

OPTIMASS

: A Package for the Minimization
of Kinematic Mass Functions with
Constraints

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ref1) arXiv:1508.00589

ref2) <http://hep-pulgrim.ibs.re.kr/optimass>

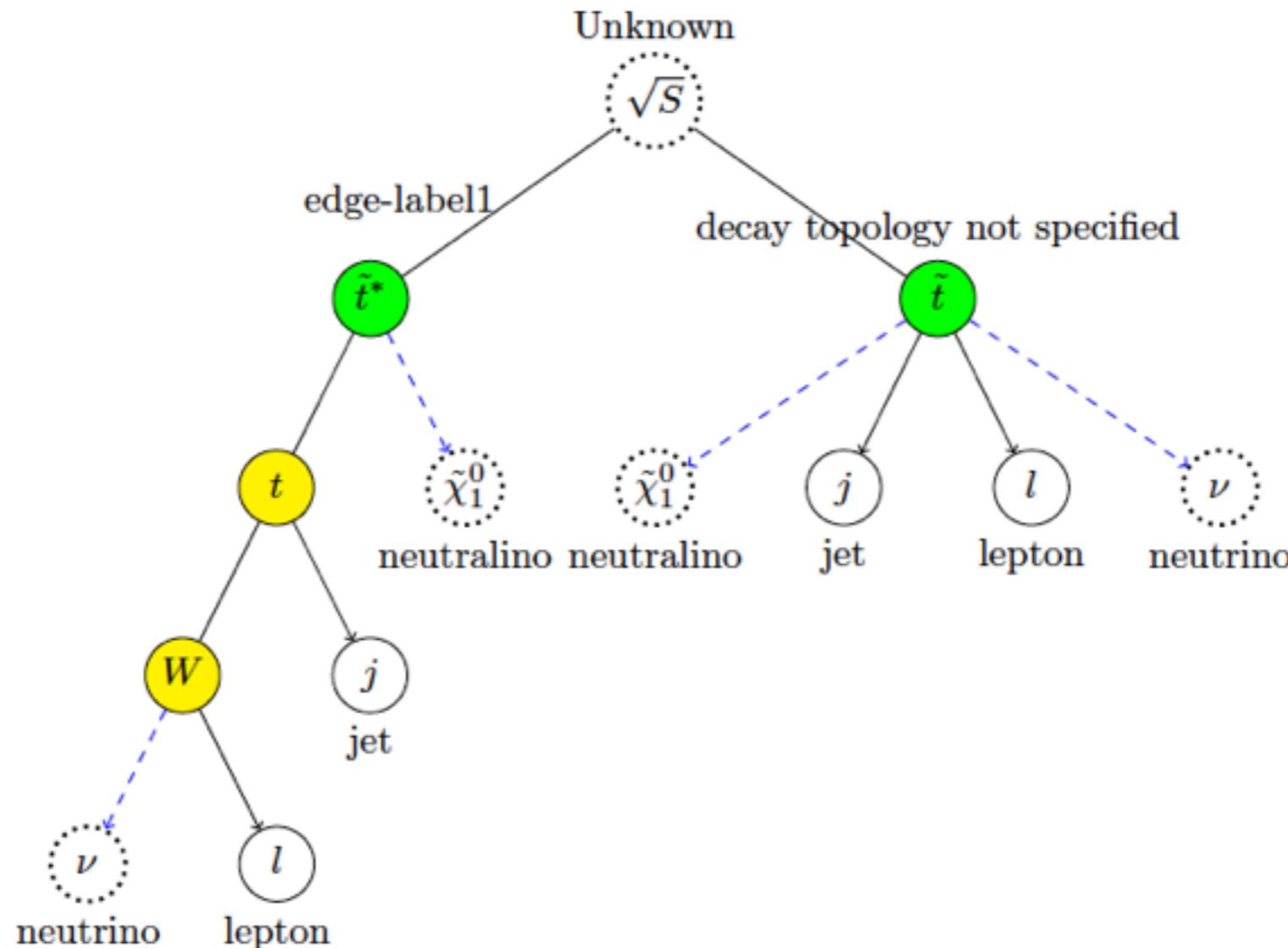
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CTPU Workshop

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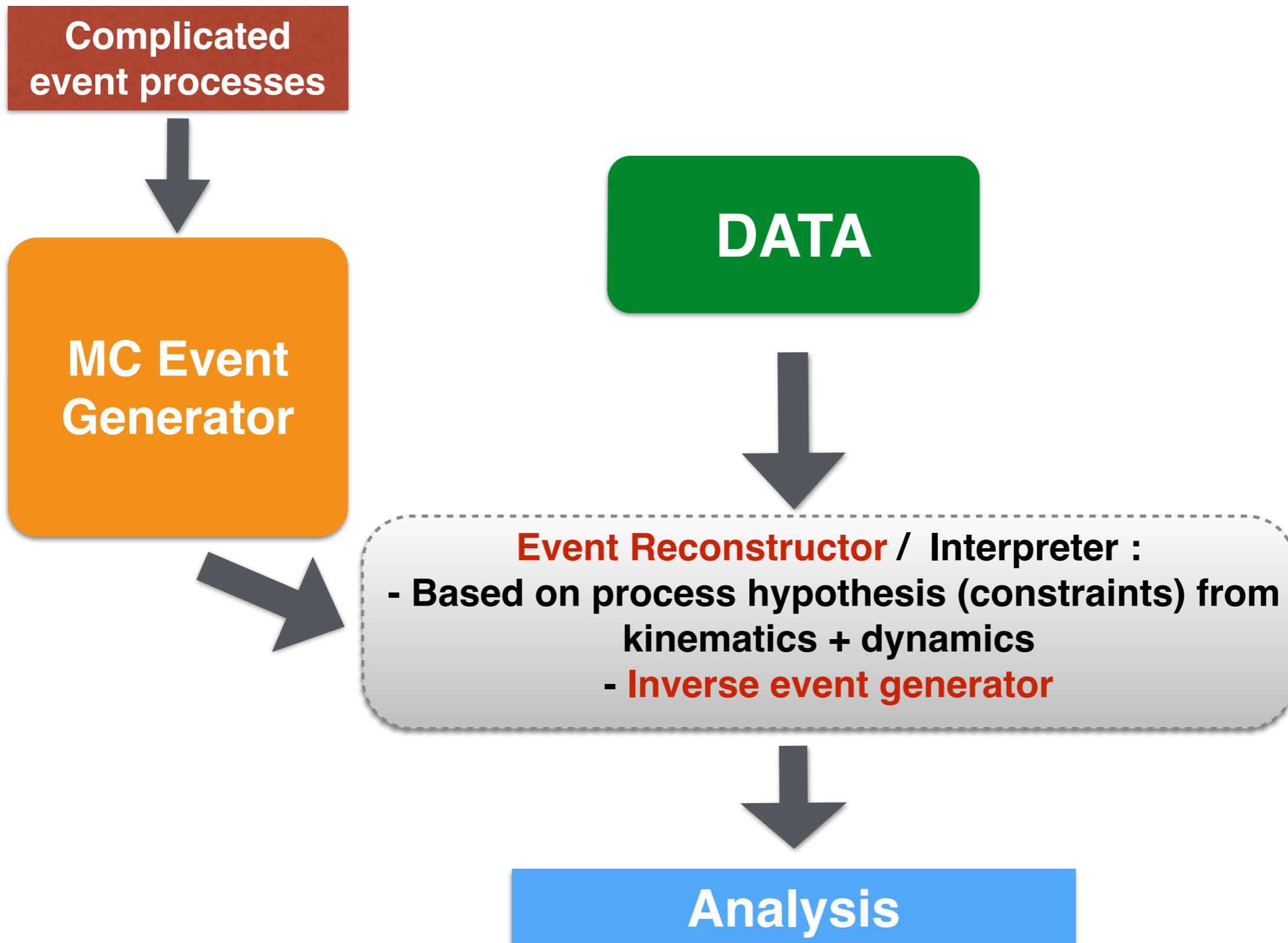
Motivation : EVENT RECONSTRUCTION

- For complex decay topologies with multiple invisible particles :



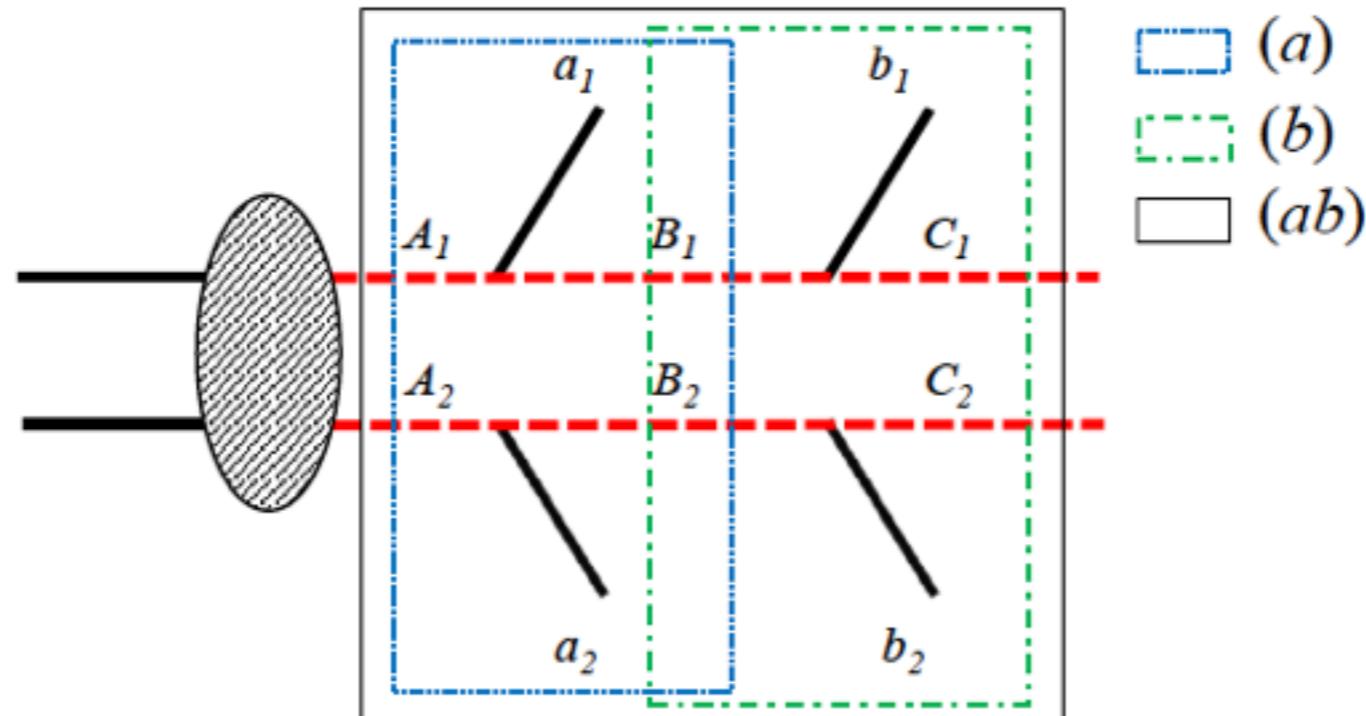
- Reconstruction of decay topology, given a general signature (n -leptons + m -jets + MET)
- Reconstruction of missing momenta
- Reconstruction of (invariant) mass variables

EVENT RECONSTRUCTION for Particle Physics



- Mass / Event reconstruction via the minimization of mass functions over unknown degrees of freedom
- Example1) MAOS momentum using MT2 minimization + OS-Mass relation
 - Phys.Rev.D79(2009)031701 [0810.4853] : WSC, K.Chi, Y.G.Kim, C.B.Park
 - Phys.Rev. D82 (2010) 113017 [1008.2690] : K.Chi, J.S.Lee, C.B.Park
 - Phys.Rev. D84 (2011) 096001 [1106.6087] : C.B.Park
 - JHEP 1111 (2011) 117 [1109.2201] : K.Chi, D.Guadagnoli, C.B.Park

- Example2) Constrained-M2 variable for ttbar-dileptonic decay chain
 - JHEP 1408 (2014)070 [1401.1449] : WSC, J.Gainer, D.Kim, K.Matchev, F.Moortgat, L.Pape, M.Park



- Power of constrained minimisation (I) : enhanced event saturation to the target mass scale to be measured

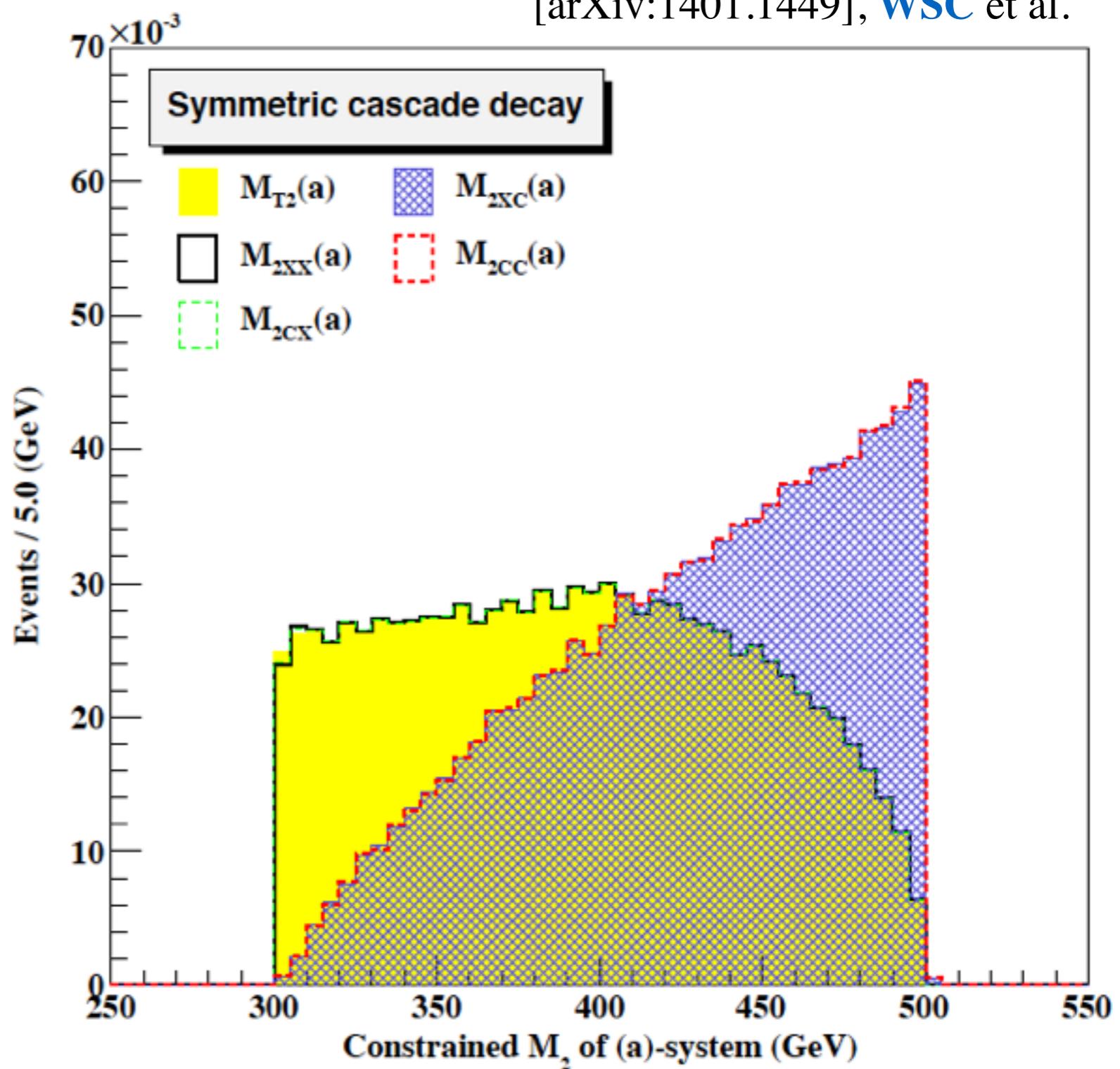
$$M_{2CC} \equiv \min_{\vec{q}_1, \vec{q}_2} \{ \max [M_{P_1}(\vec{q}_1, \tilde{m}), M_{P_2}(\vec{q}_2, \tilde{m})] \}$$

[arXiv:1401.1449], WSC et al.

$$\vec{q}_{1T} + \vec{q}_{2T} = \vec{p}_T$$

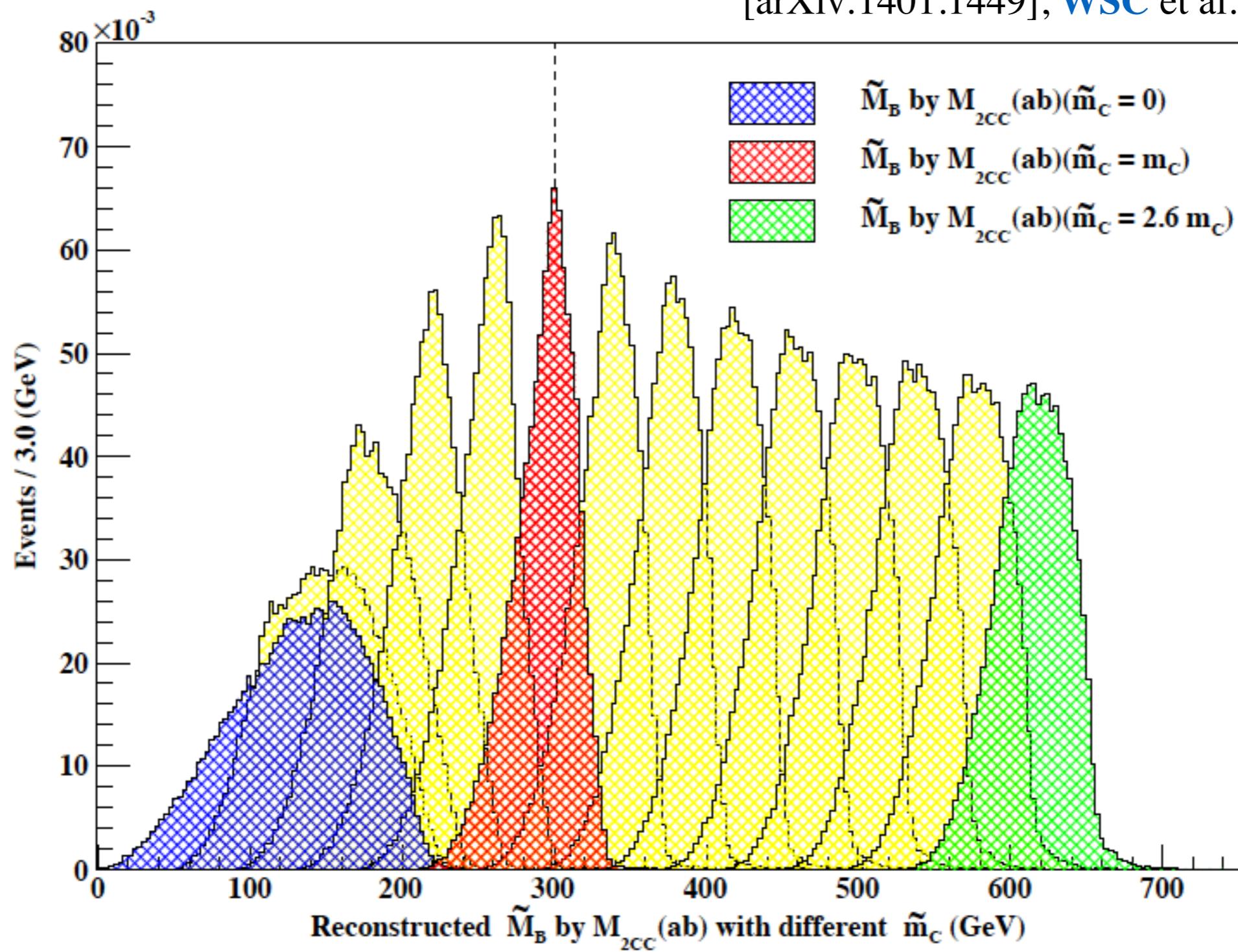
$$M_{P_1} = M_{P_2}$$

$$M_{R_1}^2 = M_{R_2}^2$$



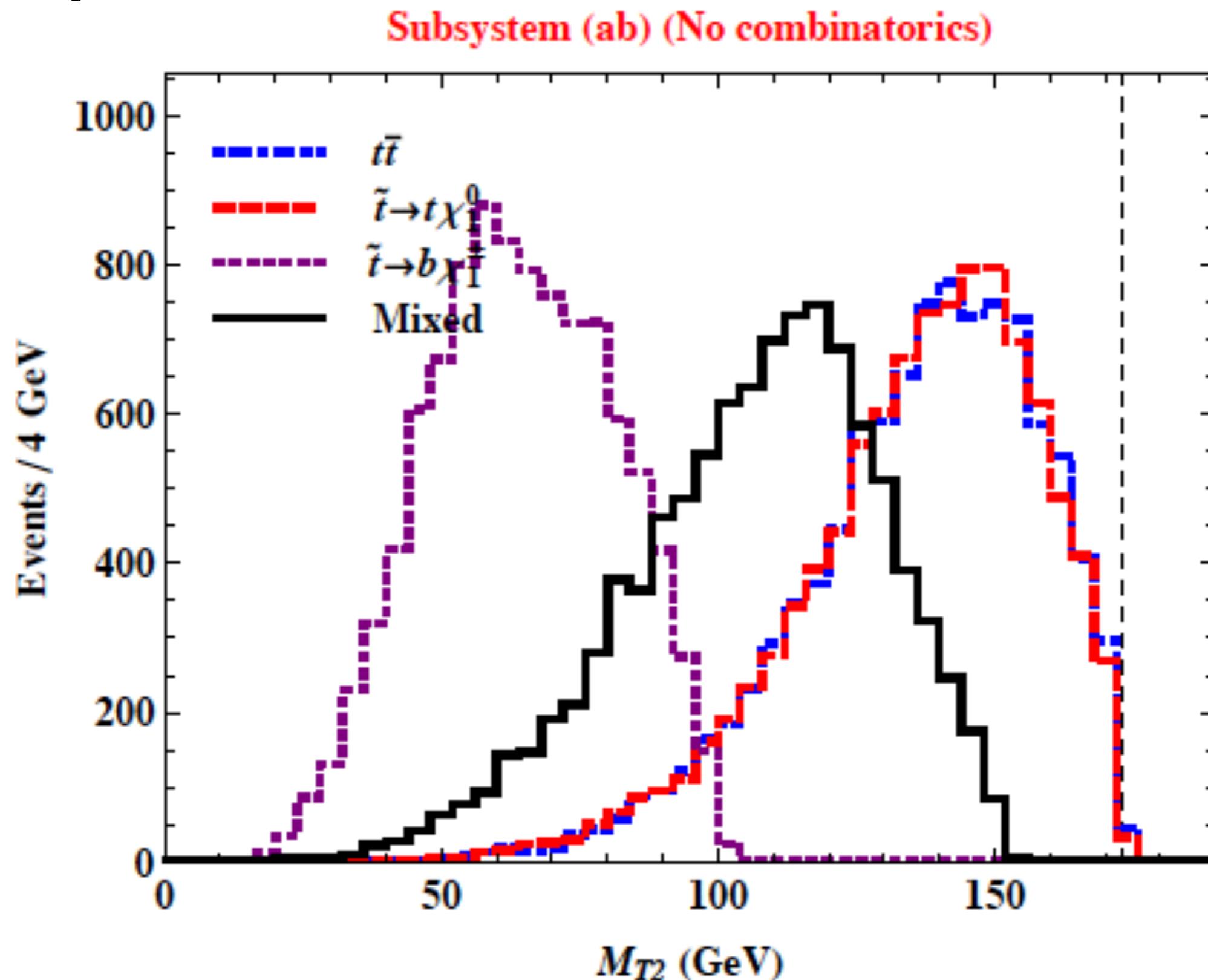
- Power of constrained minimisation (II) : mass-peak singularity (by true solution) can be restored and utilised for mass measurement, due to the restricted phase space by constraints.

[arXiv:1401.1449], WSC et al.



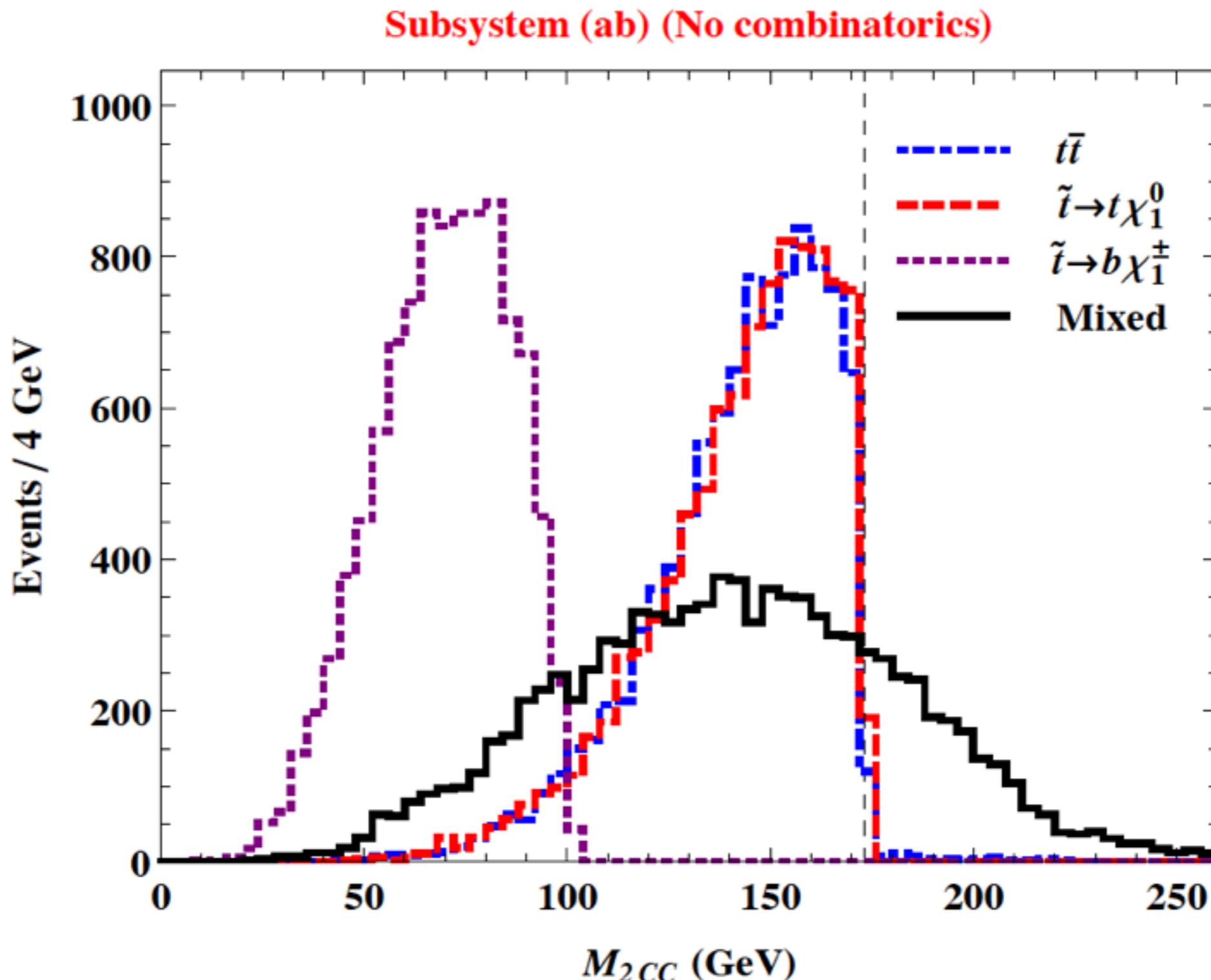
- power of constrained minimisation for signal discovery (ex: MT2 vs M2CC)

- JHEP 1505 (2015) 040 [1411.0664] D.Kim et al, on ‘Violation of the $t\bar{t}$ endpoint by stop events’



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Problem of Constrained Minimization

- of **objective functions** (>> **Sung Hak Lim's talk today!**) of **mother particle masses** :

$\tilde{M}(p, q)$ / . p: visible, q: invisible four momenta

- w.r.t **invisible momentum d.o.f** : q
- **subject to constraint functions** : $c_i(p, q)$ **involved with On-Shell / endpoint relations**

$$\bar{M} = \min_{q \in R^n} \tilde{M}(p, q) \quad \text{subject to} \quad c_{i=1..m}(p, q) = 0$$

- **For example) MT2**

- => $\tilde{M}^2 \equiv \max [(p_1 + q_1)^2, (p_2 + q_2)^2]$
- => subject to a **minimal MET constraint**.

Problem of Constrained Minimization

- **Analytically**, in principle, we can chase solutions using the method of Lagrange multipliers. However, we easily encounter usual cases where analytical approach is not effective.

$$\nabla L(x, \lambda) = 0$$

- **Numerically**, the solution is hard to be obtained by simple minimization of Lagrange function (x, λ) toward a local minimum, because the solution is extremum in (x, λ) , not stable in general.

Numerical Algorithm

- **Augmented Lagrange Method**
 - Modify the problem !
 - Constrained minimisation (in x , λ) TO Series of Unconstrained minimisation (in x),
 - => while the constraint conditions are realised by the convexification by penalty-term
 - => simultaneously, the Lagrange multipliers get updated and evolved iteration by iteration.
 - Augmented Lagrangian with the penalty parameter (μ) and augmented Lagrange parameter (λ)

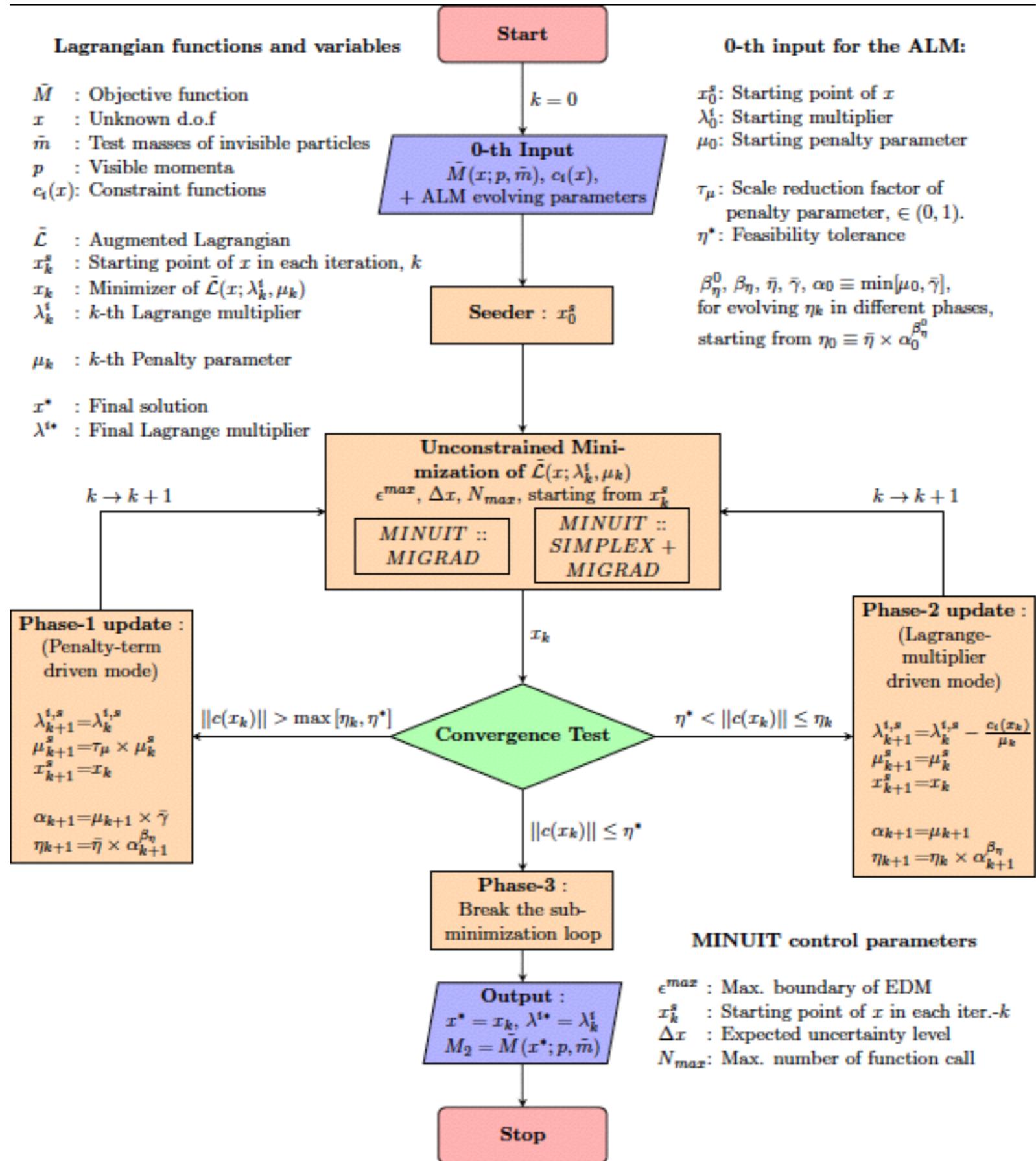
$$\tilde{\mathcal{L}}(\vec{x}; \lambda, \mu) \equiv f(\vec{x}) - \sum_a \lambda_a c_a(\vec{x}) + \frac{1}{2\mu} \sum_a c_a^2(\vec{x})$$

$$\lambda_a^{k+1} = \lambda_a^k - \frac{c_a(\vec{x}_k)}{\mu_k}$$

Our prescription for ALM

- ALM Loop
 - In each loop, unconstrained minimization by MINUIT
 - In each loop, solution phase check and convergence check
 - Optimality convergence
 - Feasibility convergence
 - Evolution in Phase1
 - Evolution in Phase2

Flowchart



Our prescription for ALM

- Utilize the MINUIT library for unconstrained minimization at each ALM iteration.
- MINUIT (by F. James) : Popular code of function minimization and data analysis for HEP Community
 - MIGRAD and SIMPLEX : Main minimization algorithms of MINUIT
 - MIGRAD - ‘Variable Metric Method’ - Gradient Based ‘Quasi-Newton Method’
 - SIMPLEX - One of the most popular ‘Stepping Method’

validation

- Simple example 1)

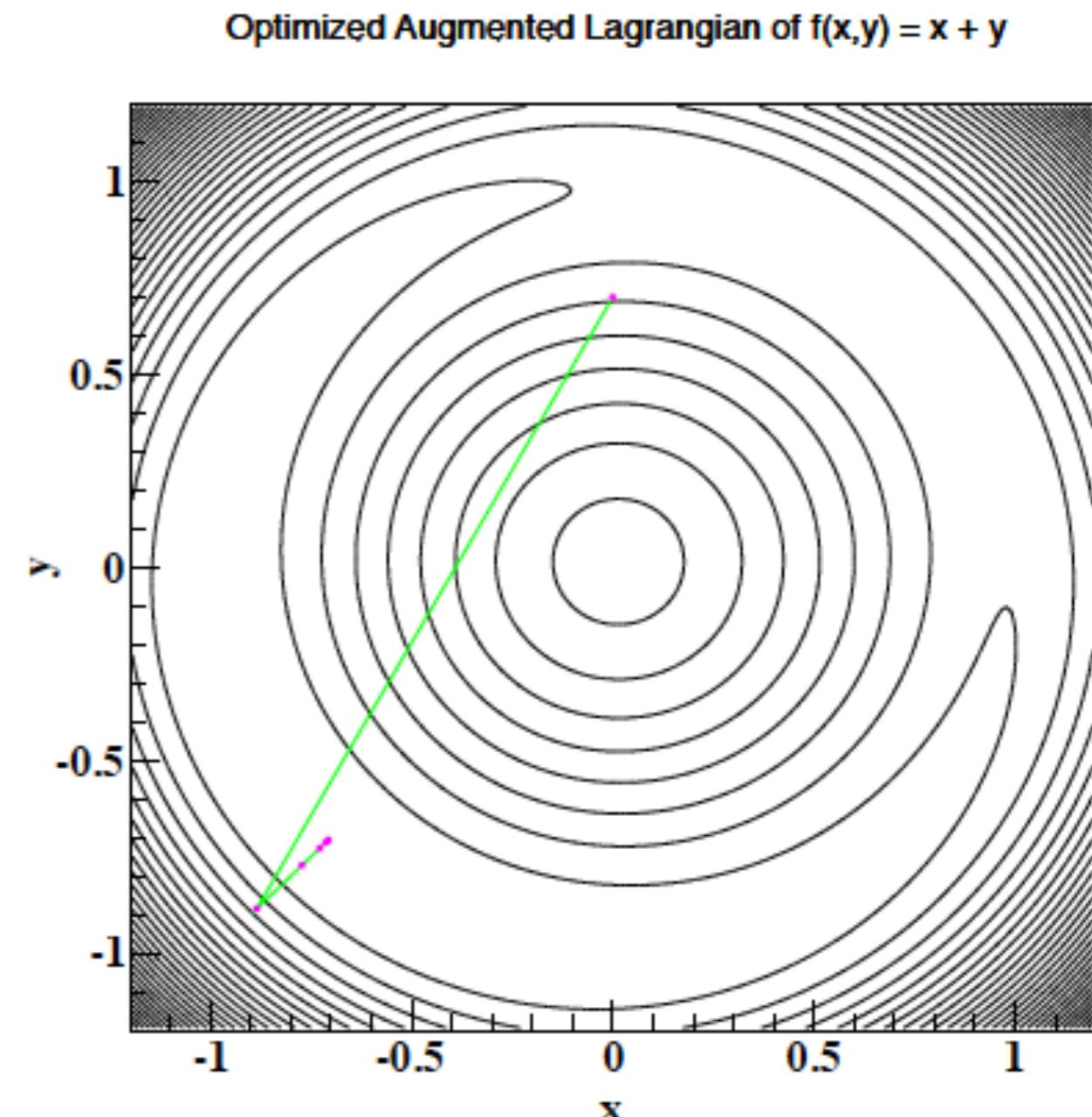
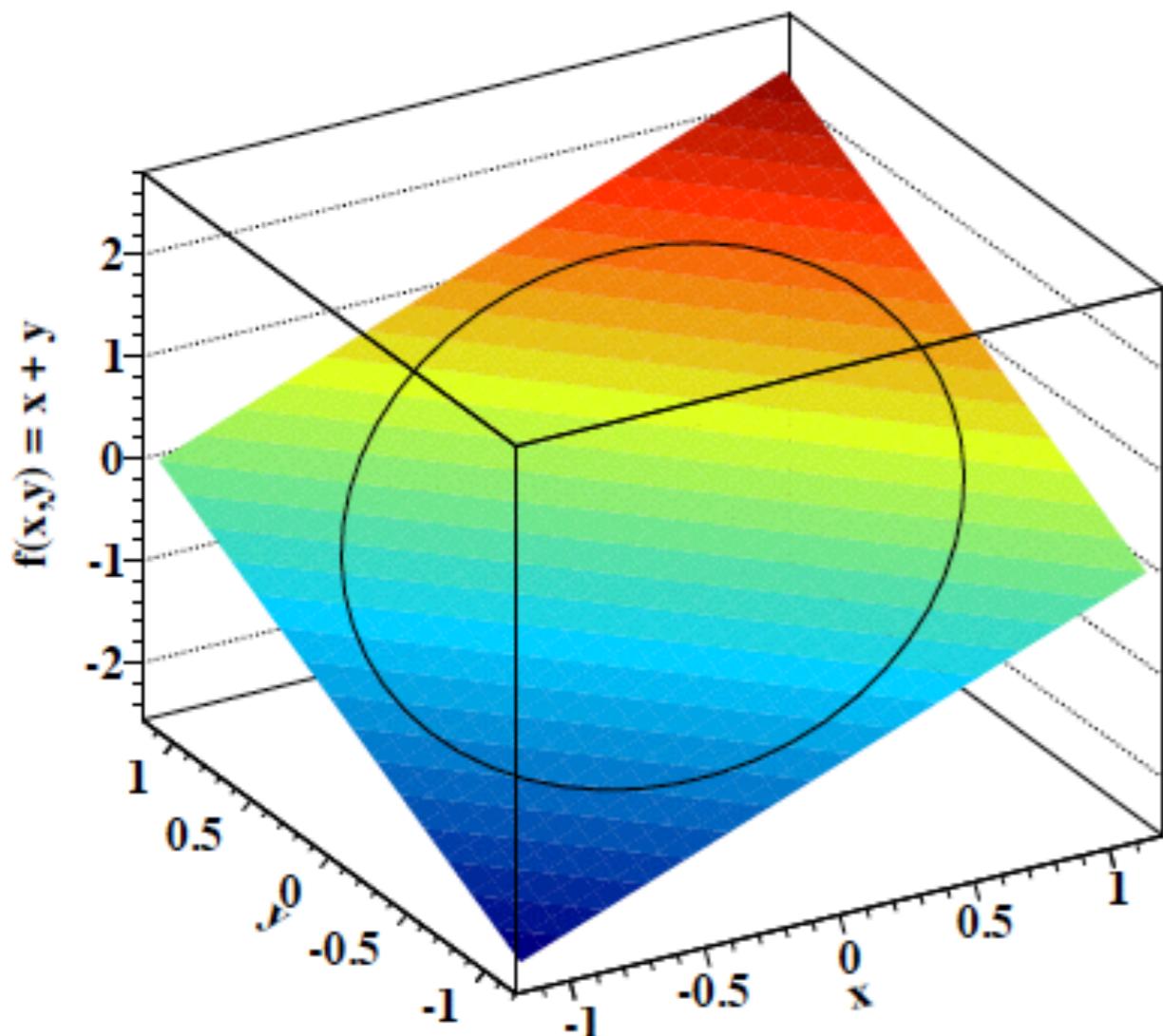


Figure 2. Test of the ALM for the objective function $f(x, y) = x + y$ subject to $x^2 + y^2 = 1$. (a) Plot of the objective function (color coded) and the constraint curve (in black). (b) Contour plot of the augmented Lagragian (3.36) for the last (fifth) iteration. The magenta points denote the minimizers, \bar{x}_k , found at each iteration.

validation

- Example 2) M2CC of ttbar dileptonic decay

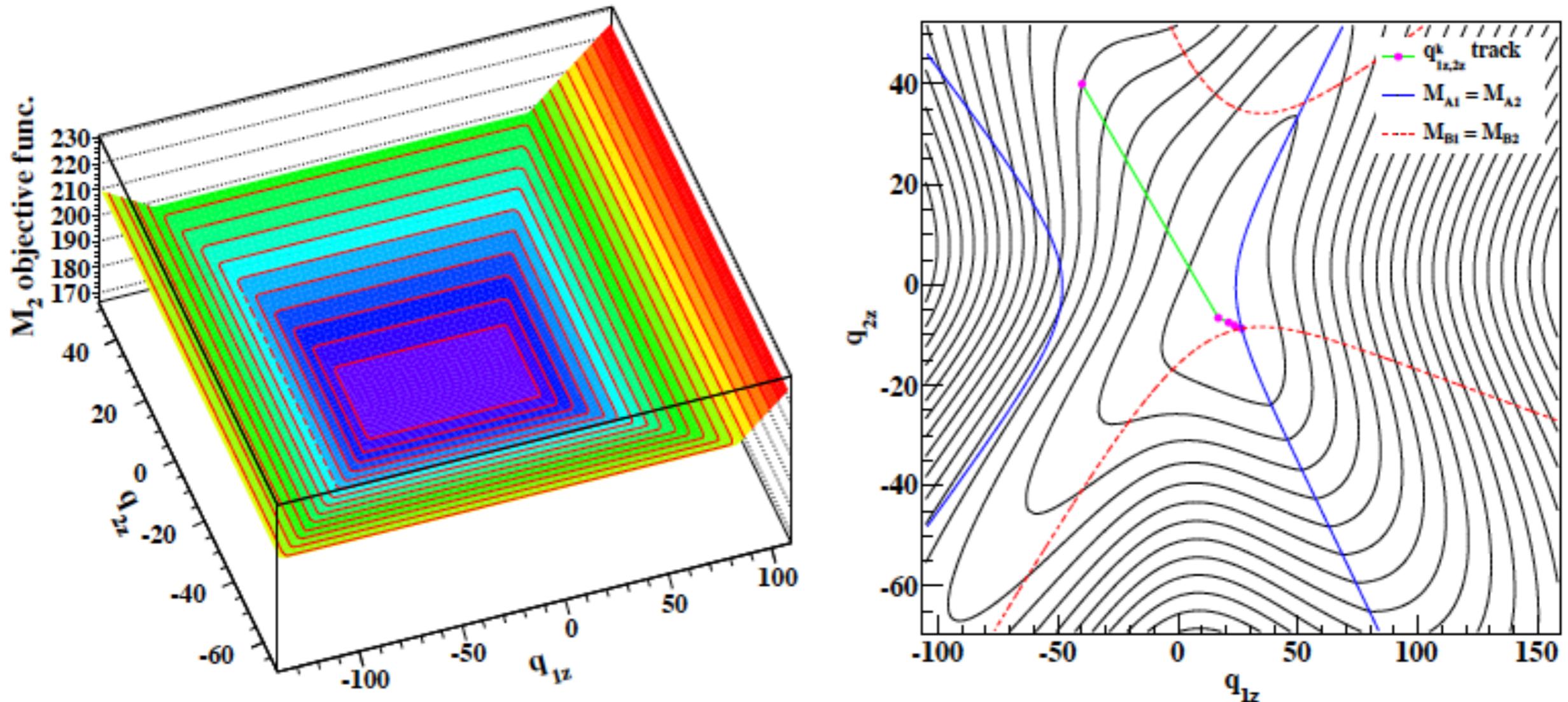


Figure 10. The same as figures 2 and 4, but for the single event considered in section 4.3. Since the objective function has four independent arguments, in order to visualize the evolution of the minimizer, we plot q_{1z} and q_{2z} , having fixed the other two variables, q_{1x} and q_{1y} , to the values which minimize the objective function for the given choice of q_{1z} and q_{2z} .

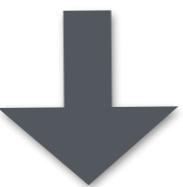
OPTIMASS-ver1.0

Released!

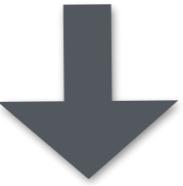
- Language : C++, Python
- Requirements : gcc(>4.4),
Python(>2.6), ROOT with MINUIT2
- Webpage (for download and installation
guide):
 - <http://hep-pulgrim.ibs.re.kr/optimass>

OPTIMASS reconstructor

DATA: $[i, j] \Rightarrow \{??\} \Rightarrow [\text{visibles}] + \{\text{invisibles}\}$



OPTIMASS with (general hypothesis
– ‘model_card.xml’ for $\{??\}$)



Physical / Unphysical
 $\{\text{invisibles}\} + \{\text{reconstructed masses}\}$

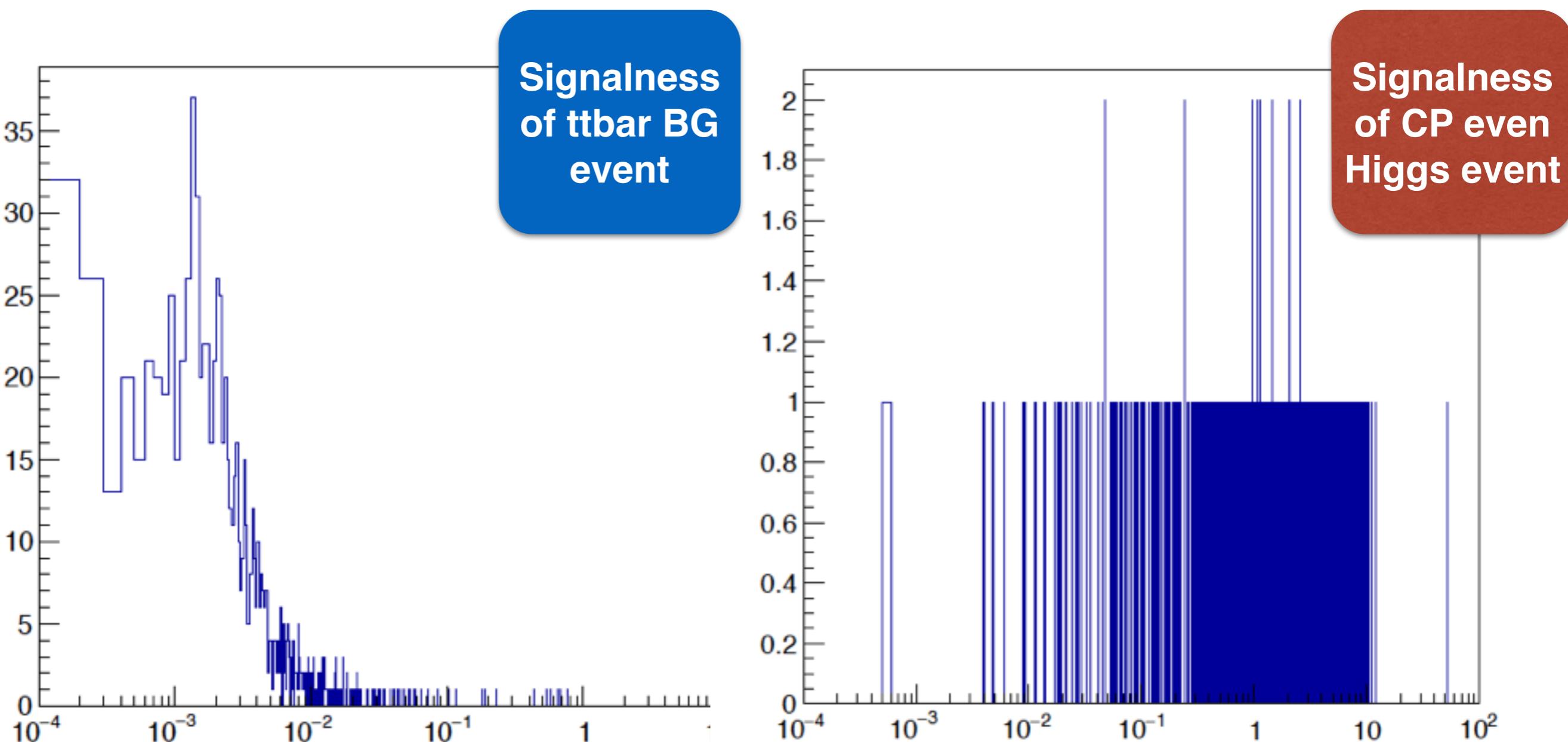
⇒ **Better discrimination power!**

An application

'Search for a Di-Higgs Resonance using OPTIMASS'

in collaboration with C.B.Park and S.H.Lim

$$H \rightarrow hh \rightarrow b\bar{b}W^+W^- \rightarrow b\bar{b} + l^+l^- + MET$$



OPTIMASS interface for user's complicated decay topology

- [FULL Decay System] define any number of decay chains, and any type of decay vertices using user's own labelling scheme!

Listing 1: Cards/ttbar-ab.xml

```
1 # XML
2 ---
3 <?xml version='1.0' encoding='utf-8'?>
4 <ProcessCard classname="TTbar_AB" debug="false" version="1.0">
5     <!-- ===== -->
6     <!-- Define event decay chain -->
7     <!-- ===== -->
8     <DecayChains>
9         <DecayChain>
10            t1 - b1 w1 , w1 - e1 v1
11        </DecayChain>
12        <DecayChain>
13            t2 - b2 w2 , w2 - e2 v2
14        </DecayChain>
15    </DecayChains>
```

- [Subsystem-Mothers] define your subsystem's head nodes easily just by listing the names (intermediate) mother particles defined in the full decay system!

```

16    <!-- ===== -->
17    <!-- Mother node particle in each decay chain to define objective function -->
18    <!-- ===== -->
19    <ParticleMassFunction>
20        <ParticleGroup mass_function="M2" group_function="max">
21            <Particle label="t1" />
22            <Particle label="t2" />
23        </ParticleGroup>
24    </ParticleMassFunction>

```

- [Subsystem-Effective Invisibles] define the effective invisible nodes by simply tagging it in the language of the full decay system!

```

41    <ParticleProperties>
42        <Particle name="top" mass="173." />
43        <Particle name="bottom" mass="4.18" />
44        <Particle name="wboson" mass="80.419" optimize_target="True" />
45        <Particle name="electron" />
46        <Particle name="neutrino" invisible="True" />
47    </ParticleProperties>

```

- [Kinematic Constraint Functions] Using the particle names in the full decay chains, their Lorentz 4 momentum d.o.f. can freely participate to define constraint functions.

```

58      <!-- ALM Constraints Configuration -->
59      <!-- ===== -->
60      <Constraints penalty_init="1.">
61          <Constraint multiplier_init="0" type="equal">
62              w1.M() - w2.M()
63          </Constraint>
64          <Constraint multiplier_init="0" type="equal">
65              t1.M() - t2.M()
66          </Constraint>
67      </Constraints>
```

- [Combined-Events System Support] via defining PT conservation groups in the full chain list

```

48      <!-- ===== -->
49      <!-- Subchains for MET conditions -->
50      <!-- ===== -->
51      <ParticleInvisibleSubsystem>
52          <Subsystem set_value="manual" >
53              <Particle label="t1" />
54              <Particle label="t2" />
55          </Subsystem>
56      </ParticleInvisibleSubsystem>
```

OPTIMASS: From Build to Final Run in 6 steps !

- The highest level bullet = top directory of OPTIMASS
- item in BLUE : srcts which need user's input
- item in GREEN : just run the commands !
- item in RED : important directories or executable
- (D,1) alm_base : ALM CORE SRCS for building LIBRARY
(just once at the first time)
 - shell> configure; make; make install
 - => Check the ROOT env. with MINUIT2
 - => Install OptiMass library (/lib, /include)

OPTIMASS: From Build to Final Run

- (D) **model** : User's model repository
 - (D,2) **example_models** : <user model>.xml : users model files
 - (F,3) **model_card.xml** : copied from one of '<user model>.xml' files above.
 - (D) **dict_src** : <user model>.cpp / .h : output dictionary srcs for the **model_card.xml**
 - (D) **main_src** : main_<user model>.cpp : output templates for the **model_card.xml**, for main.cpp
 - (D) **model_interpreter** : python interpreters/code generators
- (F,4) **build_model_dictionary** : user's model card reader and related dictionary code generator
 - **shell> build_model_dictionary** (=> default input (**model_card.xml**) to output-srcs at **dict_src**, **main_src**)

OPTIMASS: From Build to Final Run

- (F,5) `main.cpp` : customised main event interface from the skeleton `main_<user model>.cpp`
- (F,6) `Makefile` : customised Makefile for user's `main.cpp`, to include additional personal srcs
 - `shell> make`
 - `shell> ./optimass` (\Rightarrow optimass calculation!)

Conclusion

- Lots of mass functions in a huge number of event topologies are now ready to be optimised and re-interpreted by OPTIMASS
- OPTIMASS toward/as an inverse event generator for general event topologies for near(?) future