

# Fermion Pair Production in Gauge-Higgs Unification Models

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Work with M. Peskin

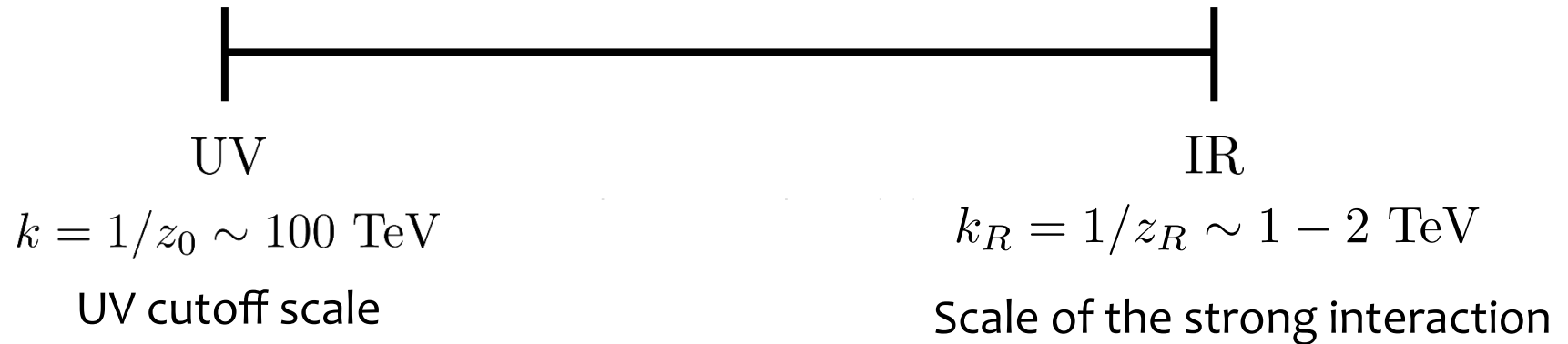
- Ideally, we would like to have a theory in which we can compute the Higgs potential and understand the EWSB.
- SUSY has been studied extensively, quantitatively.
- Higgs boson as a composite Goldstone boson of a new strong dynamics. It is not studied as thoroughly as SUSY.
- Is there a calculable, predictive approach to a new strong interaction?

- There is an interesting approach to composite Higgs based on following ideas:
  - 1) Randall-Sundrum geometry.
    - Dynamics in the 5<sup>th</sup> dimension models strong interaction.
    - 5D wavefunction models composite structure of fermions.
    - New resonances appear as KK states.
  - 2) Gauge-Higgs unification
    - Higgs as  $A_5^A$ , a part of the 5D bulk gauge symmetry.
    - We can compute the Higgs potential → predictive model.
  - 3) Dynamical EWSB by top quark condensation.
  - 4) “Little Hierarchy”  $s^2 = v^2 / f^2 \ll 1$

- These ideas were actively pursued in the early 2000's by Hall, Nomura, Agashe, Contino, Pomarol, Hosotani and others.
- Under this framework, we have studied how to generate the 2nd order phase transition in Higgs phase diagram. [arXiv:1709.07909](#)
- Also, we have built a realistic model under  $SO(5) \times U(1)$  symmetry and examined its parameter space thoroughly. [arXiv:1810.12352](#)
- In this talk, I will introduce a systematic, analytic study of fermion pair production cross sections in bulk RS models. I will focus on possible mass generation schemes for the bottom quark and its implication in  $e^+e^- \rightarrow b\bar{b}$  processes. [arXiv:1811.07877](#)

# 5D Geometry

AdS<sub>5</sub> bulk



# Fermions in RS

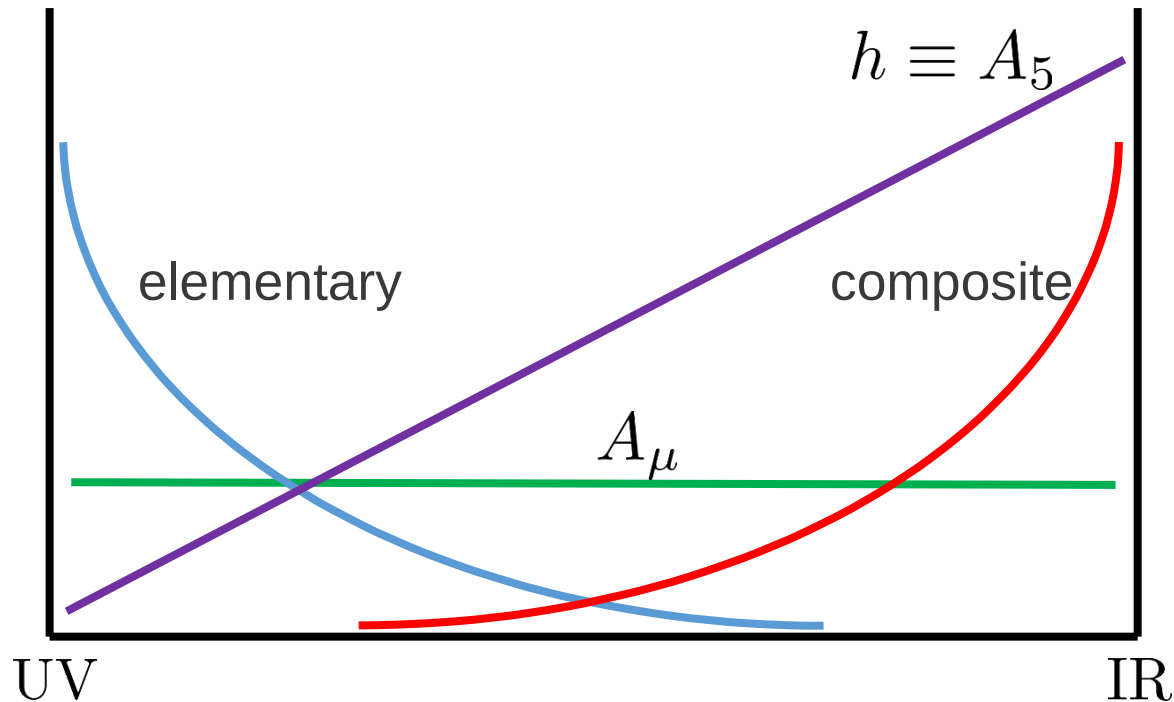
- In 5d, fermions have mass term,  $c = m/k$
- Divide the 5D fermion according to 4D chirality  $\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$
- Appropriate boundary conditions give chiral zero modes.

$$(++) : \begin{pmatrix} f_L(c, z) u_L(p) e^{-ip \cdot x} \\ 0 \end{pmatrix}$$

$$(--): \begin{pmatrix} 0 \\ f_R(c, z) u_R(p) e^{-ip \cdot x} \end{pmatrix}$$

# Fermion Partial Compositeness

- $c$  determines 5D profile of the massless mode  $f(c, z)$ .



	$c < -1/2$	$-1/2 < c < 1/2$	$1/2 < c$
<i>L</i>	<i>IR</i>	<i>IR</i>	<i>UV</i>
<i>R</i>	<i>UV</i>	<i>IR</i>	<i>IR</i>

# Warm-up: RS QED

- Consider a process  $f_1 \bar{f}_1 \rightarrow f_2 \bar{f}_2$

- The scattering amplitude of s-channel pair production is

$$i\mathcal{M} = \left( i\bar{v}_{f_1}(k_1)\gamma^m u_{f_1}(k_2) \right) \left( -i\eta_{mn}S(p) \right) \left( i\bar{u}_{f_2}(k_3)\gamma^n v_{f_2}(k_4) \right)$$

- For a massless gauge boson in a 4D theory (e.g. QED):

$$S(p) = \frac{g^2}{p^2}$$



- In the RS model, we have

$$S_{RS}(p) = \int_{z_0}^{z_R} \frac{dz_1}{(kz_1)^4} \int_{z_0}^{z_R} \frac{dz_2}{(kz_2)^4} |f_1(z_1)|^2 |f_2(z_2)|^2 \left( Q_1 \mathcal{G}(z_1, z_2, p) Q_2 \right)$$

where  $f_1(z_1), f_2(z_2)$  : 5D fermion wavefunctions

$$\eta_{mn} \mathcal{G}(z_1, z_2, p) = \langle \mathcal{A}_m(z_1, p) \mathcal{A}_n(z_2, -p) \rangle$$

Note,  $S_{RS}(p)$  is explicitly calculable!

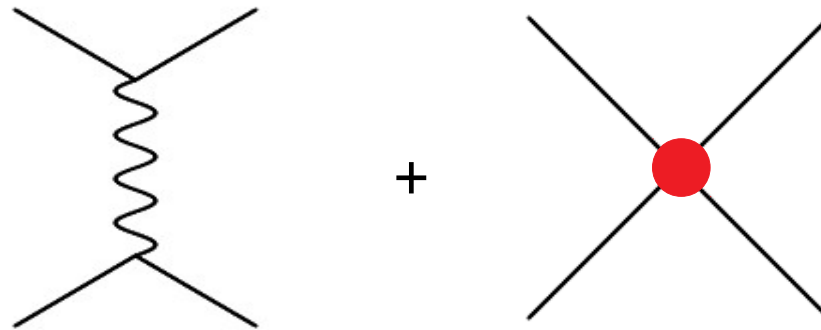
- We focus on the reactions  $e^+ e^- \rightarrow f \bar{f}$ ,  
where we assume that electrons are structureless  
i.e. their wavefunctions are localized in the UV brane.  
Then we have

$$S_{RS}(p) = \int_{z_0}^{z_R} \frac{dz}{(kz)^4} |f_f(z)|^2 \left( Q_e \mathcal{G}(z_0, z, p) Q_f \right) = Q_e \langle \mathcal{G}(z_0, z, p) \rangle Q_f$$

# Gauge Boson Propagators

- Deviations of  $S_{RS}(p) = Q_e \langle \mathcal{G}(z_0, z, p) \rangle Q_f$  away from  $\frac{g^2}{p^2}$  can be considered as a form factor for the photon or effects of the new strong resonances (KK states).
- $\mathcal{G}(z_1, z_2, p)$  is a complicated combination of Bessel functions.
- We can obtain further insight by expanding it for  $p \ll k_R$
- Four possible combinations of the gauge field boundary conditions:  $(++)$ ,  $(+-)$ ,  $(-+)$ ,  $(--)$

- First consider a gauge field with  $(++)$  boundary conditions. This includes a massless zero mode and KK states.
  - 1) a photon propagator
  - 2) a contact interaction representing the KK states



- Indeed, we have  $S_{(++)}(p) = g^2 \left[ \frac{1}{p^2} + \frac{\delta_{KK}}{k_R^2} + \dots \right]$

$$\text{where } \delta_{KK} = \frac{1}{4} \left( -\frac{1}{L_B} + \left\langle \frac{z^2}{z_R^2} \right\rangle + 2 \left\langle \frac{z^2}{z_R^2} \log \frac{z_R}{z} \right\rangle \right)$$

$\delta_{KK}$  encapsulates effects of all KK states.

Note, more composite fermion  $\rightarrow$  larger  $\delta_{KK}$

- $(+-)$  gauge field does not have a zero mode.  
→ No photon contribution

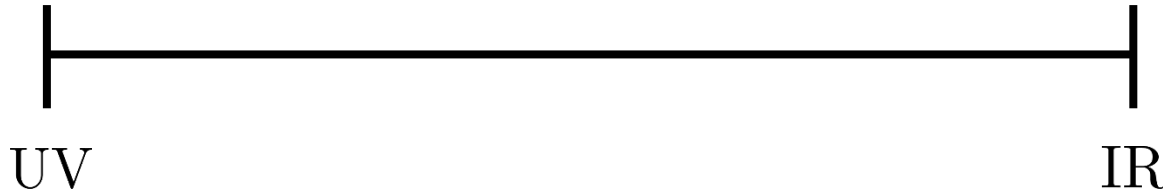
$$S_{(+-)}(p) = -\frac{g_5^2 k}{2k_R^2} \left( 1 - \left\langle \frac{z^2}{z_R^2} \right\rangle \right)$$

- $(-+), (--)$  gauge fields vanish on the UV boundary.  
→ do not couple to UV-localized electron.

$$S_{(-+)}(p) = S_{(--)}(p) = 0$$

# **SO(5) x U(1) Model** Agashe, Contino, Pomarol

AdS<sub>5</sub> bulk



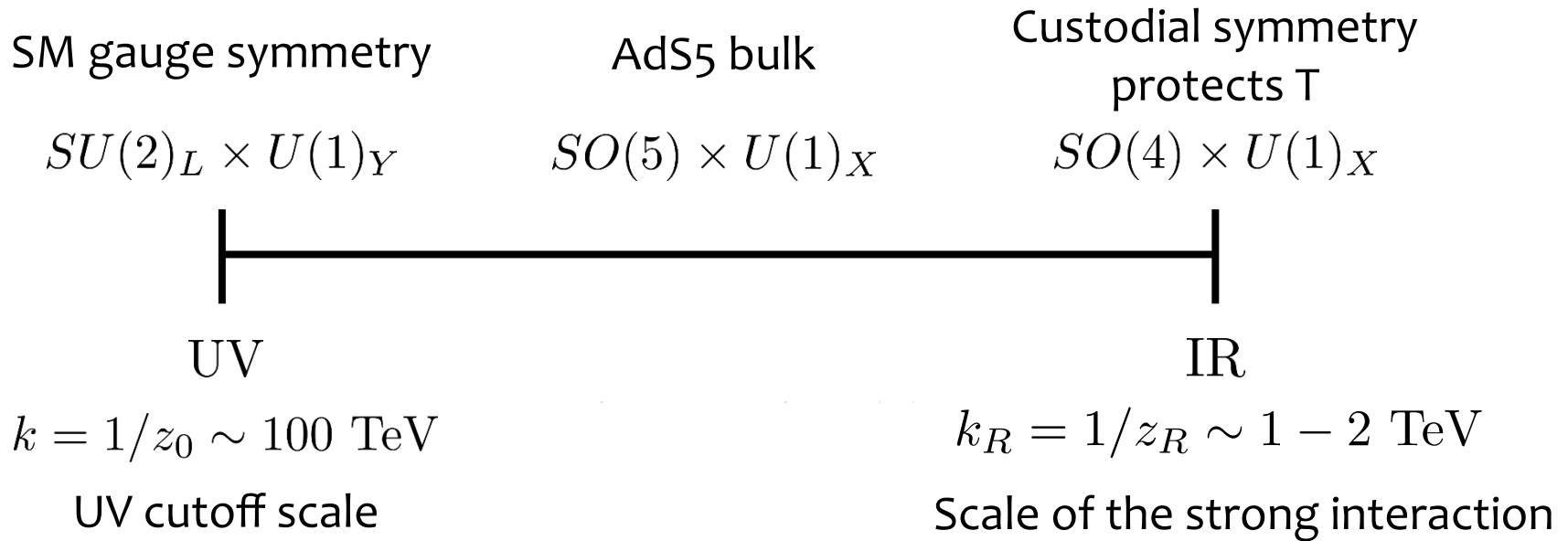
$$k = 1/z_0 \sim 100 \text{ TeV}$$

UV cutoff scale

$$k_R = 1/z_R \sim 1 - 2 \text{ TeV}$$

Scale of the strong interaction

# SO(5) x U(1) Model Agashe, Contino, Pomarol



$SO(5)/SO(4)$  : Higgs as Goldstone bosons ( $A_5$  zero mode)

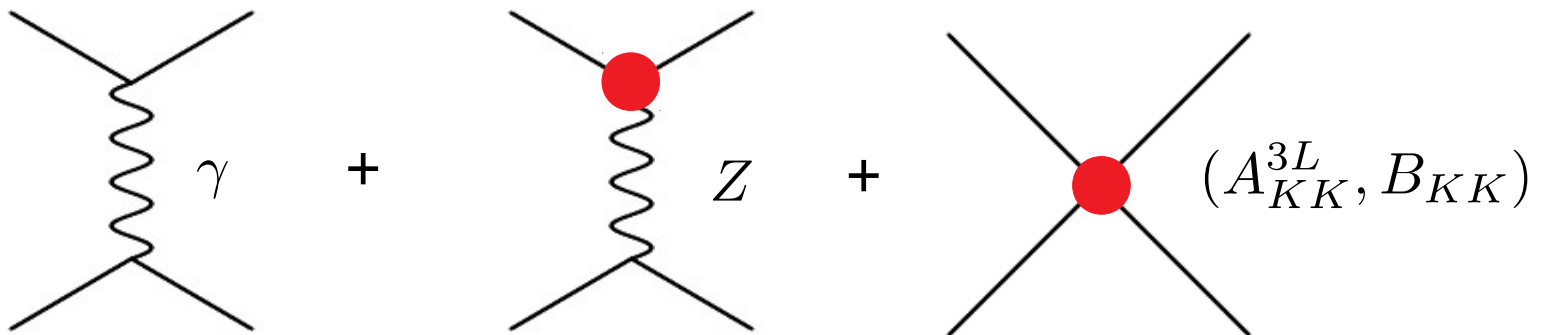
$$Y = T_R^3 + X \quad \text{and} \quad Q = T_L^3 + T_R^3 + X$$

Typical masses in this setting:  $Z' \sim 2.4k_R$

- Neutral gauge field boundary conditions :

$$A^{3L} (++) , B (++) , Z' (-+) , A^{35} (--)$$

- Two zero modes of  $(A^{3L}, B)$  give  $(\gamma, Z)$ .
- KK states of  $(A^{3L}, B)$  generate contact interactions, but those of  $(Z', A^{35})$  do not.
- After EWSB,  $Z$  gets massive and can have a form factor related to the “Little Hierarchy” parameter  $s^2 = v^2/f^2$



- The expansion of the neutral boson propagator in small  $s^2, p^2/k_R^2$

$$S(p) = \frac{e^2}{p^2} Q_e Q_f + \frac{g_{eff}^2}{c_w^2} \frac{1}{p^2 - m_Z^2} (T_e^{3L} - s_*^2 Q_e) (T_f^{3L} - s_*^2 Q_f + \delta Q_f) \\ + \frac{g^2}{k_R^2} \left[ \delta_{KK}^W T_e^{3L} T_f^{3L} + \frac{s_w^2}{c_w^2} \delta_{KK}^B Y_e Y_f \right]$$

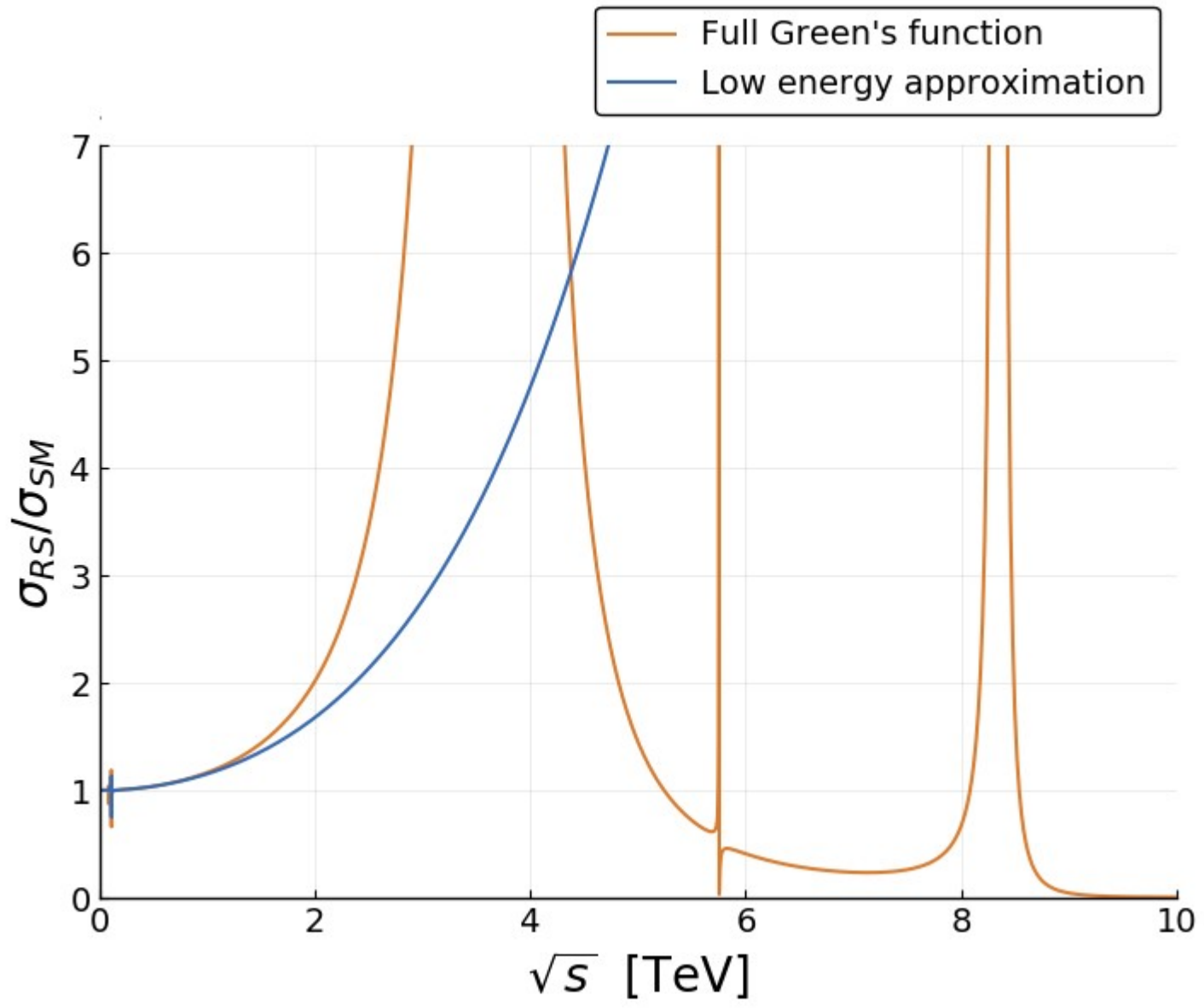
- Deviations in  $(g_{eff}, s_*^2)$  are of order  $\frac{m_Z^2}{4k_R^2} = 0.1\%$  for  $k_R = 1.5$  TeV
- Two dominant sources of deviations:

$$\delta Q = \left( \frac{s^2}{2} (-T^{3L} + T^{3R}) + \frac{s}{\sqrt{2}} T^{35} \right) \left\langle \frac{z^2}{z_R^2} \right\rangle$$

$$\delta_{KK}^{W,B} = \frac{1}{4} \left( -\frac{1}{L_{W,B}} + \left\langle \frac{z^2}{z_R^2} + 2 \frac{z^2}{z_R^2} \log \frac{z_R}{z} \right\rangle \right)$$

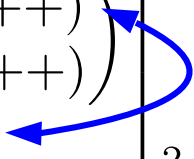
- Except in  $\delta Q$ , the entire expression is in terms of  $(T^{3L}, Y)$





# Dynamical EWSB by the Top Quark

- In the most attractive models, the top quark condensation drives the EWSB.
- Then,  $t_L, t_R$  should be in the same multiplet. For example, the top quark embedding in 5 of  $SO(5)$  should be

$$\Psi_t = \left[ \begin{array}{cc} \left( \begin{array}{cc} \chi_t(-+) & t_L(++), \\ \chi_b(-+) & b_L(++), \end{array} \right) & \\ & t_R(--) \end{array} \right]_{2/3}$$


Higgs in  $SO(5)/SO(4)$  pairs up the left and right-handed zero-modes and make the top massive.

- Tune  $v^2/f^2$  using a competing top partner multiplet; see [arXiv:1709.07909](https://arxiv.org/abs/1709.07909) [arXiv:1810.12352](https://arxiv.org/abs/1810.12352)

# Natural Mass Hierarchy between top & bottom

- The bottom quark is massless at this stage.

$$\Psi_t = \left[ \begin{array}{cc} \chi_t(-+) & t_L(++ ) \\ \chi_b(-+) & b_L(++ ) \\ & t_R(-- ) \end{array} \right]_{2/3}$$

# Natural Mass Hierarchy between top & bottom

- Consider an additional multiplet  $\Psi_b$  with  $X = -1/3$ .

$$\Psi_t = \left[ \begin{array}{cc} \left( \begin{array}{cc} \chi_t(-+) & t_L(++), \\ \chi_b(-+) & b_L(++), \end{array} \right) & \\ & t_R(--), \end{array} \right]_{2/3} \quad \Psi_b = \left[ \begin{array}{cc} \left( \begin{array}{cc} t'(-+) & \chi_b(-+), \\ b'(-+) & \psi_b(-+), \end{array} \right) & \\ & b_R(--), \end{array} \right]_{-1/3}$$

# Natural Mass Hierarchy between top & bottom

- Consider an additional multiplet  $\Psi_b$  with  $X = -1/3$ .

$$\Psi_t = \begin{bmatrix} \left( \begin{array}{cc} \chi_t(-+) & t_L(+ +) \\ \chi_b(-+) & b_L(+ +) \end{array} \right) \\ t_R(- -) \end{bmatrix}_{2/3} \quad \Psi_b = \begin{bmatrix} \left( \begin{array}{cc} t'(-+) & \chi_b(-+) \\ b'(-+) & \psi_b(-+) \end{array} \right) \\ b_R(- -) \end{bmatrix}_{-1/3}$$

$\sin \beta$

- Mix it with  $\Psi_t$  on the UV boundary, so that the Higgs pairs up  $b_L$  and  $b_R$  through  $\sin \beta$ .

$$\frac{m_b^2}{m_t^2} \sim \tan^2 \beta \times \left( \frac{z_0}{z_R} \right)^{2(c_b - c_t)}$$

- $\beta$  of order-one can still give a large mass hierarchy, if  $c_b > c_t$ .
- Large  $c_b$  means a very composite  $b_R$   
 → possibly, sizable cross section deviations

# Natural Mass Hierarchy between top & bottom

- Consider an additional multiplet  $\Psi_b$  with  $X = -1/3$ .

$$\Psi_t = \left[ \begin{array}{cc} \chi_t(-+) & t_L(++ ) \\ \chi_b(-+) & b_L(++ ) \\ & t_R(-- ) \end{array} \right]_{2/3} \quad \Psi_b = \left[ \begin{array}{cc} t'(-+) & \chi_b(-+) \\ b'(-+) & \psi_b(-+) \\ & b_R(-- ) \end{array} \right]_{-1/3}$$

- Custodial symmetry for  $Z \rightarrow bb$  Agashe, Contino, Da Rold, Pomarol

$$\begin{aligned} b_L : T^{3L} = T^{3R} = -\frac{1}{2} & \implies \delta Q = 0 \\ b_R : T^{3L} = T^{3R} = 0 & \end{aligned}$$

- The effects of the RS structure come only from the contact term.

## Cross Sections of $e^+e^- \rightarrow b\bar{b}$

- At a linear collider with polarized beams, we can measure independently all four helicity cross sections.

$$\frac{d\sigma}{d\cos\theta}(e_L^- e_R^+ \rightarrow b\bar{b}) = \Sigma_{LL}(s) (1 + \cos\theta)^2 + \Sigma_{LR}(s)(1 - \cos\theta)^2$$

$$\frac{d\sigma}{d\cos\theta}(e_R^- e_L^+ \rightarrow b\bar{b}) = \Sigma_{RL}(s) (1 - \cos\theta)^2 + \Sigma_{RR}(s)(1 + \cos\theta)^2$$

- Study deviations of the RS models compared to the SM.

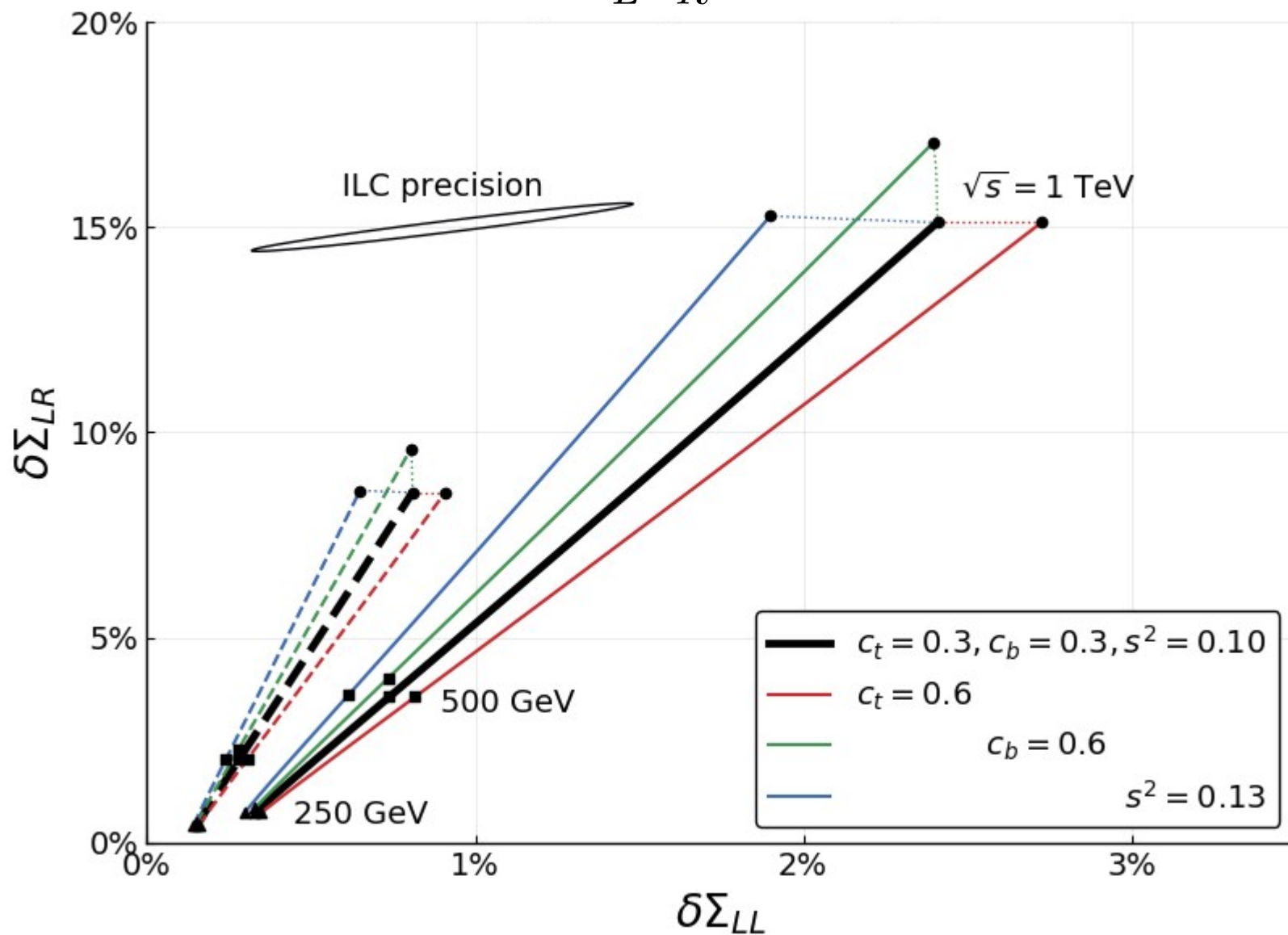
$$\delta\Sigma = \frac{\Delta\Sigma}{\Sigma_{SM}}$$

# Cross Sections of $e^+e^- \rightarrow b\bar{b}$

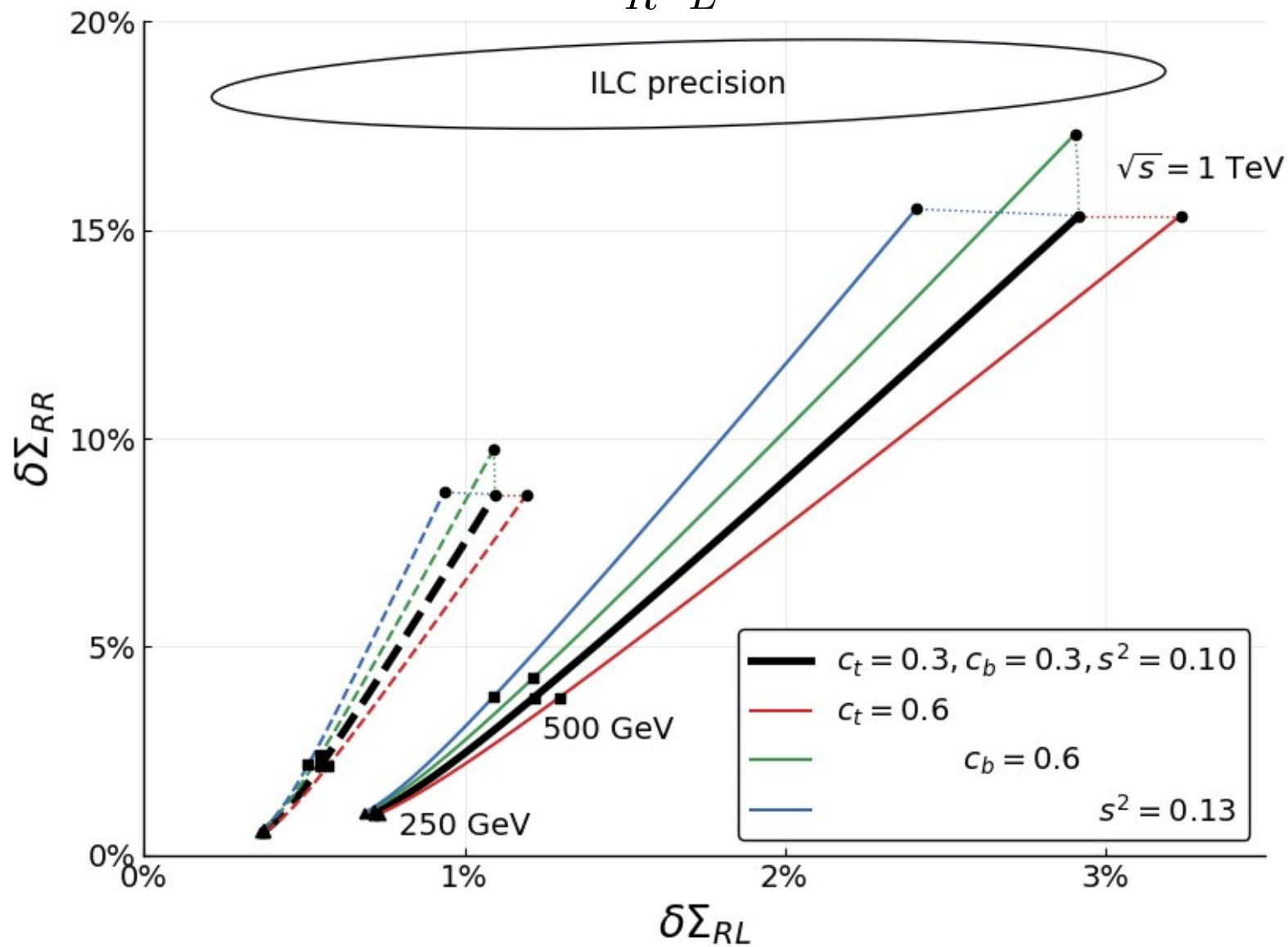
- The parameter space depends on four parameters:  $k_R, c_b, c_t, s^2$ 
  - $k_R$  : scale of the new strong interaction.  
The most optimistic choice from S parameter:  $k_R = 1.5$  TeV  
larger  $k_R \rightarrow$  suppressed deviations  $\delta\Sigma$
  - $c_b$  : determines the 5D shape of  $b_R$   
larger  $c_b \rightarrow$  more composite  $b_R \rightarrow$  larger  $\delta\Sigma_{LR,RR}$
  - $c_t$  : determines the 5D shape of  $b_L$   
larger  $c_t \rightarrow$  more composite  $b_L$  to fit  $m_t \rightarrow$  larger  $\delta\Sigma_{LL,RL}$
  - $s^2$  : proportional to the strength of the new strong interaction  
larger  $s^2 \rightarrow$  more elementary  $b_L$  to fit  $m_t \rightarrow$  smaller  $\delta\Sigma_{LL,RL}$



$$e_L^- e_R^+ \rightarrow b\bar{b}$$



$$e_R^- e_L^+ \rightarrow b \bar{b}$$



## $b_R$ in 4 of SO(5)

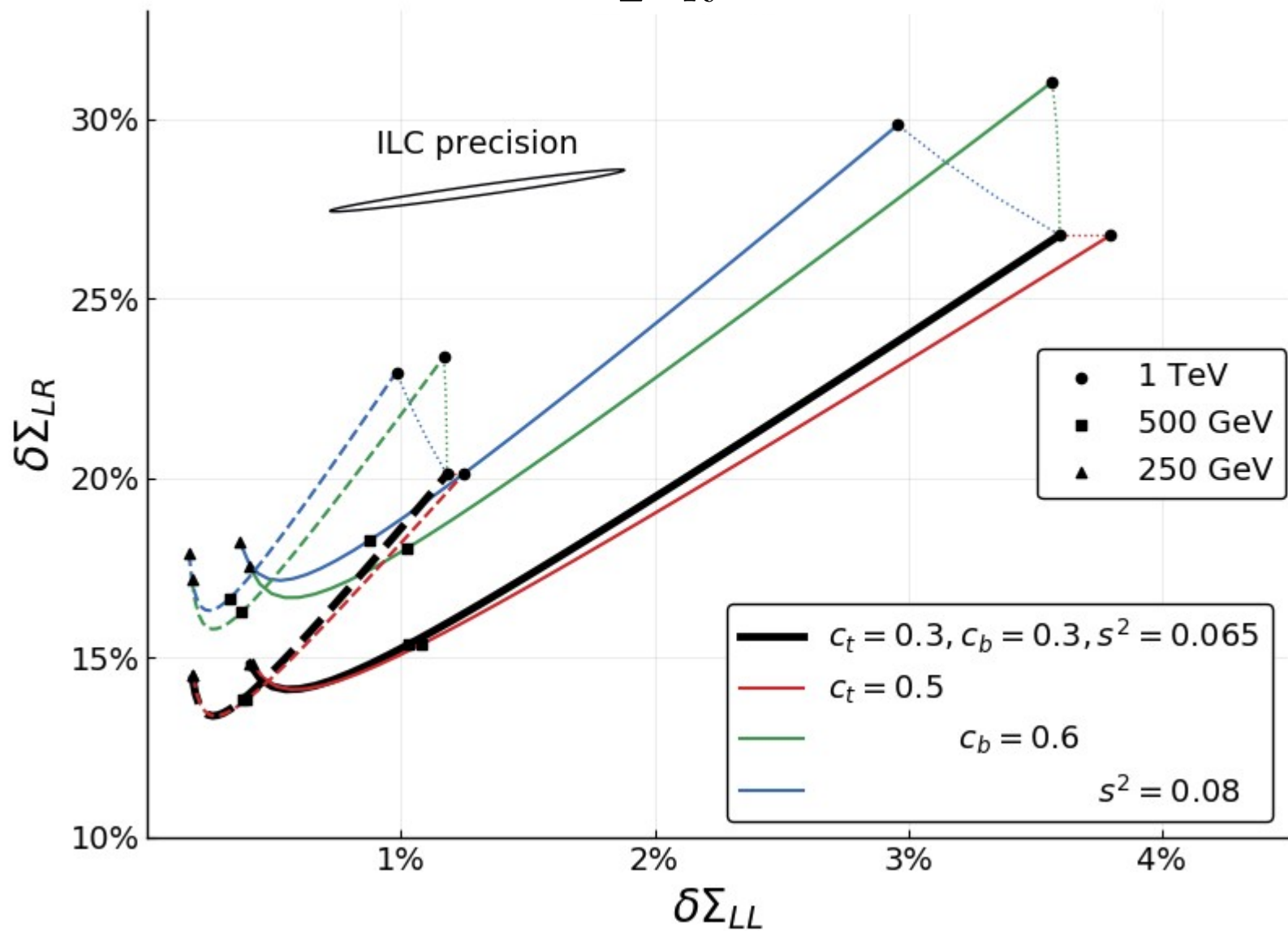
- Nonzero  $\delta Q$  if  $b_L$  or  $b_R$  is embedded in 4 of SO(5).
- $b_L$  in 4 is disfavored by  $Z \rightarrow bb$  constraints unless  $k_R > 3$  TeV.
- $b_R$  in 4 is also constrained, but a substantial parameter space remains still at  $k_R = 1.5$  TeV.

$$\delta Q_{b_R}^4 = -\frac{s^2}{4} \left( \frac{1 + 2c_b}{3 + 2c_b} \right)$$

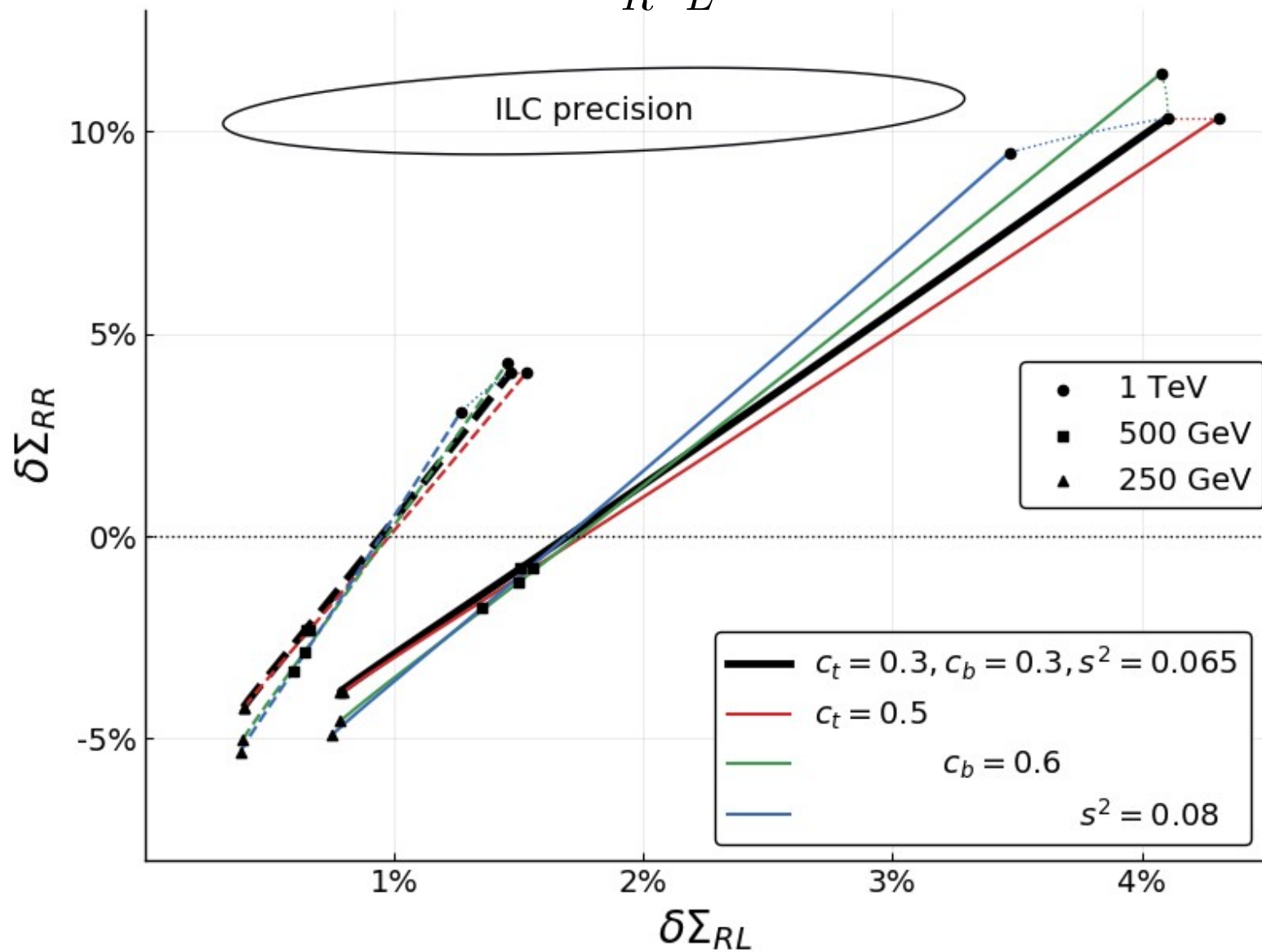
- The effect of nonzero  $\delta Q$  is largest on the Z pole.
- We consider a case with  $b_L$  in 5 and  $b_R$  in 4 of SO(5).

$$\Psi_t = \left[ \begin{array}{cc} \left( \begin{array}{cc} \chi_t(-+) & t_L(++), \\ \chi_b(-+) & b_L(++), \end{array} \right) & \\ & t_R(--) \end{array} \right]_{2/3} \quad \Psi_b = \left[ \begin{array}{c} t'_L(-+) \\ b'_L(-+) \\ t'_R(-+) \\ b_R(--) \end{array} \right]_{1/6}$$

$$e_L^- e_R^+ \rightarrow b\bar{b}$$

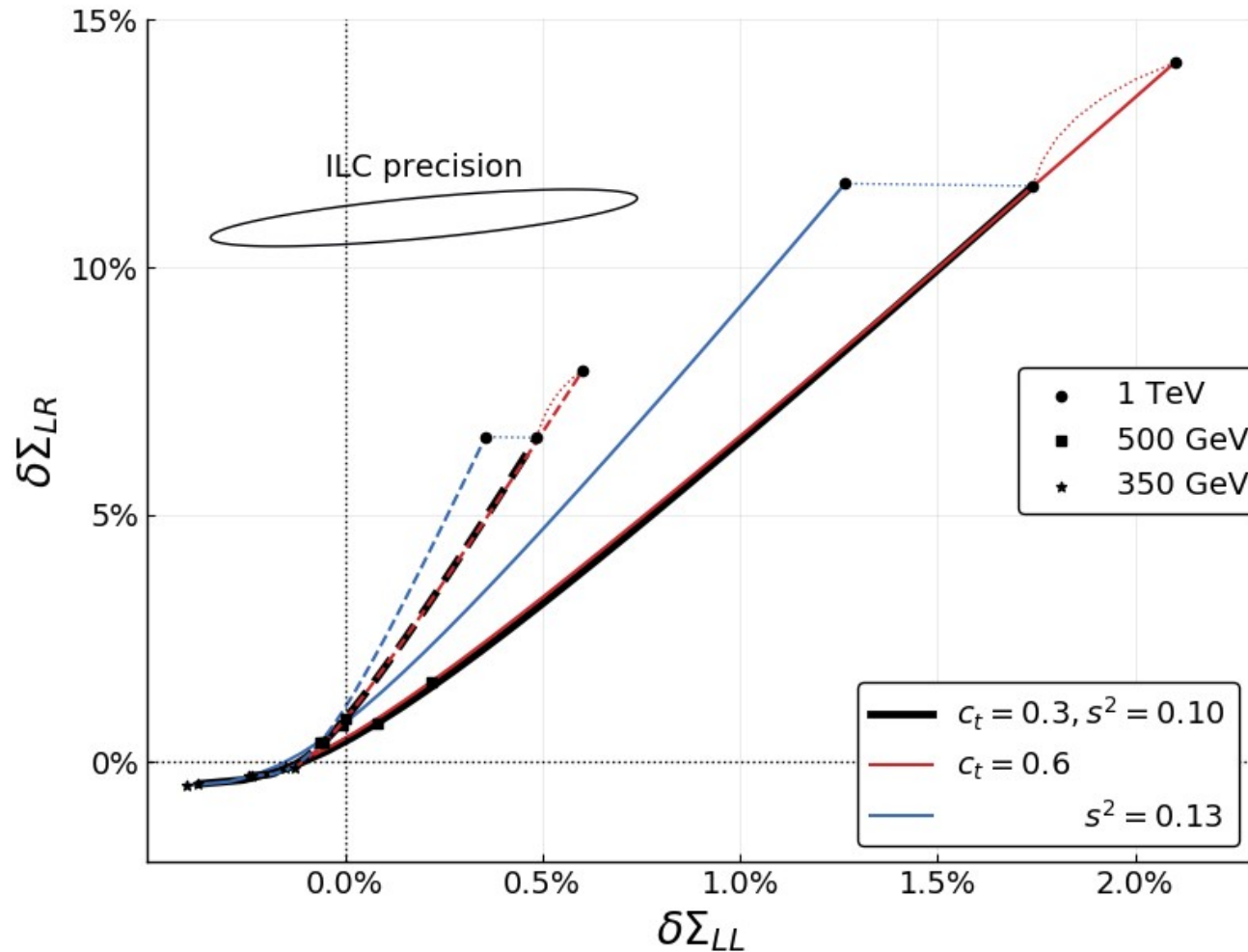


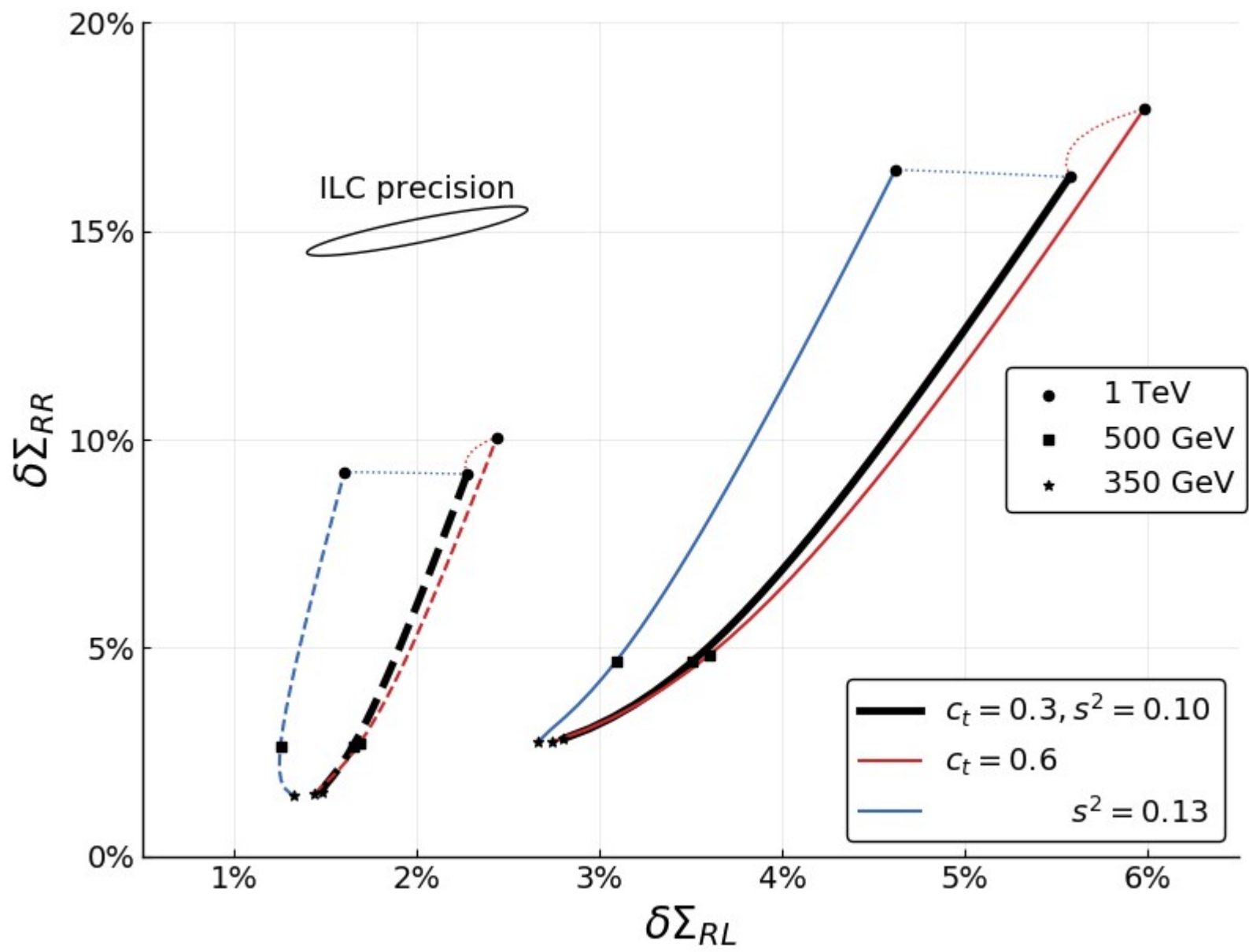
$$e_R^- e_L^+ \rightarrow b \bar{b}$$



# Cross Sections of $e^+e^- \rightarrow t\bar{t}$

- Further complications due to the top quark mass.



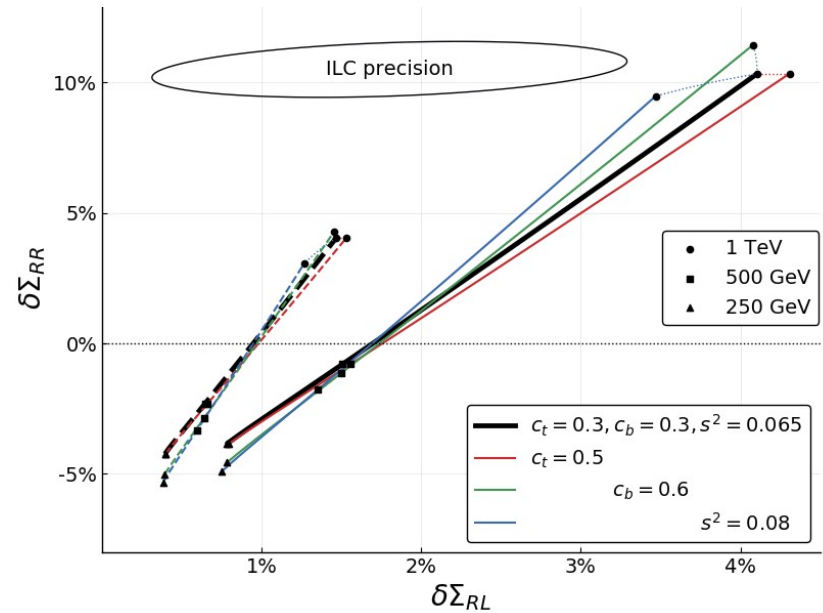
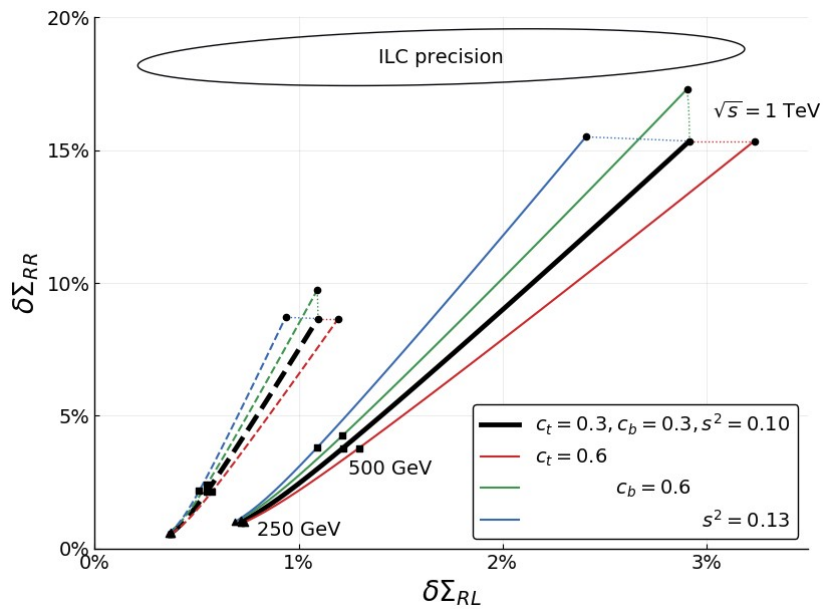
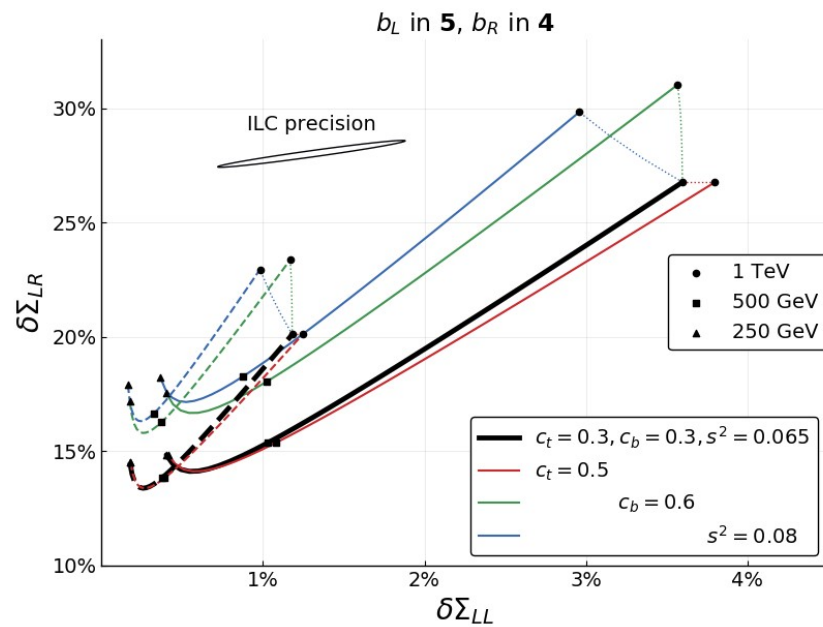
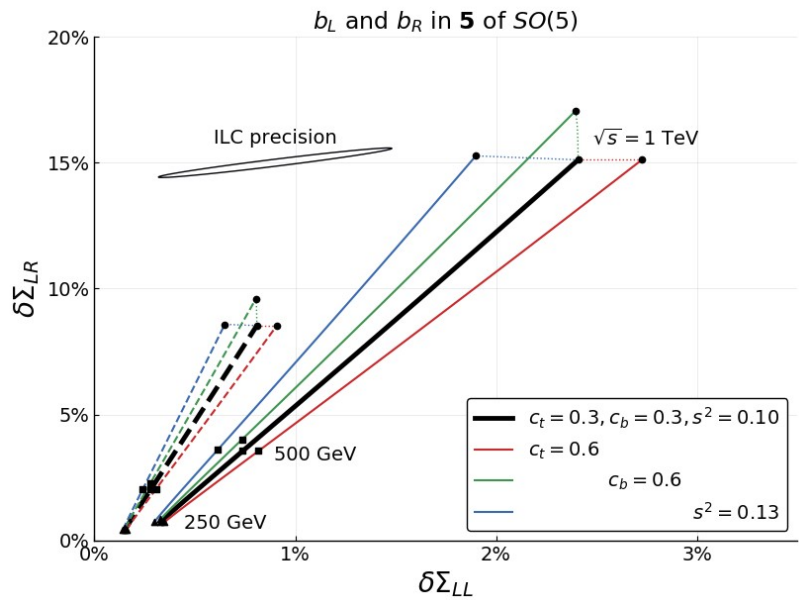


# Summary

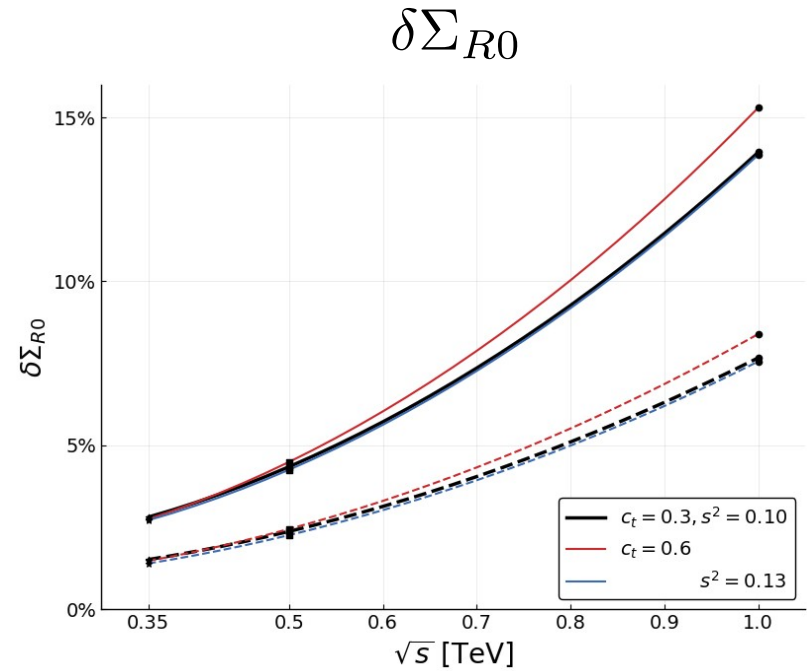
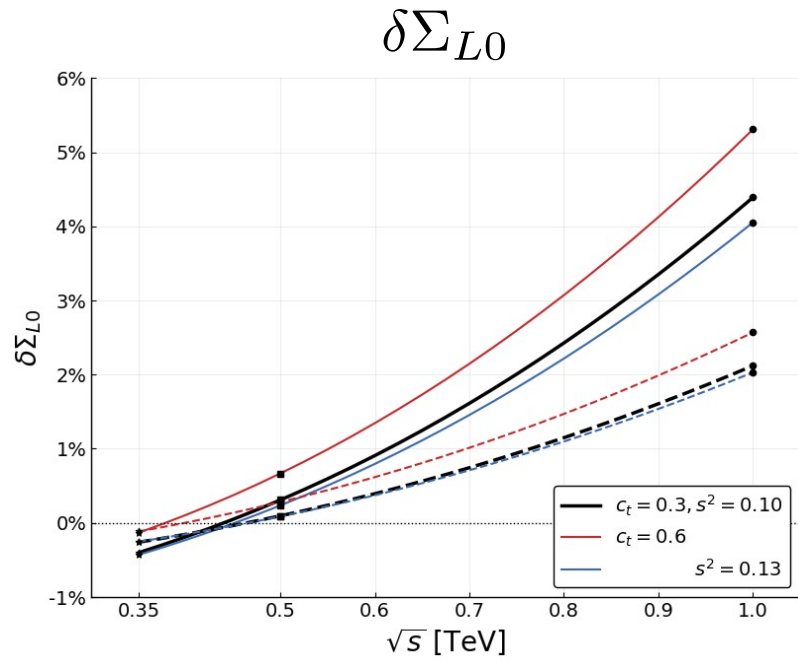
- Gauge-Higgs unification models in AdS<sub>5</sub> gives a calculable and predictive approach to 4D composite Higgs models.
- $e^+e^- \rightarrow f\bar{f}$  processes at linear colliders offer a unique window into new physics, due to its simple cross section, polarized beams, and excellent heavy flavor identification.
- Through an analytic understanding of the RS propagators, we can identify the main sources of deviations from the SM and study quantitatively effect of each parameter.
- There are scenarios in which the expected deviation is already sizable at  $\sqrt{s} = 250$  GeV.



**THANK YOU**



# Cross section deviations in the helicity-flip final states $t_L\bar{t}_L, t_R\bar{t}_R$



## Cross Sections of $e^+e^- \rightarrow t\bar{t}$

- Three further effects due to the top quark mass:
  - 1) The mass term mixes the left- and right-chirality 5D wavefunctions.
  - 2) Each helicity amplitude is a combination of both of the 5D Dirac fermions  $(t_L, t_R)$ .
  - 3) Nonzero matrix element from  $T^{35}$