Gravitational waves from first-order phase transitions: an analytic approach

Ryusuke Jinno (IBS-CTPU)

1605.01403 / 1707.03111 with Masahiro Takimoto (Weizmann Institute)

1708.01253 with Hyeonseok Seong, Sangjun Lee (KAIST), M.Takimoto

2018/12/5 @ IBS workshop

SELF INTRODUCTION

- Ryusuke (隆介) Jinno (神野)

  - 2016/3 : Ph.D. @ Univ. of Tokyo (particle physics group, supervised by Takeo Moroi)

  - 2016/4-8 : JSPS fellow (PD) @ KEK, Japan

  - 2016/9- : Research Fellow @ IBS-CTPU, Korea

  - 2019/4- : DESY, Germany (planned)

SELF INTRODUCTION

- Research interests & recent works

  - Machine learning : Application of machine learning to QFT tunneling problem

  - Gravitational waves : Analytic approach to GW production in phase transitions

  - (P)reheating : Preheating in Higgs inflation (discovery of new "spike preheating" channel)

  - Inflation : Hillcliming inflation (small-field inflationary attractor)

    Hillclimbing Higgs inflation (new realization of Higgs inflation)

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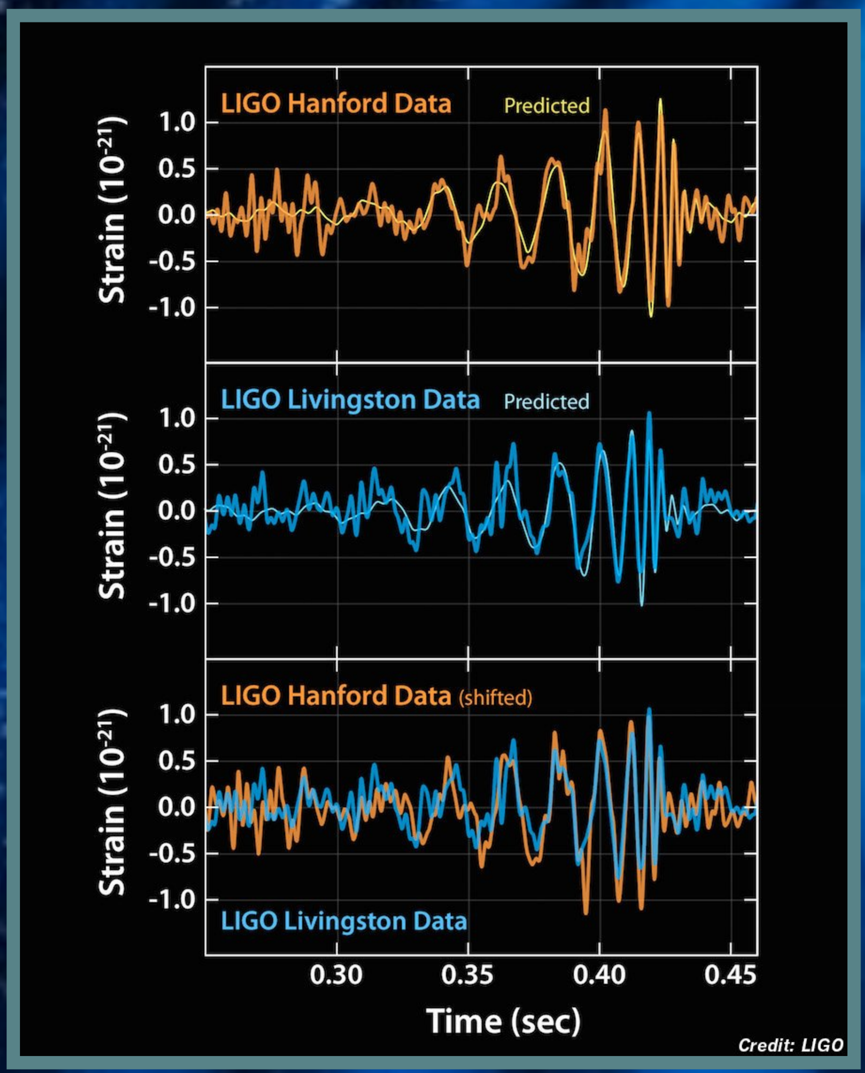
Introduction

ERA OF GRAVITATIONAL WAVES

- Detection of GWs from BH & NS binaries → **GW astronomy** has started



[LIGO]

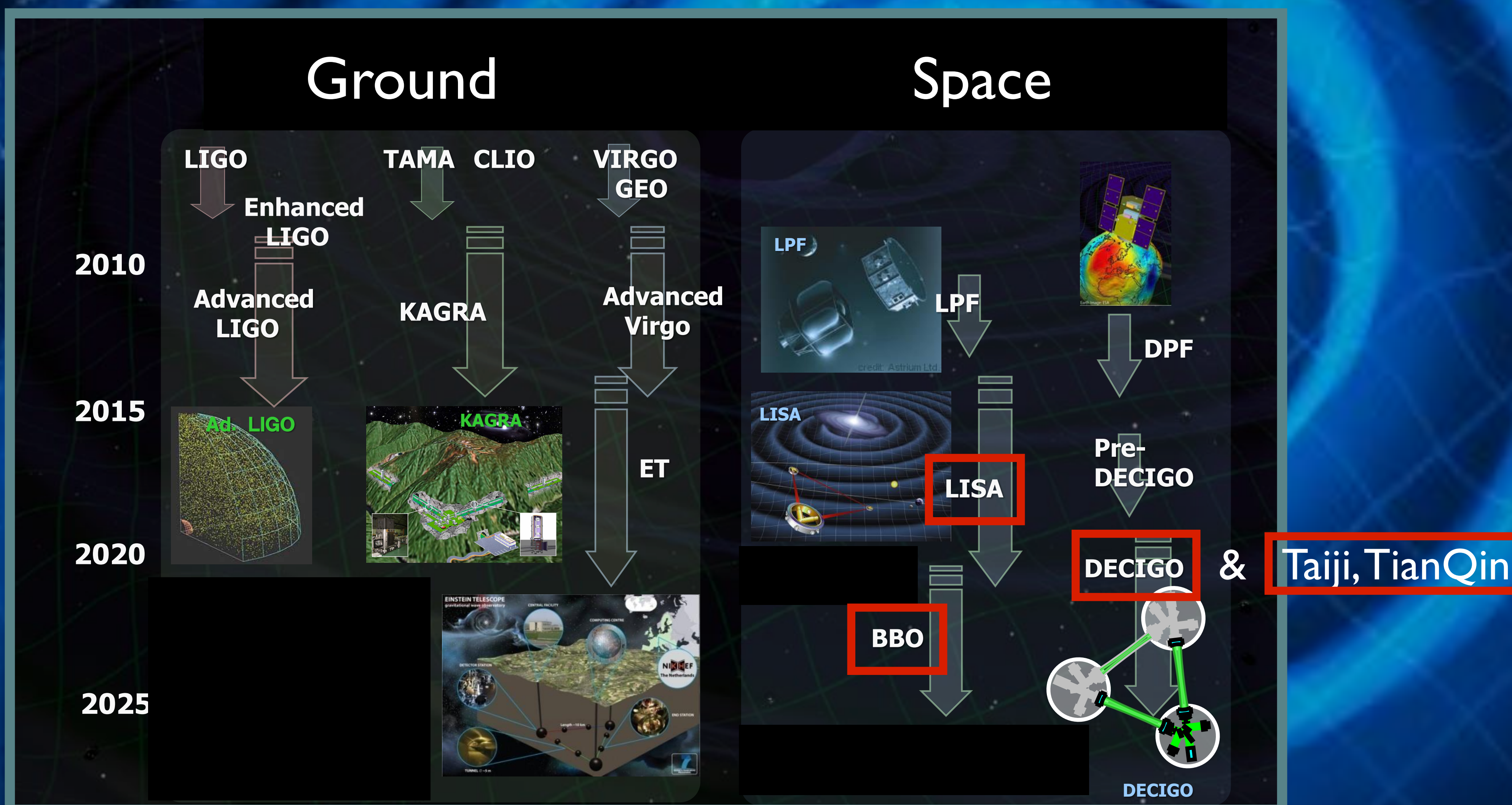
- Black hole binary 36M⊙ + 29M⊙ →62M⊙

- Frequency ~ 35 to 250 Hz

- Significance > 5.1σ

ERA OF GRAVITATIONAL WAVES

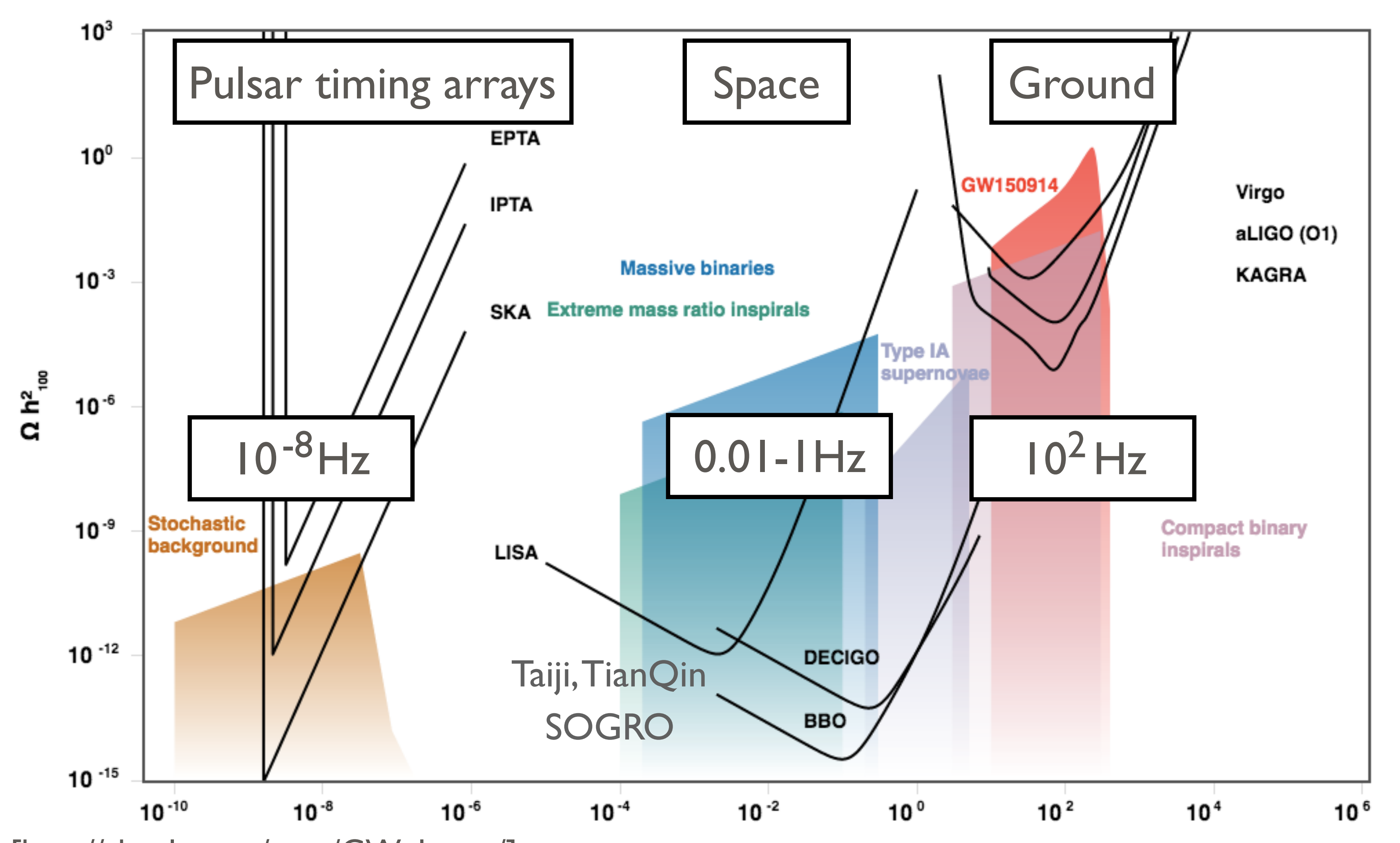
- Next will be **GW cosmology** with space interferometers



From Ando-san’s talk @ JPS meeting 2014

ERA OF GRAVITATIONAL WAVES

- Sensitivity curves for current & future experiments



[http://rhcole.com/apps/GWplotter/]

SOURCES FOR COSMOLOGICAL GWS

- Inflationary quantum fluctuations ("primordial GWs")
- Preheating (particle production just after inflation)
- Cosmic strings, Domain walls
- First-order phase transition can occur when a symmetry breaks:

  - Electroweak sym. breaking (w/ extension)
  - B-L breaking
  - PQ sym. breaking
  - Breaking of GUT group
  - Strong dynamics

  ...

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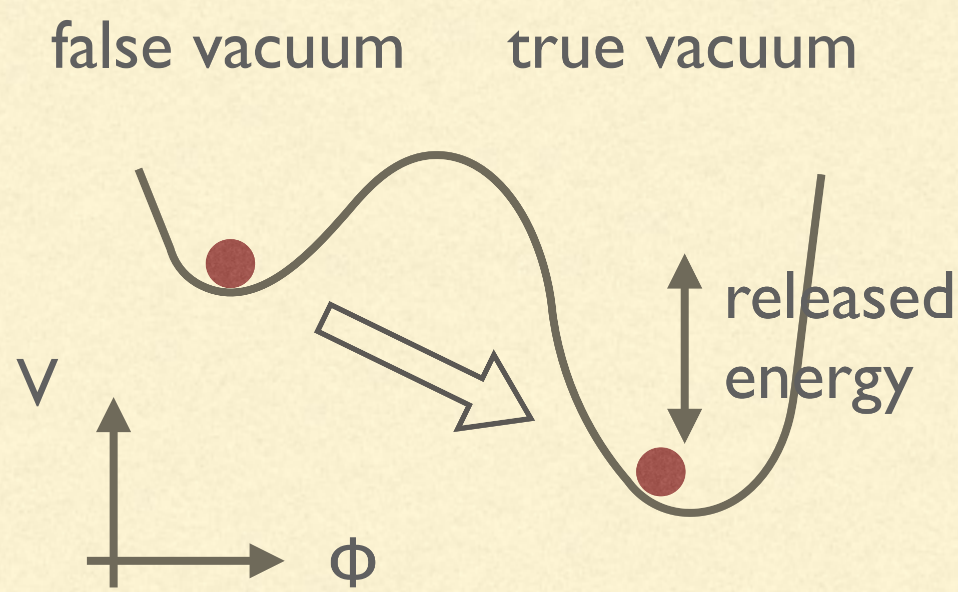
ROUGH SKETCH OF PHASE TRANSITION & GW PRODUCTION

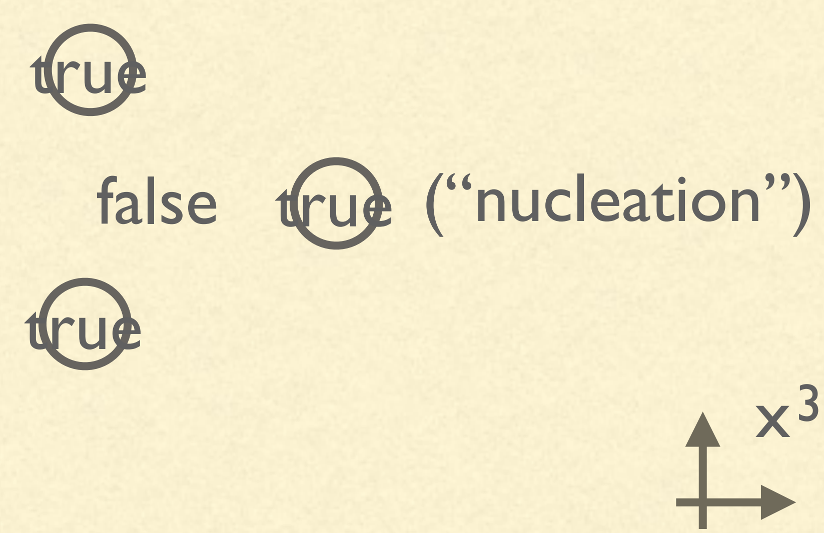
- How thermal first-order phase transition produces GWs

Field space



Quantum tunneling

Position space



Bubble formation & GW production

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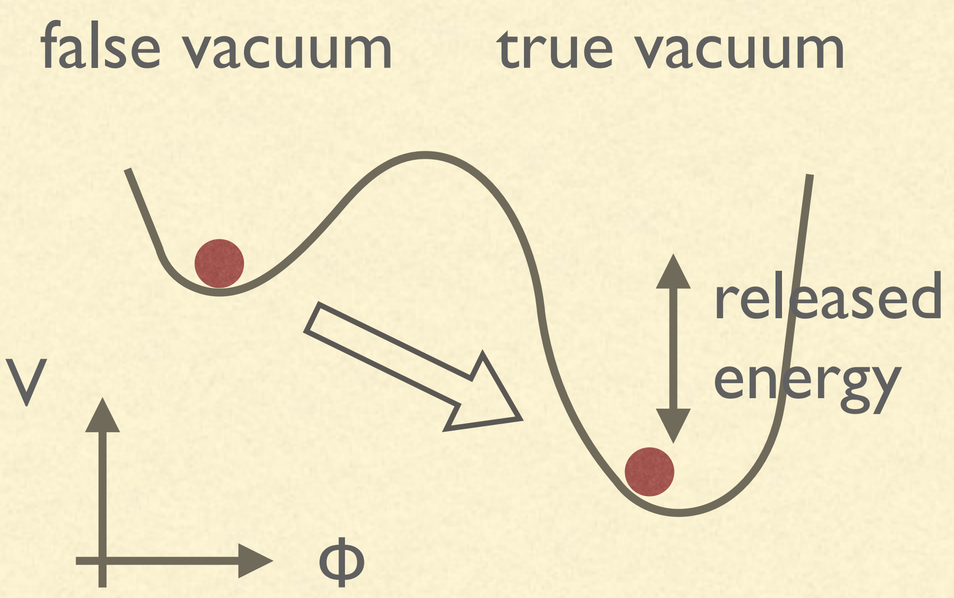

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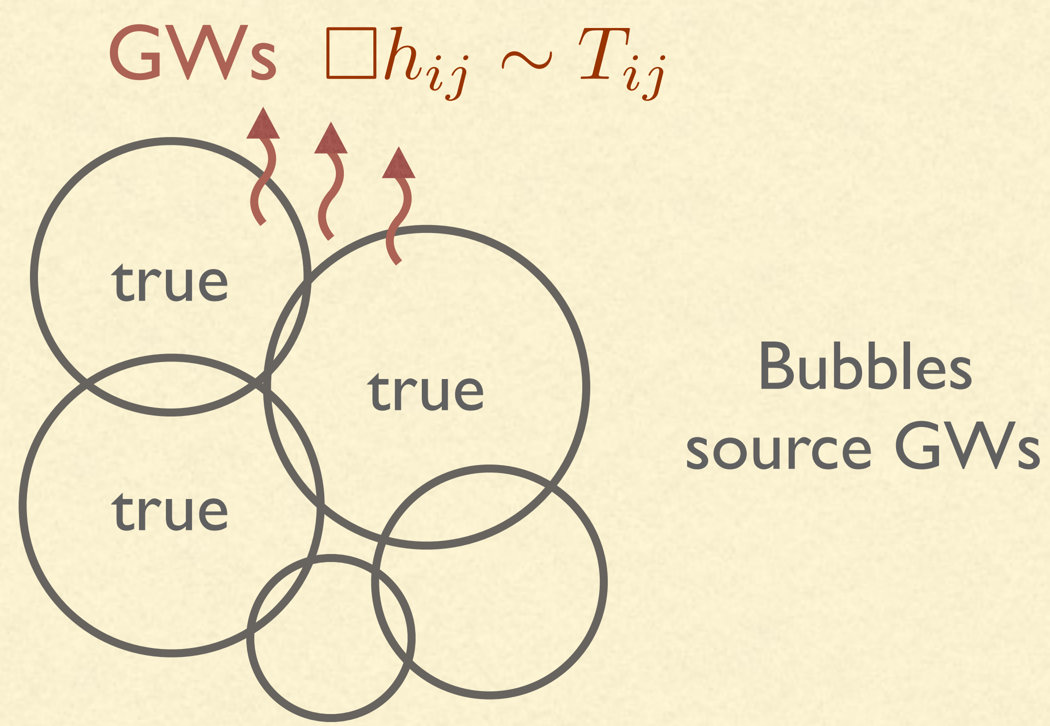


Bubble formation & GW production

TALK PLAN

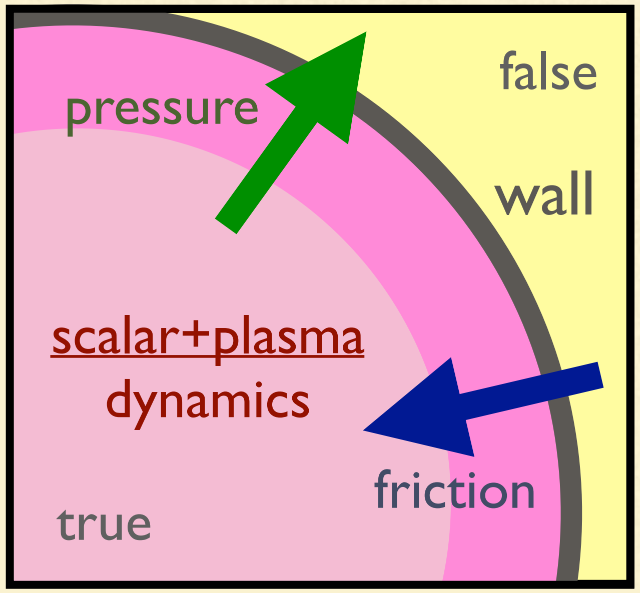
✔0. Introduction

1. First-order phase transition and GW production

2. GWs from phase transitions: an analytic approach

BEHAVIOR OF BUBBLES

- Two main players : **scalar field & plasma**



- Walls (where the scalar field value changes) want to expand ("pressure")

Controlled by $\alpha \equiv \frac{\rho_{\rm released}}{\rho_{\rm plasma}}$

- Walls are pushed back by plasma ("friction")

Controlled by coupling η between scalar field and plasma

- Let's see how bubbles behave for different α (with fixed η)

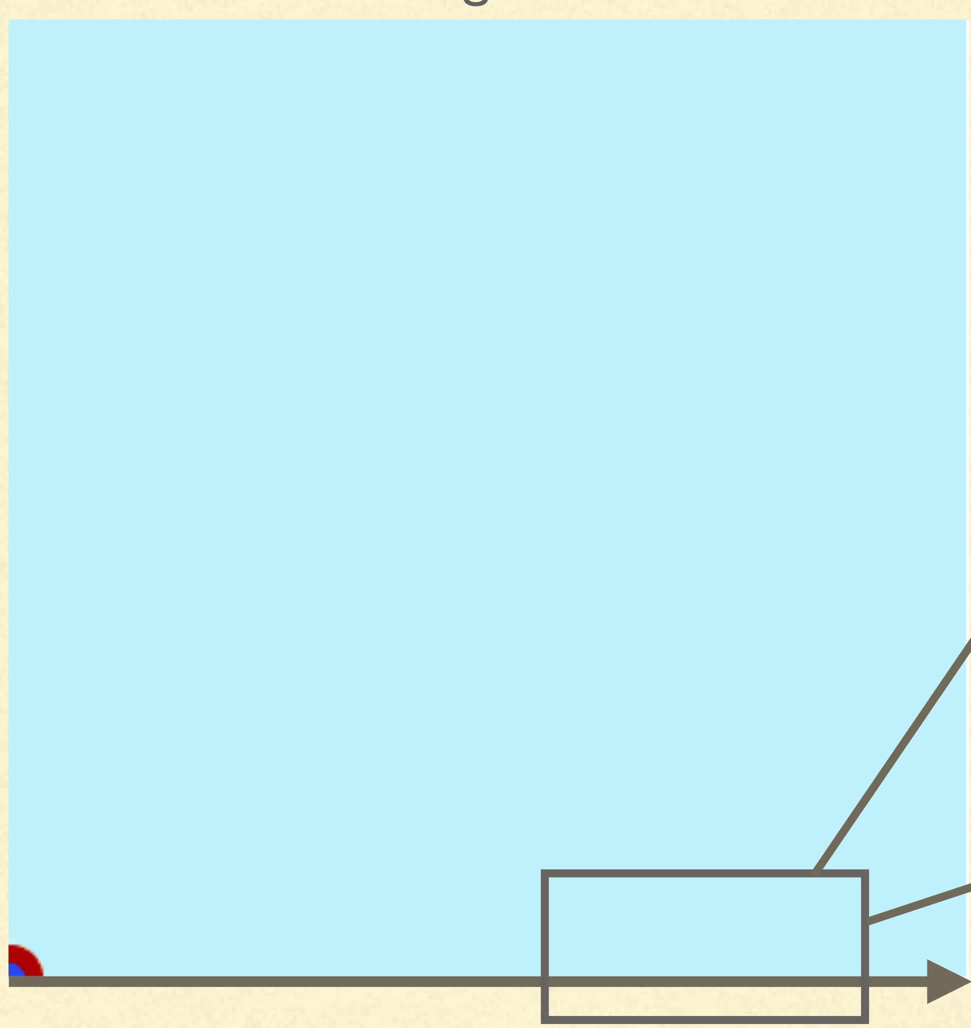
BEHAVIOR OF BUBBLES

$$\alpha \equiv \frac{\rho_{\rm released}}{\rho_{\rm plasma}}$$

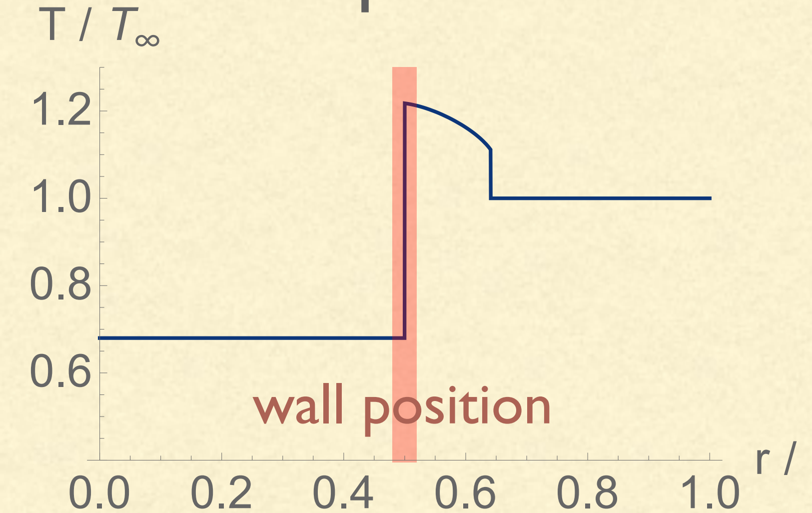
[Espinosa, Konstandin, No, Servant '10]

- Small α ($\lesssim \mathcal{O}(0.1)$)

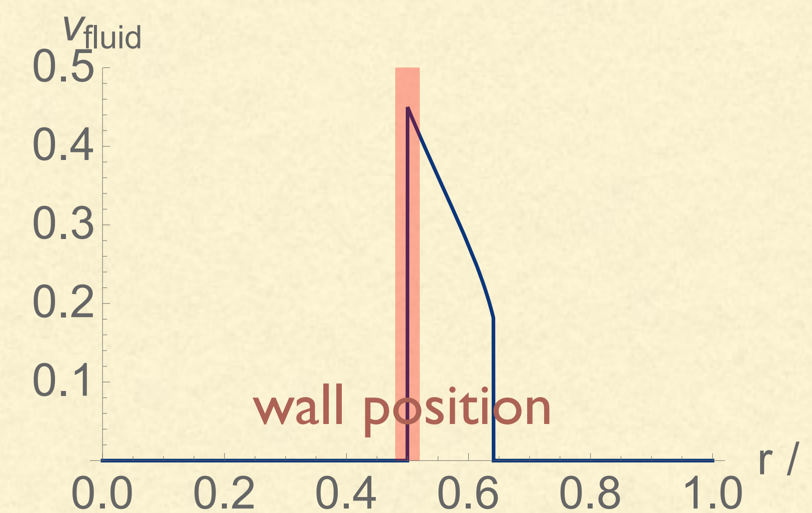
"deflagration"



Temperature



Fluid outward velocity



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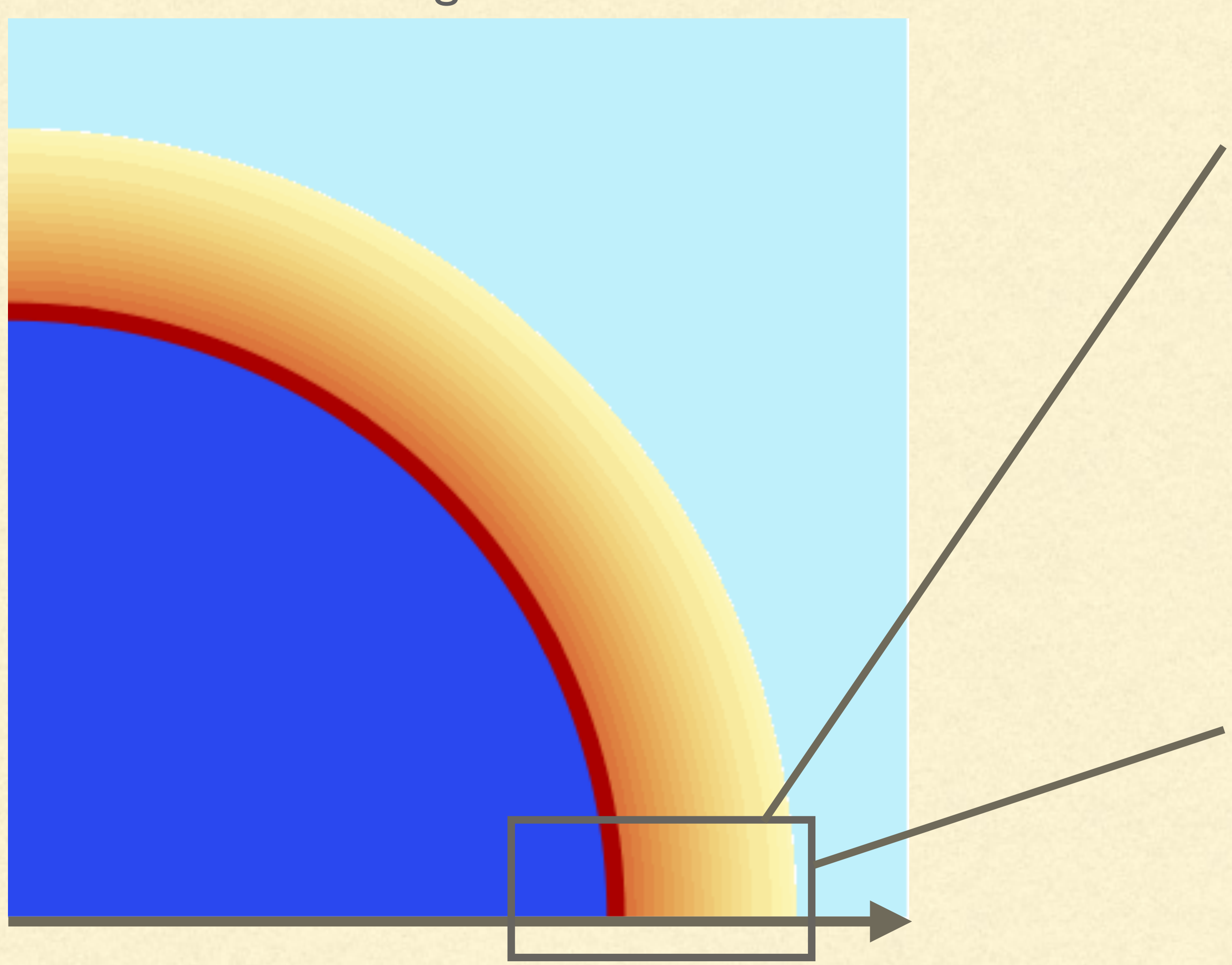

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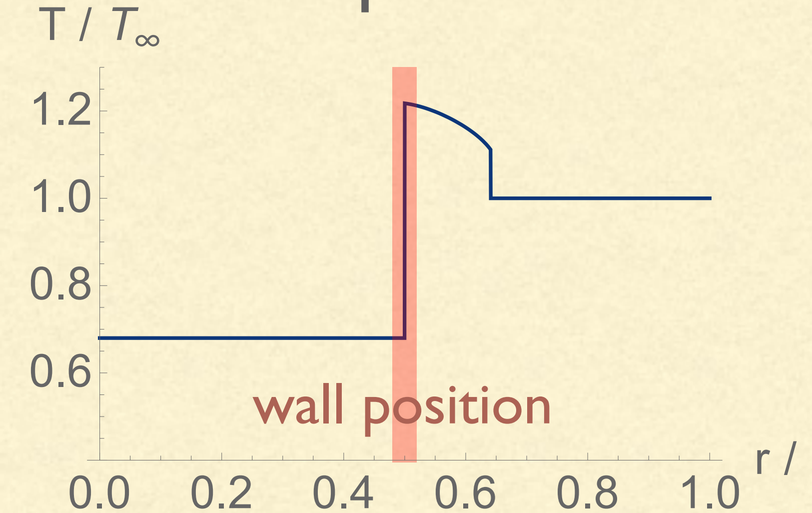

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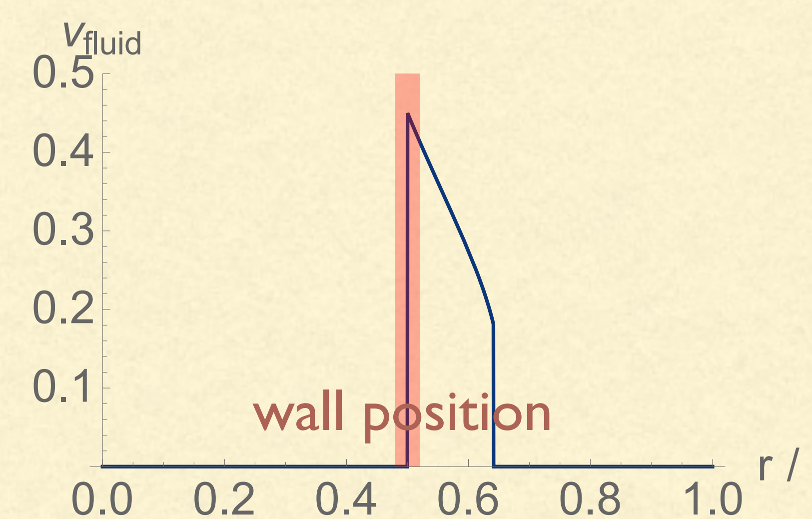

BEHAVIOR OF BUBBLES

$$\alpha \equiv \frac{\rho_{\rm released}}{\rho_{\rm plasma}}$$

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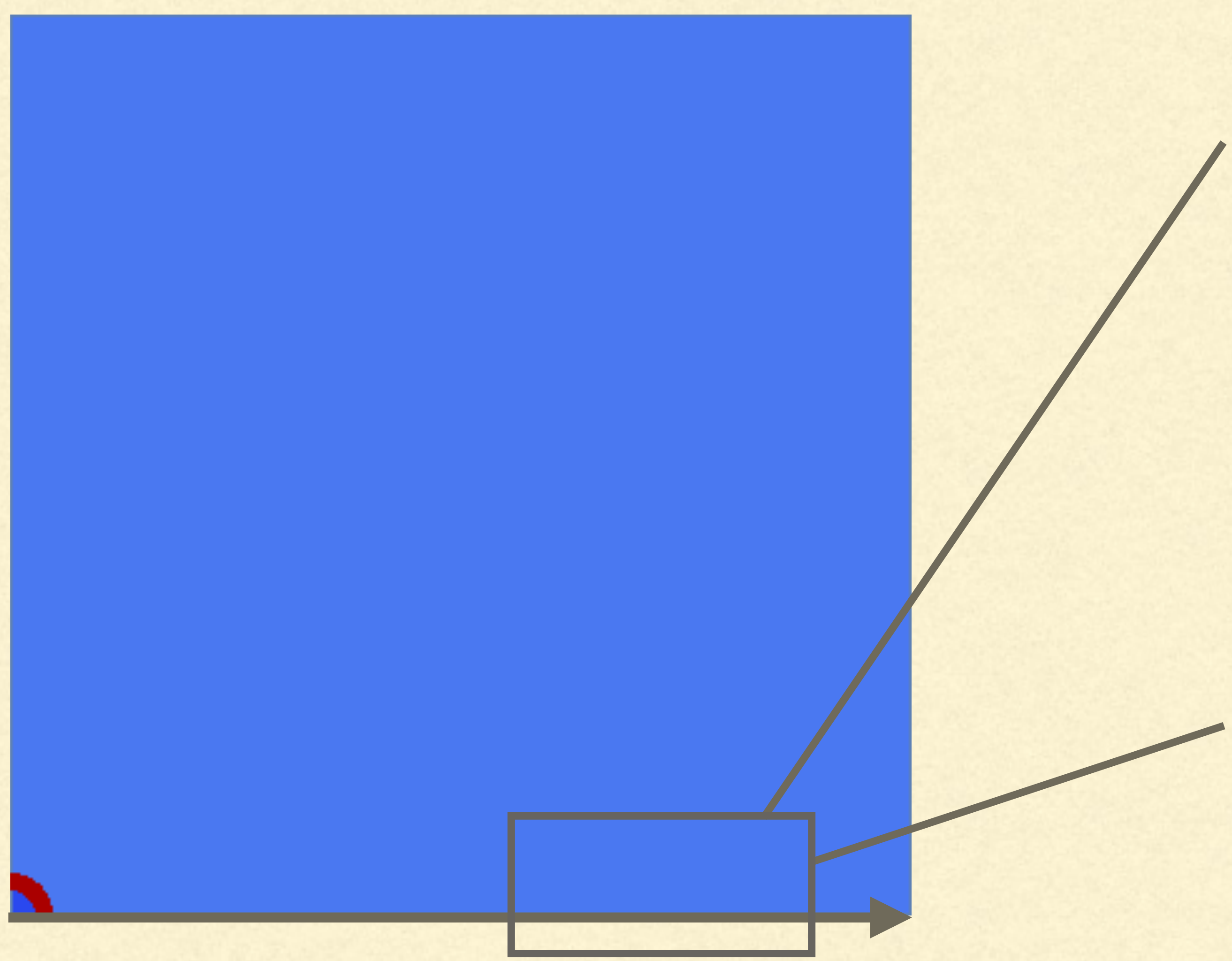
- Small but slightly increased $\alpha \ (\lesssim \mathcal{O}(0.1))$

"detonation"



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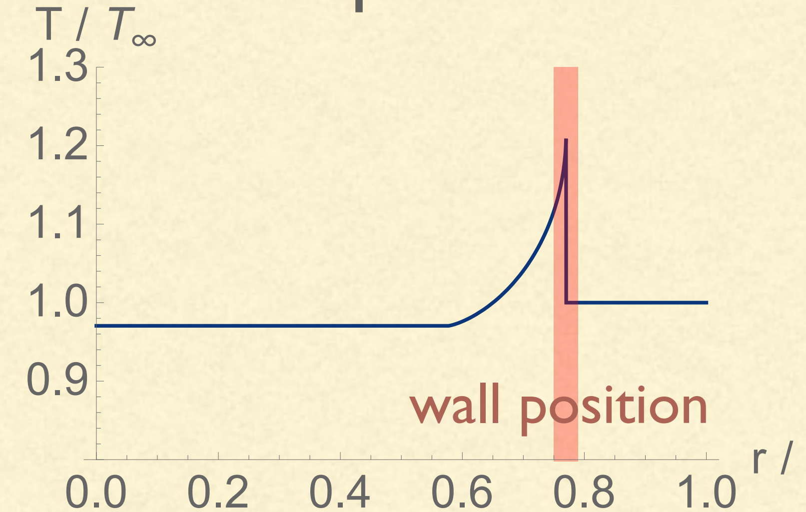

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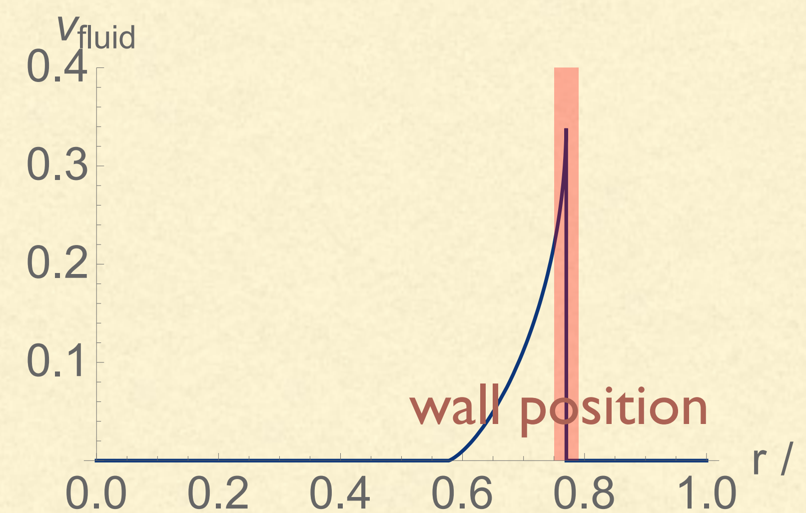

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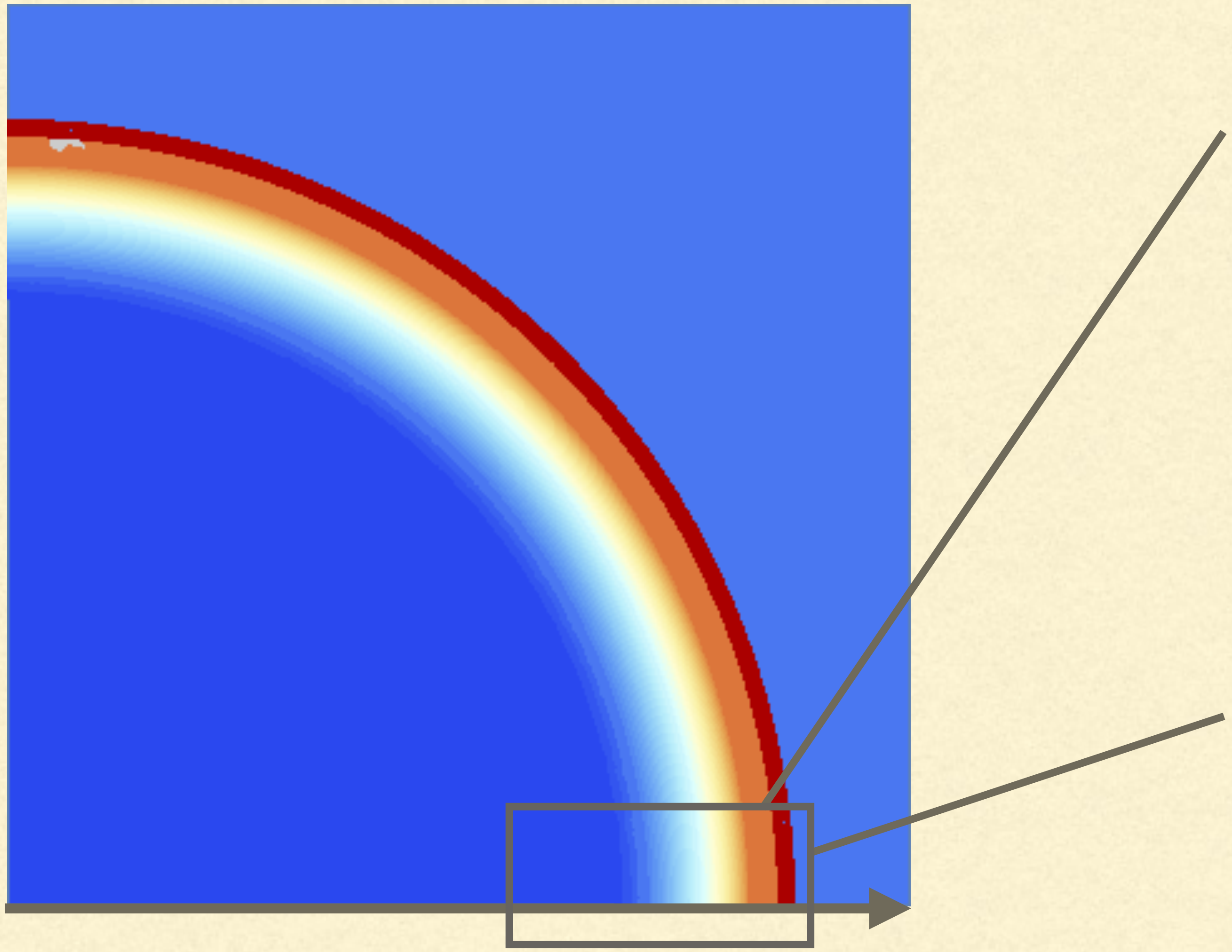

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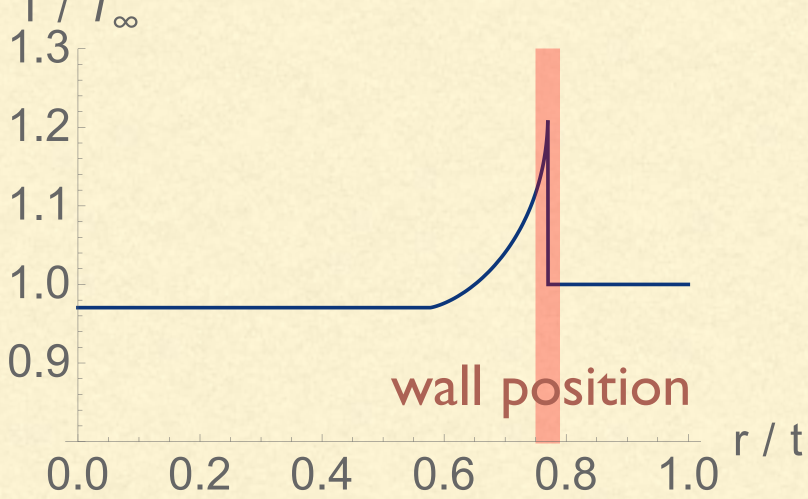

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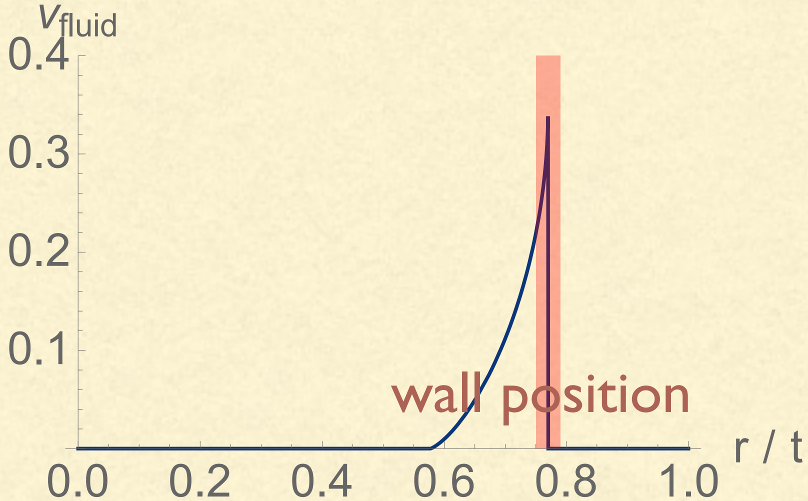

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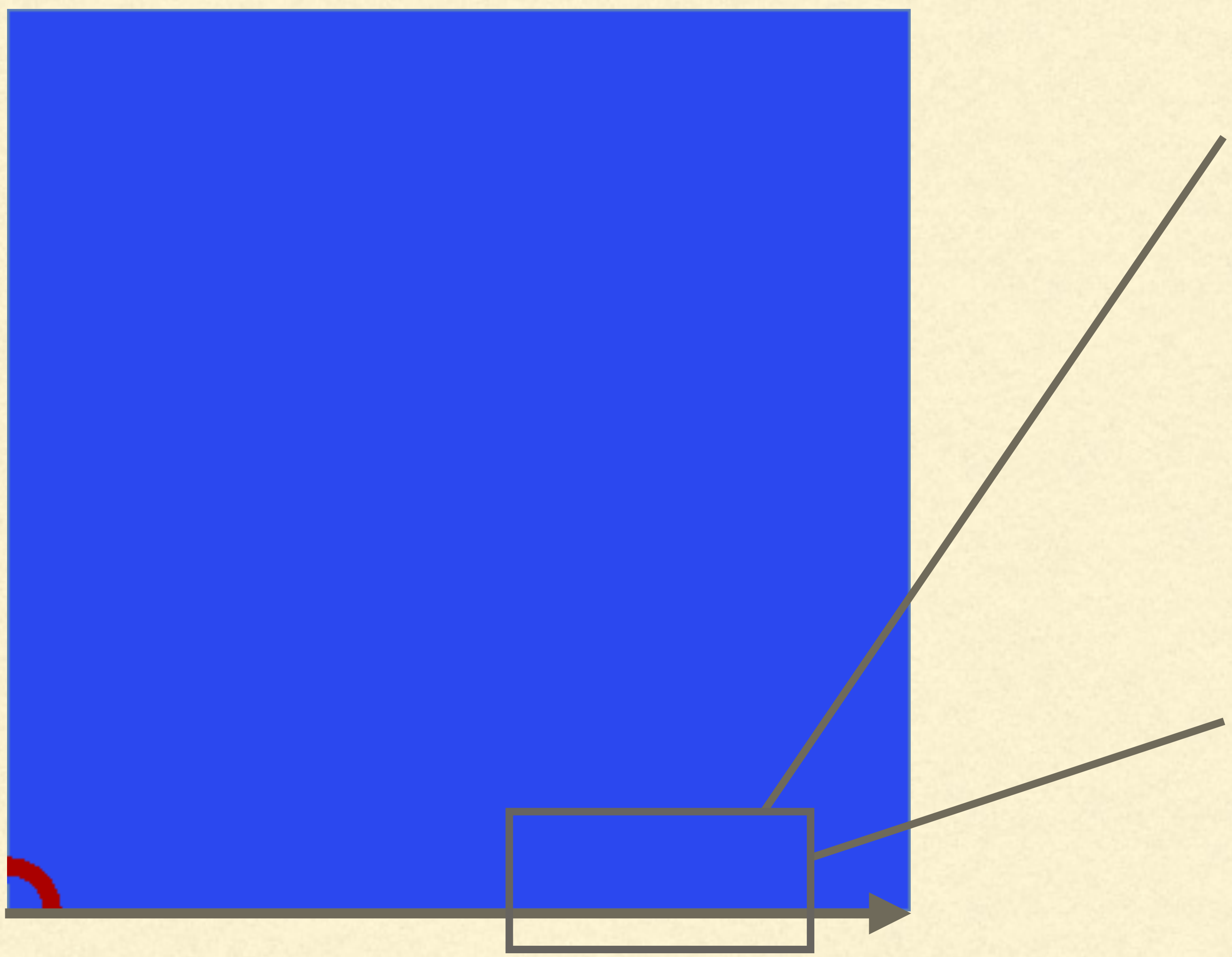
- Large α $(\gg 1)$

"strong detonation"



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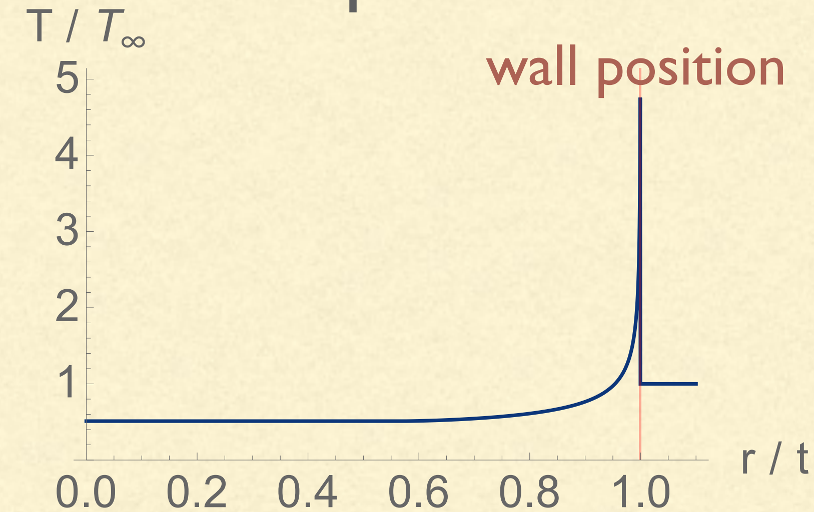

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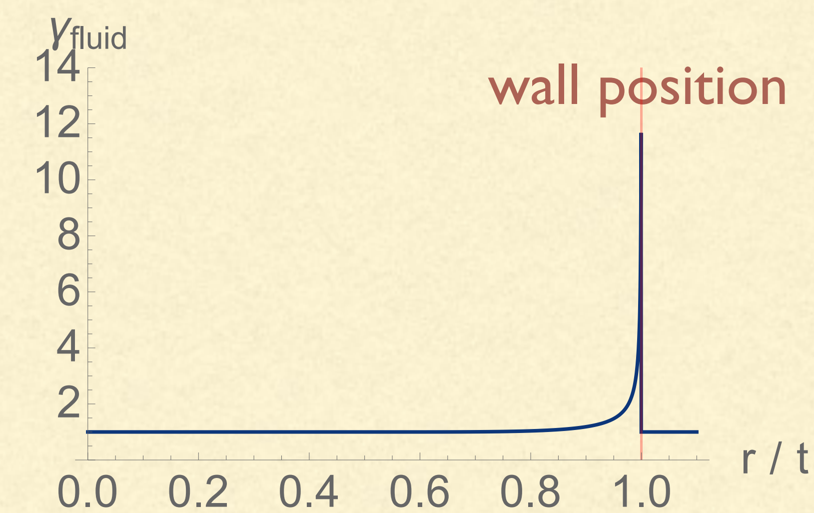

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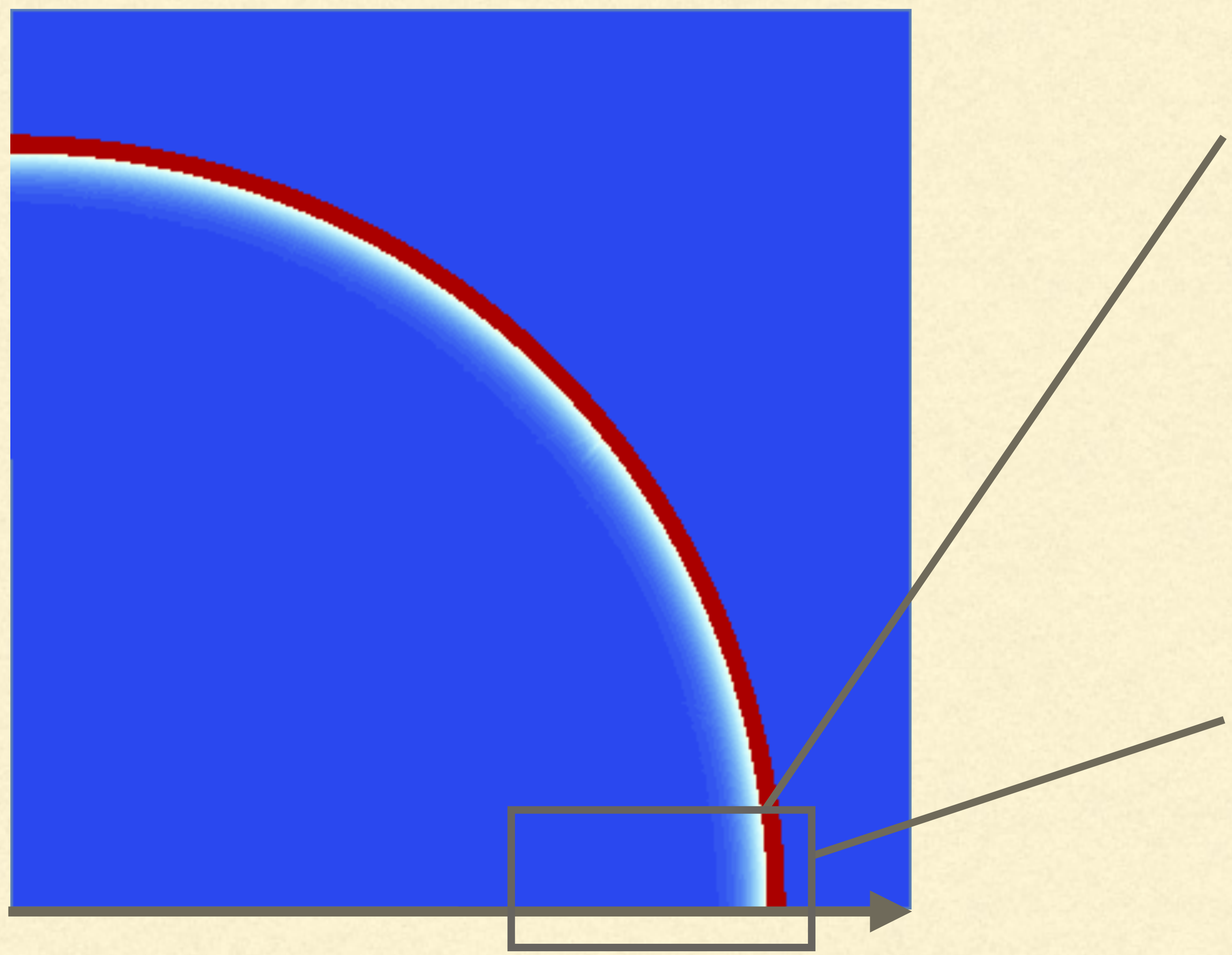

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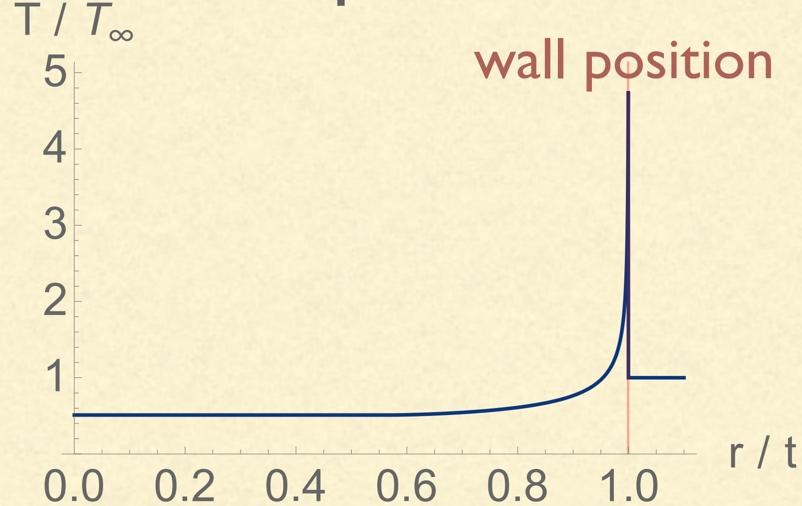

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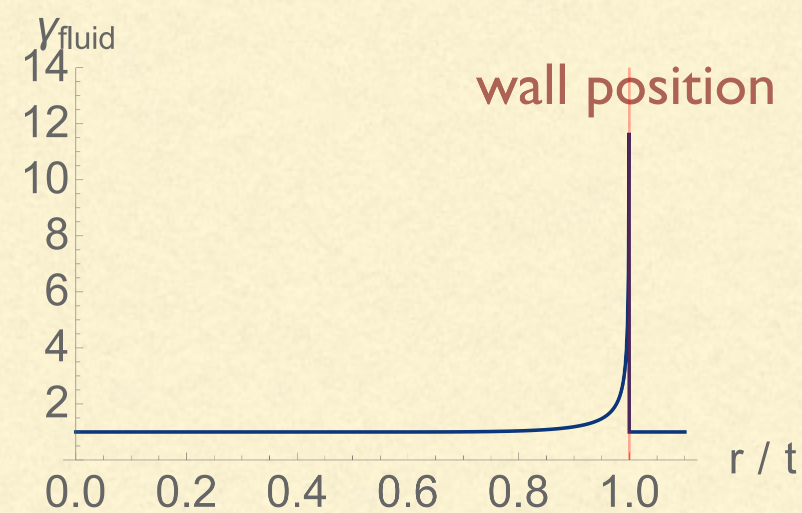
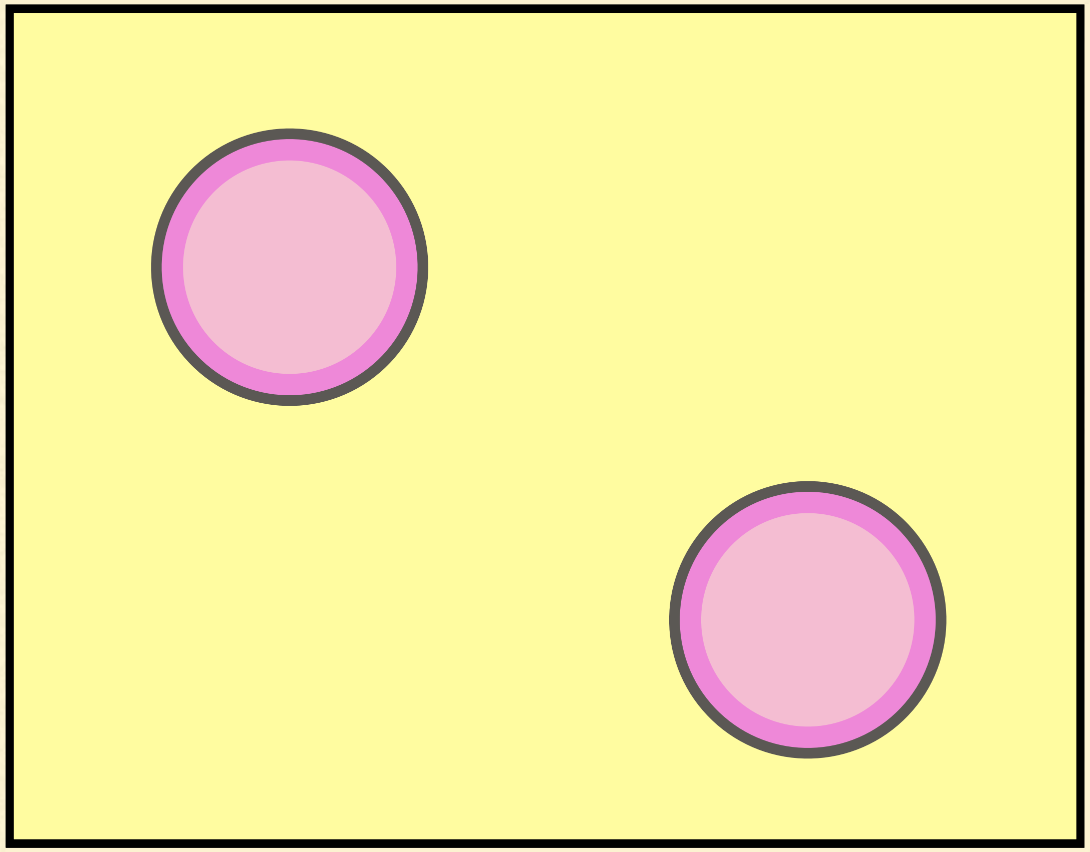


DYNAMICS AFTER COLLISION

Bubbles nucleate & expand



- Nucleation rate (per unit time & vol)

$$\Gamma(t) \propto e^{\beta t} \quad \text{with} \quad \beta \text{ : some const.}$$

- We assume that released energy is mainly carried by plasma bulk motion [Bodeker & Moore '17]

- Typically collide after $\Delta t \sim 1/\beta$ expansion

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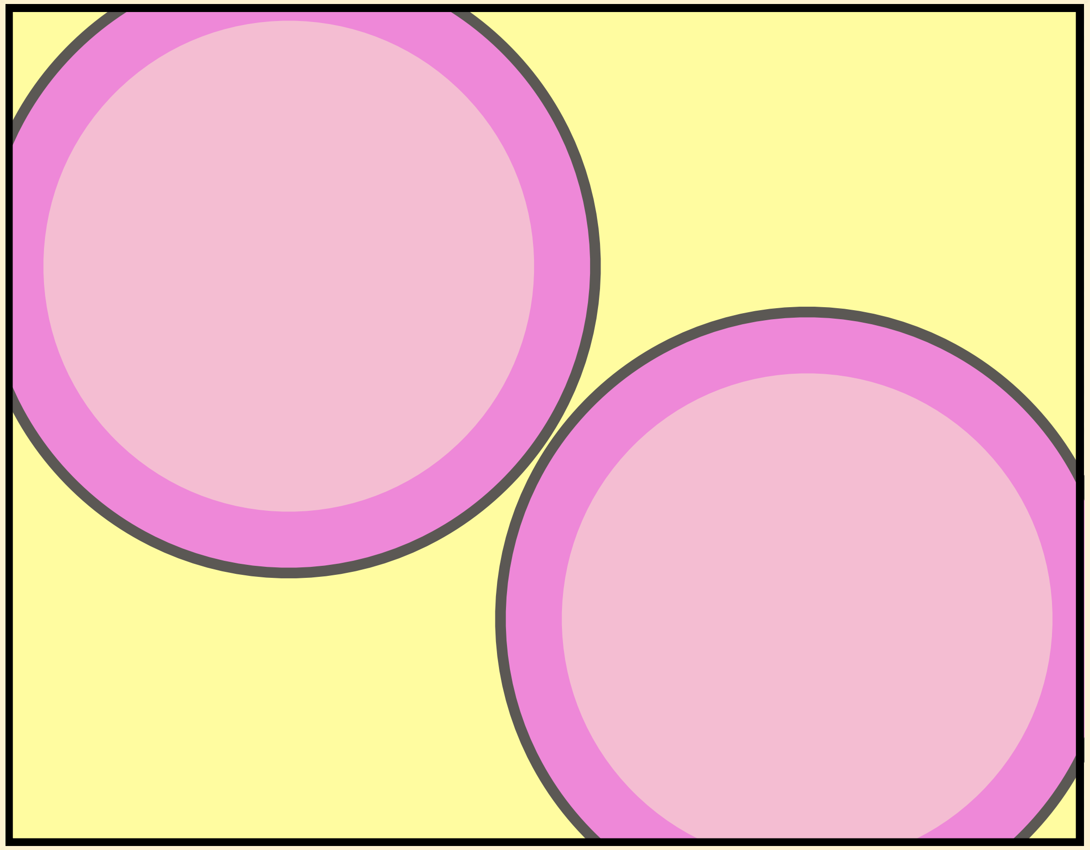

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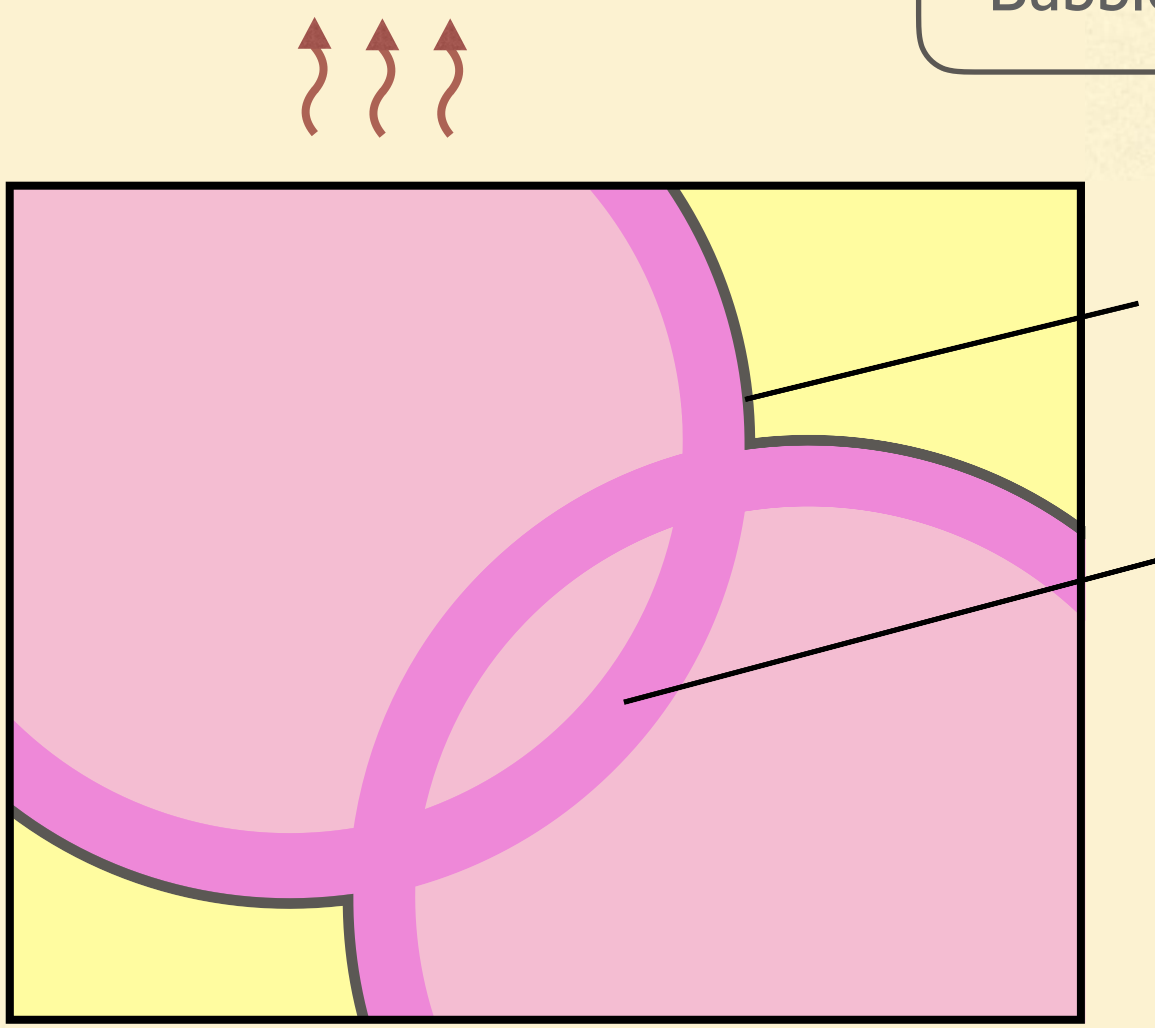
DYNAMICS AFTER COLLISION

GWs $\Box h_{ij} \sim T_{ij}$

Bubbles collide



- Scalar field damps soon after collision
- For small α ($\lesssim \mathcal{O}(0.1)$) case, propagation of plasma bulk motion is well described by linear approximation:

$\left(\partial_t^2 - c_s^2 \nabla^2\right) \vec{v} \simeq 0$ "sound waves"

DYNAMICS AFTER COLLISION

GWs $\Box h_{ij} \sim T_{ij}$

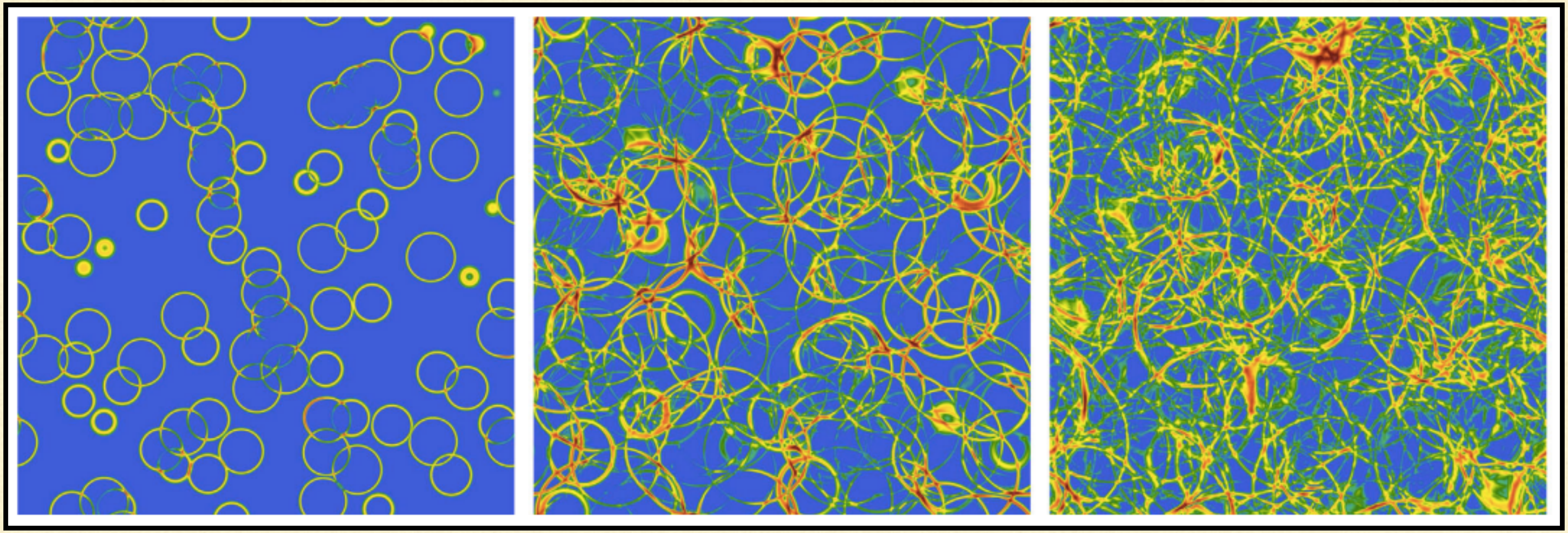
Turbulence develops

- Nonlinear effects appear at late times

“turbulence”

SIMULATIONS ARE DRIVING OUR UNDESTANDING

- Example of numerical simulations ($\alpha \lesssim \mathcal{O}(0.1)$)

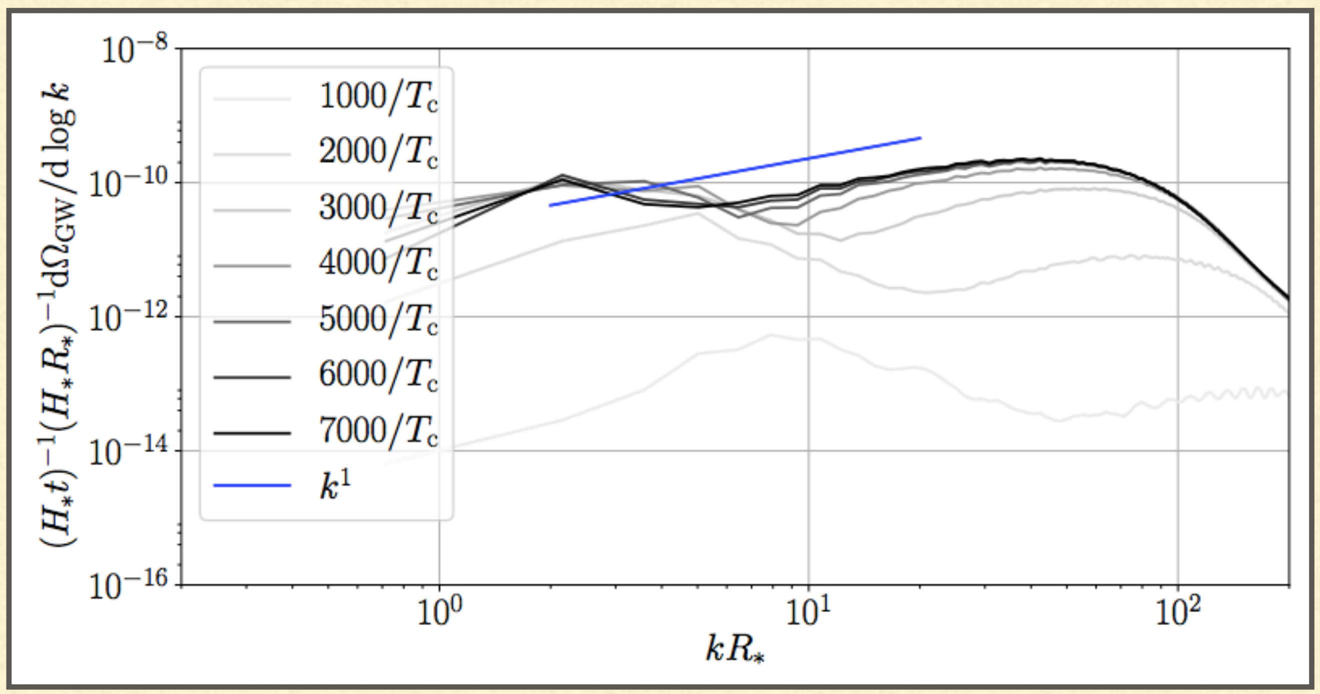


[Hindmarsh, Huber, Rummukainen, Weir '15]

SIMULATIONS ARE DRIVING OUR UNDESTANDING, BUT...

- Resulting GW spectrum ($\alpha \lesssim \mathcal{O}(0.1)$)

GW amplitude



[Hindmarsh et al. '17]

GW wavenumber

1) Everything is summed up: difficult to understand relevant physics each by each

2) Number of bubbles is limited & Full time evolution is difficult to follow

3) Simulation for large α $(\gg 1)$ is challenging due to shock waves & hierarchies

NECESSITY FOR ALTERNATIVE UNDERSTANDING

- What we do when we predict GWs from particle physics models:

Particle physics model → Parameters relevant to phase transition → Prediction on GWs

$\mathcal{L}$

- Released energy (i.e. α)
- Nucleation rate (i.e. β)
- Transition temperature ... and so on

$\rho_{\rm GW}$

- To prepare for future observations, we have to understand [ ] well

- Currently, understanding on [ ] is mainly driven by numerical simulations

- However, numerical approach alone does not give good understanding of the system

→ We would like to develop an alternative approach e.g. CMB, Lattice QCD, ...

TALK PLAN

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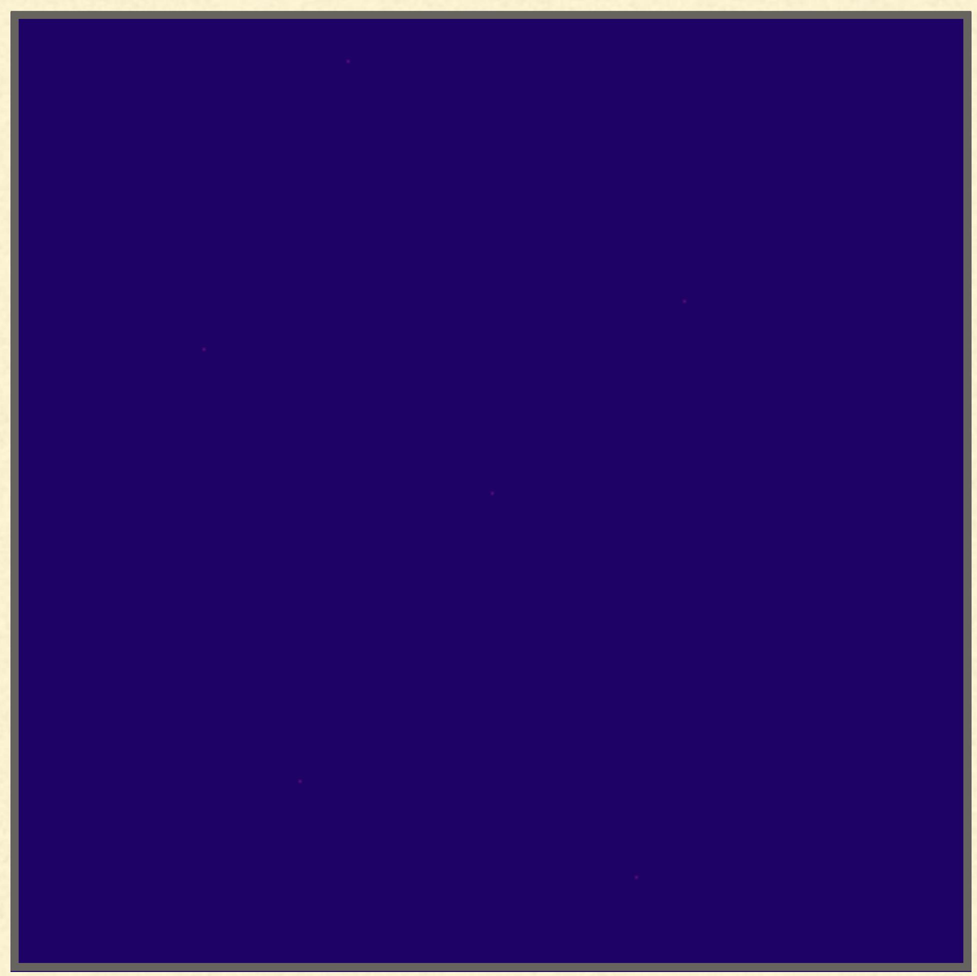
✔1. First-order phase transition and GW production

2. GWs from phase transitions: an analytic approach

THE SYSTEM WE WANT TO UNDERSTAND

- We propose the following modeling as a first step



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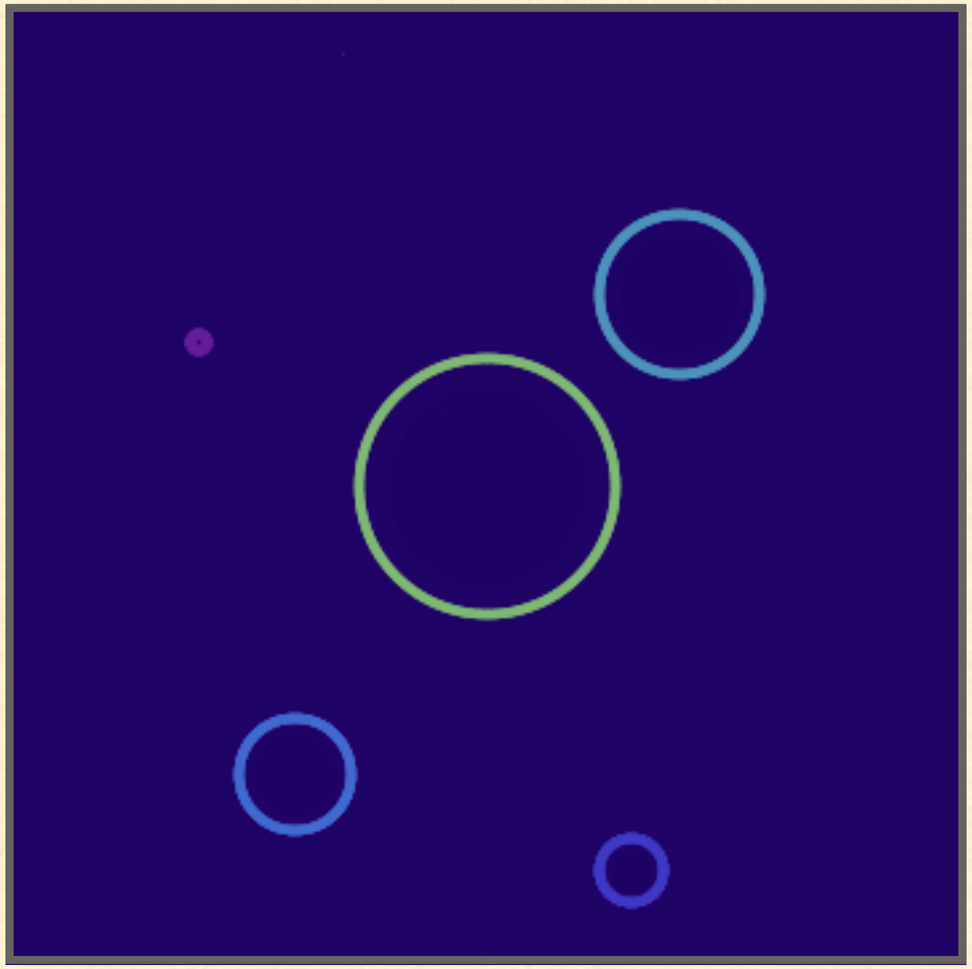

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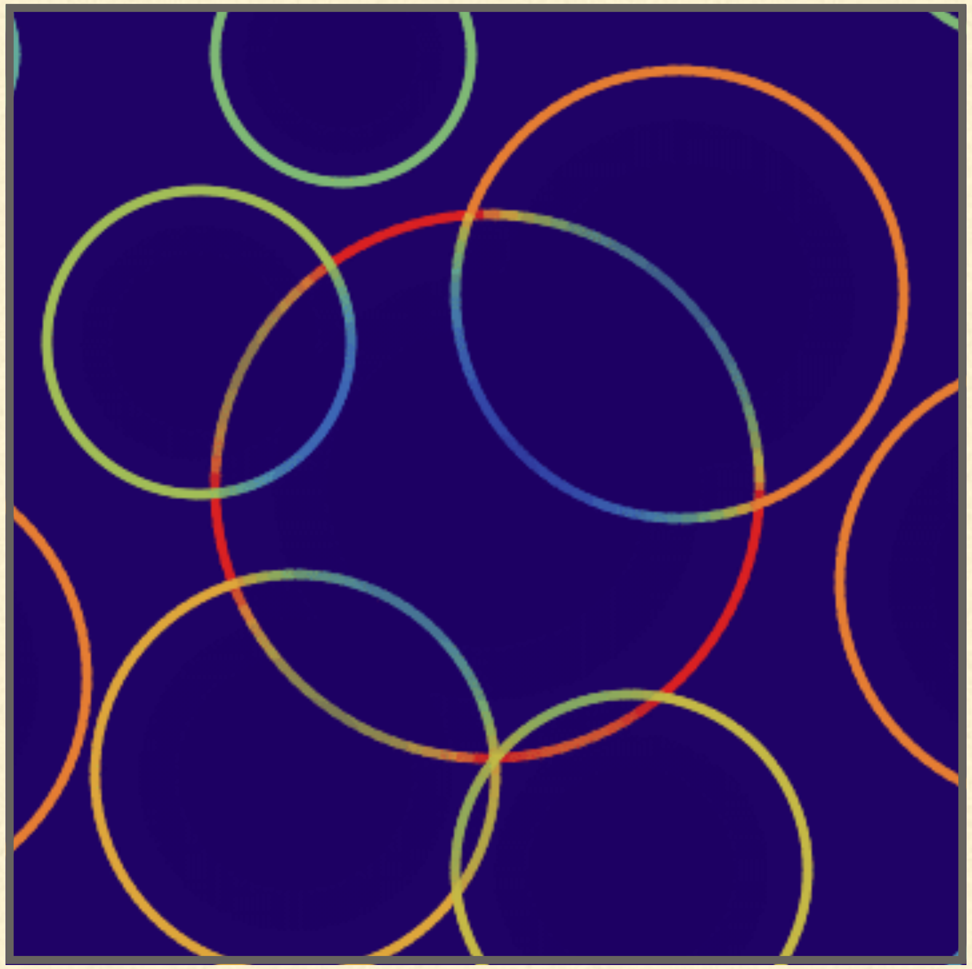

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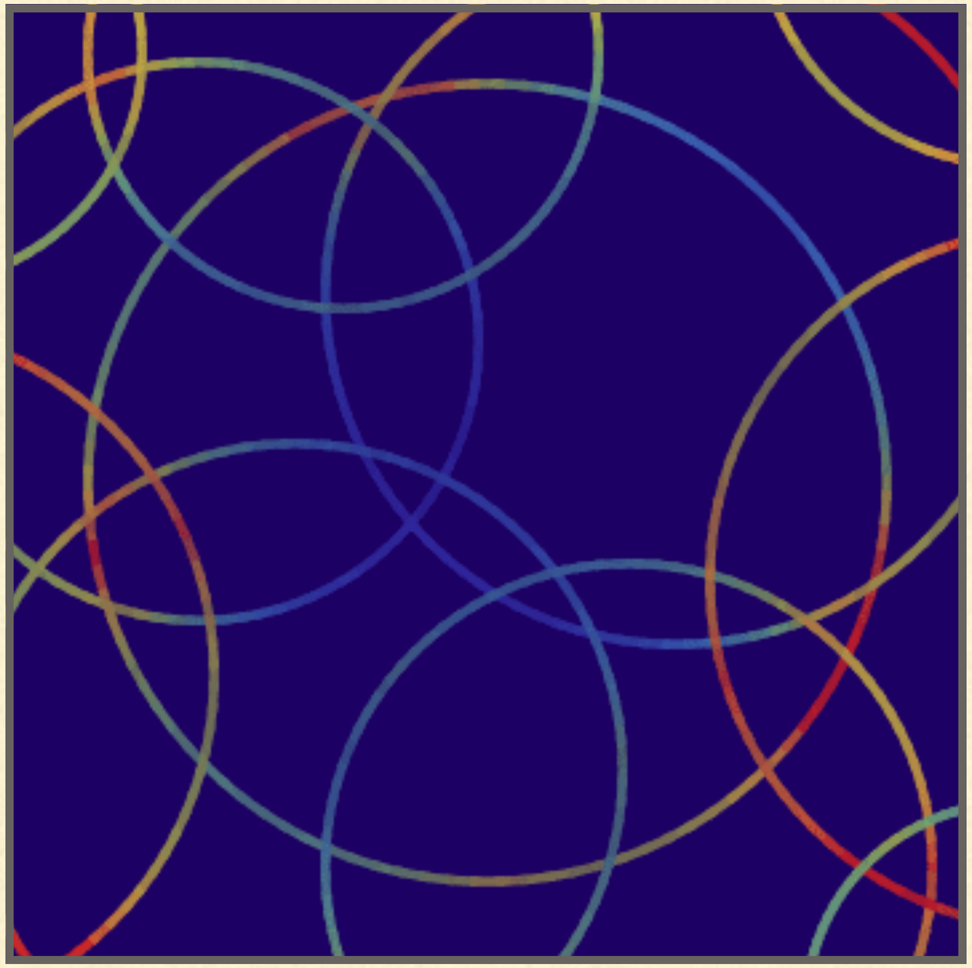

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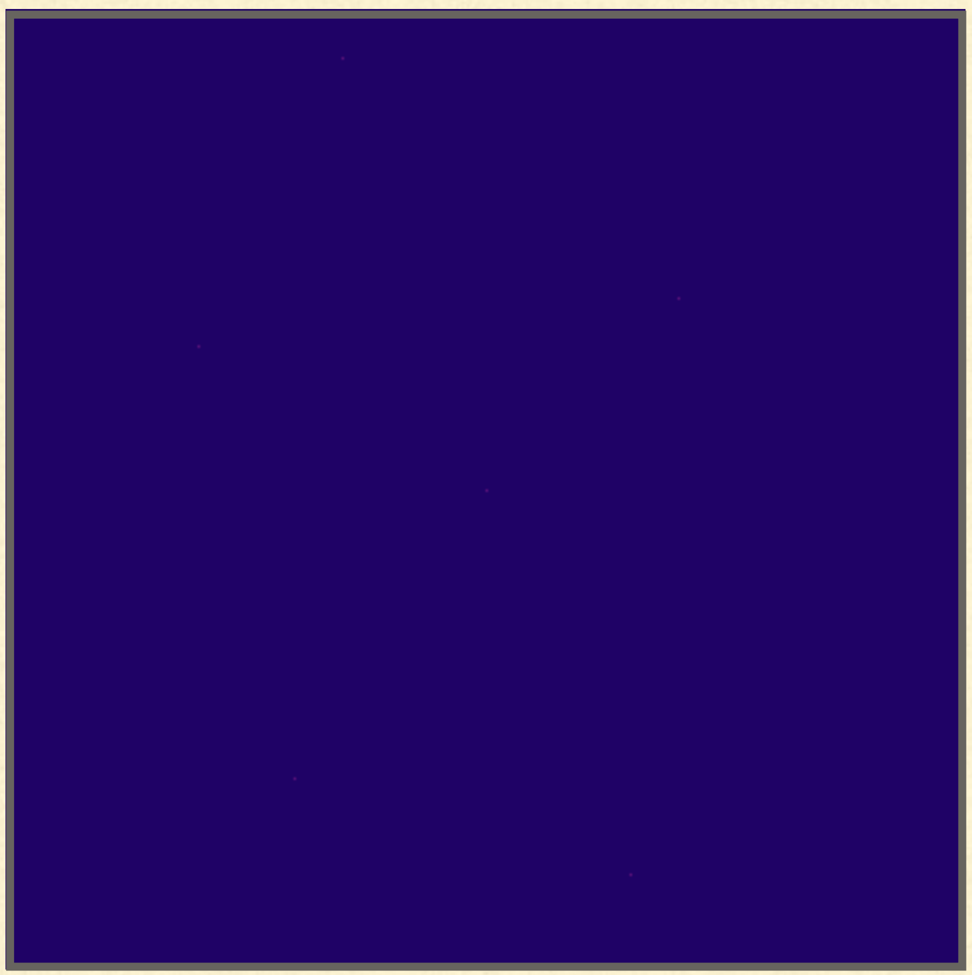

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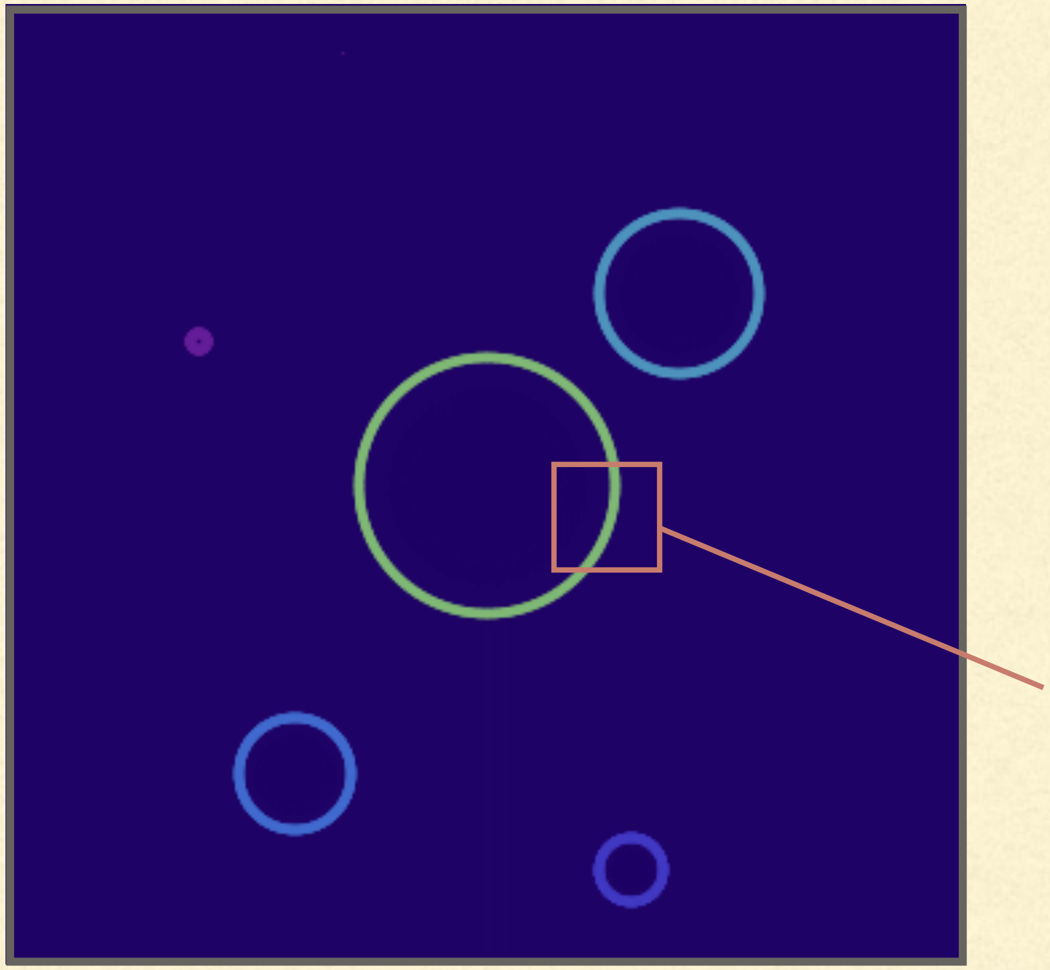

- Cosmic expansion neglected

- Bubbles nucleate with rate Γ
  (Typically $\Gamma \propto e^{\beta t}$ in thermal transitions)

- Bubbles are approximated to be thin

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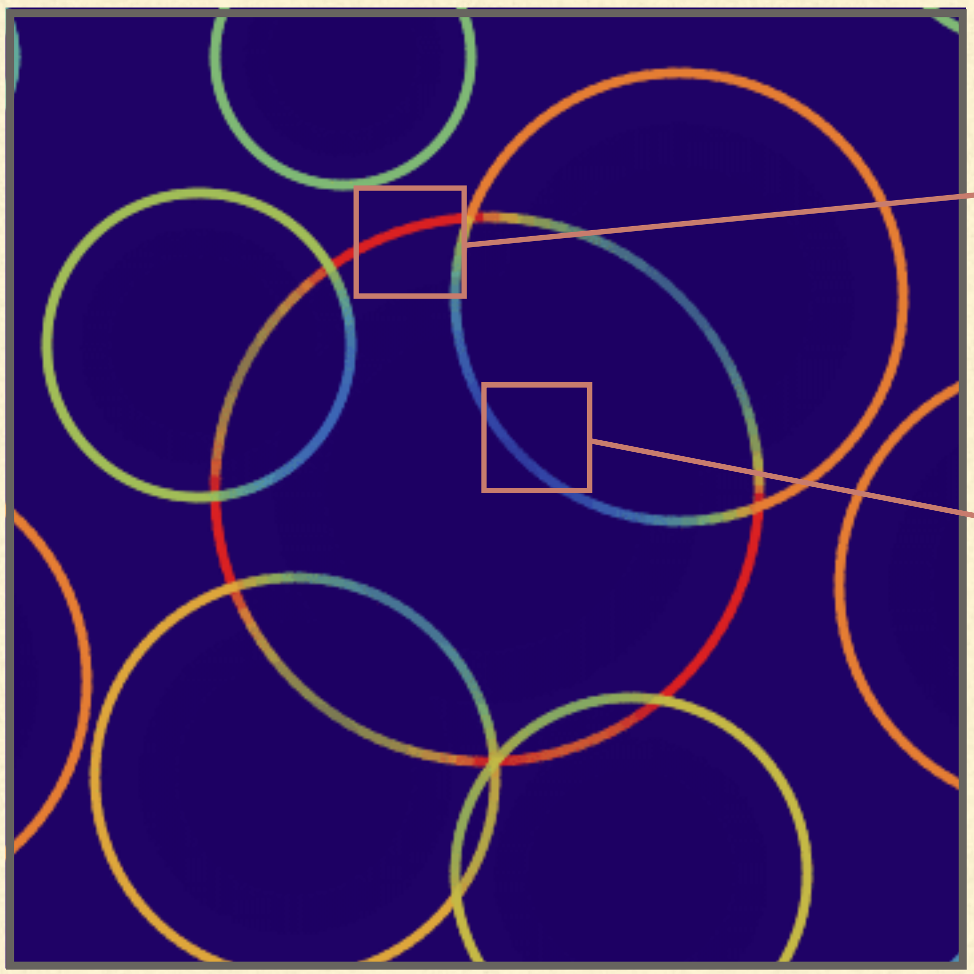

- Shells become more and more energetic

$$T_{ij} \propto \text{(bubble radius)}$$

- They lose energy & momentum after first collision

$$T_{ij} = T_{ij} \text{@ collision} \times \frac{\text{(bubble radius @ collision)}^2}{\text{(bubble radius)}^2}$$

$$\times \text{ (arbitrary damping func. D)}$$

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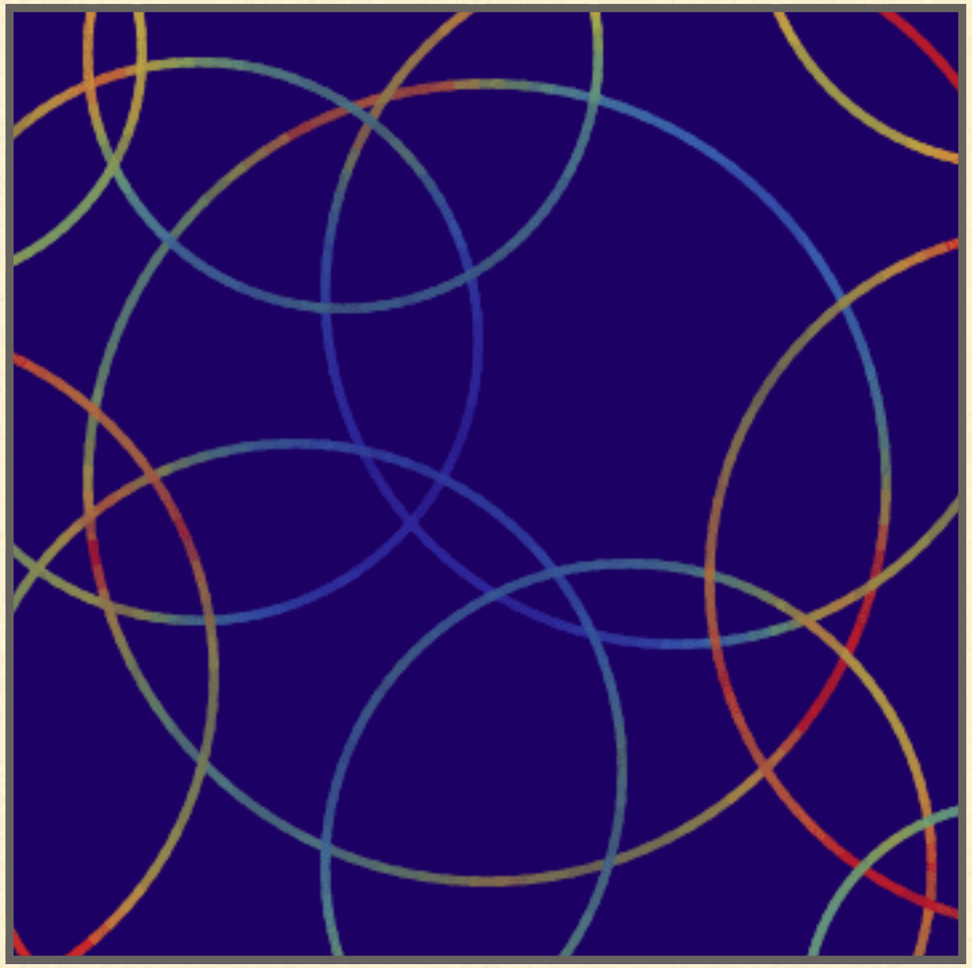


GW SPECTRUM FROM CONSIDERATION ON CAUSALITY

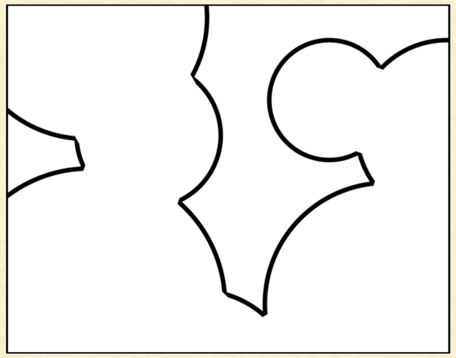
- This system, if solved, will serve as a good benchmark for real systems

- We wrote down GW spectrum in this system analytically, essentially from causality

[Jinno & Takimoto '16, '17]

- Full derivation takes too long, so we illustrate the derivation in a simplified setup:

Envelope approximation



proposed long time ago to model scalar-only system [Kosowsky & Turner '93]

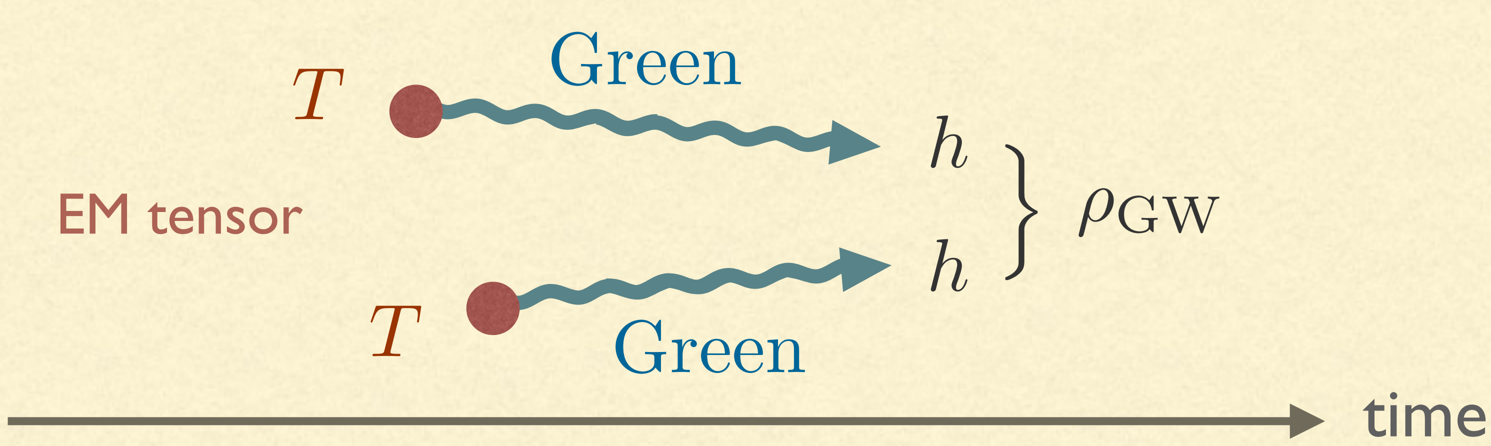
GW SPECTRUM AS EM TENSOR CORRELATOR

- Master formula: [e.g. Caprini et al., PRD77 (2008)]

$$\rho_{\rm GW}(k) \sim \int dt_x \int dt_y \cos(k(t_x - t_y)) {\rm F.T.} \langle T_{ij}(t_x, \vec{x}) T_{ij}(t_y, \vec{y}) \rangle$$

GW energy density per each log wavenumber k — Green function — EM tensor

- Why? GW EOM : $\Box h \sim T$ → solution : $h \sim \int dt \; {\rm Green} \times T$



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$$\rho_{\rm GW}(k) \sim \int dt_x \int dt \ldots \langle T_{ij}(t_x, \vec{x}) T_{ij}(t_y, \vec{y}) \rangle$$

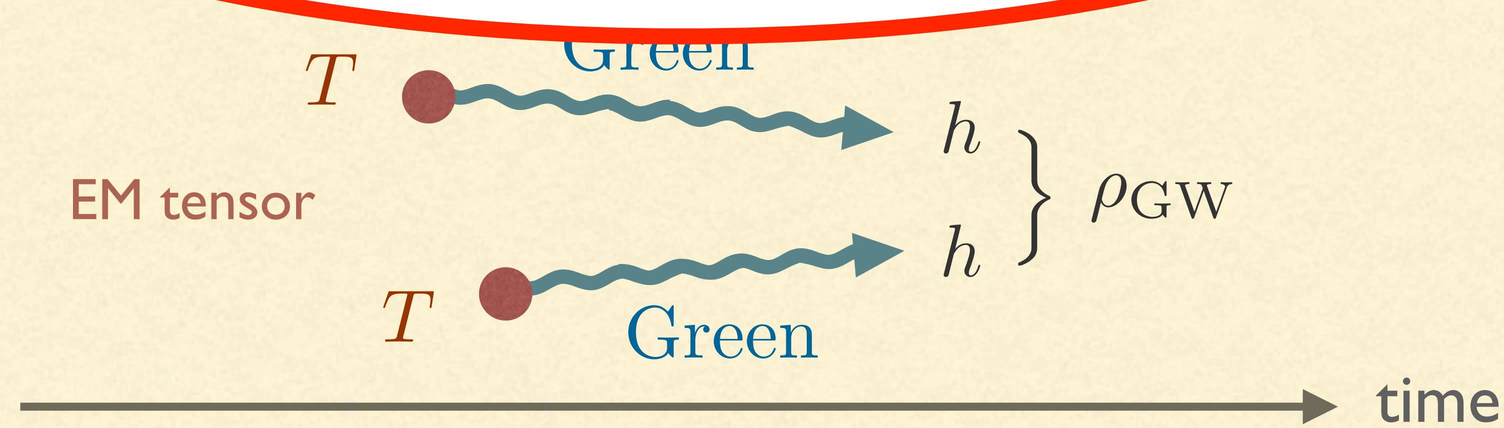
GW …

per e…

- Why? G… T

GW spectrum is essentially two-point ensemble average

< T T >



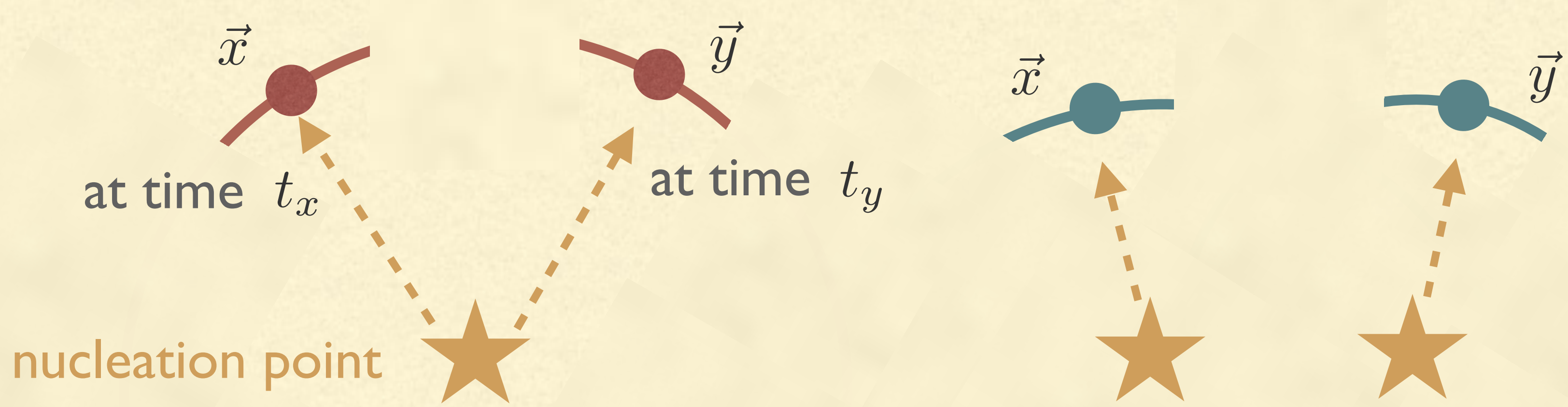
CALCULATION OF $\langle TT \rangle$

[Jinno & Takimoto '16 & '17]

- Calculating $\langle T(t_x,\vec{x})T(t_y,\vec{y})\rangle_{\rm ens}$ means ...

  - Fix spacetime points $x = (t_x, \vec{x})$ and $y = (t_y, \vec{y})$

  - Find bubble configurations s.t. EM tensor T is nonzero at x & y



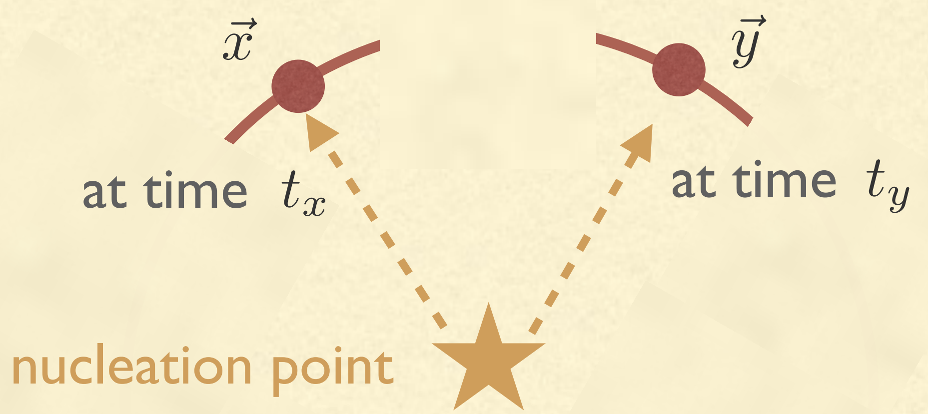
  - Calculate $\left\{ \begin{array}{c} \text{probability} \\ \text{value of } T(t_x,\vec{x})T(t_y,\vec{y}) \end{array} \right\}$ for such configurations and sum up

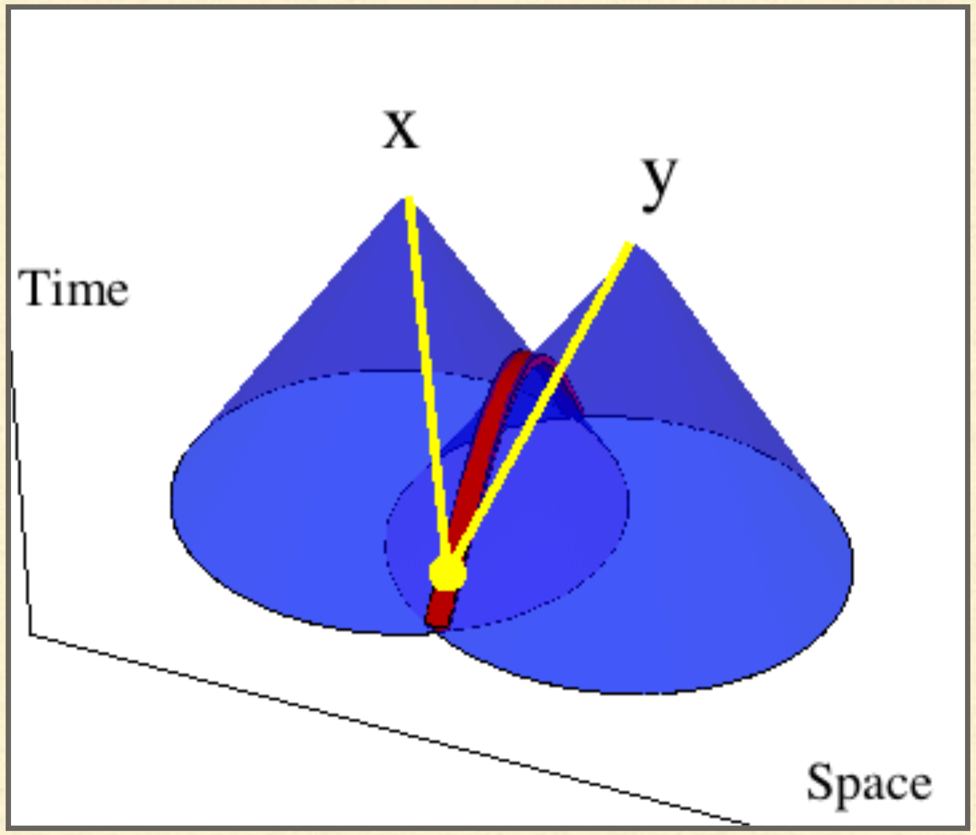
CALCULATION OF $\langle TT \rangle$

[Jinno & Takimoto '16 & '17]

- Only two types of configurations exist :

  - Single-bubble





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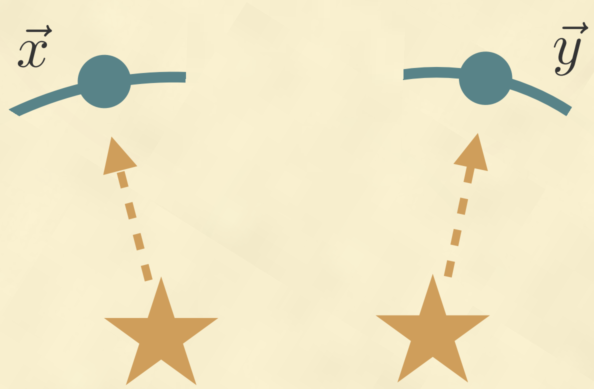

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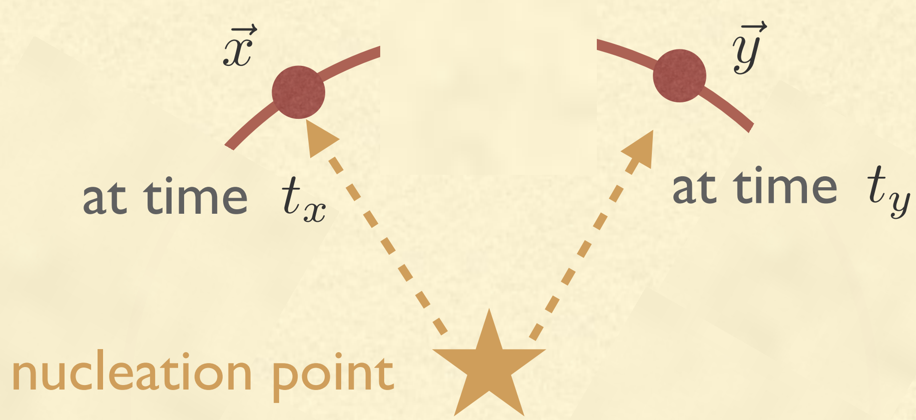
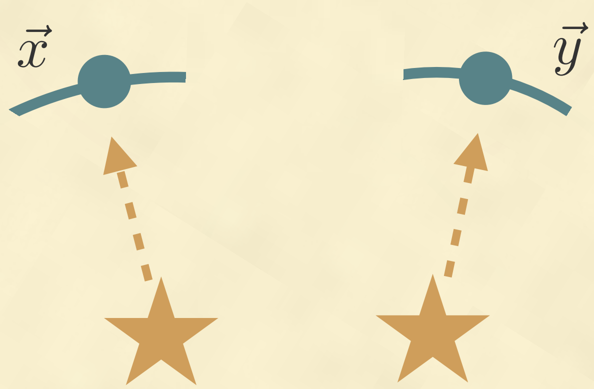


  - Double-bubble



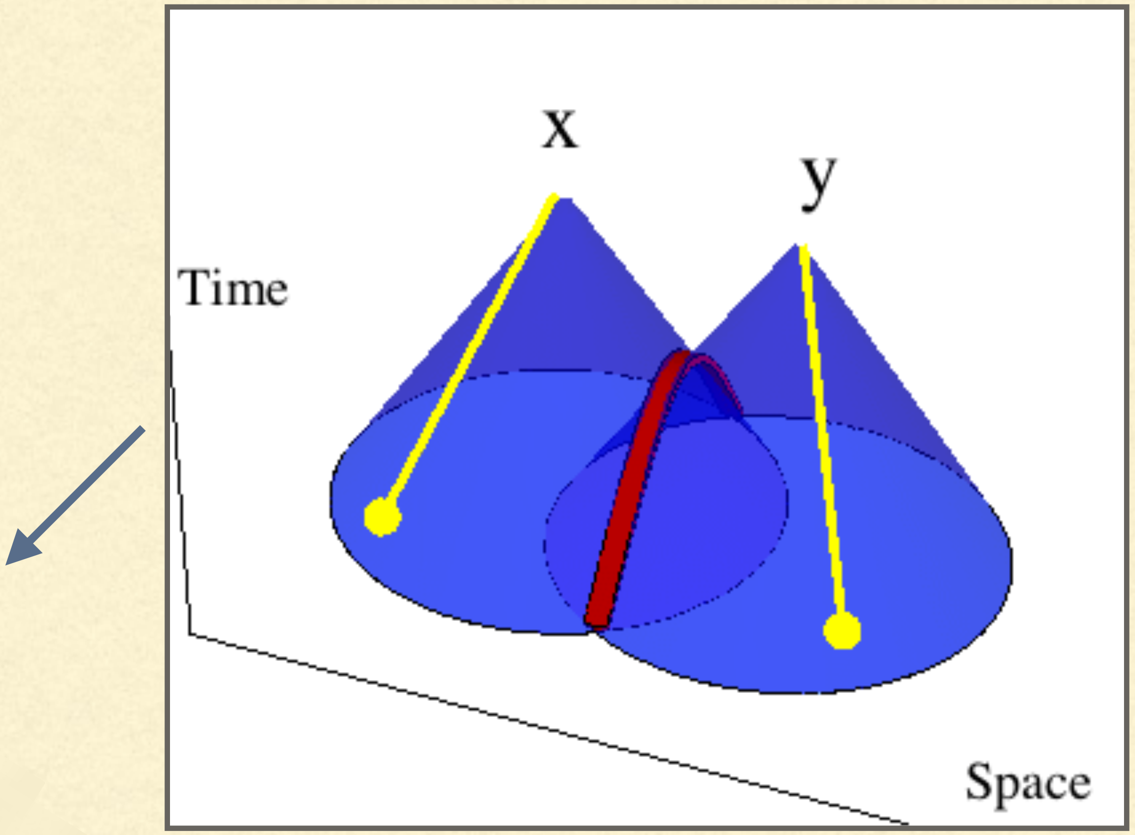


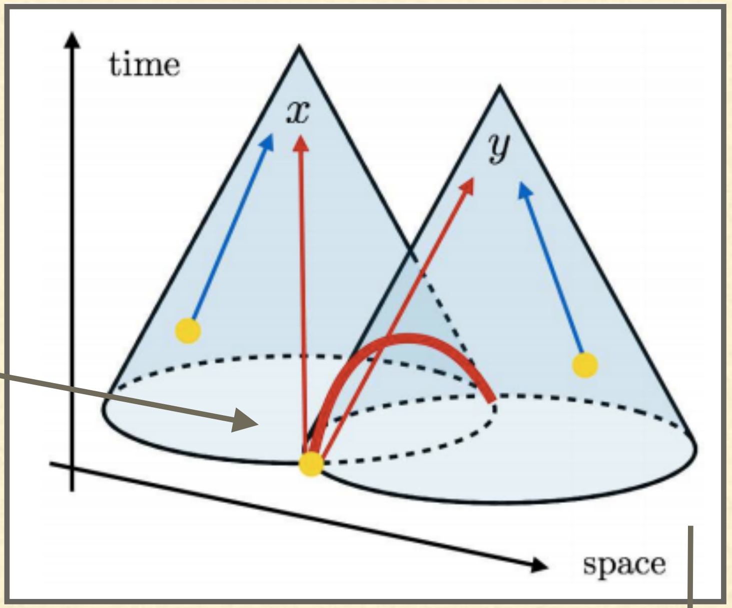
ILLUSTRATION: SINGLE-BUBBLE SPECTRUM

- Necessary and sufficient conditions

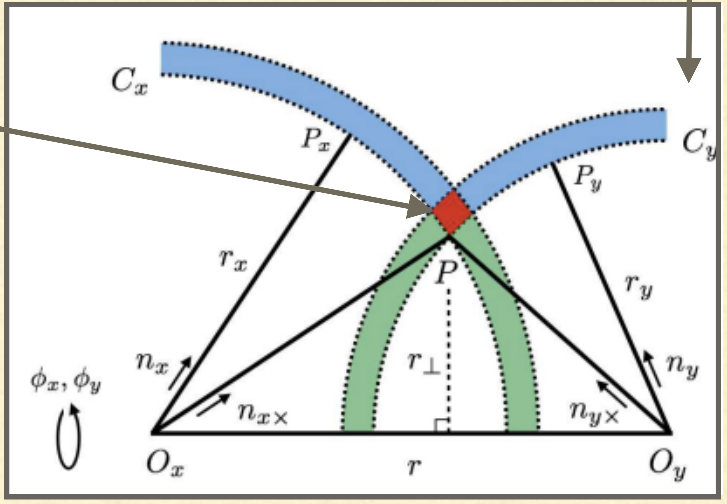
  - No bubbles nucleate inside past cones

  - One bubble nucleates inside the red diamond

    within infinitesimal time interval $t_n \sim t_n + dt_n$



$t = t_n$ slice



- Resulting expression

$$\langle T(x)T(y)\rangle^{(s)}_{\text{ens}}$$

$$= P(x,y) \int dt_n \left(\begin{array}{l} \text{prob. for one bubble} \\ \text{to nucleate} \\ \text{in the red diamond} \end{array} \right) \left(\begin{array}{l} \text{value of } T(x)T(y) \\ \text{realized in each case} \end{array} \right)$$

ENVELOPE: FULL EXPRESSIONS

- The spectrum becomes sum of two contributions

$$\rho_{\rm GW}(k) \propto \Delta^{(s)} + \Delta^{(d)}$$

Each term is just an integration of polynomials, exponentials and spherical Bessels

$$\Delta^{(s)} = \int_{-\infty}^{\infty} dt_{x,y} \int_{|t_{x,y}|}^{\infty} dr_v \; v^3 k^3 \frac{e^{-\frac{r_v}{2}}}{\mathcal{I}(t_{x,y}, r_v)} \left[j_0(vkr_v)\mathcal{S}_0 + \frac{j_1(vkr_v)}{vkr_v}\mathcal{S}_1 + \frac{j_2(vkr_v)}{(vkr_v)^2}\mathcal{S}_2 \right] \cos(kt_{x,y})$$

$$\Delta^{(d)} = \int_{-\infty}^{\infty} dt_{x,y} \int_{|t_{x,y}|}^{\infty} dr_v \; v^3 k^3 \frac{e^{-r_v}}{\mathcal{I}(t_{x,y}, r_v)^2} \frac{j_2(vkr_v)}{(vkr_v)^2} \; \mathcal{D}(t_{x,y}, r_v)\mathcal{D}(-t_{x,y}, r_v) \cos(kt_{x,y})$$

ENVELOPE:

F

Maggiore "Gravitational Waves: Astrophysics and Cosmology" (book)

shape of the spectrum near the peak. Jinno and Takimoto (2017) show how to obtain analytic results in the envelope approximation by computing the two-point correlator of the energy–momentum tensor, using the formalism that we have discussed in Section 22.2. Different regimes of bubble evo-

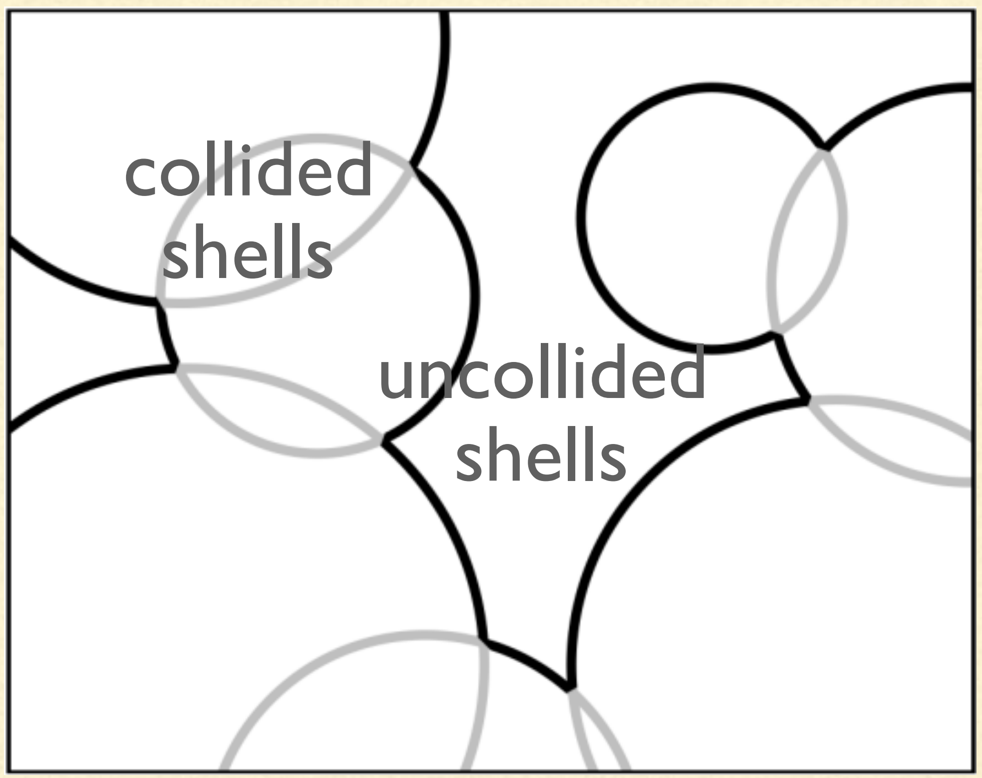
Mazumdar & White 1811.01948

"Cosmic phase transitions: their applications and experimental signatures" (recent review)

$$f = 1.65 \times 10^{-5}\mathrm{Hz} \left(\frac{f_*}{\beta}\right)\left(\frac{\beta}{H_*}\right)\left(\frac{T}{100\mathrm{GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6} \times \frac{0.35}{1+0.069v_w+0.69v_w^4},$$

$$\Omega_{GW}h^2 = 1.67 \times 10^{-5}\left(\frac{g_*}{100}\right)^{-1/3}\kappa^2\left(\frac{\beta}{H_*}\right)\left(\frac{\alpha}{1+\alpha}\right)^2 \times \frac{0.48v_w^3}{1+5.3v_w^2+5.0v_w^4}\Delta\ .$$

BEYOND THE ENVELOPE



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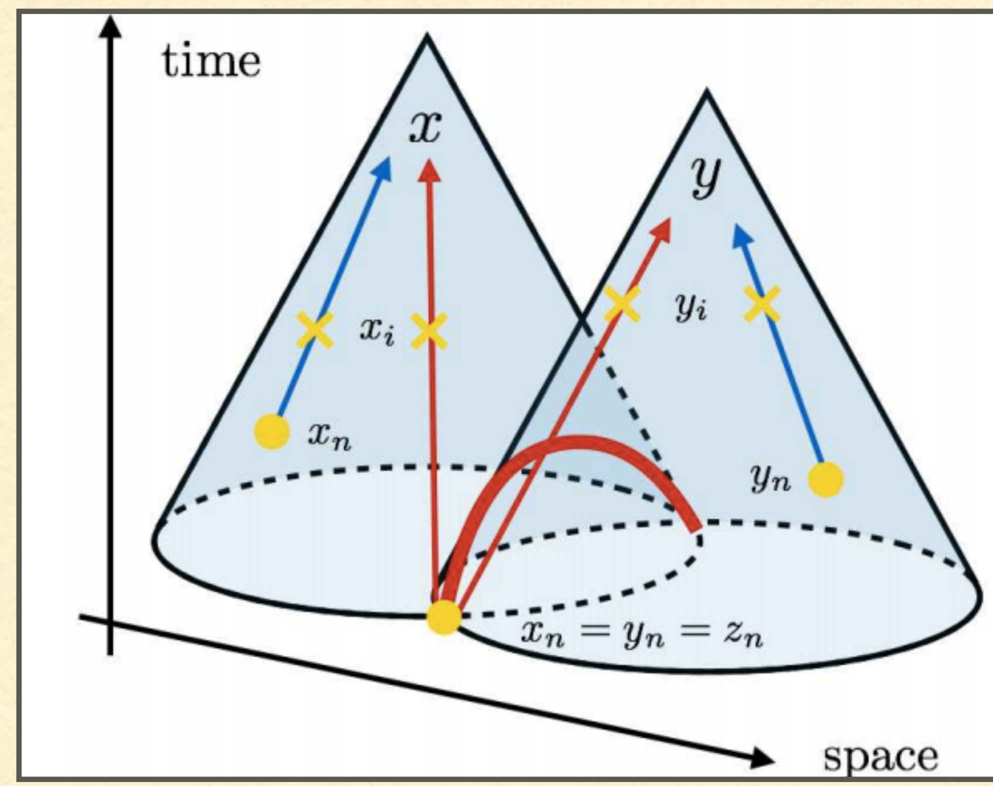


BEYOND THE ENVELOPE: FULL EXPRESSIONS

- Full expression reduces to only ~10-dim. integration [Jinno & Takimoto '17]

$\rho_{\rm GW}(k) \propto$ 1. single-bubble spectrum $\Delta^{(s)}$ + 2. double-bubble spectrum $\Delta^{(d)}$

$$\Delta^{(s)} = \int_{-\infty}^{\infty} dt_x \int_{-\infty}^{\infty} dt_y \int_{v|t_{x,y}|}^{\infty} dr \int_{-\infty}^{t_{\rm max}} dt_n \int_{t_n}^{t_x} dt_{xi} \int_{t_n}^{t_y} dt_{yi}$$

$$\frac{k^3}{3} \left[\begin{array}{l} e^{-I(x_i,y_i)}\, \Gamma(t_n)\, \dfrac{r}{r_{xn}^{(s)} r_{yn}^{(s)}} \\ \times \left[j_0(kr)\mathcal{K}_0(n_{xn\times}, n_{yn\times}) + \dfrac{j_1(kr)}{kr}\mathcal{K}_1(n_{xn\times}, n_{yn\times}) + \dfrac{j_2(kr)}{(kr)^2}\mathcal{K}_2(n_{xn\times}, n_{yn\times}) \right] \\ \times\, \partial_{txi} \left[r_B(t_{xi}, t_n)^3 D(t_x, t_{xi}) \right] \partial_{tyi} \left[r_B(t_{yi}, t_n)^3 D(t_y, t_{yi}) \right] \cos(k t_{x,y}) \end{array} \right]$$

BEYOND THE ENVELOPE: FULL EXPRESSIONS

- Full expression reduces to only ~10-dim. integration [Jinno & Takimoto '17]

$\rho_{\rm GW}(k) \propto$ 1. single-bubble spectrum $\Delta^{(s)}$ + 2. double-bubble spectrum $\Delta^{(d)}$

$$
\begin{aligned}
\Delta^{(s)} = &\int_{-\infty}^{\infty} dt_x \int_{-\infty}^{\infty} dt_y \int_{v|t_{x,y}|}^{\infty} dr \int_{-\infty}^{t_{\rm max}} dt_n \int_{t_n}^{t_x} dt_{xi} \int_{t_n}^{t_y} dt_{yi} \\
&\frac{k^3}{3} \left[\begin{aligned} & e^{-I(x_i,y_i)} \Gamma(t_n) \frac{r}{r_{xn}^{(s)} r_{yn}^{(s)}} \\ & \times \left[j_0(kr) \mathcal{K}_0(n_{xn\times}, n_{yn\times}) + \frac{j_1(kr)}{kr} \mathcal{K}_1(n_{xn\times}, n_{yn\times}) + \frac{j_2(kr)}{(kr)^2} \mathcal{K}_2(n_{xn\times}, n_{yn\times}) \right] \\ & \times \partial_{t_{xi}} \left[r_B(t_{xi}, t_n)^3 D(t_x, t_{xi}) \right] \partial_{t_{yi}} \left[r_B(t_{yi}, t_n)^3 D(t_y, t_{yi}) \right] \cos(k t_{x,y}) \end{aligned} \right]
\end{aligned}
$$

General nucleation rate

General shell velocity

General damping function after collision

$T_{ij} \propto$ (bubble radius)$^{-2}$ × D

BEYOND THE ENVELOPE: FULL EXPRESSIONS

- Full expression reduces to only ~10-dim. integration [Jinno & Takimoto '17]

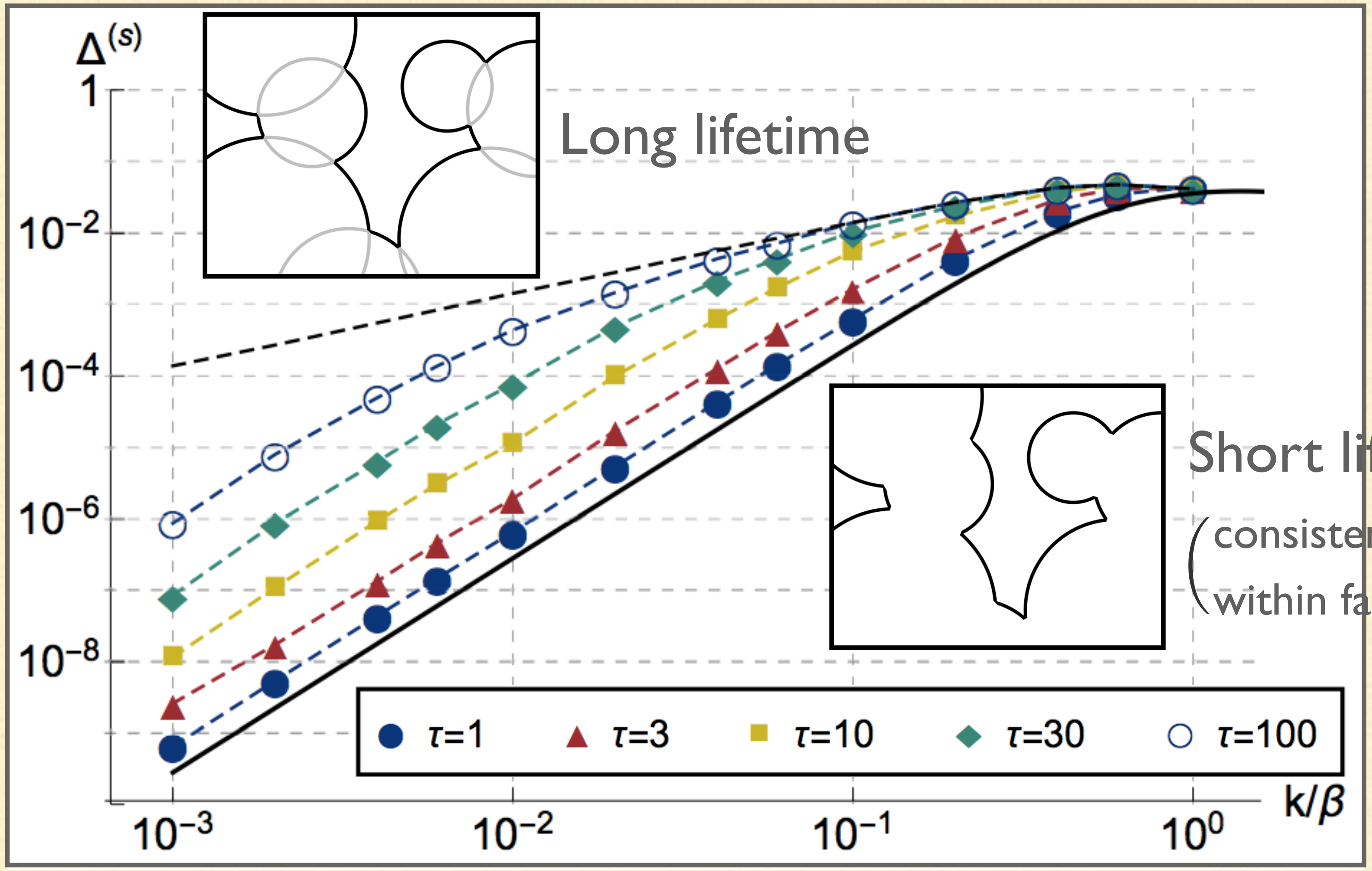
$\rho_{\rm GW}(k) \propto$ 1. single-bubble spectrum $\Delta^{(s)}$ + 2. double-bubble spectrum $\Delta^{(d)}$

$$\Delta^{(d)} = \int_{-\infty}^{\infty} dt_x \int_{-\infty}^{\infty} dt_y$$
$$\int_0^{\infty} dr \int_{-\infty}^{t_x} dt_{xn} \int_{-\infty}^{t_y} dt_{yn} \int_{t_{xn}}^{t_x} dt_{xi} \int_{t_{yn}}^{t_y} dt_{yi} \int_{-1}^{1} dc_{xn} \int_{-1}^{1} dc_{yn} \int_0^{2\pi} d\phi_{xn,yn}$$
$$\frac{k^3}{3} \left[\begin{aligned} &\Theta_{\rm sp}(x_i, y_n)\Theta_{\rm sp}(x_n, y_i) e^{-I(x_i,y_i)} \Gamma(t_{xn})\Gamma(t_{yn}) \\ &\times r^2 \left[j_0(kr)\mathcal{K}_0(n_{xn}, n_{yn}) + \frac{j_1(kr)}{kr}\mathcal{K}_1(n_{xn}, n_{yn}) + \frac{j_2(kr)}{(kr)^2}\mathcal{K}_2(n_{xn}, n_{yn}) \right] \\ &\times \partial_{txi}\left[r_B(t_{xi}, t_{xn})^3 D(t_x, t_{xi}) \right] \partial_{tyi}\left[r_B(t_{yi}, t_{yn})^3 D(t_y, t_{yi}) \right] \cos(kt_{x,y}) \end{aligned} \right]$$

NUMERICAL PLOT

- Single-bubble spectrum $\Delta^{(s)}$ (Damping function $D = e^{-(t-t_i)/\tau}$, τ : shell lifetime)

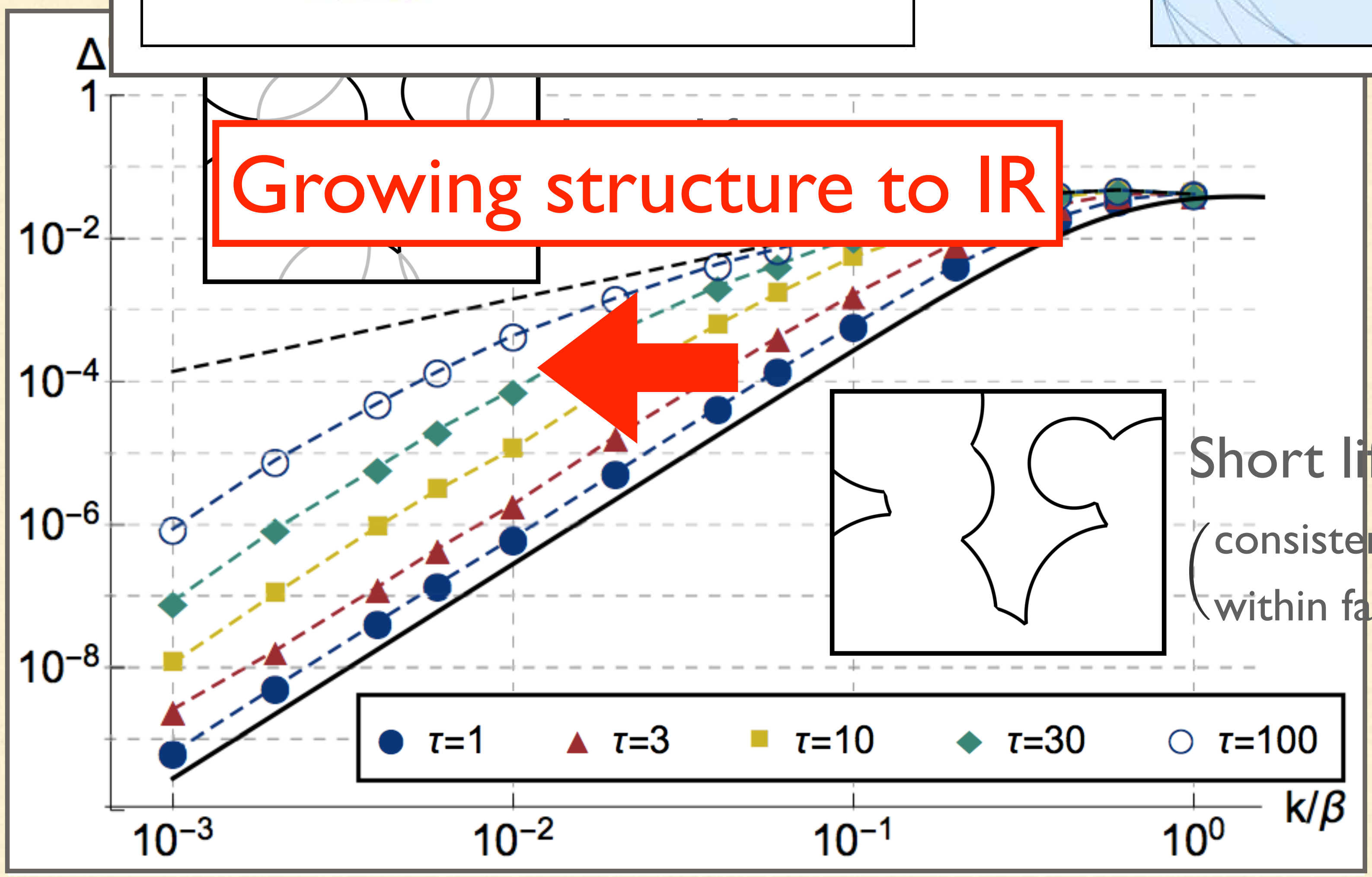
GW spectrum

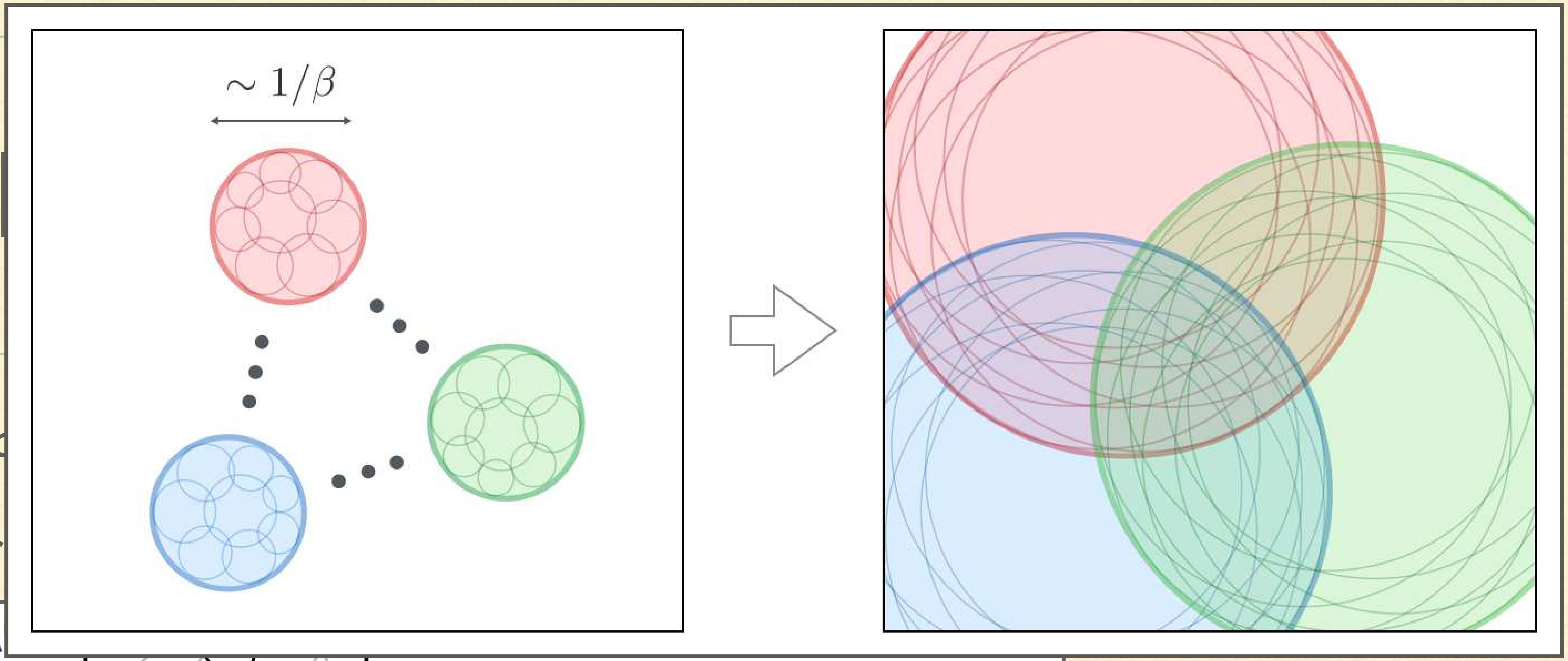


NUM

- Single-bubb

GW spectr



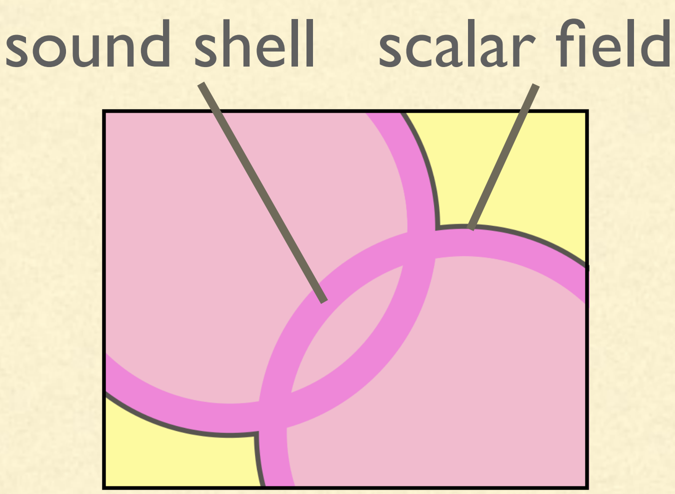


Short lifetime = Envelope

(consistent with [Huber & Konstandin '08] within factor 2)

GW wavenumber

IMPLICATIONS



- Our modeling ...

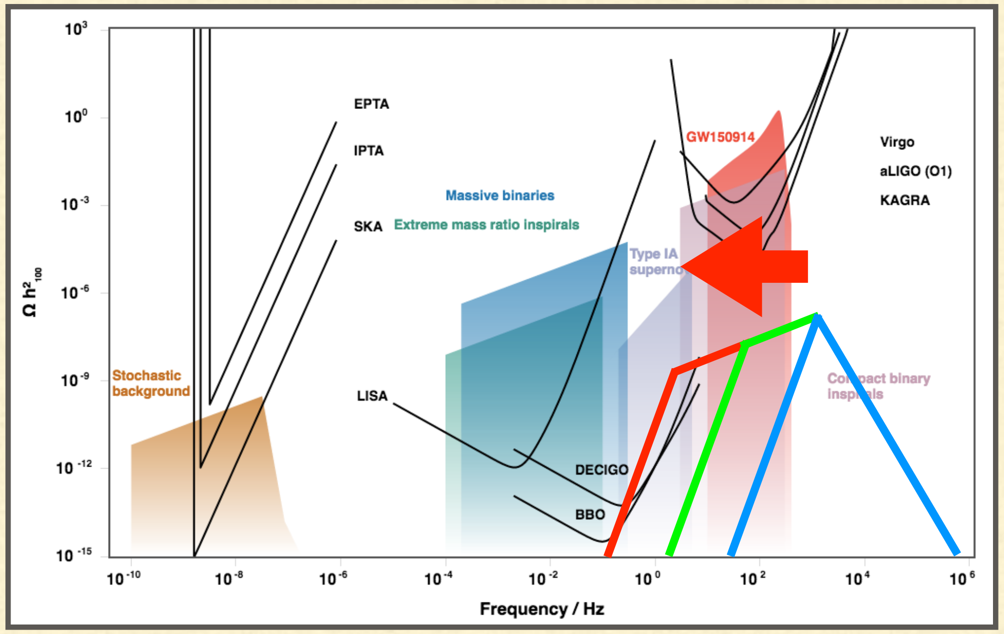
  - will represent the scalar field contribution well,

    and may represent the bubble-like structure of sound shells

- Our results ...

  - imply that GW spectrum has

    a growing structure to IR



- Applicability & limitations of our modeling are being studied

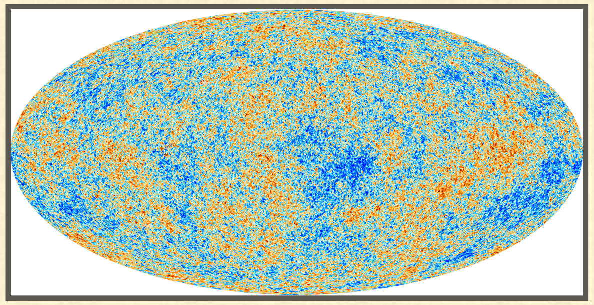
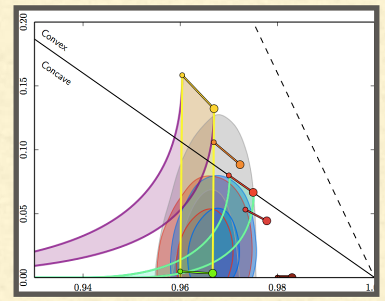
TALK PLAN

✔0. Introduction

✔1. First-order phase transition and GW production

2. GWs from phase transitions: an analytic approach

ANOTHER IMPLICATION: MODEL SELECTION FROM GW SPECTRUM

- Imagine CMB  

  - Many inflationary models realize observed scalar amplitude $\mathcal{P}_\zeta \ (\sim V/\epsilon)$ = "Leading"

  - What contributes to model selection is spectral index $n_s \ (\supset \epsilon, \eta)$ = "Next-leading"

- Can we do the same in GWs from first-order phase transitions?

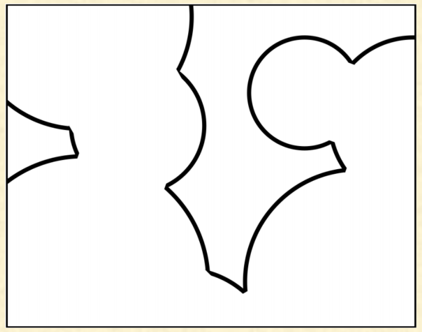
  - Current precision of numerical simulations is far from answering this question

  - Analytic approach can provide a quantitative estimate

MODEL SELECTION FROM GW SPECTRUM

[RJ, Hyeonseok Seong, Sangjun Lee, Masahiro Takimoto '17]

- Let's take previous "envelope" modeling as a working example



- Each particle physics model predicts different nucleation rate

$$\Gamma \propto e^{\beta t + \gamma^2 t^2 + \cdots}$$

"Leading" (β) : Does not appear in GW spectral shape (as long as cosmic expansion is neglected)

"Next-leading" (γ^2) : Generically nonzero but often neglected. Gives slight difference in GW spectrum

MODEL SELECTION FROM GW SPECTRUM

[RJ, Hyeonseok Seong, Sangjun Lee, Masahiro Takimoto '17]

- We can again calculate the spectrum analytically

$$\rho_{\mathrm{GW}}(k) \propto \Delta^{(s)} + \Delta^{(d)}$$

$$\Delta^{(s)} = \beta^2 v^6 k^3 \, \Gamma_* \int_{-\infty}^{\infty} dt_{\langle x,y \rangle} \int_{-\infty}^{\infty} dt_{x,y} \int_{|t_{x,y}|}^{\infty} dr_v \; e^{-I(x,y)} \left[j_0(vkr_v)\mathcal{S}_0 + \frac{j_1(vkr_v)}{vkr_v}\mathcal{S}_1 + \frac{j_2(vkr_v)}{(vkr_v)^2}\mathcal{S}_2 \right] \cos(kt_{x,y})$$

MODEL SELECTION FROM GW SPECTRUM

[RJ, Hyeonseok Seong, Sangjun Lee, Masahiro Takimoto '17]

- We can agai

$$\mathcal{S}_0 = c^{(s)}_{\mathrm{Exp},0}\mathrm{Exp}\left[-\left(t_{\langle x,y\rangle}-\frac{r_v}{2}\right)^2\right]+c^{(s)}_{1+\mathrm{Erf},0}\left(1+\mathrm{Erf}\left[t_{\langle x,y\rangle}-\frac{r_v}{2}\right]\right)$$

$$\mathcal{S}_1 = c^{(s)}_{\mathrm{Exp},1}\mathrm{Exp}\left[-\left(t_{\langle x,y\rangle}-\frac{r_v}{2}\right)^2\right]+c^{(s)}_{1+\mathrm{Erf},1}\left(1+\mathrm{Erf}\left[t_{\langle x,y\rangle}-\frac{r_v}{2}\right]\right)$$

$$\mathcal{S}_2 = c^{(s)}_{\mathrm{Exp},2}\mathrm{Exp}\left[-\left(t_{\langle x,y\rangle}-\frac{r_v}{2}\right)^2\right]+c^{(s)}_{1+\mathrm{Erf},2}\left(1+\mathrm{Erf}\left[t_{\langle x,y\rangle}-\frac{r_v}{2}\right]\right)$$

$$\Delta^{(s)} = \beta^2 v^6 k^3\,\Gamma_*\int_{-\infty}^{\infty}dt_{\langle x,y\rangle}\int_{-\infty}^{\infty}dt_{x,y}\int_{|t_{x,y}|}^{\infty}dr_v\; e^{-I(x,y)}\left[j_0(vkr_v)\mathcal{S}_0+\frac{j_1(vkr_v)}{vkr_v}\mathcal{S}_1+\frac{j_2(vkr_v)}{(vkr_v)^2}\mathcal{S}_2\right]\cos(kt_{x,y})$$

MODEL SELECTION FROM GW SPECTRUM

[RJ, Hyeonseok Seong, Sangjun Lee, Masahiro Takimoto '17]

- We can agai

$$\mathcal{S}_0 = c^{(s)}_{\mathrm{Exp},0}\mathrm{Exp}\left[-\left(t_{\langle x,y\rangle} - \frac{r_v}{2}\right)^2\right] + c^{(s)}_{1+\mathrm{Erf},0}\left(1 + \mathrm{Erf}\left[t_{\langle x,y\rangle} - \frac{r_v}{2}\right]\right)$$

$$\mathcal{S}_1 = c^{(s)}_{\mathrm{Exp},1}\mathrm{Exp}\left[-\left(t_{\langle x,y\rangle} - \frac{r_v}{2}\right)^2\right] + c^{(s)}_{1+\mathrm{Erf},1}\left(1 + \mathrm{Erf}\left[t_{\langle x,y\rangle} - \frac{r_v}{2}\right]\right)$$

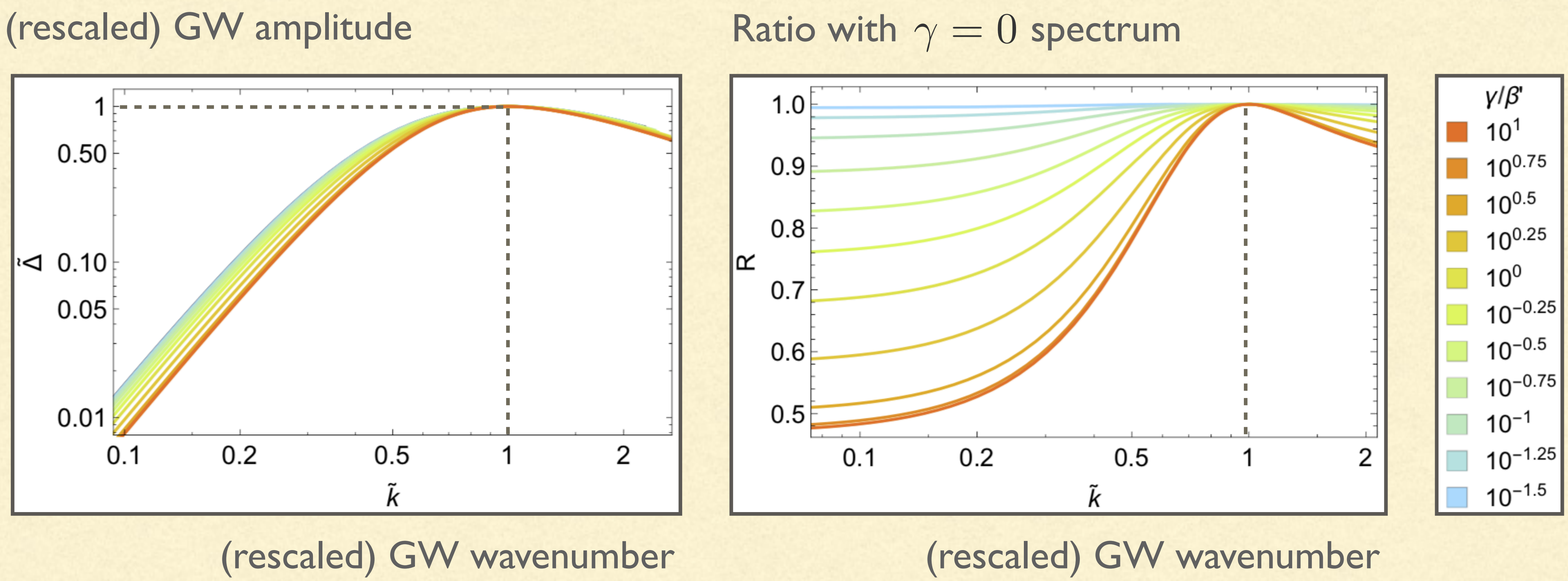
$$\mathcal{S}_2 = c^{(s)}_{\mathrm{Exp},2}\mathrm{Exp}\left[-\left(t_{\langle x,y\rangle} - \frac{r_v}{2}\right)^2\right] + c^{(s)}_{1+\mathrm{Erf},2}\left(1 + \mathrm{Erf}\left[t_{\langle x,y\rangle} - \frac{r_v}{2}\right]\right)$$

$$\Delta^{(s)} = \beta^2 v^6 k^3\, \Gamma_* \int_{-\infty}^{\infty} dt_{\langle x,y\rangle} \int_{-\infty}^{\infty} dt_{x,y} \int_{|t_{x,y}|}^{\infty} dr_v \; e^{-I(x,y)}\left[j_0(vkr_v)\mathcal{S}_0 + \frac{j_1(vkr_v)}{vkr_v}\mathcal{S}_1 + \frac{j_2(vkr_v)}{(vkr_v)^2}\mathcal{S}_2\right]\cos(kt_{x,y})$$

$$c^{(s)}_{\mathrm{Exp},2} = \frac{1}{96}\frac{1}{r_v^3}\begin{pmatrix} 1 \\ t_{\langle x,y\rangle} \\ t^2_{\langle x,y\rangle} \\ t^3_{\langle x,y\rangle} \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} 36r_v^5 + 10r_v^7 & 0 & -360r_v^3 - 36r_v^5 & 0 & 420r_v + 10r_v^3 \\ 120r_v^4 + 20r_v^6 & 0 & -1200r_v^2 - 72r_v^4 & 0 & 1400 + 20r_v^2 \\ 24r_v^5 & 0 & -240r_v^3 & 0 & 280r_v \\ 48r_v^4 & 0 & -480r_v^2 & 0 & 560 \end{pmatrix} \begin{pmatrix} 1 \\ t_{x,y} \\ t^2_{x,y} \\ t^3_{x,y} \\ t^4_{x,y} \end{pmatrix}$$

MODEL SELECTION FROM GW SPECTRUM

- How GW spectrum changes as the next-leading term γ increases



- This is the ***first quantitative study*** on the information imprinted in GW spectrum through bubble nucleation rate

SUMMARY

- 1st-order phase transitions may be explored with GWs in (near) future

- Currently numerical simulations are driving our understanding.

  We have to develop alternative ways to understand the system.

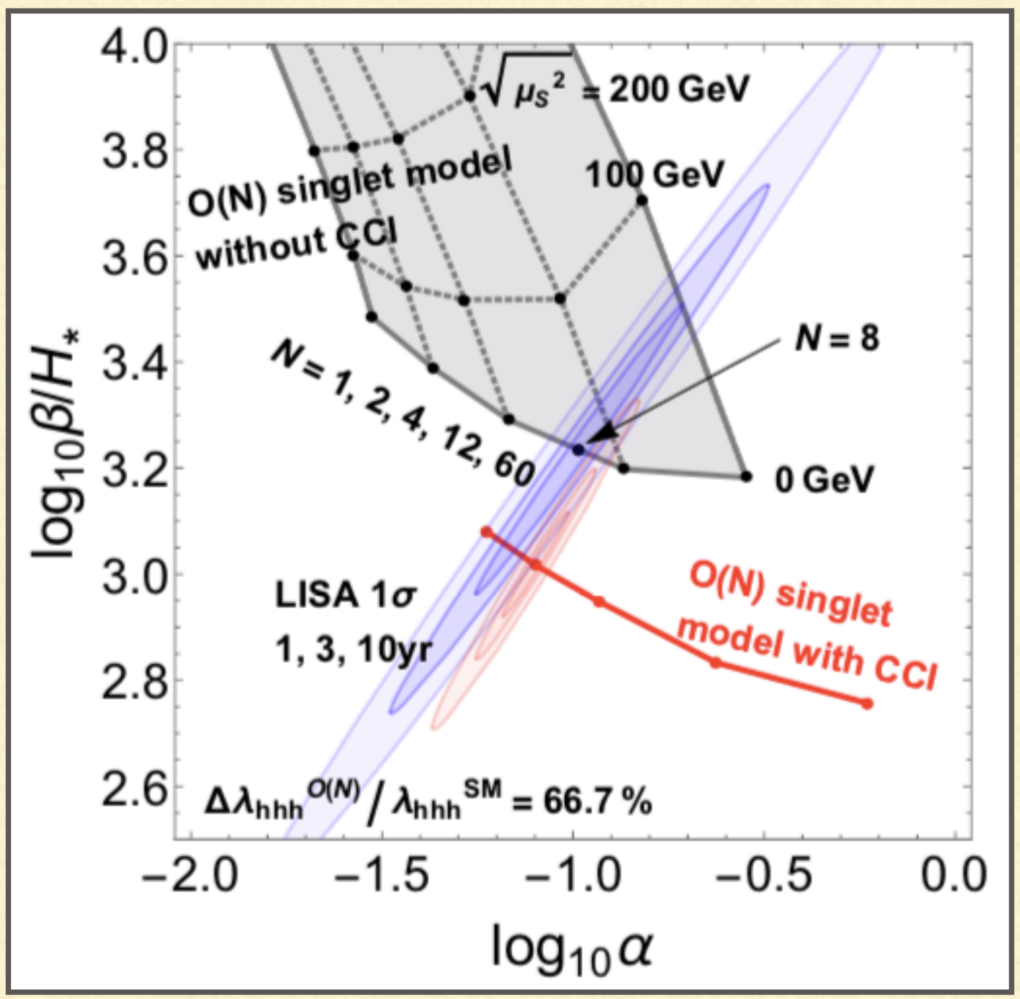
- We proposed one possible solution: analytic approach.

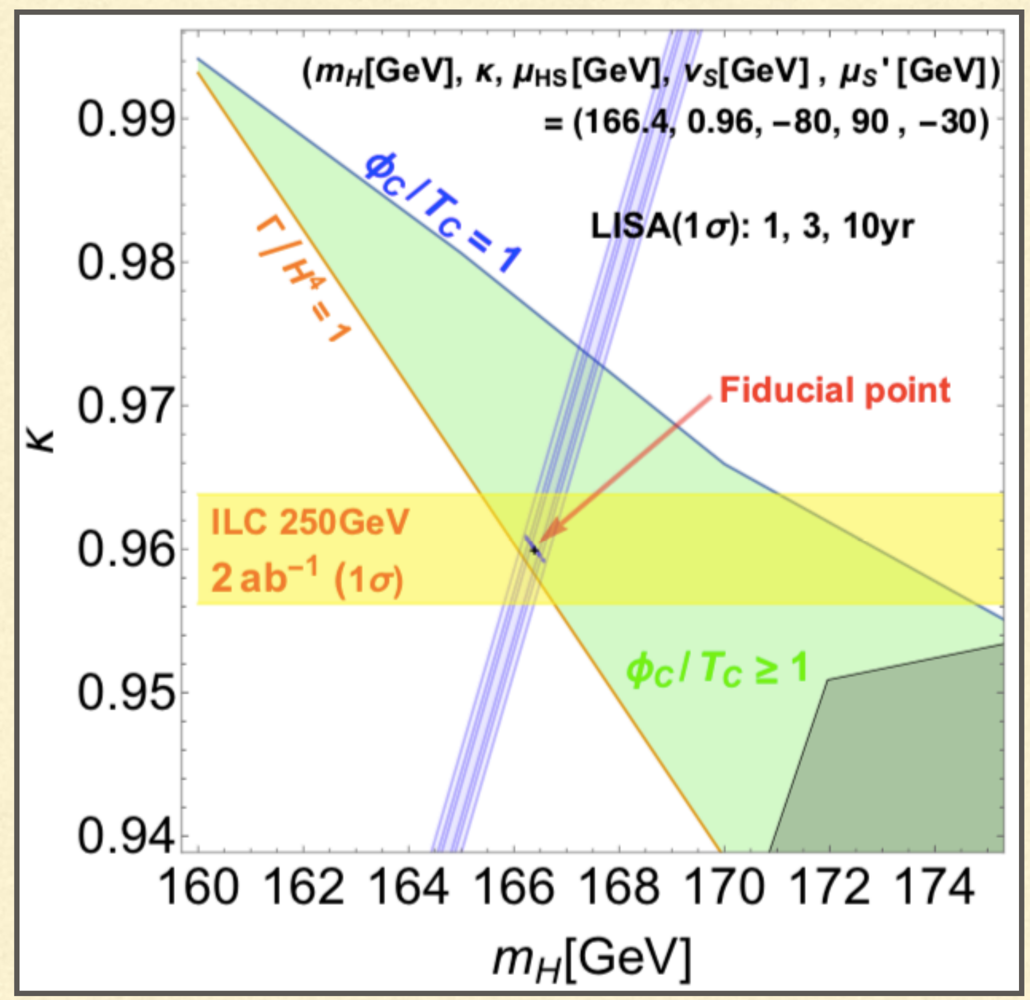
  We modeled the system with thin & free-propagating bubbles, and analytically solved it.

  Implication is a structure in GW spectrum which grows to IR.

SYNERGY BTWN. COLLIDER AND GWS: FIRST LIKELIHOOD ANALYSIS

- Collider constraints + GW likelihood analysis





[Hashino, RJ, Kakizaki, Kanemura, Takahashi, Takimoto '18]