou de cette vitesse à l'instant où l'*onde gravifique* est partie de ce corps; quand nous parlerons de la position ou de la vitesse du corps attiré, il s'agira de cette position ou de cette vitesse à l'instant où ce corps attiré a été atteint par l'onde gravifique émanée de l'autre corps; il est clair que le premier instant est antérieur au second.



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フランス語▼



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Quand nous parlerons donc de la position ou de la vitesse du corps attirant, il s'agira de cette position ou de cette vitesse à l'instant où l'onde gravifique est partie de ce corps; quand nous parlerons de la position ou de la vitesse du corps attiré, il s'agira de cette position ou de cette vitesse à l'instant où ce corps attiré a été atteint par l'onde gravifique émanée de l'autre corps; il est clair que le premier instant est antérieur au second.

編集

英語▼



It was important to examine this hypothesis more closely and especially to find out what modifications it would require us to make to the laws of gravitation. This is what I sought to determine; I was first of all led to suppose that the propagation of gravitation is not instantaneous, but is done with the speed of light.

...

When, therefore, we bet on the position or velocity of the moving body, it will be this position or this velocity at the moment when the gravitational wave is part of this body; when we speak of the position or velocity of the attracted body, it will be this position or this velocity at the moment when this attracted body has been reached by the gravitational wave emanating from the other body; it is clear that the first moment is before the second.



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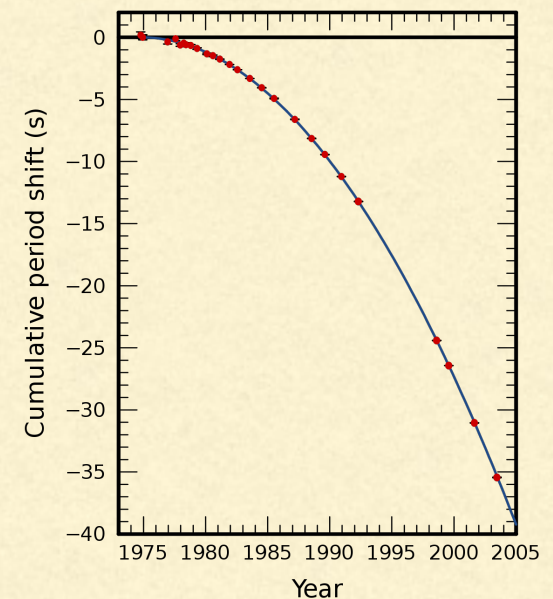
- 1956, Pirani remedied the confusion caused by various coordinate systems
- 1950's, Discussions on whether GWs carry energy  
→ settled by Feynman, "sticky bead" thought experiment

- 1969, First detection announced by Joseph Weber



→ though not confirmed by others,  
experimentally this was an important step

- 1974, Hulse & Taylor confirmed the existence of GWs  
(though indirectly) by orbital decay of binary pulsar





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# SUMMARY SO FAR

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- In General Relativity, there are degrees of freedom

propagating with speed of light: Gravitational Waves



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- What's gravitational waves?
- Some history

## astrophysical GWs

- New probe to the Universe  
(focussing on recent aLIGO)

## cosmological GWs

- What's interesting?
- "Standard" cosmic history
- Various sources

## Summary

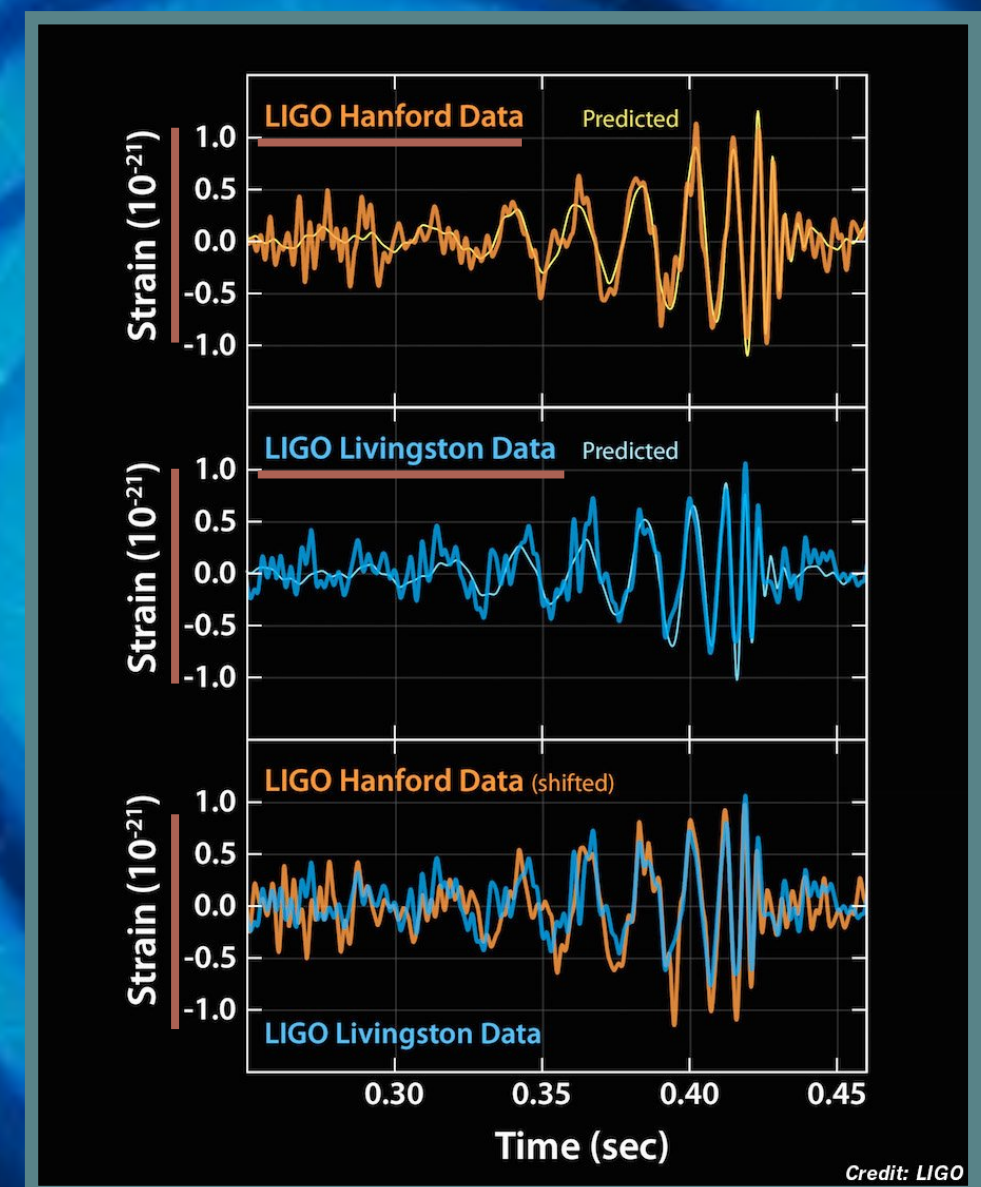
My own  
work





# FIRST GW DETECTION

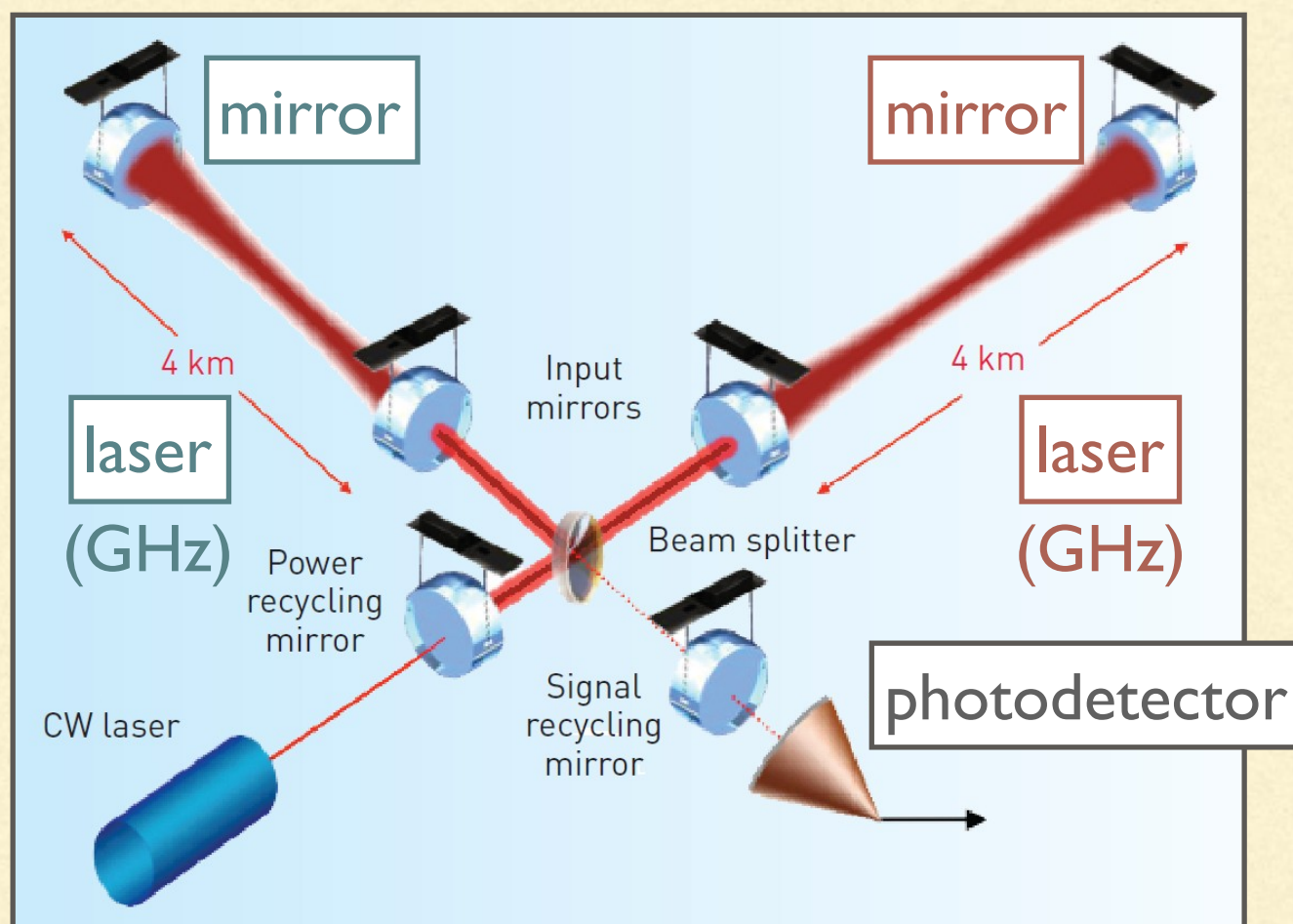
- Advanced LIGO announced first detection of GWs in 2016/2
- Event itself was in 2015/9/14
- Black hole binary  $36M_{\odot} + 29M_{\odot} \rightarrow 62M_{\odot}$   
( $3M_{\odot}$  radiated as GWs)
- Strain  $\sim 10^{-21}$ , Frequency  $\sim 250\text{Hz}$





# FIRST GW DETECTION

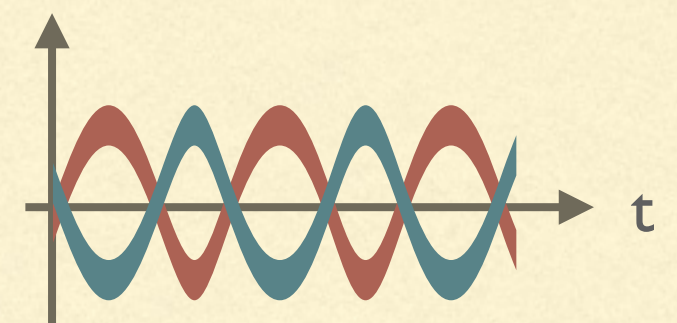
- Advanced LIGO?
  - Ground interferometer experiment in US



Total power  $\sim$  electric field<sup>2</sup>



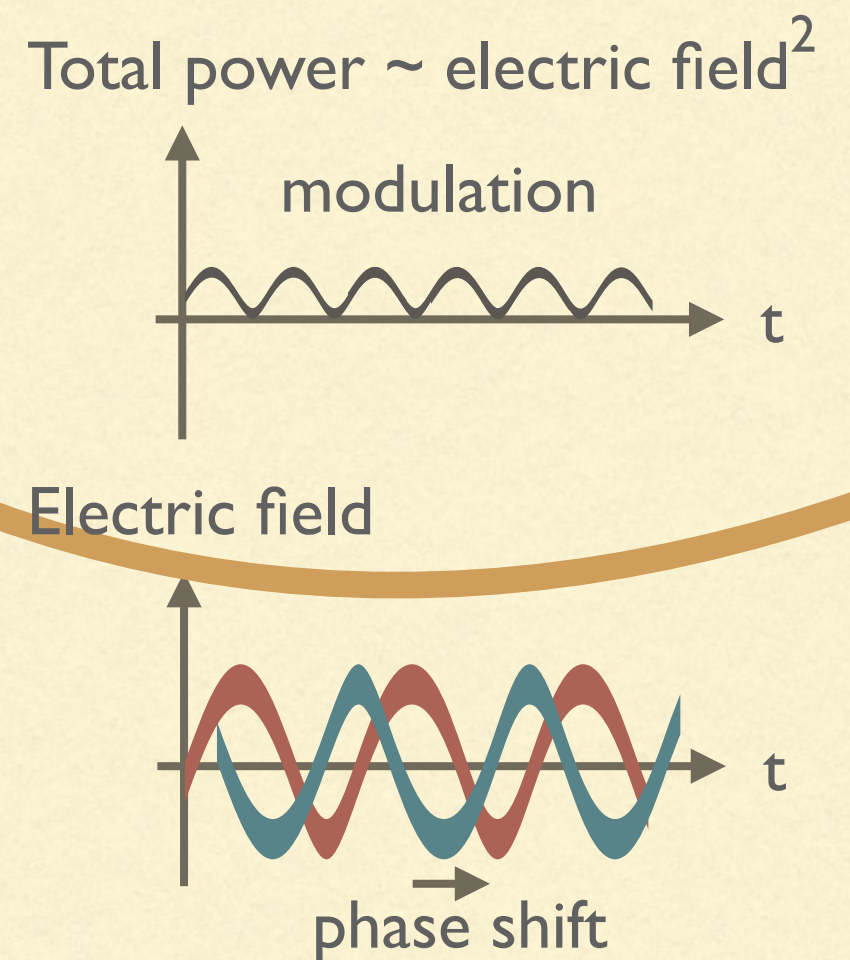
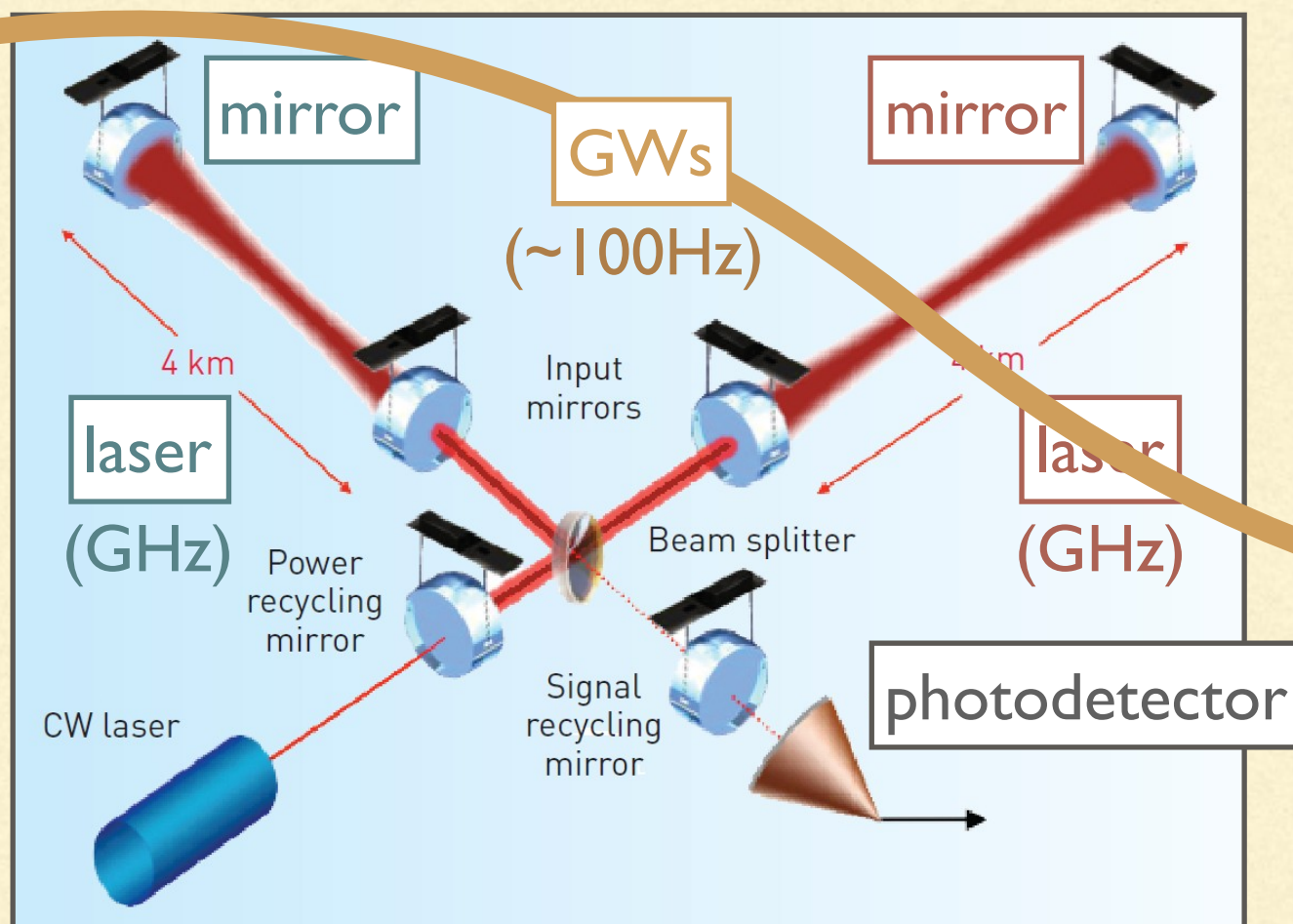
Electric field





# FIRST GW DETECTION

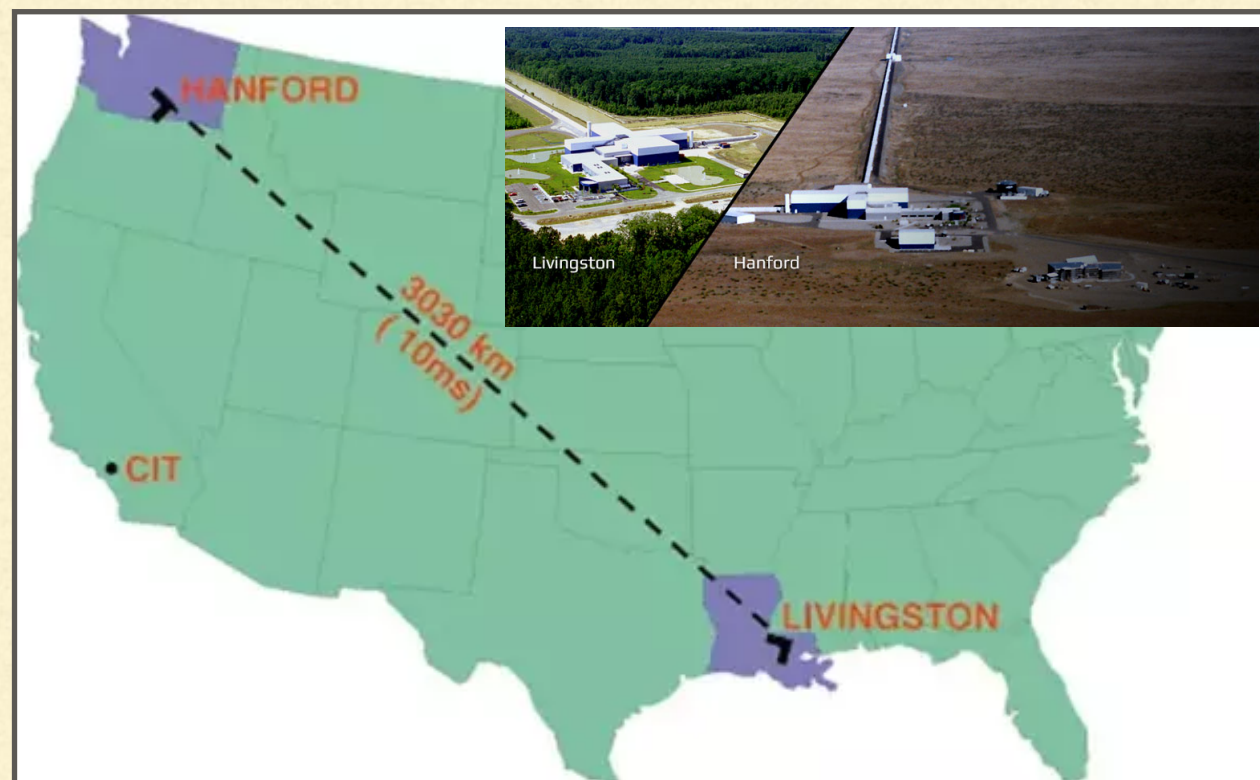
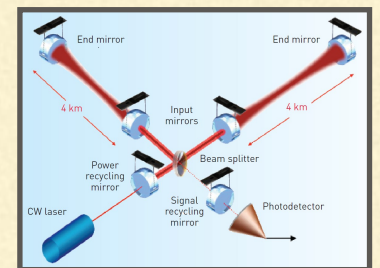
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# FIRST GW DETECTION

- Advanced LIGO?
  - Ground interferometer experiment in US
  - 2 interferometers: Hanford & Livingston





# FIRST GW DETECTION

## ■ Advanced LIGO?

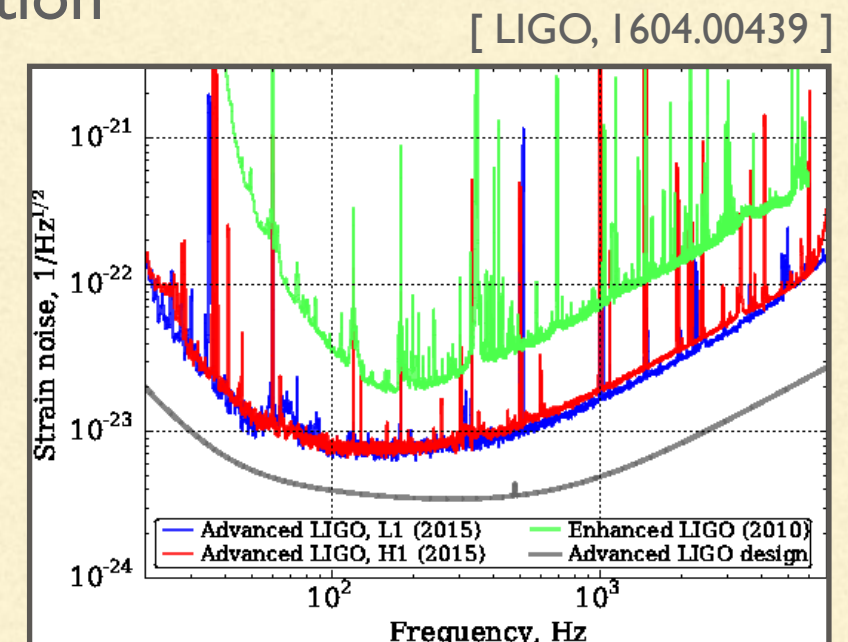
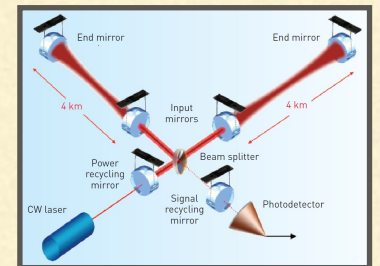
- Ground interferometer experiment in US
- 2 interferometers: Hanford & Livingston
- Brief history

2002-2010 : data collection by LIGO → no detection

2010- : major update to Advanced LIGO

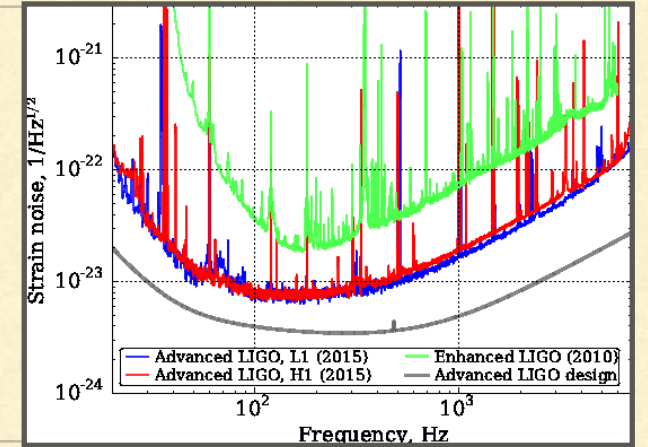
2015- : first detection of GWs from BH binary

→ announced in 2016





# FIRST GW DETECTION



## ■ Strain?

- Typically shown in unit  $\text{Hz}^{-1/2}$ , as a function of frequency  $f$
- Rough meaning: typical contribution to  $h_{ij}(t, \vec{x})$  caused by GWs of frequency  $f$
- More rigorously, ...

$$\langle h_{ij}^2(t, \vec{x}) \rangle_{\text{long time average}} \sim \int df \text{Strain}(f)^2$$

- So, strain  $10^{-22} \text{Hz}^{-1/2}$  around frequency 100Hz means...

$$\text{typical } h_{ij} \sim \sqrt{(\text{freq. band } \Delta f) \times (\text{typical strain})^2} \sim 10^{-21}$$

- How crazy?      spacial metric  $\sim \delta_{ij} + h_{ij} \rightarrow$  distance  $\sim 4\text{km} \pm 10^{-18}\text{m}$



# ADVANCED LIGO: DETECTION SO FAR

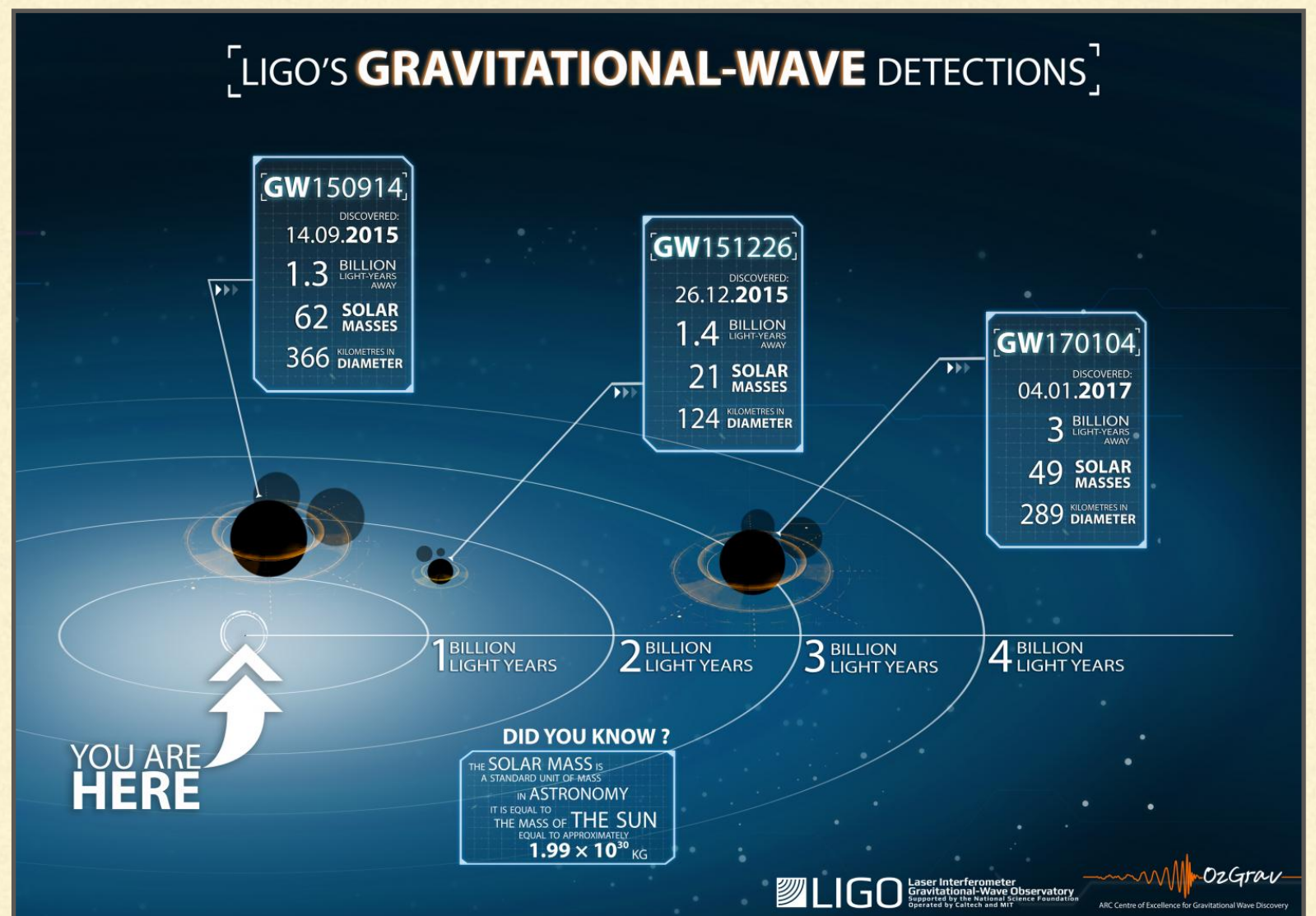
- Advanced LIGO results so far

- 6 detections:

- 5 BH binaries, 1 NS binary

BH { GW150914  
GW151226  
GW170104  
GW170608  
GW170814

NS { GW170817

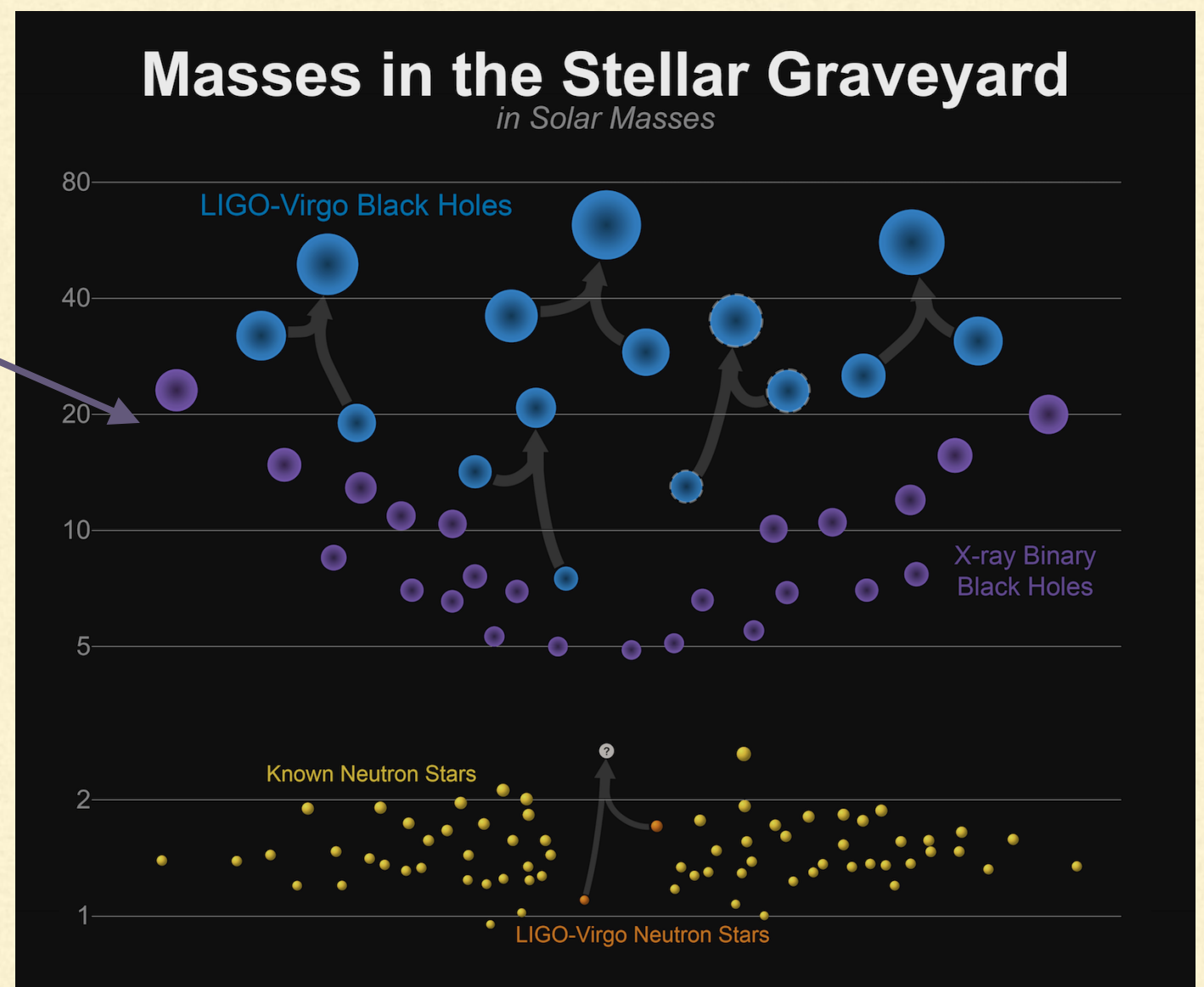




# ADVANCED LIGO: NEW POPULATION OF BHS

- Black holes

- Smaller mass ( $\lesssim 10\text{-}20 M_{\odot}$ ) BHs already found by X-ray studies
- GWs revealed new population of BHs ( $\gtrsim 20 M_{\odot}$ )



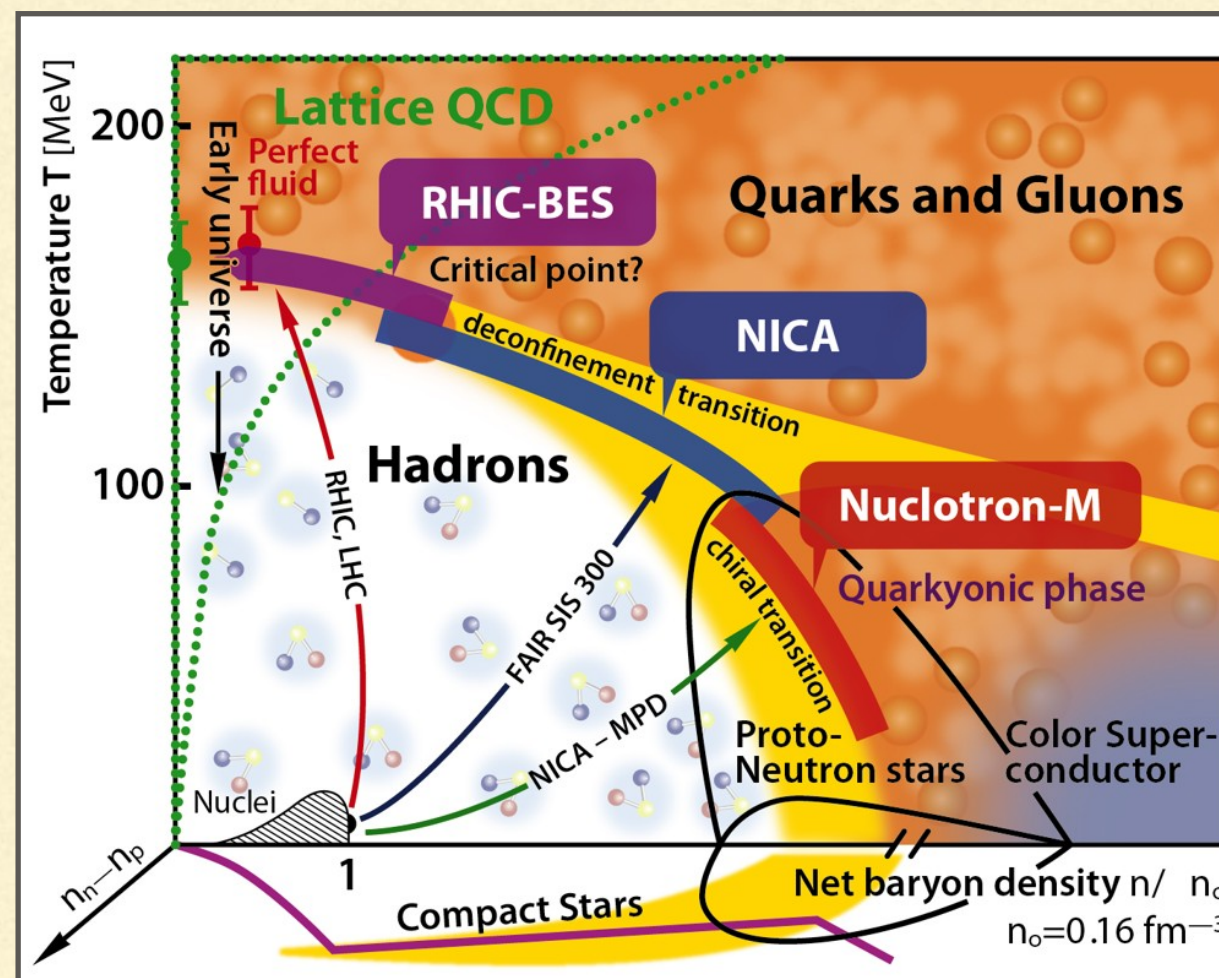
[ LIGO ]



# ADVANCED LIGO: NEUTRON STAR EOS

- Neutron star equation of state  $p = p(\rho)$ 
  - QCD phase diagram: colliders cannot access high  $\mu$  (chemical potential) regions

Temperature



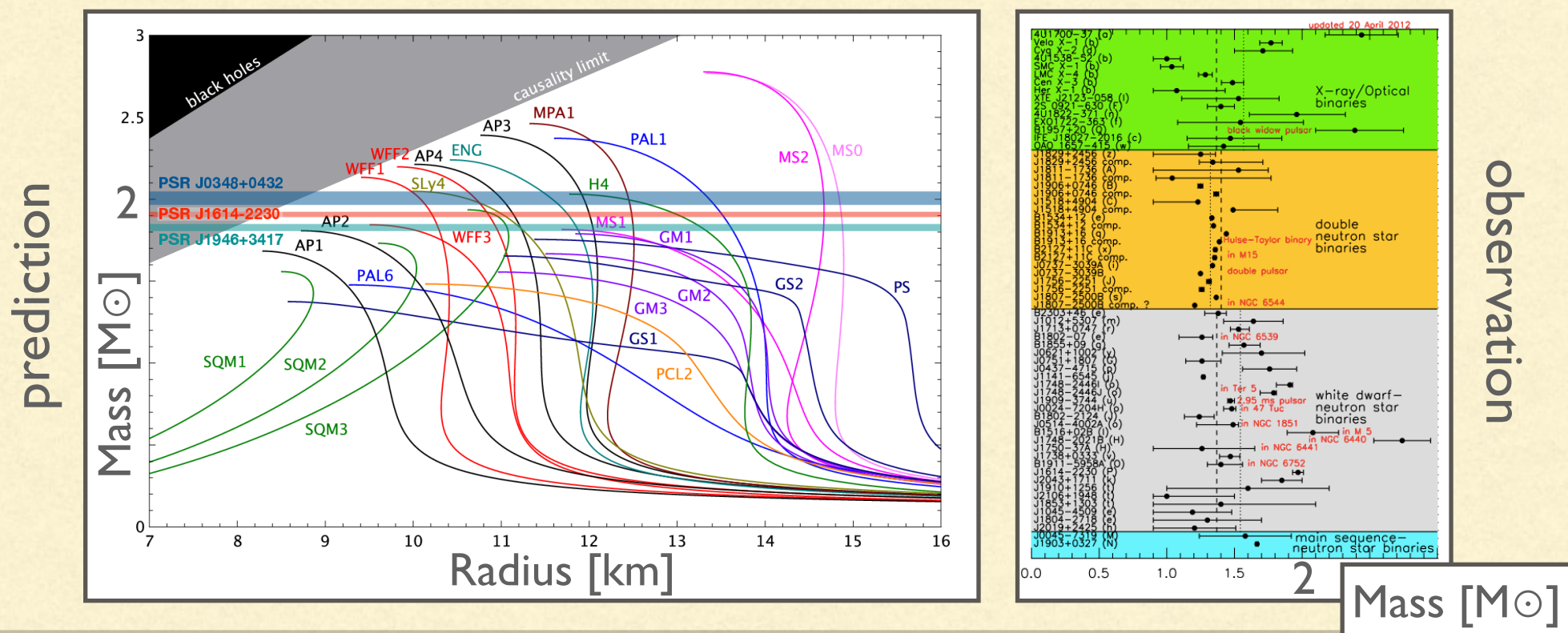
Chemical potential

[ <http://nica.jinr.ru/physics.php> ]



# ADVANCED LIGO: NEUTRON STAR EOS

- Neutron star equation of state  $p = p(\rho)$ 
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  - Different eos predicts different mass-radius relation
  - NS with mass  $> 2M_{\odot}$  found  $\rightarrow$  some eos models dead already

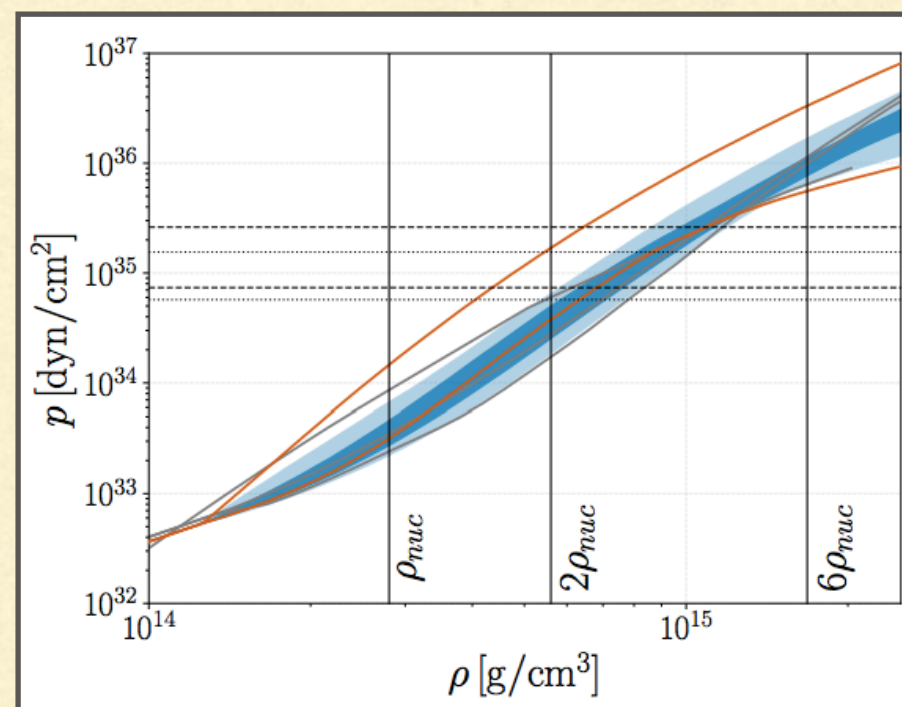




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  - GWs already

start to constrain eos



[ LIGO, 1805.11581 ]



# MULTI-MESSENGER ASTRONOMY

- Era of multi-messenger astronomy: EM, CR, GWs, Neutrinos

[ Wikipedia ]

Event type	Electromagnetic	Cosmic rays	Gravitational waves	Neutrinos	Example
Solar flare	yes	yes	-	-	Flare in 1940 <sup>[5]</sup>
Supernova	yes	-	predicted <sup>[6]</sup>	yes	SN 1987A
Neutron star merger	yes	-	yes	predicted <sup>[7]</sup>	GW170817
Blazar	yes	-	-	yes	TXS 0506+056

**August 2017:** A [neutron star collision](#) in the galaxy [NGC 4993](#) produced the gravitational wave signal [GW170817](#), which was observed by the [LIGO/Virgo](#) collaboration. After 1.7 seconds, it was observed as the [gamma ray burst](#) GRB 170817A by the [Fermi Gamma-ray Space Telescope](#) and [INTEGRAL](#), and its optical counterpart SSS17a was detected 11 hours later at the [Las Campanas Observatory](#), then by the [Hubble space telescope](#) and the [Dark Energy Camera](#). Ultraviolet observations by the [Neil Gehrels Swift Observatory](#), X-ray observations by the [Chandra X-ray Observatory](#) and [radio](#) observations by the [Karl G. Jansky Very Large Array](#) complemented the detection. This was the first gravitational wave event observed with an electromagnetic counterpart, thereby marking a significant breakthrough for multi-messenger astronomy.<sup>[10]</sup> Non-observation of neutrinos was attributed to the jets being strongly off-axis.<sup>[11]</sup> On 9 December 2017, astronomers reported a brightening of X-ray emissions from GW170817/GRB 170817A/SSS17a.<sup>[12][13]</sup>



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# STANDARD "SIREN"

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Skipped due to time constraint

Please google it  $m(\_\_)m$



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# ASTROPHYSICAL GWS: SUMMARY

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- GW detection opens up a new era of GW astronomy:
  - New population of black holes
  - Neutron star equation of state
  - Multi-messenger astronomy
  - Standard "siren" ...



## Introduction

- What's gravitational waves?
- Some history

## astrophysical GWs

- New probe to the Universe  
(focussing on recent aLIGO)

## cosmological GWs

- What's interesting?
- "Standard" cosmic history
- Various sources

## Summary

My own  
work





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# COSMOLOGICAL GWS: SUMMARY

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- Early Universe = High energy physics

- GWs are good probe to the early Universe

because of their suppressed interaction with other particles

- GWs with different frequencies have information on different energy scales:

Frequency-Energy scale correspondence



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# EARLY UNIVERSE & HIGH-ENERGY PHYSICS

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- Observed / strongly suggested "Beyond the Standard Model" in particle physics

- Baryon asymmetry
- Dark energy
- Dark matter
- Grand unification
- Hierarchy problem
- Inflation
- Neutrino mass
- Strong CP ...

...alphabetical order,  
to avoid religious conflict

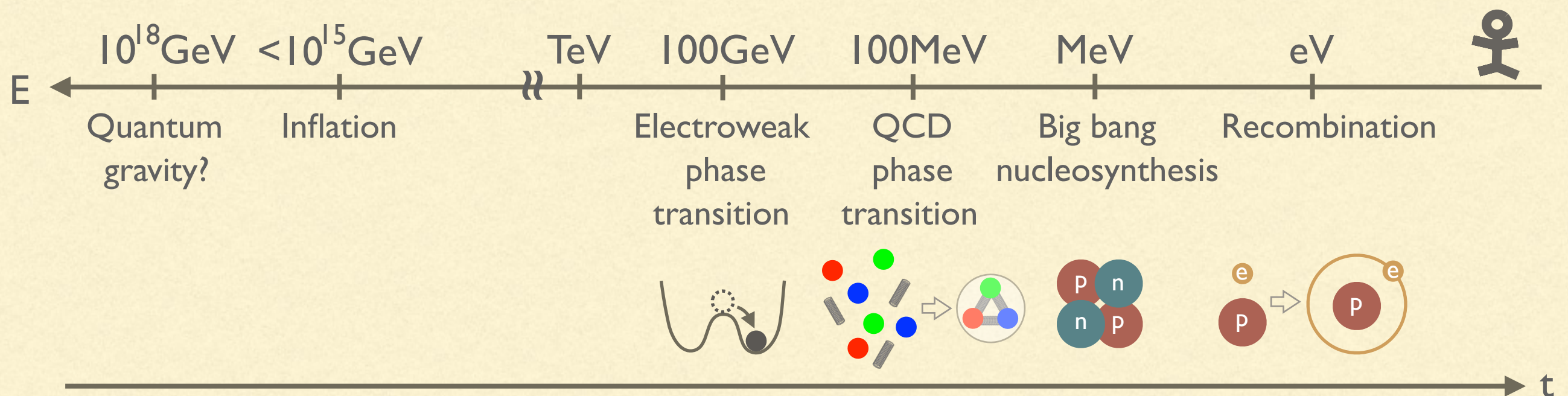


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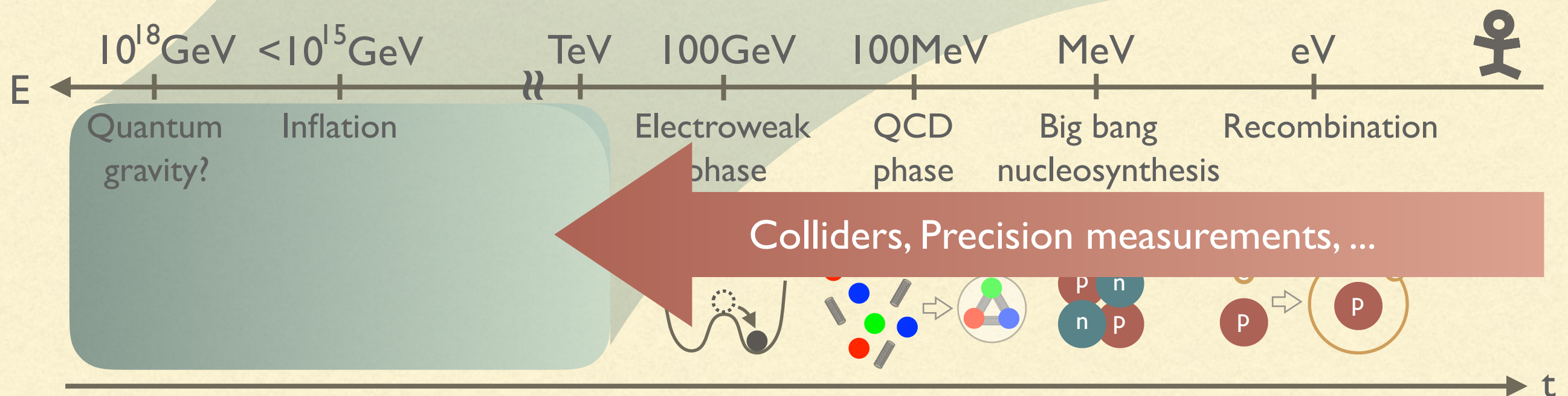


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# EARLY UNIVERSE & HIGH-ENERGY PHYSICS

- Early Universe = High energy thermal plasma

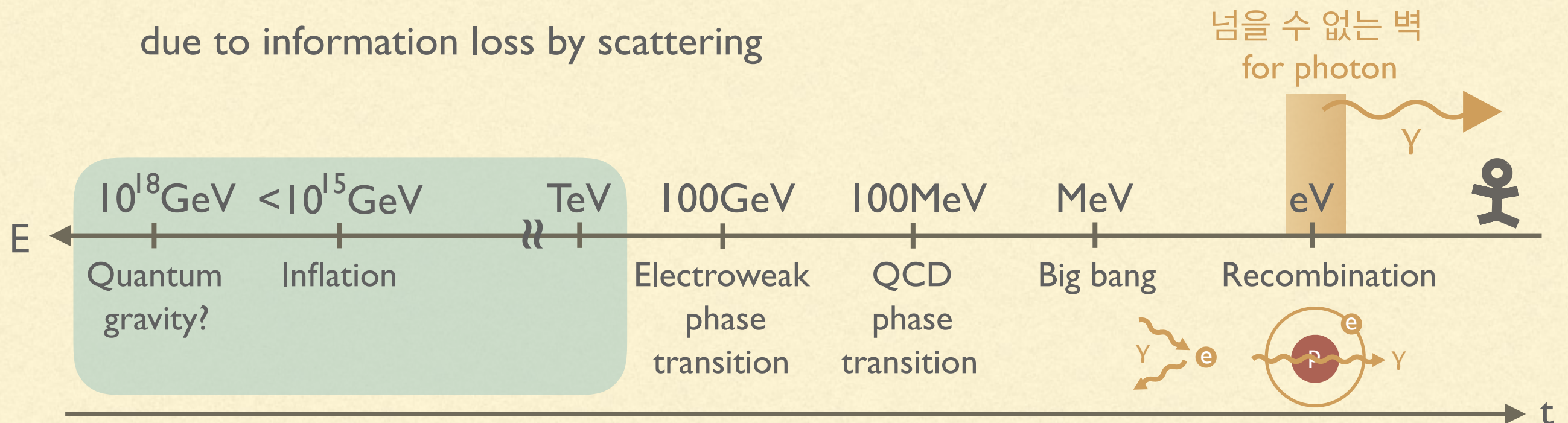
→ Cosmic relics are clues to unknown high-energy physics

- The most well-known relic is CMB (cosmic microwave background)

→ However, CMB basically probes only up to  $O(\text{eV})$  causal Universe

[ Note: inflationary physics  
by CMB is exceptional,  
due to over-horizon  
conservation of  $\zeta$  ]

due to information loss by scattering



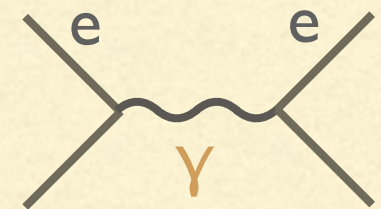


# EARLY UNIVERSE & HIGH-ENERGY PHYSICS

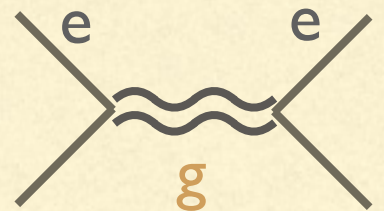
## ■ Gravitational waves

- All possible interactions suppressed by  $M_P \equiv 1/\sqrt{8\pi G} \sim 10^{18}$  GeV:

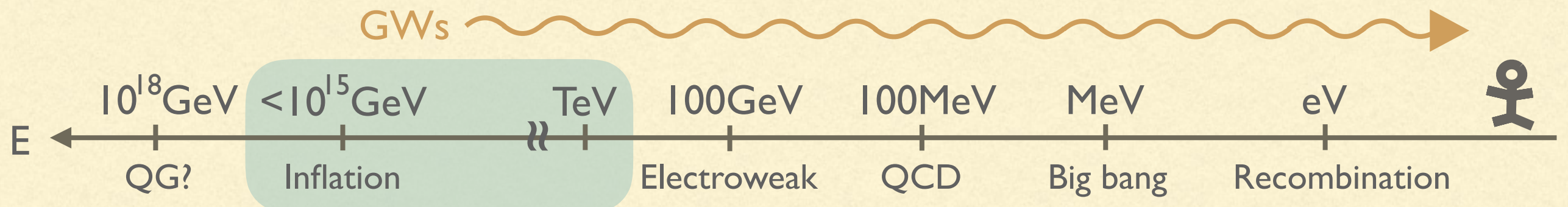
EM interaction between electrons:  $F \sim \frac{\alpha}{r^2} \sim \frac{1/137}{r^2}$



Gravitational interaction between electrons:  $F \sim \frac{Gm_e^2}{r^2} \sim \frac{10^{-42}}{r^2}$



- As a result, GWs propagate until present **without being scattered**, if produced





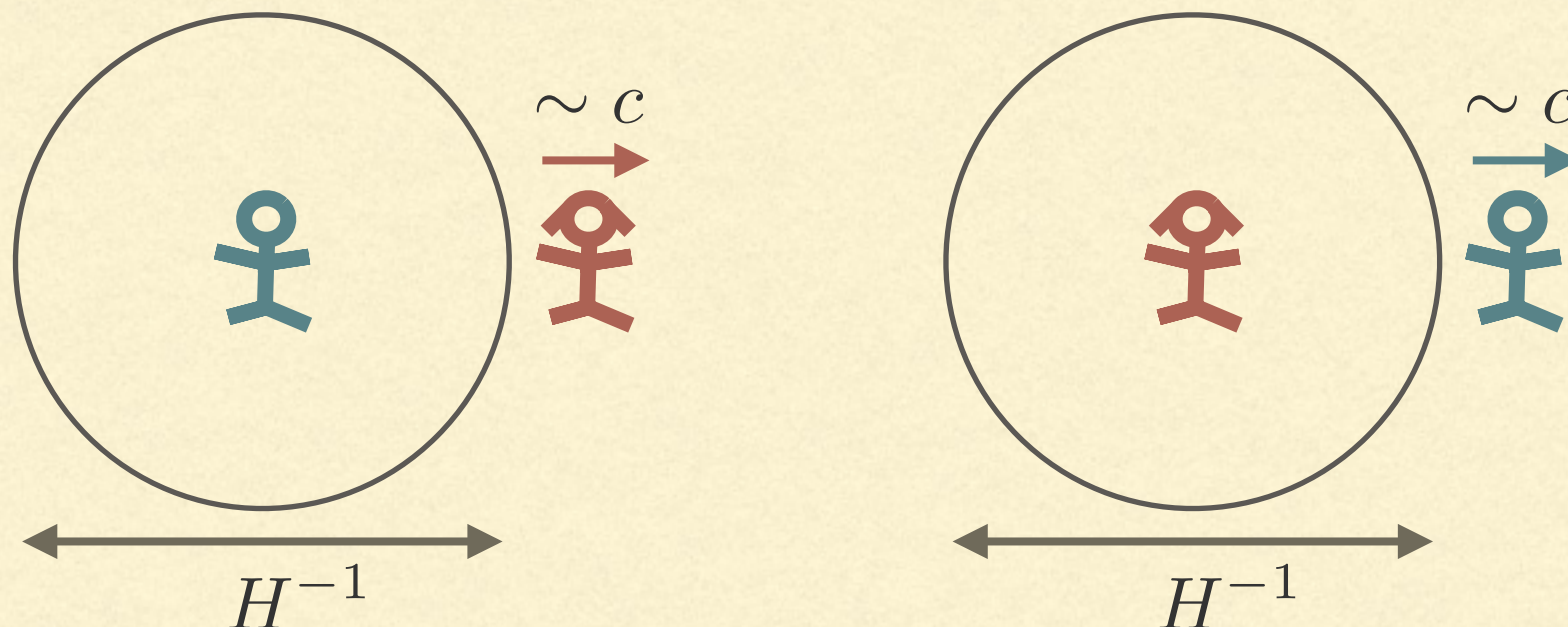
# FREQUENCY-ENERGY CORRESPONDENCE

## ■ Hubble horizon?

- Hubble parameter  $H \equiv \frac{\dot{a}}{a}$  ← Remember  $ds^2 = -dt^2 + a^2(t)dx^i dx^i$   
"scale factor"  
~ size of the Universe
- Inverse Hubble parameter  $H^{-1}$  determines

typical size of causally accessible region at that time

- Why?





# FREQUENCY-ENERGY CORRESPONDENCE

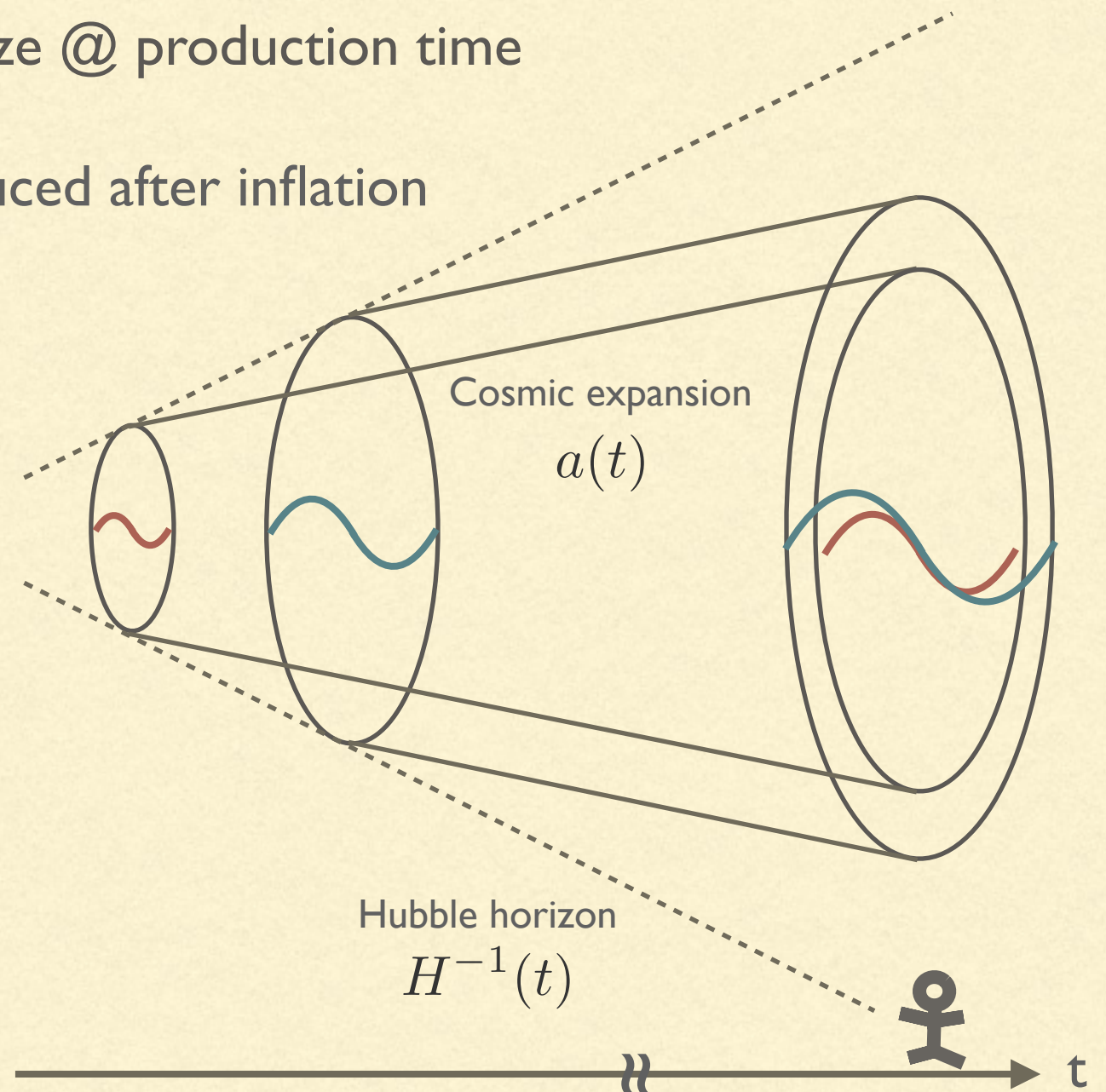
- GW present frequency vs. Horizon size @ production time

- Suppose horizon-size GWs are produced after inflation

- GWs are stretched in proportion to  $a(t)$  (redshift)

- Hubble horizon expand faster than  $a(t)$

- As a result, GWs produced earlier has shorter wavelength (higher freq.)





# FREQUENCY-ENERGY CORRESPONDENCE

- GW present frequency vs. Energy scale of the Universe @ production time
  - Energy of the Universe is higher at earlier times
  - As a result : GW present frequency  $\Leftrightarrow$  Energy scale of the Universe @ prod.

$$f_0 \sim 1 \text{ Hz} \times \left( \frac{T}{10^7 \text{ GeV}} \right)$$

- If sub-horizon GWs are produced, they shift toward higher frequency accordingly:

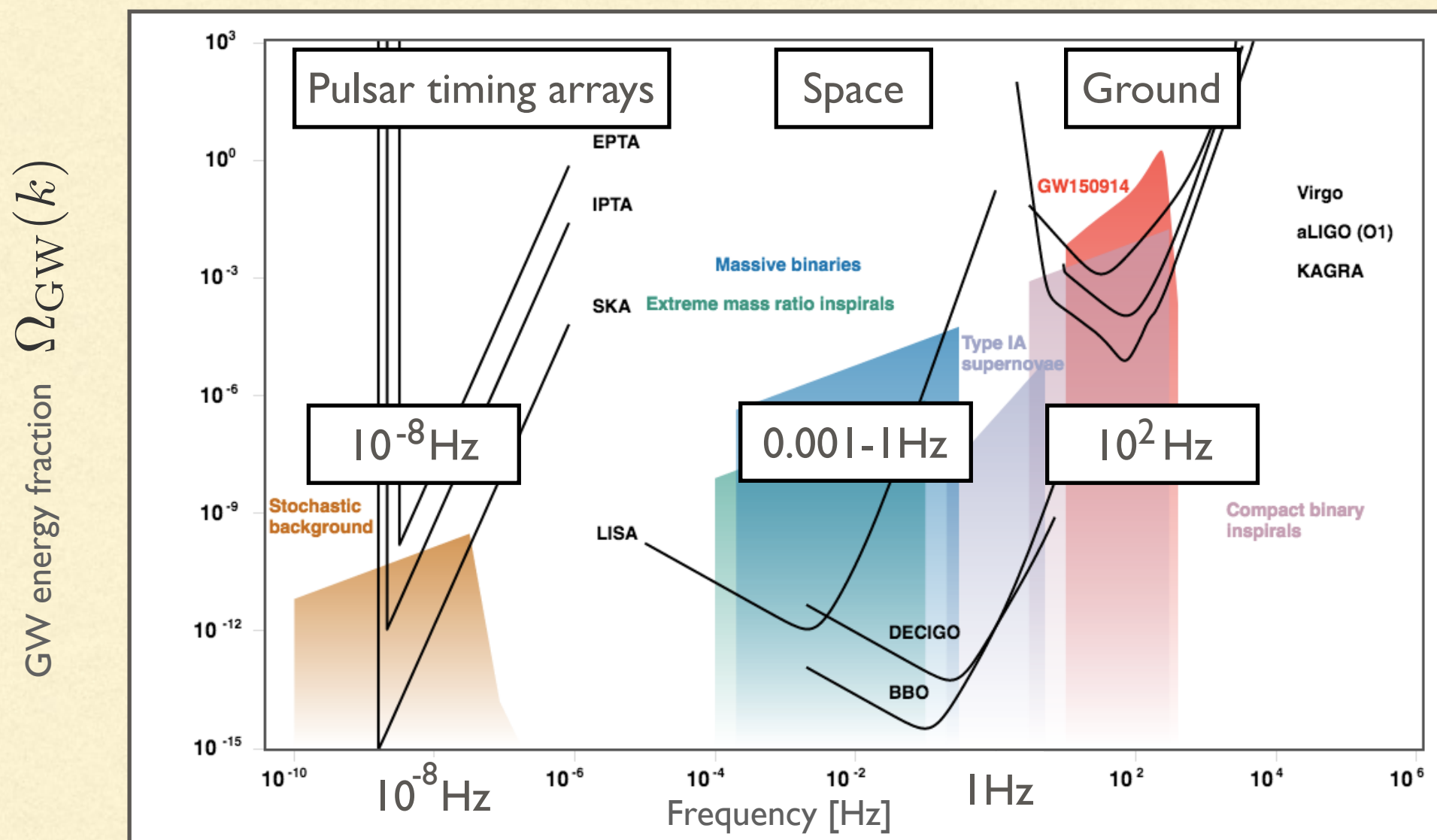
$$f_0 \sim 1 \text{ Hz} \times \frac{1}{R} \times \left( \frac{T}{10^7 \text{ GeV}} \right)$$

$R$  : (GW wavelength) / (Horizon size) @ prod.



# VARIOUS GW EXPERIMENTS

[<http://rhcole.com/apps/GWplotter/>]



100 MeV

10<sup>7</sup> GeV

T

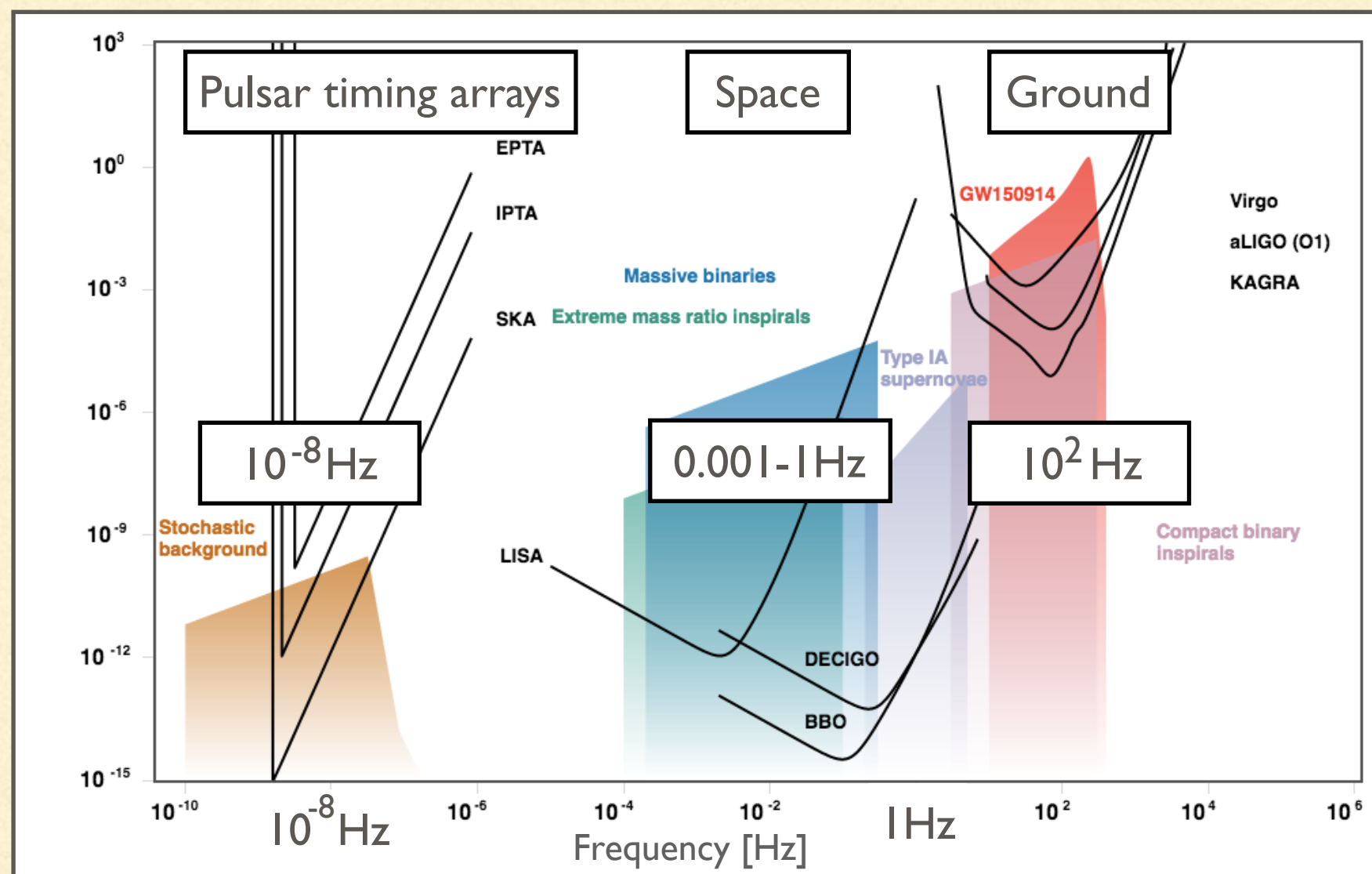
If produced  
with horizon size



# VARIOUS GW EXPERIMENTS

[<http://rhcole.com/apps/GWplotter/>]

GW energy fraction  $\Omega_{\text{GW}}(k)$



100 MeV

$10^7$  GeV

T

If produced  
with horizon size



# ENERGY DENSITY OF GWS

- Energy density of GWs

- Quadratic action of GWs

FRW background

Up to quadratic

$$S_{\text{grav}} = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R \right] \quad \downarrow \quad \simeq \quad \frac{M_P^2}{4} \int d^4x \, a^3 \left[ \frac{1}{2} (\dot{h}_{ij})^2 - \frac{1}{2} (\partial_k h_{ij})^2 \right]$$

- Looks similar to canonical scalar field, if we identify  $(M_P/2)h_{ij}$  as canonical field

- Energy density  $\rho_{\text{GW}} = \frac{M_P^2}{4} \left[ \frac{1}{2} (\dot{h}_{ij})^2 + \frac{1}{2} (\partial_k h_{ij})^2 \right]$

Abuse of notation

$$\Omega_{\text{GW}}(t, k)$$

- Energy density of GWs compare to the total energy density

$$\Omega_{\text{GW}}(t) \equiv \frac{\rho_{\text{GW}}(t)}{\rho_{\text{tot}}(t)} \quad \begin{array}{c} \text{Decomposition into} \\ = \\ \text{various } \ln k \text{ contributions} \end{array} \quad \int d \ln k \quad \frac{d\Omega_{\text{GW}}(t, k)}{d \ln k}$$

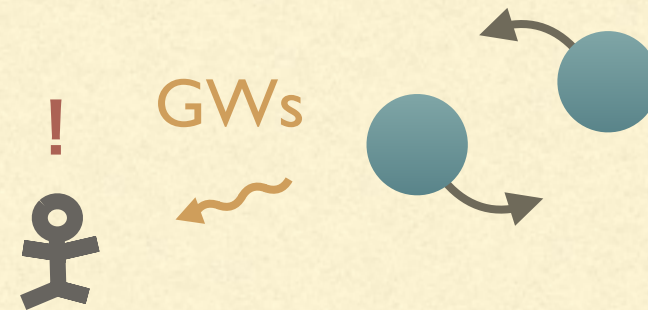


# POINT SOURCE vs. STOCHASTIC

- Main difference between astrophysical & cosmological GWs

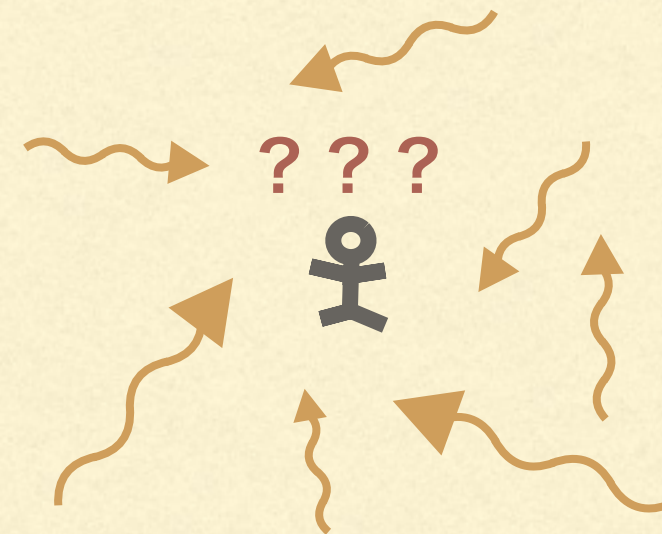
Astrophysical GWs : point-source

- Event-by-event
- Definite direction & frequency



Cosmological GWs : stochastic

- Cannot identify "event"
- Various directions & frequencies mixed up





# VARIOUS GW EXPERIMENTS

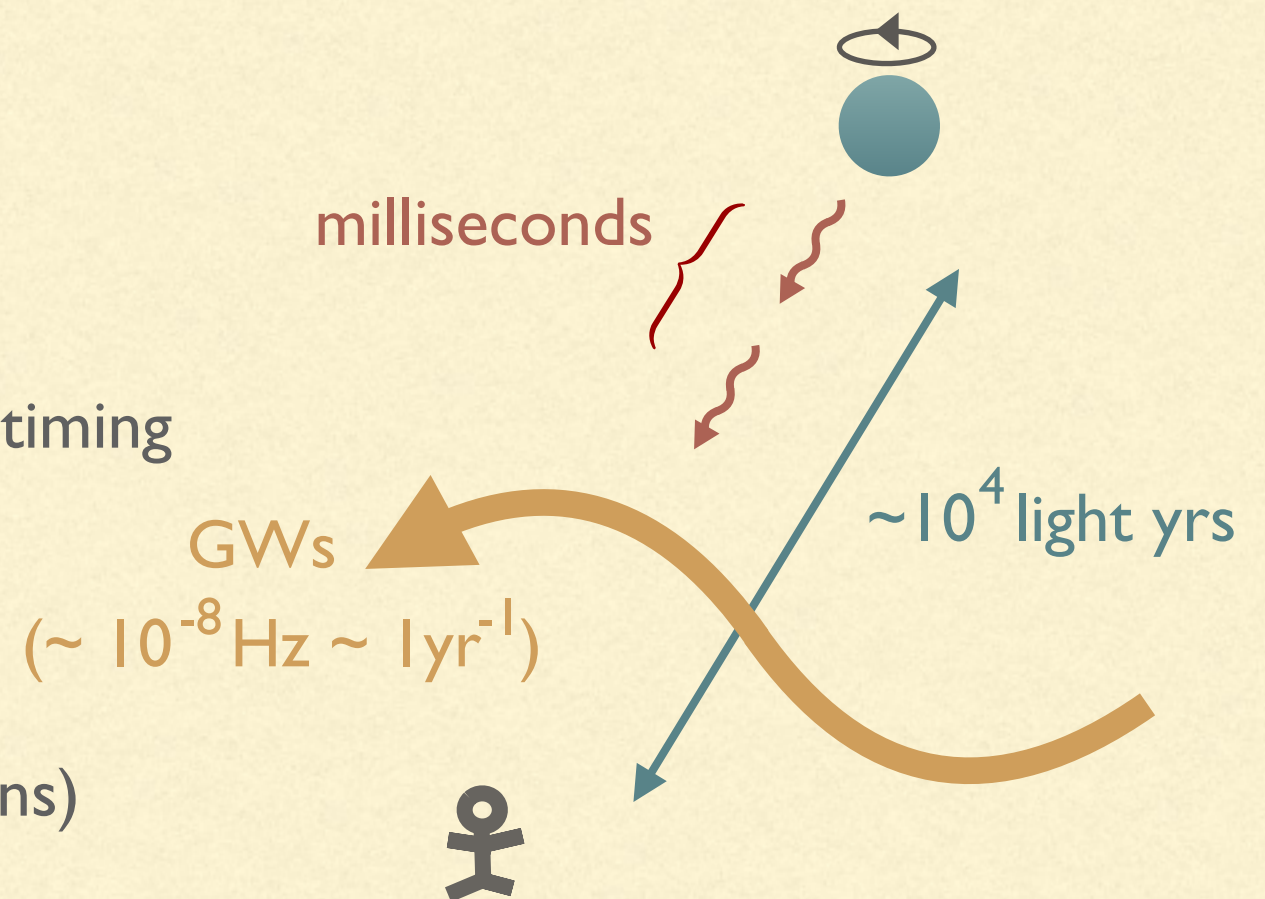
- Pulsar timing array

- Millisecond pulsar : pulsar with rotational period 1-10 milliseconds

(presumably, old & rapidly rotating NS)

- If GWs  $\sim 10^{-8}$  Hz pass through,  
they cause modulations in pulse arrival timing  
with a period of O(months)

- Pulse arrival timing measurable with O(ns)





# VARIOUS GW EXPERIMENTS

- Space interferometer

- Interferometers launched in space

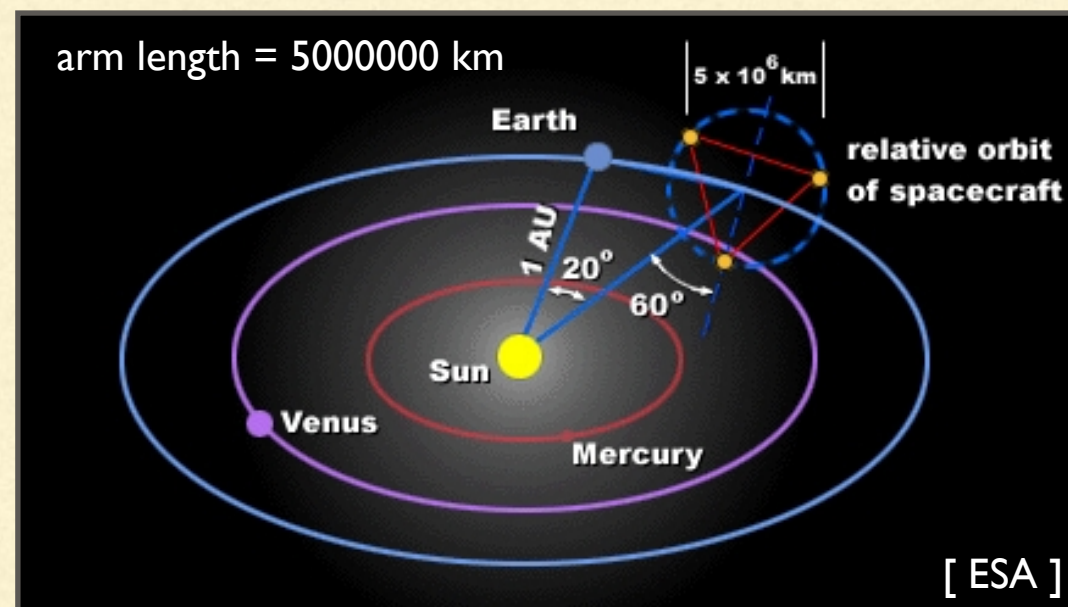
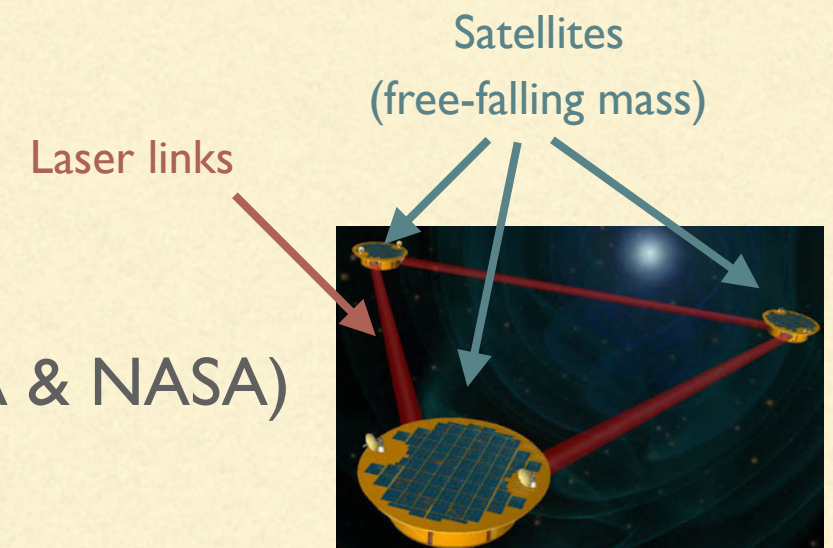
e.g. LISA = Laser Interferometer Space Antenna (ESA & NASA)

Launch scheduled in 2034

e.g. DECIGO = DECIherz Interferometer Gravitational wave Observatory (Japan)

e.g. Taiji, TianQin (China)

- "Arm" length corresponds to observable GW frequency





# COSMOLOGICAL GW SOURCES

- Cosmic string

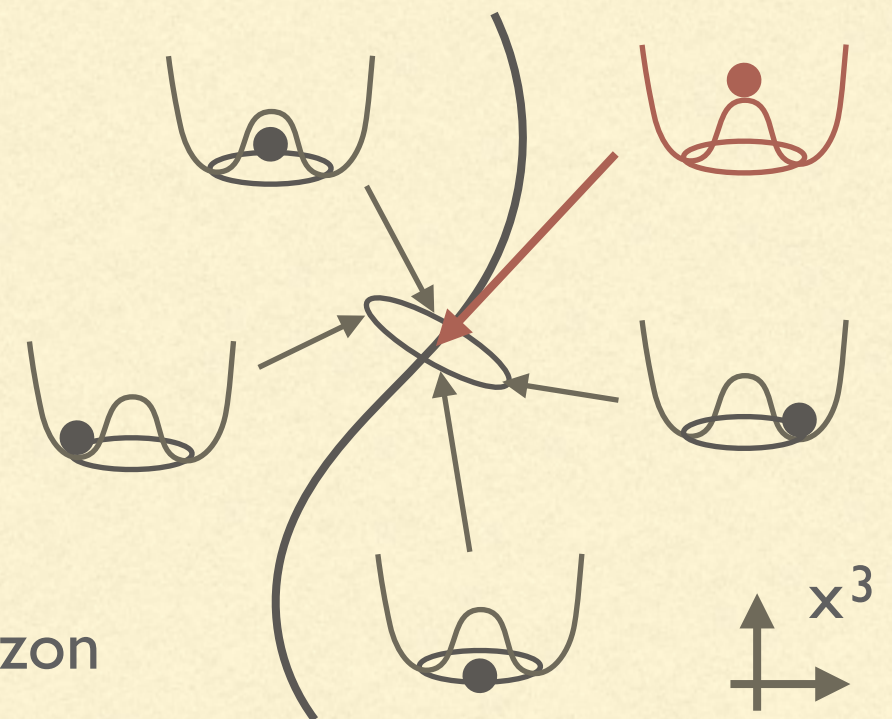
- Remnant of  $U(1)$  symmetry breaking

- Known to follow scaling law :

After production,  $O(1)$  strings per one Hubble horizon

at any time in radiation-dominated Universe

- Cusps and loops continuously emit GWs





# COSMOLOGICAL GW SOURCES

## ■ Cosmic string

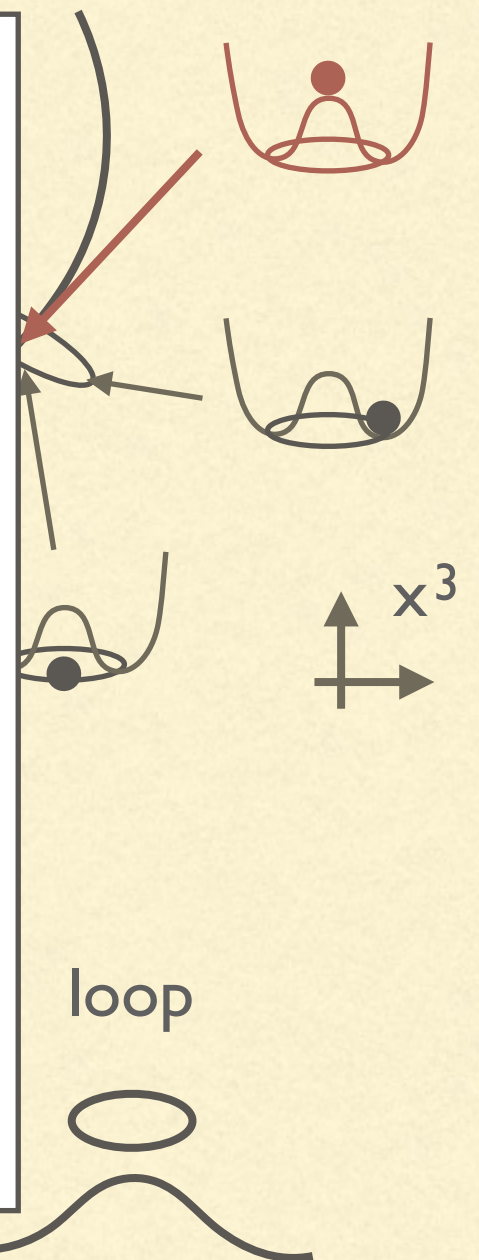
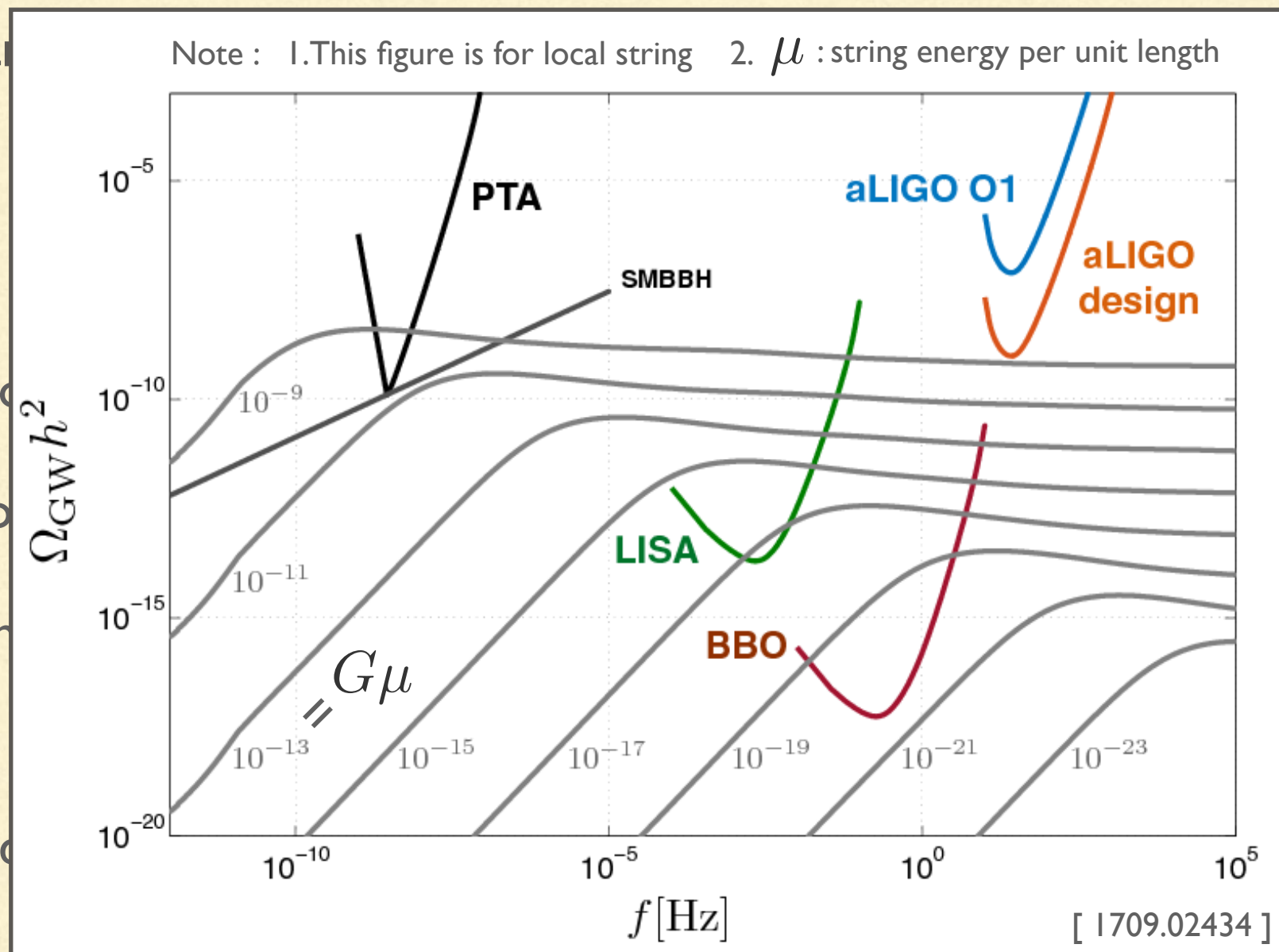
- Remnant

- Known to

After pro

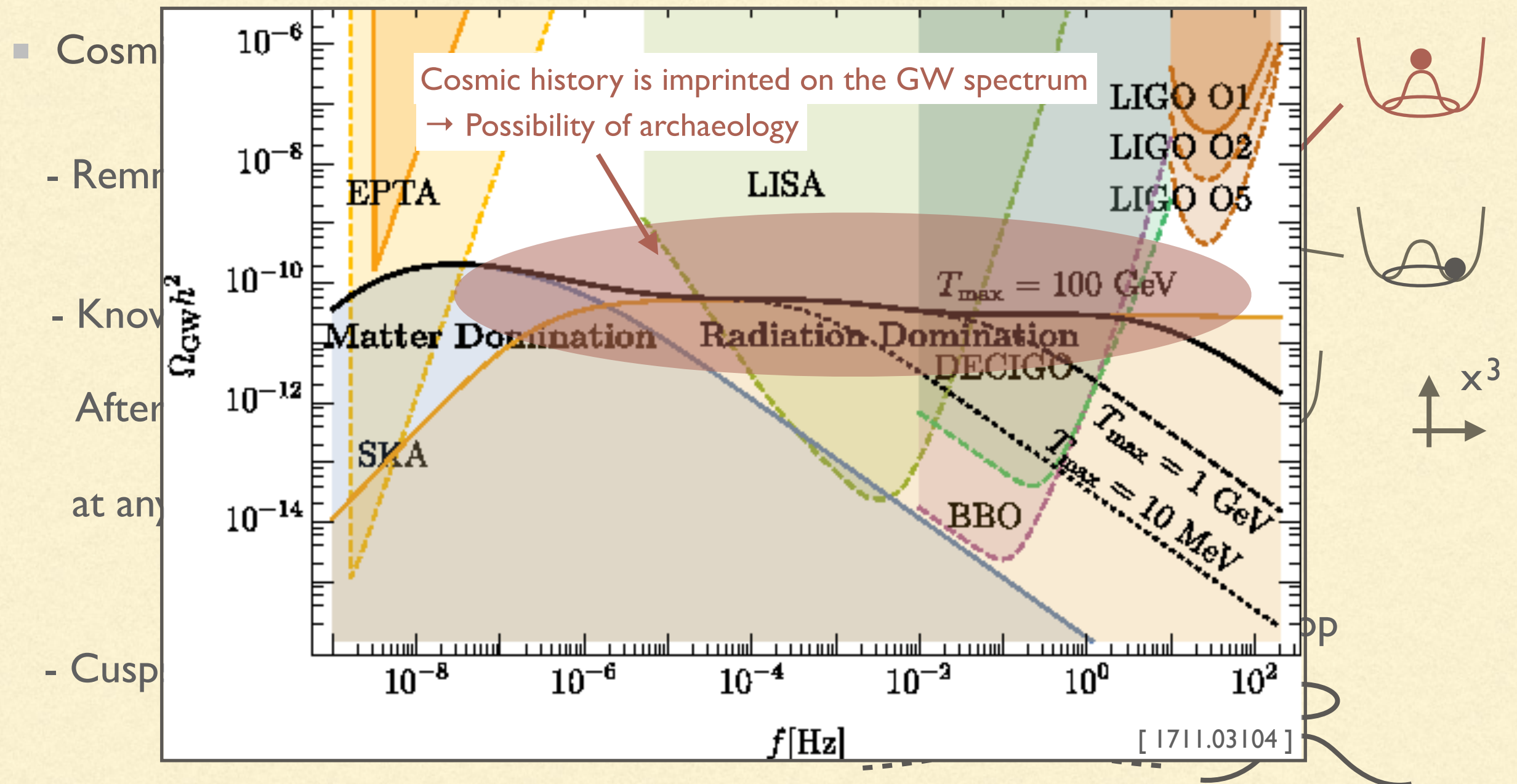
at any tim

- Cusps and





# COSMOLOGICAL GW SOURCES





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# COSMOLOGICAL GW SOURCES

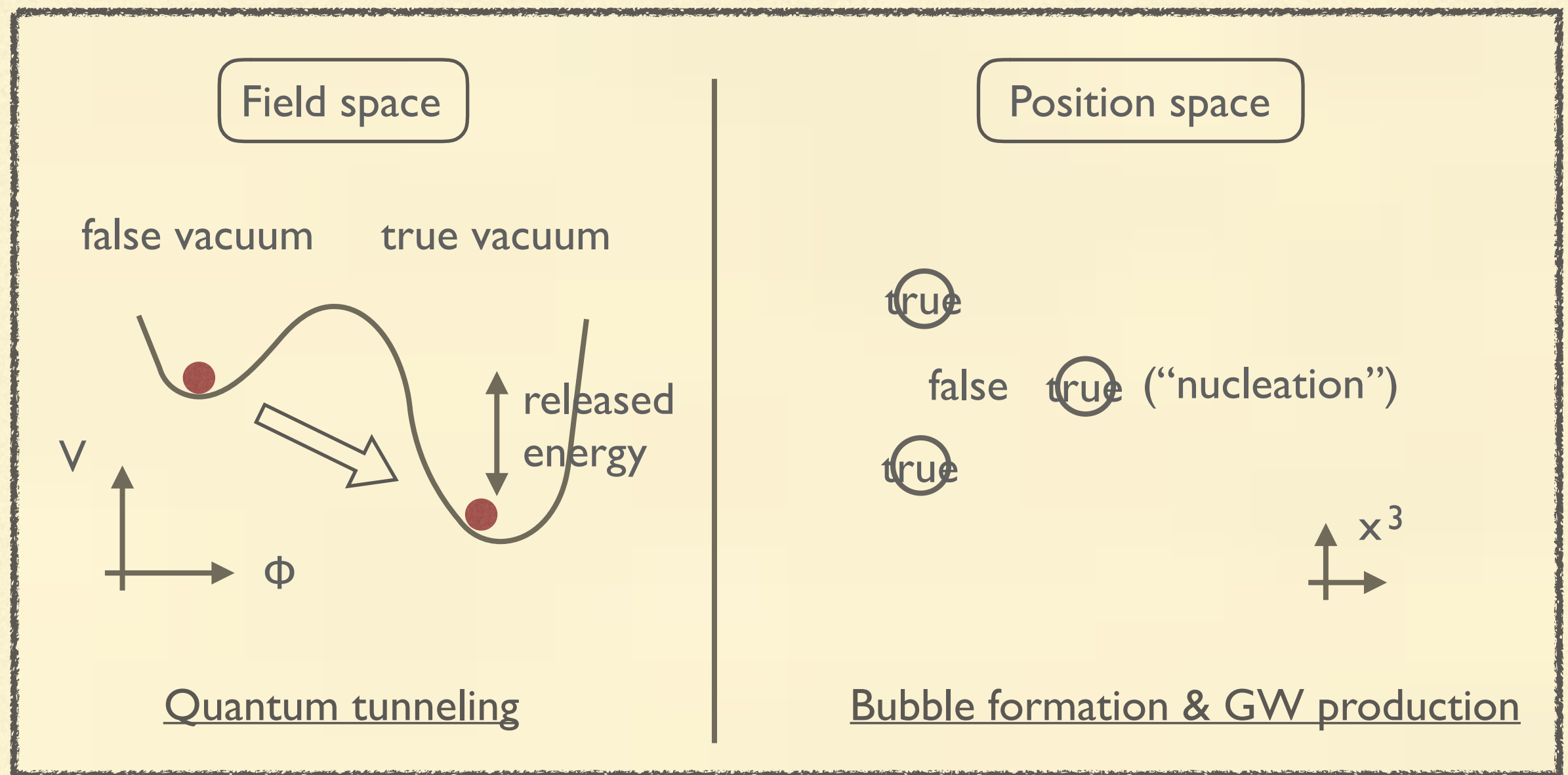
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- First-order phase transition
  - In some particle physics models (including extensions of EW sector), quantum tunneling of scalar field occurs in the history of the Universe
  - The transition proceeds with bubble formation and expansion



# COSMOLOGICAL GW SOURCES

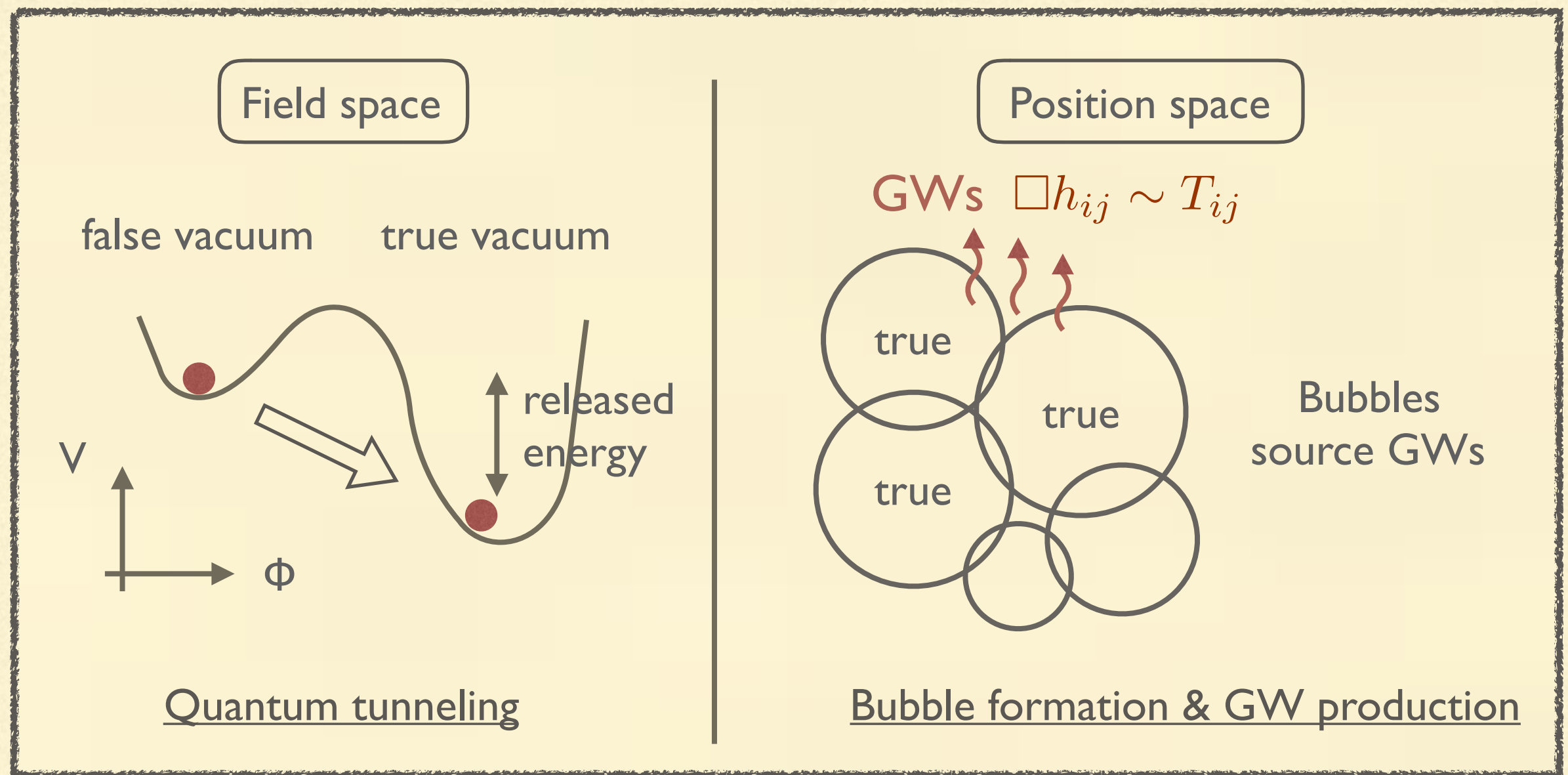
- First-order phase transition





# COSMOLOGICAL GW SOURCES

- First-order phase transition





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# COSMOLOGICAL GW SOURCES

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- First-order phase transition
  - In some particle physics models (including extensions of EW sector), quantum tunneling of scalar field occurs in the history of the Universe
  - The transition proceeds with bubble formation and expansion
  - GWs are produced by the scalar field bubbles and surrounding plasma bulk motion
  - GW frequency roughly corresponds to the bubble size, which is typically  $1/100 \sim 1/1000$  of the Hubble horizon
    - EW scale transition corresponds to sub-1 Hz



# COSMOLOGICAL GW SOURCES

- First-order

- In some

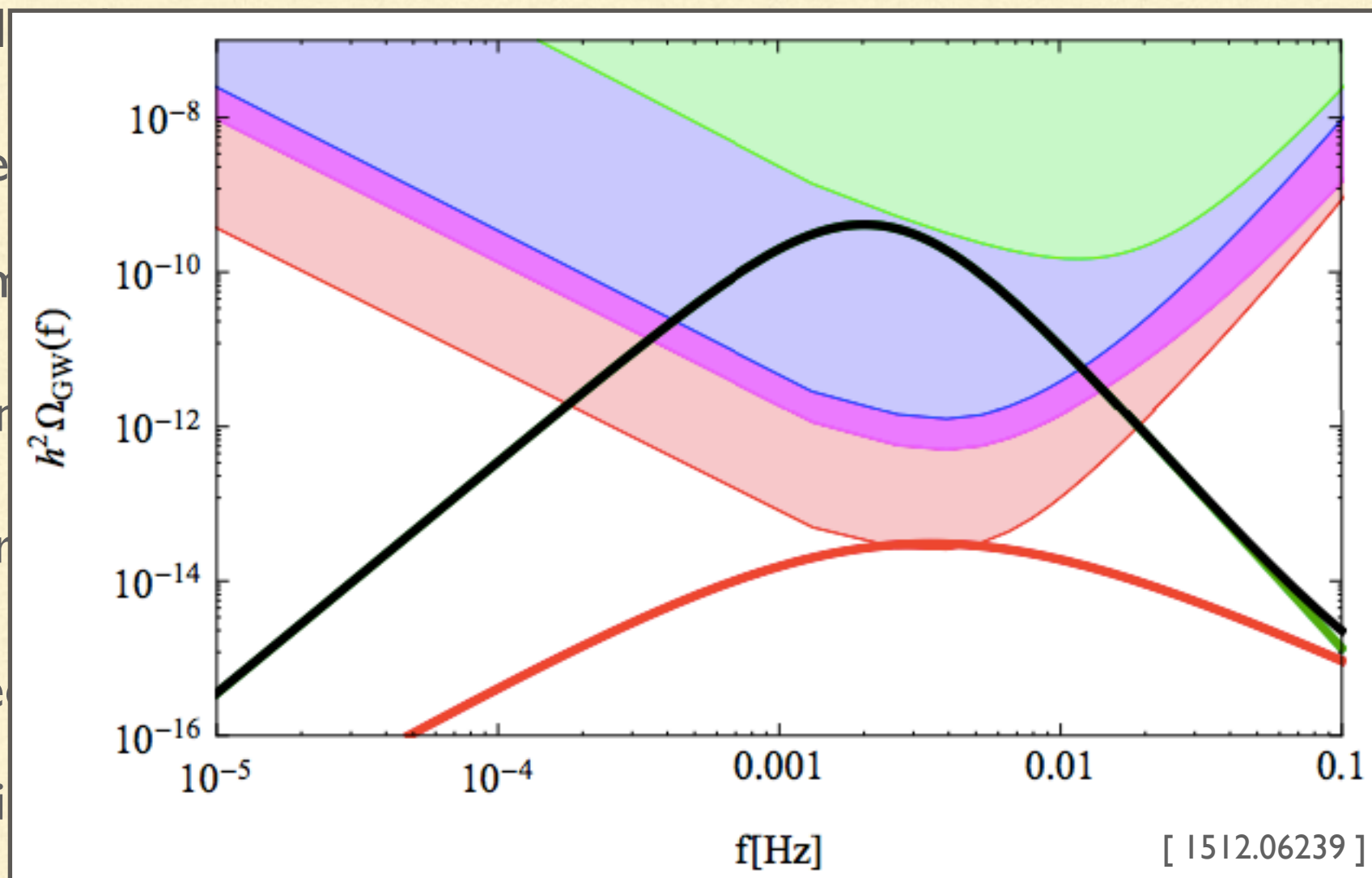
- quantum

- The tran

- GWs are

- GW fre

- which i

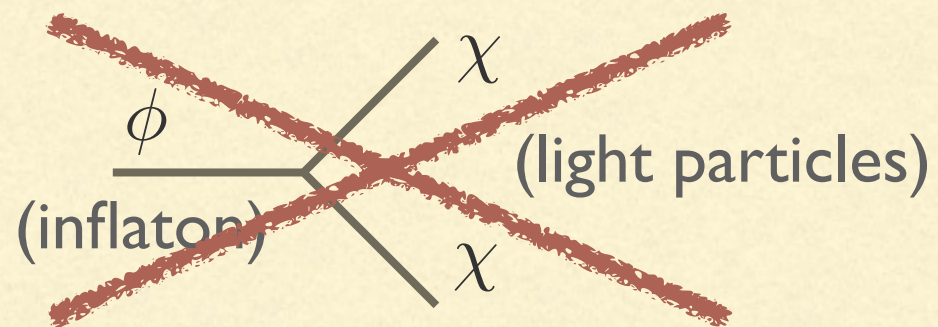


→ EW scale transition corresponds to sub-1 Hz



# COSMOLOGICAL GW SOURCES

- Preheating = dynamics before reheating
  - At the end of inflation (= accelerated expansion of the Universe at very early stage), inflaton (= scalar field which drives inflation) decays into light particles, and light particles eventually thermalize (= reheating)
  - This decay process is explosive : not like perturbative decay



- This explosive dynamics produces GWs typically at high frequencies



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# COSMOLOGICAL GW SOURCES

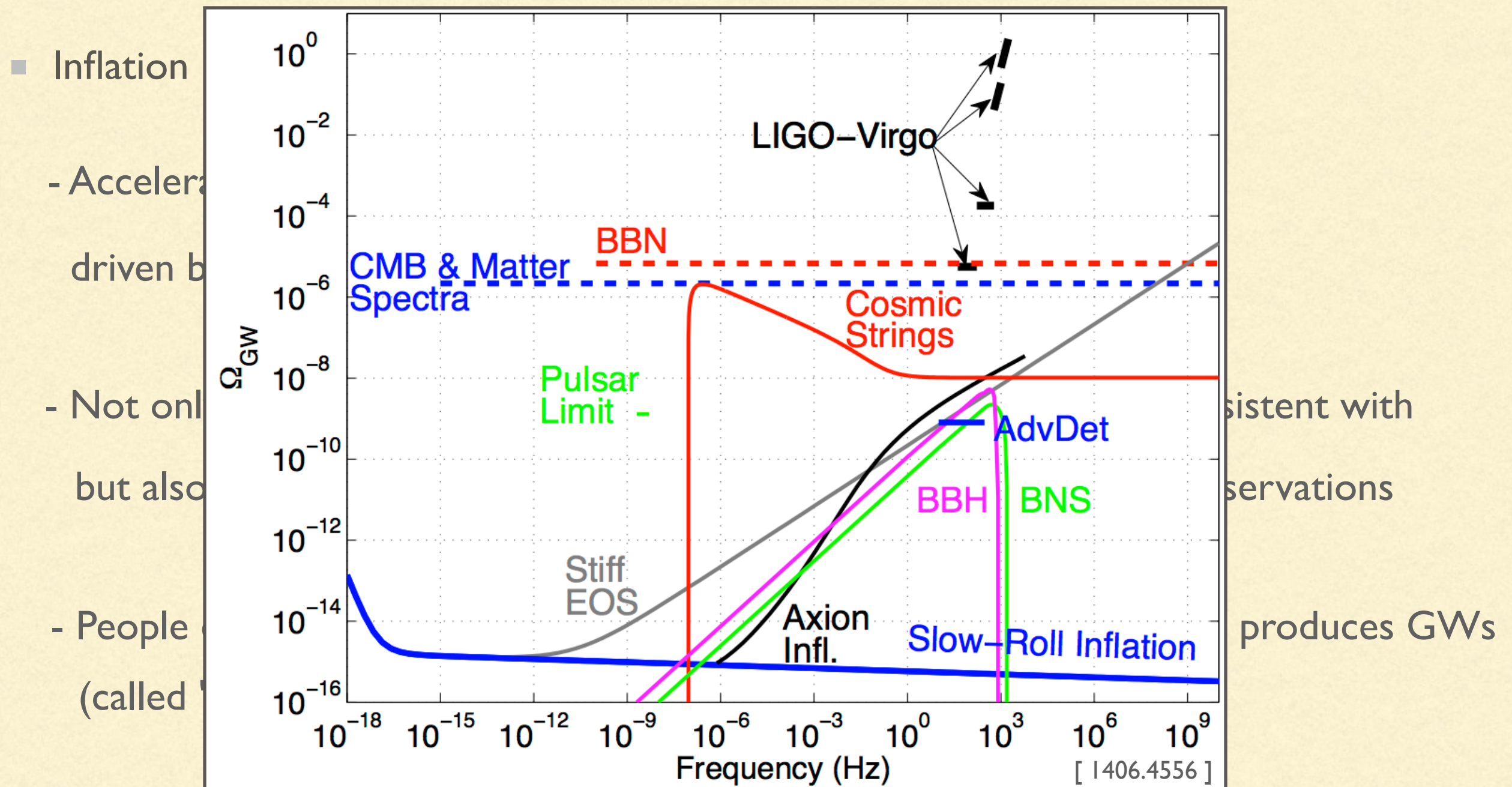
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- Inflation

- Accelerated expansion of the Universe at very early stage,  
driven by some scalar field (called inflaton)
- Not only solves notorious horizon & flatness problems,  
but also produces late-time seeds of galaxies → consistent with  
observations
- People expect that the same mechanism as this seed generation also produces GWs  
(called "primordial GWs")



# COSMOLOGICAL GW SOURCES





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# SUMMARY

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- In General Relativity, there are degrees of freedom  
propagating with speed of light : Gravitational Waves
- Detection of GWs opened up new era of **GW astronomy**
- **GW cosmology** may play a key role to explore  
unknown high-energy physics



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# Back up

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# COSMOLOGICAL GW SOURCES

- Cosmological GW production : Rules of thumb

Integration over  
Source duration time

$$\square h \sim GT \rightarrow \dot{h} \sim GT \Delta t$$

$$\rho_{\text{GW}} \sim M_P^2 \dot{h}^2 \sim \frac{T^2}{M_P^2} \Delta t^2$$

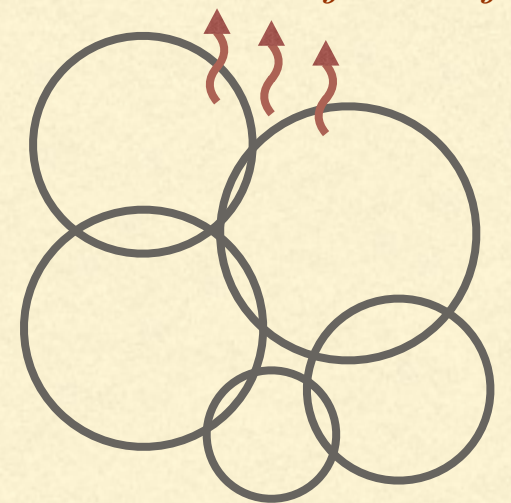
- $T$  means  $T_{ij}$  : Energy density only does not produce GWs.

We need (quadrupole of) momentum.

- Long-lasting sources are favored for GW production

Cosmological 1st-order  
phase transition

GWs  $\square h_{ij} \sim T_{ij}$



bubbles expanding with  $\sim c$

→ momentum

Note: 1. This does not apply to inflationary GWs! 2. This applies only to  $\Delta t \lesssim$  typical length scale of the source



# COSMOLOGICAL GW SOURCES

- Cosmological GW evolution

- Roughly : Just redshifted as radiation component

Note: except for  $g_*$

