Constant-roll Inflation

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Introduction

- Inflation is currently the dominant paradigm providing the initial seeds for the Cosmic Microwave Background(CMB) anisotropies and the Large-Scale-Structure (LSS) formation.
- In its most common form, inflation is driven by a scalar field rolling slowly down a <u>not</u> very steep potential. This is known as the slow-roll scenario.
- Since the scenario leads to nearly scale-invariant spectrum of density perturbations, it is widely thought to be necessary for agreement with observations.
- However, slow roll by itself is not a necessary condition for inflationary models to be viable hence it is interesting to study models which break slow-roll restriction.
- As a phenomenological way to parameterize deviations from the slow-roll scenario, Motohashi, Starobinsky, Yokoyama first introduced the idea of constant-roll inflation.

Single field inflation

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] ,$$

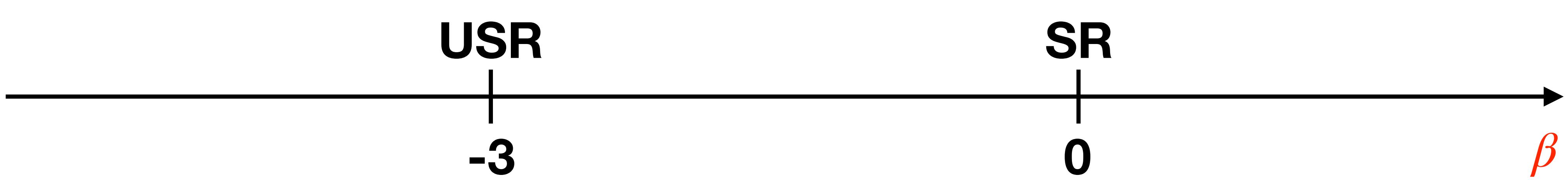
In a flat FRW universe with metric $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$, **one arrives at equations:**

$$H^2 = \frac{1}{3} \left(\frac{1}{2} \dot{\phi}^2 + V(\phi) \right) ,$$

$$\dot{H} = -\frac{1}{2} \dot{\phi}^2 ,$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 .$$

1. **Slow-roll inflation:** $|\ddot{\phi}| \ll 3H|\dot{\phi}|$,
2. **Ultra slow-roll inflation:** $|\ddot{\phi}| = -3H|\dot{\phi}|$,
3. **Constant-roll inflation:** $|\ddot{\phi}| = \beta H|\dot{\phi}|$,



$$\epsilon_1 \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2H^2} , \qquad \epsilon_{n+1} \equiv \frac{\dot{\epsilon}_n}{H\epsilon_n} .$$

Constant-roll potential

In addition to the constant-roll condition, if we consider $H = H(\phi)$ **and** $t = t(\phi)$ **, the evolution equations give**

$$\dot{H} = -\frac{1}{2}\dot{\phi}^2,$$

$$\frac{d\phi}{dt}\frac{dH}{d\phi} = -\frac{1}{2}\dot{\phi}^2,$$

$$\dot{\phi} = -2\frac{dH}{d\phi},$$

$$\ddot{\phi} = \beta H\dot{\phi},$$

$$\frac{d^2H}{d\phi^2} + \frac{\beta}{2}H = 0,$$

$$H(\phi) = c_1 e^{\sqrt{-\frac{\beta}{2}}\phi} + c_2 e^{-\sqrt{-\frac{\beta}{2}}\phi}.$$

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V\right)$$

$$V(\phi) = 3H^2 - 2\left(\frac{dH}{d\phi}\right)^2$$

for each $V(\phi)$, **one can get** $\phi(t), H(t)$, **and** $a(t)$!

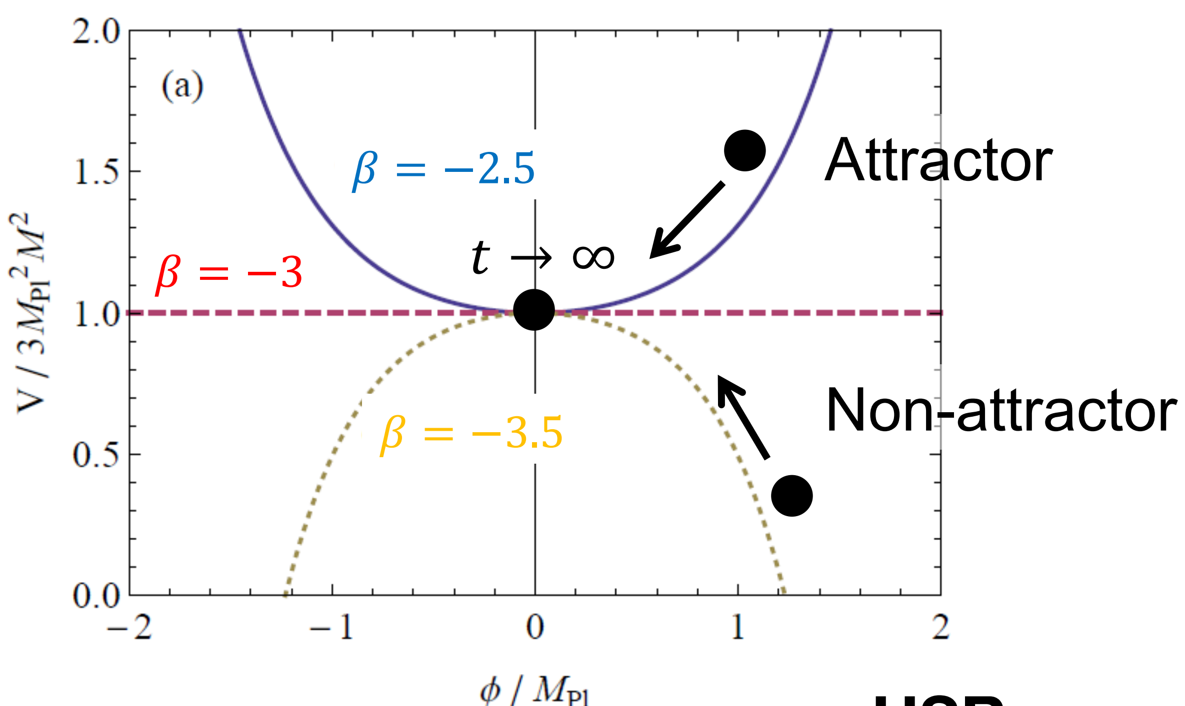
Constant-roll potential

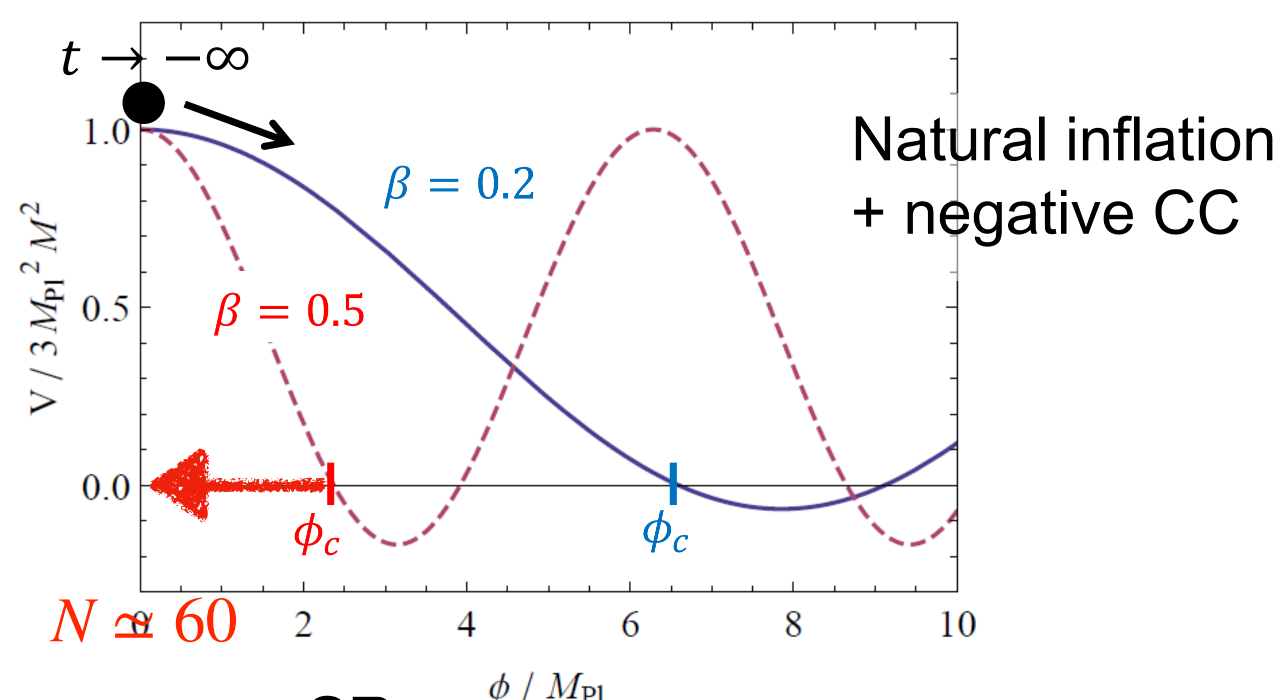
$1.V(\phi) = V_0 e^{\sqrt{-2\beta}\phi}$ $\beta < 0$, **Power-law inflation with** $r = 8(1 - n_s) \simeq 0.28$

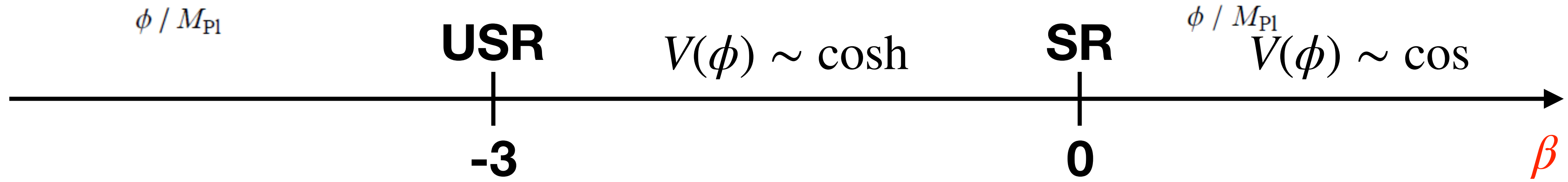
$$2.V(\phi) = 3M^2 M_p^2 \left[1 - \frac{\beta+3}{6}\left(1 - \cosh\left(\sqrt{-2\beta}\frac{\phi}{M_p}\right)\right)\right], \qquad \beta < 0,$$

$$N = \int_{\phi}^{\phi_0} \frac{V}{V'} d\phi$$

$$3.V(\phi) = 3M^2 M_p^2 \left[1 - \frac{\beta+3}{6}\left(1 - \cos\left(\sqrt{2|\beta|}\frac{\phi}{M_p}\right)\right)\right], \qquad \beta > 0,$$

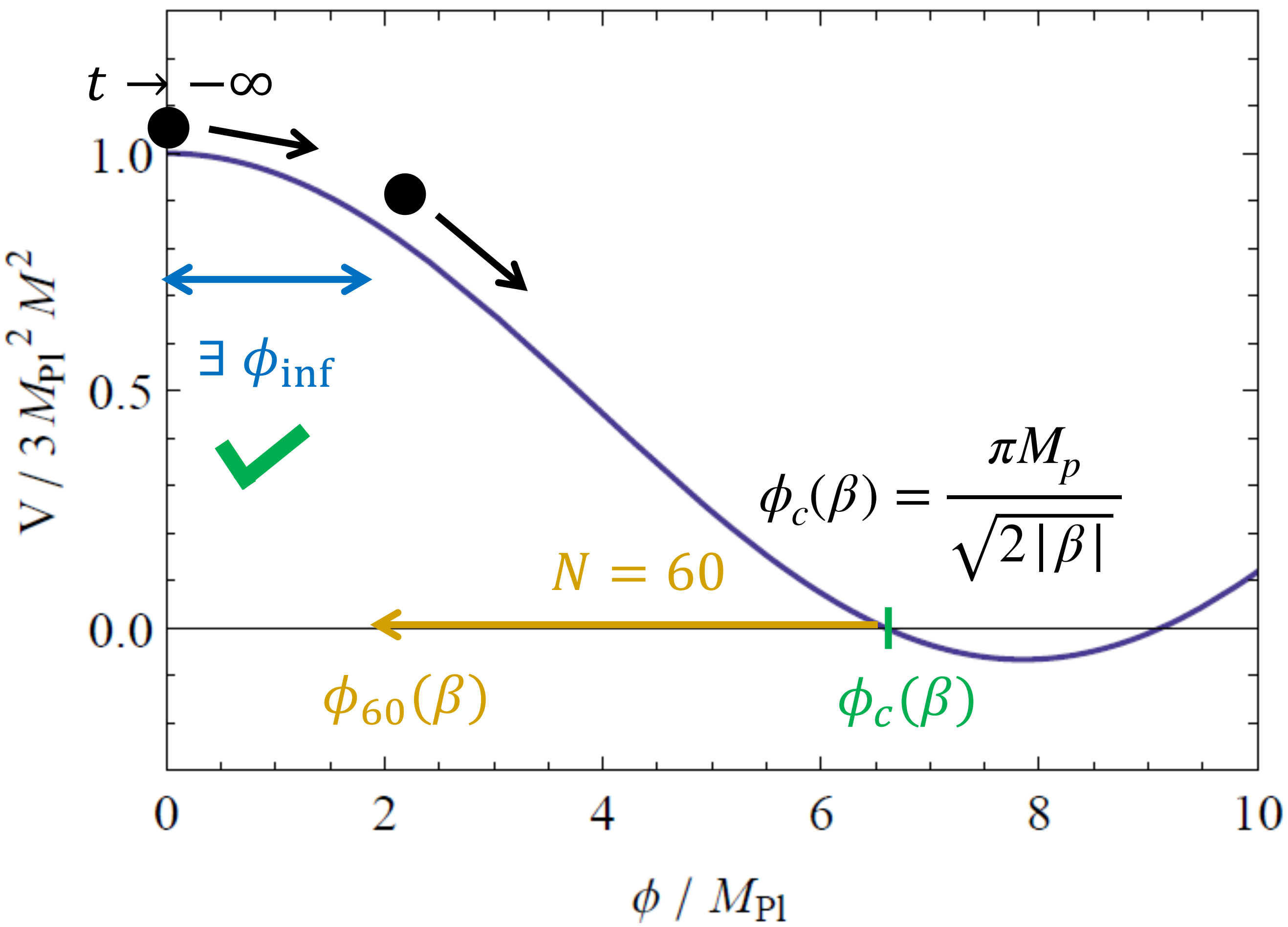






Natural inflation + negative CC

$$V(\phi) = 3M^2 M_p^2 \left[1 - \frac{\beta+3}{6}\left(1 - \cos\left(\sqrt{2|\beta|}\frac{\phi}{M_p}\right)\right)\right]$$



$$N = \int_\phi^{\phi_0} \frac{V}{V'} d\phi = \frac{3}{\beta(\beta+3)} \ln\left(\frac{\phi_0}{\phi}\right) \approx 50 \ln\left(\frac{\phi_0}{\phi}\right) .$$

β **has to be small for any reasonable value of** ϕ_0.

Scalar and tensor pert.

Curvature perturbation ζ_k relates to the metric perturbation as $\delta g_{ij} = a^2(1-2\zeta)\delta_{ij}$,

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0$$

where $v_k = \sqrt{2} z \zeta_k$, **with** $z = a\sqrt{\epsilon_1}$, $\frac{z''}{z} = a^2 H^2 \left(2 - \epsilon_1 + \frac{3}{2}\epsilon_2 + \frac{1}{4}\epsilon_2^2 - \frac{1}{2}\epsilon_1\epsilon_2 + \frac{1}{2}\epsilon_2\epsilon_3\right)$,

$$\epsilon_1 \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2H^2}, \qquad \epsilon_{n+1} \equiv \frac{\dot{\epsilon}_n}{H\epsilon_n}.$$

On the sub-horizon limit $k^2 \gg z''/z$, **we obtain** $\epsilon_1 = \epsilon_{2n+1} \to 0$ **and** $\epsilon_{2n} \to 2\beta$

$$\frac{z''}{z} \simeq \frac{\nu^2 - 1/4}{\tau^2}$$ **with** $\nu = \left|\beta + \frac{3}{2}\right|$ → $v_k(\tau) = \frac{\sqrt{-\pi\tau}}{2} H_\nu^{(1)}(-k\tau)$.

The power spectrum of the curvature perturbation:

$$\Delta_S^2(k) \equiv \frac{k^3}{2\pi^2}|\zeta_k|^2 = \frac{H^2}{8\pi^2 M_p^2 \epsilon_1} \frac{2^{2\nu-1}|\Gamma(\nu)|^2}{\pi} \left(\frac{k}{aH}\right)^{3-2\nu},$$

$n_S - 1 = 3 - 2\nu$ → $\beta = \frac{n_S - 7}{2}$ **or** $\frac{1 - n_S}{2}$.

Scalar and tensor pert.

For the tensor perturbation $\delta g_{ij} = a^2 h_{ij}$,

$$u''_{k,\lambda} + \left(k^2 - \frac{a''}{a}\right) u_{k,\lambda} = 0 \qquad \textbf{with} \quad \frac{a''}{a} = (aH)^2(2-\epsilon_1) \simeq \frac{2+3\epsilon_1}{\tau^2} \simeq \frac{2}{\tau^2},$$

where $u_{k,\lambda} = a h_{k,\lambda}/2$ **and** $\lambda = +\,,\times$ **the two polarization modes of GWs.**

We obtain

$$\Delta_T^2(k) \equiv \frac{k^3}{\pi^2} |\, h_{k,\lambda} |^2 = \frac{2H^2}{\pi^2 M_p^2}$$

and the tensor-to-scalar ratio reads

$$r \equiv \frac{\Delta_T}{\Delta_S} \approx 16\epsilon_1 = 32 M_p^2 \left(\frac{d\ln H(\phi)}{d\phi}\right)^2 \approx 8 M_p^2 \left(\frac{d\ln V(\phi)}{d\phi}\right)^2 .$$

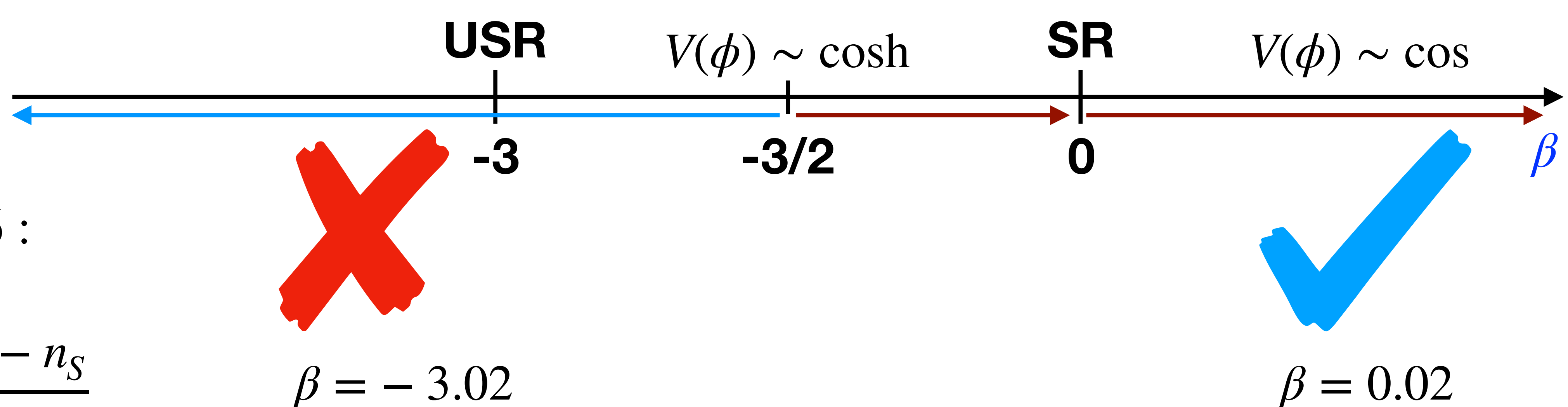
Super-horizon evolution

On the super-horizon limit $k^2 \ll z''/z\,,$

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0 \qquad \textbf{with} \qquad v_k = \sqrt{2} z \zeta_k$$

$$\zeta_k = A_k + B_k \int \frac{dt}{a^3 \epsilon_1}\,,$$

1. **Slow-roll inflation with** $\epsilon_1 \simeq$ **const.** $\ll 1$ **;** **decaying mode**
2. **Ultra slow-roll inflation with** $\epsilon_1 \sim a^{-6}$ **;** **growing mode**
3. **Constant-roll inflation with** $\epsilon_1 \sim a^{2\beta}$ **;** ***both***



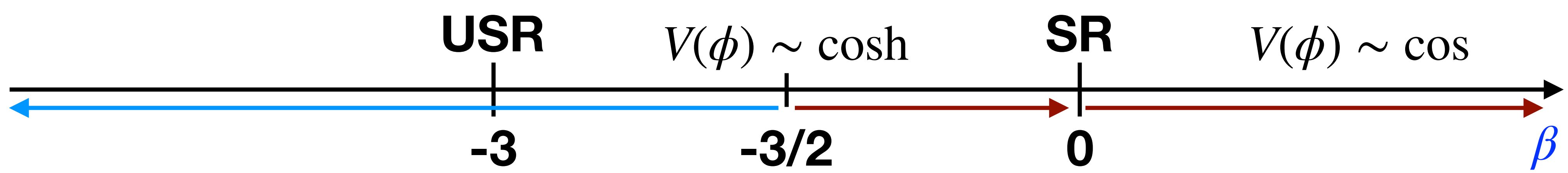
For example, $n_S = 0.96$:

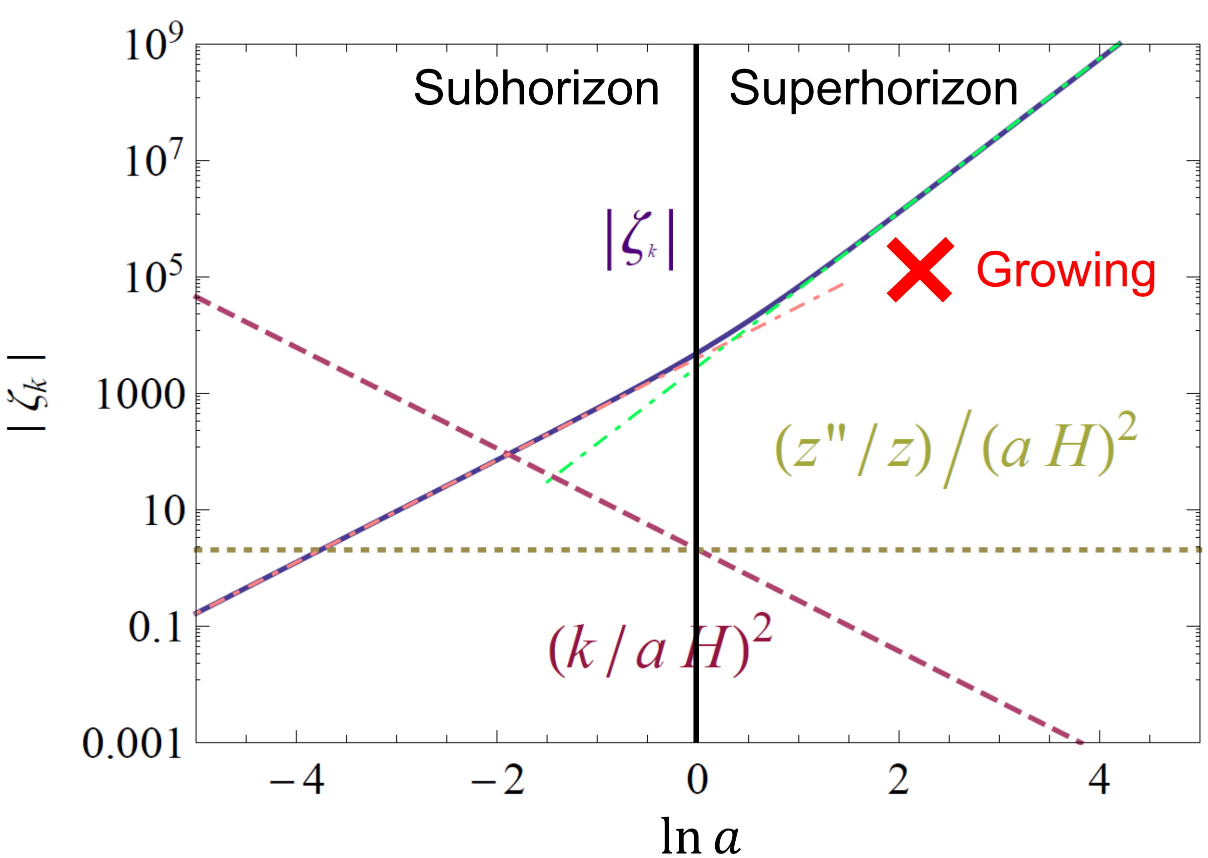
$$\beta = \frac{n_S - 7}{2} \quad \textbf{or} \quad \frac{1 - n_S}{2}$$

ζ_k – **is conserved on super-horizon scales.**

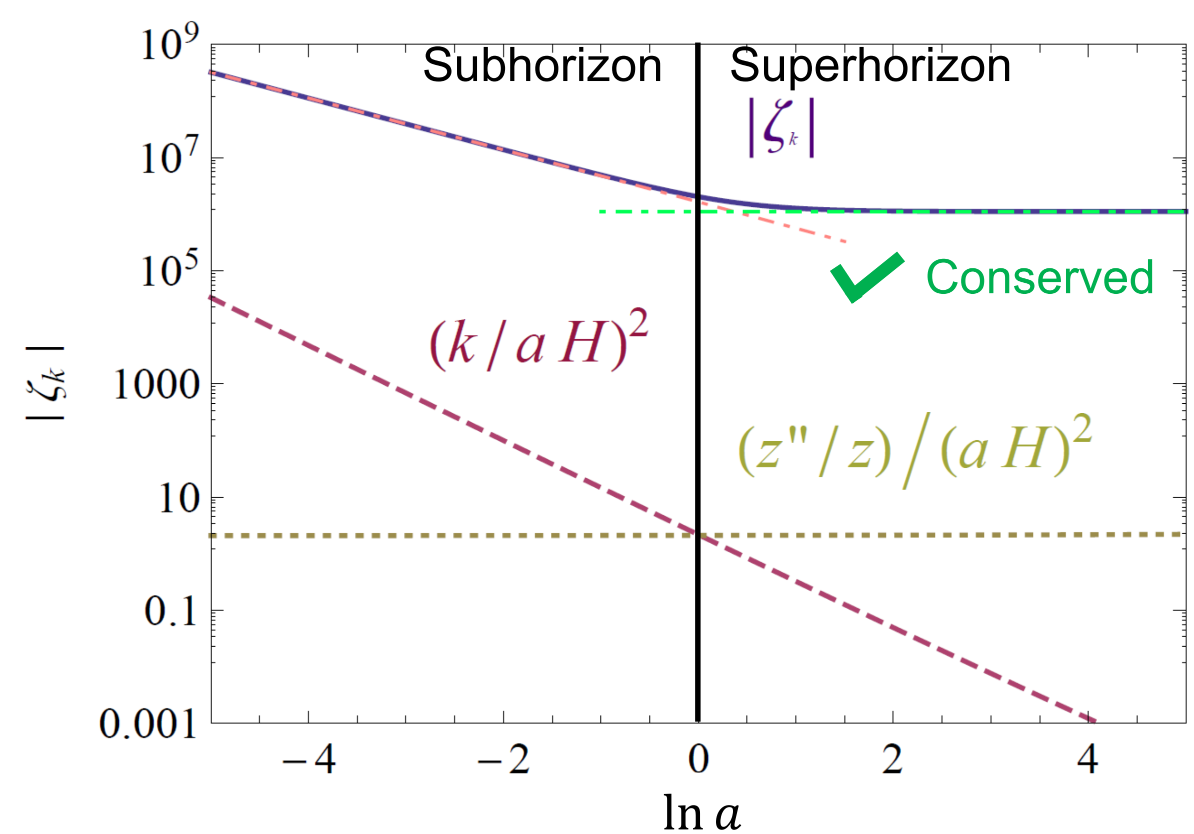
Super-horizon evolution

$$\zeta_k = A_k + B_k \int \frac{dt}{a^{-3}\epsilon_1},$$





$V(\phi) \sim \cosh$ **with** $\beta = -3.02$



$V(\phi) \sim \cos$ **with** $\beta = 0.02$

Conclusion

- Exact solutions satisfying the constant-roll condition has been found,
- Power-law inflation and particular (and somehow modified) cases of hilltop inflation and natural inflation,
- It seems difficult for the model with $\beta < 0$ to explain the observed universe due to the super-horizon evolution of the curvature perturbation,
- On the other hand, the constant-roll inflation model with $\beta > 0$ has desired constant + decaying mode on super-horizon scale,
- Therefore, the model with $\beta > 0$ is observationally viable inflationary model with constant rate of roll, which possesses an attractor background evolution, slightly red-tilted scalar spectrum, and conservation of the curvature perturbation on super-horizon scales.
- However, for a realistic model, we have to cut the potential before it becomes negative…