Axions Relic Density and Instanton

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Outline

- Introduction
- Instanton contribution to the Free Energy
- $\chi(T)$ At Intermediate Temperatures: Model Building
- Axion Relic Density from Misalignment
- Conclusion

Strong CP Problem

In Standard Model, there is one term allowed that could break CP-symmetry

$$\theta \frac{g_s^2}{32\pi^2} \epsilon^{\mu\nu\alpha\beta} F^a_{\mu\nu} F^a_{\alpha\beta}$$

- Since CP is violated in Standard Model, there is no reason to forbid this term.
- The current bound on the neutron electric dipole moment gives that $\theta < 10^{-10}$.

The smallness of θ is known as the strong CP problem.

Axions

- A new global chiral U(1) symmetry called Peccei-Quinn symmetry which is anomolous.
- Once symmetry is spontaneously broken, it will generate pseudo-Goldstone boson *a*, with coupling to QCD

$$\mathcal{L}_{axion} = (\partial_\mu a)^2 + \frac{a/f_a + \theta}{32\pi^2} F\tilde{F}$$

where f_a is known as the axion decay constant.
One can calculate the axion contribution to the vacuum energy, which gives

$$E = f_\pi^2 m_\pi^2 cos(\theta - \frac{a(x)}{f_a})$$

Minimizing the potential solve the strong CP problem.

Axion as dark matter

The axion mass is givin by

$$m_a = \sqrt{\frac{m_u m_d}{(m_u + m_d)^2}} \frac{m_\pi f_\pi}{f_a} \approx \mu eV(\frac{10^{12} GeV}{f_a})$$

- Presently astrophysical constraints bound f_a between 10^8 GeV and 10^{17} GeV.
- axion may explain the observed dark matter of the universe.

Axion Relic Density

In the early universe, axion field is governed by the equation of motion

$$\ddot{a} + 3H\dot{a} + V'(a) = 0$$

The axion field starts to oscillate coherently when its thermal mass $m(T)$ becomes comparable to Hubble scale.

$$m_a(T_{osc}) = 3H(T_{osc})$$

Soon after that the comoving number density $n_a =< m_a a^2 >$ becomes an adiabatic invariant and the axion behaves as cold dark matter.

To calculate relic density, it's important to decide the thermal mass $m_a(T)$. This mass is related to the QCD topological susceptibility by

$$m_a^2(T) f_a^2 = \chi(T), \quad \chi = \int d^4x < F\tilde{F}(x) F\tilde{F}(0) >= \partial_\theta^2 F(\theta, T)|_{\theta=0}$$

- At high temperatures, $F(\theta, T)$ can be calculated by standard instanton methods.
- At low temperatures, $F(\theta, T)$ is known from chiral perturbation theory, and in fact converges rapidly to its T = 0 limit below the confining phase transition

At high temperature, the complete expression for the free energy in the presence of a single instanton at one loop is given by [GPY 1981]

$$F(\theta, T) = -\int \frac{d\rho}{\rho^5} (\frac{4\pi^2}{g^2})^{2N} e^{-\frac{8\pi^2}{g^2(\rho)}} C_N \prod_{i=1}^{N_f} (\xi \rho m_i)$$

$$\times e^{-1/3\lambda^2(2N+N_F)-12A(\lambda)[1+\frac{1}{6}(N-N_f)]+i\theta}$$

where

$$A(\lambda) = -\frac{1}{12}\ln(1+\lambda^2/3) + \alpha(1+\gamma\lambda^{-2/3})^{-8}$$

$$\lambda = \pi\rho T \quad C_N = 0.097163; \quad \xi = 1.3388 \quad \alpha = -.01290 \quad \gamma = 0.1586$$

and $N_f = 3$ in temperature regimes where three quarks are excited. At a temperature of $T = 1.5 GeV$ and using a renormalization scale $\mu = T$, we obtain $F_0(1.5) = -3.7 \times 10^{-14} GeV^{-4}$

Theoretical Uncertainty

- Because the dominant instanton size is of order $(\pi T)^{-1}$, $\mu = \pi T$ is another natural choice for the renormalization scale.
- At $T = 1.5 GeV$, πT is substantially above the charm threshold and near the bottom quark mass. We might therefore include at least the charm quark in the free energy.
- A complete two-loop calculation of F is not available at present.
- The IR cutoff on the instanton size can be qualitatively associated with the Debye mass term in the effective action, which receive large corrections beyond leading order.

We therefore recompute the free energy using $\mu = [1, \sqrt{\pi}, \pi] \times T$, three or four active flavors, and including or not including the UV-divergent two-loop corrections. For the change in renormalization scale, we use the program RunDec to accurately determine $\alpha_s(\mu)$ with different numbers of active flavors.

3F, 1L, T	3.6
3F, 2L, T	10
3F, 1L, $\sqrt{\pi}$T	4.9
3F, 2L, $\sqrt{\pi}$T	7.2
4F, 1L, $\sqrt{\pi}$T	3.2
4F, 2L, $\sqrt{\pi}$T	5.2
3F, 1L, πT	6.0
3F, 2L, πT	5.5
4F, 1L, πT	4.0
4F, 2L, πT	3.8

The instanton-induced free energy in units of $-10^{-14} GeV^{-4}$ at $\theta = 0$ and $T = 1.5 GeV$

The infrared divergence in the ρ integral is cut off by the effective mass of the A_4 field

$$m_D^2 = \frac{1}{3} g^2 T^2 (N + \frac{N_f}{2})$$

It has been argued that there are large, infrared divergent corrections to this mass, already at one loop. The NLO Debye mass has the form[Rebhan 1993; Arnold and Yaffe 1995]

$$m_D^2 = (m_D)_0^2 + \frac{2Ng^2}{4\pi} T (m_D)_0 \ln(m_D/g^2 T) + \ldots .$$

Numerically, it gives rise to

$$\frac{(m_D)_1}{(m_D)_0} \simeq 0.6$$

at $T = 1.5$ GeV. We are led to associate an uncertainty in the free energy due to two-loop finite temperature corrections,

$$\frac{\Delta F_0(1.5)}{F_0(1.5)} \simeq 20 \; .$$

$\chi(T)$ At Intermediate Temperatures: Model Building

At scales below 1 GeV, the coupling rapidly becomes strong, and the instanton calculation unreliable. At very low temperatures, we know the dependence of the vacuum energy on θ reliably from current algebra,

$$V(\theta) = -3.6 \times 10^{-5}\ \mathrm{GeV}^4 \cos\theta.$$

We would like a model which interpolates between these regimes. More precisely, we would like a class of models, with a parameter we can vary, so as to get some sense of the sensitivity to QCD uncertainties in the intermediate regime.

Considering simple models that interpolate between the ChPT and instanton regimes, we will adopt the following class of models for $F(\theta, T)$

$$F(\theta, T) = \begin{cases} -\chi(0)\cos\theta & 0 < T < T_2 \\ -\chi(T_0)\left(\frac{T_0}{T}\right)^n \cos\theta & T_2 < T < T_0 \\ -\chi(T_0)\left(\frac{T_0}{T}\right)^8 \cos\theta & T > T_0 \end{cases}$$

Here T_0 is the anchor point for the instanton regime and will be taken to be 1.5 GeV. T_2 is given by

$$T_2^n = T_0^n \times \frac{\chi(T_0)}{\chi(0)}$$

We will vary n such that T_2 varies between 100 and 500 MeV.

The axion obeys the equation

$$\ddot{a} + 3H\dot{a} + V'(a) = 0.$$

A good approximation is obtained by treating the axion as frozen until a temperature, T_{osc}:

$$m_a(T_{osc}) = 3H(T_{osc}).$$

At this point, the axion begins to oscillate with a time (temperature) dependent mass. Calling

$$\rho(t) = \frac{1}{2}\dot{a}^2 + \frac{1}{2}m_a^2(T)a^2$$

one can show:

$$\rho(T) = \rho(T_{osc})\left(\frac{R^3(T_{osc})}{R^3(T)}\right)\frac{m_a(T)}{m_a(T_{osc})}.$$

The relic density Ω_a can then be expressed as a function of the parameters $\chi(T_0)$ and n (or T_2). The result is:

$$\Omega_{axion} = 0.13 \times (7.3)^{\frac{2}{4+n}} (\frac{m_a}{30\mu eV})^{-\frac{6+n}{4+n}} (\frac{\chi_0(1.5)}{3.7 \times 10^{-14} GeV^4})^{-\frac{1}{4+n}} (\frac{\theta_0}{2.155})^2$$

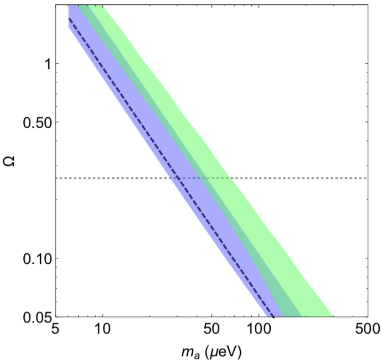
where θ_0 is the initial misalignment angle and m_a is the zero-temperature axion mass.

Ω

n	$\chi_0 = 1/10$	$\chi_0 = 1$	$\chi_0 = 10$
8	0.22	0.18	0.15
14	0.18	0.16	0.14
20	0.17	0.15	0.14

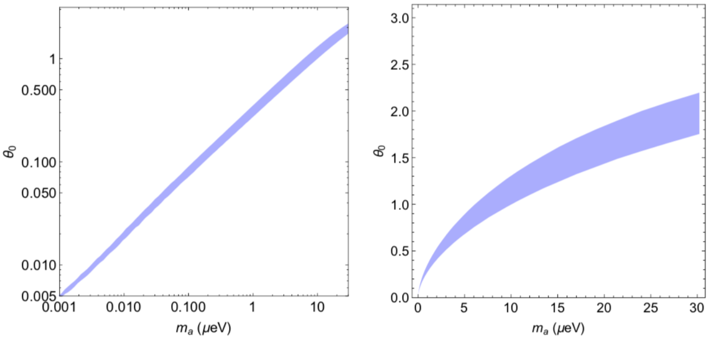
Table: Axion relic density as a function of model parameters.

Here $m_a(0) = 30\mu eV$, χ_0 is given in units of $3.7 \times 10^{-14} GeV^4$, and the misalignment angle is set to the value appropriate for post-inflationary breaking of the Peccei-Quinn symmetry, $\theta_0 = 2.16$.



- We solve the full axion equation of motion numerically through the time where it starts to oscillate.
- This figure shows the relic density obtained in this way for two values of T_2 and a range of χ_0, in the post-inflationary PQ-breaking scenario ($\theta_0 = 2.155$).

Even with the factor of 5 variation in T_2 and the factor of 20^2 variation in χ_0, we find that the axion mass required to account for all of dark matter varies by only a factor of 2-3.



In the case of symmetry breaking before inflation, θ_0 is a free parameter. In figures above we show the sensitivity of the misalignment angle required to saturate the relic density to the uncertainty in χ_0. As in the post-inflationary case studied above, we find that the theoretical uncertainties have essentially no qualitative impact on the required parameters. Furthermore, for a wide range of O(1) values for θ_0, the relevant axion masses are compatible with current and next-generation cavity experiments

Conclusion

- Within the conventional picture of axion cosmology, we have found that the standard computation of the axion relic density is relatively robust against theoretical uncertainties stemming from the dilute gas computation of the QCD free energy at high temperatures and the behavior of the free energy at strong coupling.
- An improved determination of the finite-temperature topological susceptibility would lead to improvement in the precision of the (relic density, axion mass) relation, it is not expected to lead to qualitative (order-of-magnitude) changes