# The muon $(g-2)$ in SM and MSSM: precision corrections and novel phenomenological scenario

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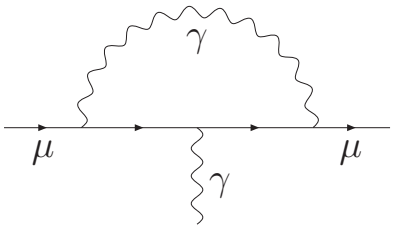
# Muon Anomalous Magnetic Moment

## Theory

$$\vec{\mu} = g\frac{-e}{m_\mu}\vec{s}$$

$$a_\mu = \frac{(g-2)_\mu}{2}$$

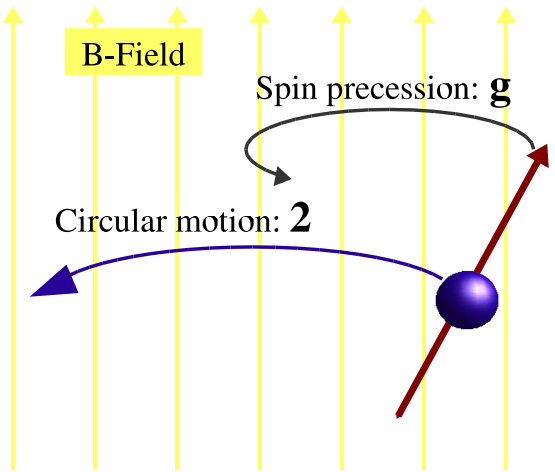
$$\Gamma_\mu = \gamma_\mu F_1(q^2) + \frac{i}{2m_\mu} a_\mu \sigma_{\mu\nu} q^\nu + \cdots$$




$$a_\mu{}^{\rm QED,1L} = \frac{\alpha}{2\pi}$$

Schwinger '48

## Experiment

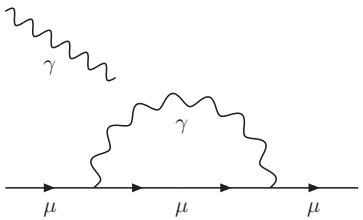



circular motion: $\omega_c$

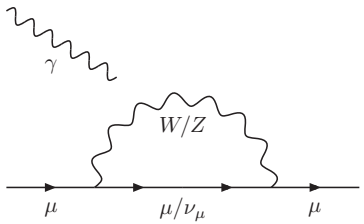
spin precession: $\omega_s$

$$\omega_s - \omega_c = a_\mu \frac{-e}{m_\mu} B$$

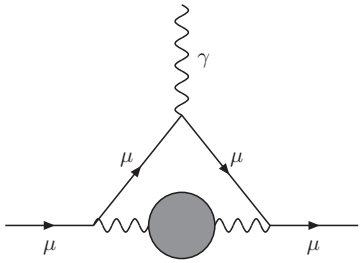
# $a_\mu^{\rm SM}$



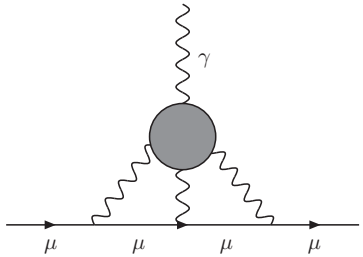

$a_\mu^{\rm QED}$




$a_\mu^{\rm EW}$




$a_\mu^{\rm Had.VP}$




$a_\mu^{\rm Had.lbl}$

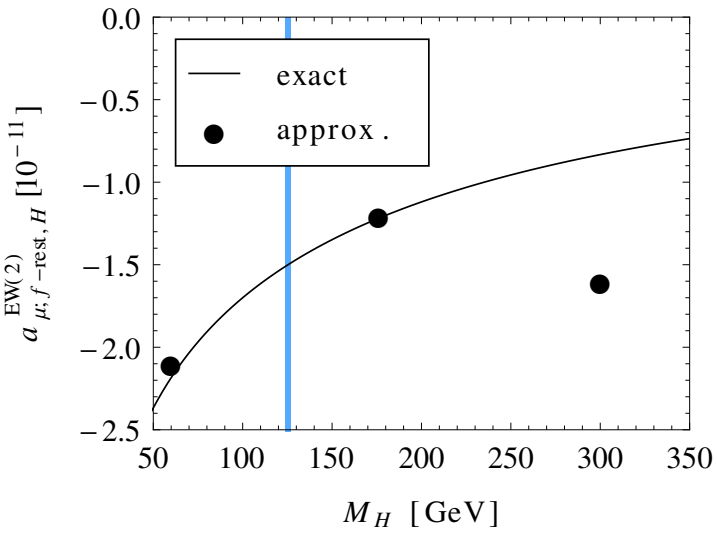
- QED 5-loop calculation completed. [Kinoshita et al '12]
- Convergence of hadronic contributions.

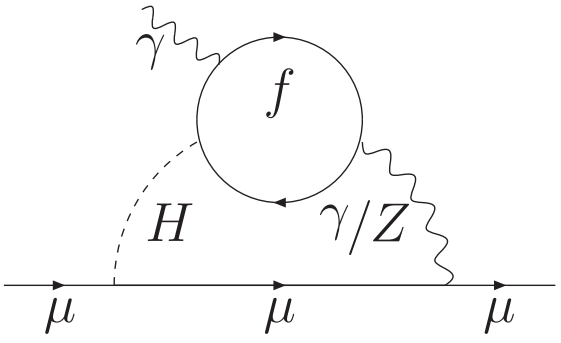
  [Davier et al; Hagiwara et al; Benayoun et al]
- New BESIII results [1507:08188]
- Precision improvement in $a_\mu^{\rm EW}$ [Gnendiger, Stöckinger, S-K '13]

  : uncertainty caused by $M_{\rm H}$ eliminated $\Rightarrow$

# $a_\mu^{\rm EW}$ after $M_{\rm H}$ measurement

- The only contribution depending on $M_{\rm H}$
- Estimation before $M_{\rm H}$ measurement:
  $a_\mu^{\rm EW} = (154 \pm 1 \pm 2) \times 10^{-11}$ [Czarnecki, Krause, Marciano]
- Electroweak (full 2-loop) contributions:
  $a_\mu^{\rm EW} = (153.6 \pm 1.0) \times 10^{-11}$
  with $M_{\rm H} = 125.6 \pm 1.5\,{\rm GeV}$
  [1306.5546][Phys. Rev. D 88 (2013) 053005][Gnendiger, Stöckinger, S-K '13]



# $a_\mu^{\rm Exp}$

- The latest result from BNL [Bennett et al. '06]
  $a_\mu^{\rm E821} = (11659208.9 \pm 6.3) \times 10^{-10}$
- New experiment at Fermilab E989:
  0.54 $\rightarrow$ 0.14 ppm.
  Current uncertainty in $a_\mu^{\rm SM}$: 0.42 ppm

# Current status of $a_\mu^{\rm Exp} - a_\mu^{\rm SM}$

$$\Delta a_\mu({\rm E821} - {\rm SM}) = \begin{cases} (28.7 \pm 8.0) \times 10^{-10} & \text{[Davier et al.]} \\ (26.1 \pm 8.0) \times 10^{-10} & \text{[Hagiwara et al.]} \end{cases}$$

$$\boxed{\sim 3...4\,\sigma}$$

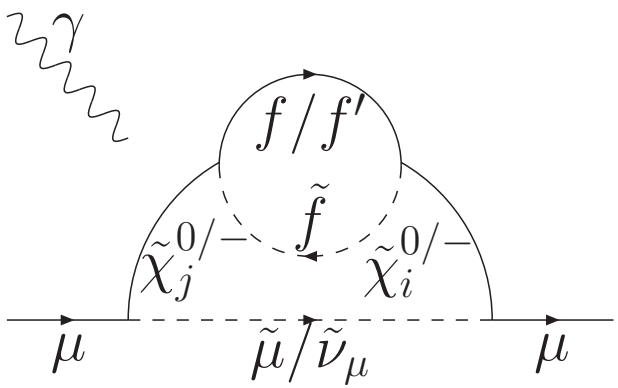
- This deviation motivates New Physics: SUSY can explain it.
- $a_\mu$ is an important constraint on SUSY
- Still, large contributions possible, e.g. if sleptons $<<$ squarks (non-traditional models)

[Endo, Hamaguchi, Iwamoto, Yanagida, D.P. Roy, et al]

# $a_\mu$ in MSSM

1. MSSM loop corrections:

$f\tilde{f}$ two-loop




2. Radiative muon mass generation:

$$\tan\beta \equiv \frac{v_u}{v_d}\Big|_{v_d=0} \to \infty$$

# MSSM loop corrections

## 1. MSSM one-loop correction

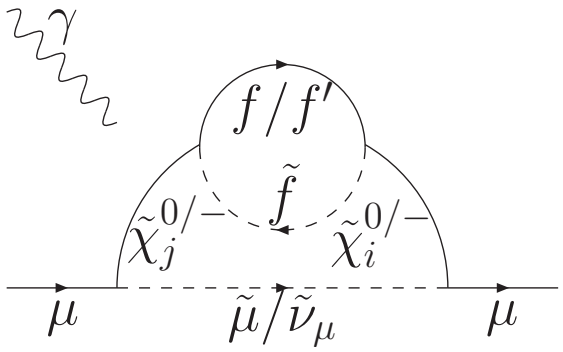
- Parameter dependence: $\mu, M_1, M_2, M_L, M_E, \tan\beta$

$$a_\mu^{\rm SUSY,1L} \approx 13 \times 10^{-10}\, {\rm sgn}(\mu) \tan\beta \left(\frac{100\,{\rm GeV}}{M_{\rm SUSY}}\right)^2$$

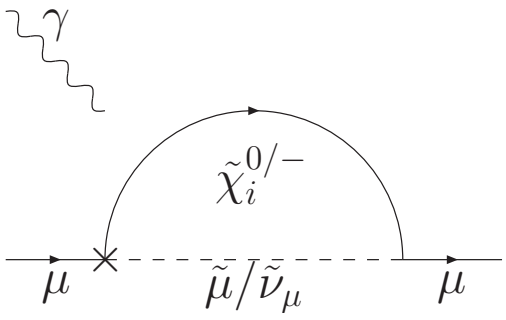
## 2. $f\tilde{f}$ two-loop corrections

[Fargnoli, Gnendiger, Paßehr, Stöckinger, S-K '13]

- Maximum complexity:
  5 heavy scales + 2 light scales
- Additional parameter dependence:
  $M_{Q_i}, M_{U_i}, M_{D_i}, M_{L_i}, M_{E_i},\ i \in \{1,2,3\}$
- solve the one-loop ambiguity caused by $\alpha$ $\Rightarrow$
- Non-decoupling behaviour $\Rightarrow$



# $\alpha$ ambiguity solved




$$= a_\mu^{1\rm L} \times \left( \frac{\delta(e^2/s_W^2)}{e^2/s_W^2} + \ldots \right)$$
$$= a_\mu^{1\rm L} \times \left( \Delta\alpha_{f,\tilde{f}} - \frac{c_W^2}{s_W^2} \Delta\rho_{f,\tilde{f}} + \ldots \right)$$

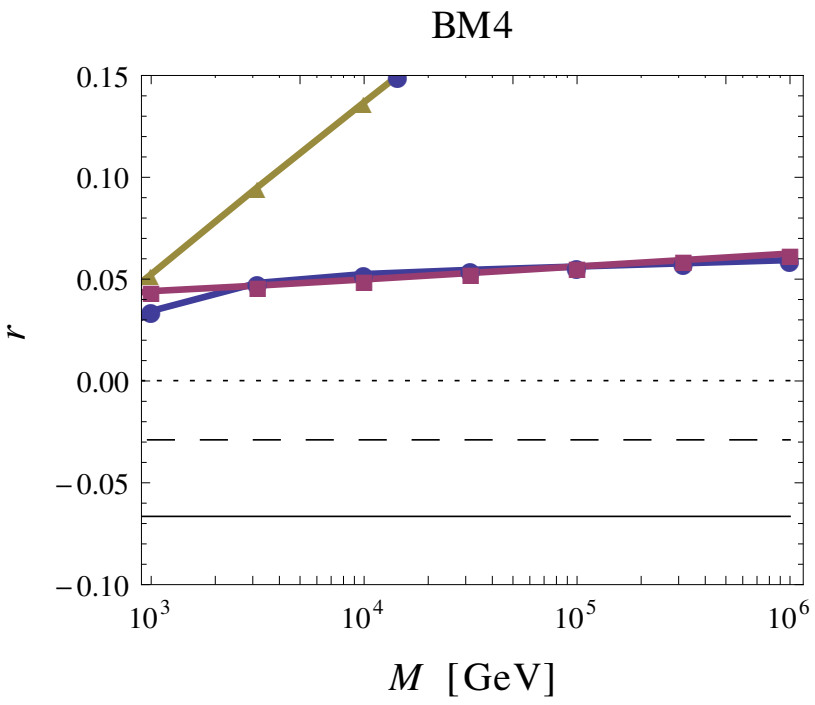
1-loop ambiguity fixed by full 2-L $f\tilde{f}$

$$a_\mu^{1\rm L}|\alpha(0) = 29.4 \times 10^{-10}$$
$$a_\mu^{1\rm L}|\alpha(M_Z) = 31.6 \times 10^{-10}$$
$$a_\mu^{1\rm L}|\alpha(G_F) = 30.5 \times 10^{-10}$$

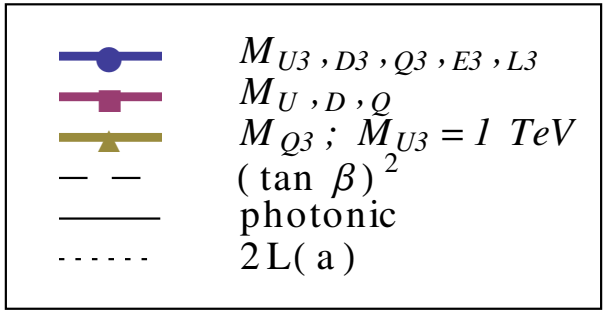
$$\Longrightarrow a_\mu^{1\rm L+2L f\tilde{f}} = 32.2 \times 10^{-10}$$

Difference from $\Delta\alpha$ and $\Delta\rho$: 2-loop $f\tilde{f}$ terms.

# $f\tilde{f}$-loop corrections: Non-decoupling behaviour




- $M_2, m_{\tilde{\mu}_L} >> M_1, m_{\tilde{\mu}_R}$
- $M_1 = 140\text{GeV}$
- $m_{\tilde{\mu}_R} = 200\text{GeV}$
- $M_2 = m_{\tilde{\mu}_L} = 2000\text{GeV}$
- $\mu = -160,\ \tan\beta = 40$
- $\mathcal{O}(10...30\%)$



# MSSM: Radiative muon mass generation

$$\tan\beta \to \infty$$

Novel phenomenological approach

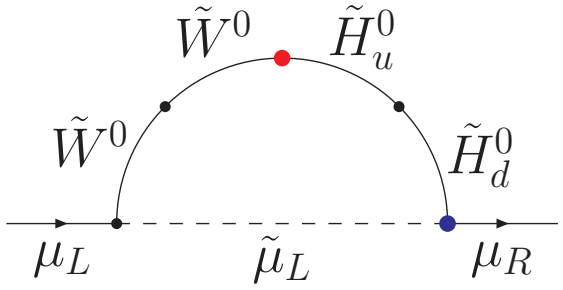
# Radiative muon mass generation: $\tan\beta \to \infty$

$$m_\mu^{\rm tree} = y_\mu v_d \quad \Rightarrow \quad 0$$

- $v_d \to 0$, $\tan\beta \equiv \frac{v_u}{v_d} \to \infty$,
  $m_\mu$ generated via coupling to $v_u$

  [Dobrescu, Fox '10][Altmannshofer, Straub '10]

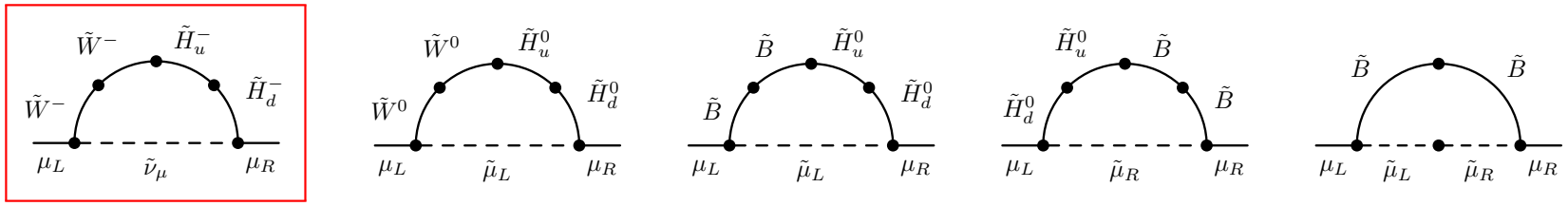
- $m_\mu \equiv \cancel{y_\mu v_d} + y_\mu v_u \Delta_\mu^{\rm red}$:
  $y_\mu$ obtained from one-loop self energy.
- $a_\mu^{\rm SUSY} = \frac{y_\mu v_u}{m_\mu} a_\mu^{\rm red}$
- $a_\mu^{\rm SUSY} \propto y_\mu$ and $m_\mu \propto y_\mu$




$$\Rightarrow \quad a_\mu^{\rm SUSY} = \frac{a_\mu^{\rm red}}{\Delta_\mu^{\rm red}}$$

[1504.05500][Bach, Park, Stöckinger,S-K]

# The limit of $\tan\beta \to \infty$



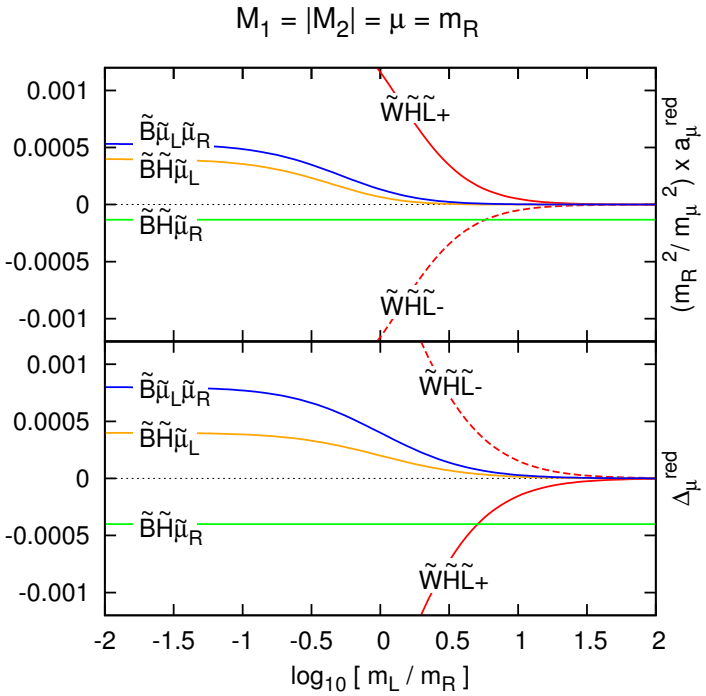

$$
\begin{aligned}
a_\mu^{\rm red} &= a_\mu^{\rm red}(\tilde{W}\tilde{H}\tilde{\nu}) + a_\mu^{\rm red}(\tilde{W}\tilde{H}\tilde{\mu}_L) + a_\mu^{\rm red}(\tilde{B}\tilde{H}\tilde{\mu}_L) + a_\mu^{\rm red}(\tilde{B}\tilde{H}\tilde{\mu}_R) + a_\mu^{\rm red}(\tilde{B}\tilde{\mu}_L\tilde{\mu}_R) \\
\Delta_\mu^{\rm red} &= \Delta_\mu^{\rm red}(\tilde{W}\tilde{H}\tilde{\nu}) + \Delta_\mu^{\rm red}(\tilde{W}\tilde{H}\tilde{\mu}_L) + \Delta_\mu^{\rm red}(\tilde{B}\tilde{H}\tilde{\mu}_L) + \Delta_\mu^{\rm red}(\tilde{B}\tilde{H}\tilde{\mu}_R) + \Delta_\mu^{\rm red}(\tilde{B}\tilde{\mu}_L\tilde{\mu}_R)
\end{aligned}
$$

- $a_\mu^{\rm SUSY}$ sign depends on the mass ratios.
- $\mathrm{sgn}(\mu)$ and $\tan\beta$ dependence disappears.
- $a_\mu^{\rm red}(\tilde{W}\tilde{H}\tilde{\nu})$ and $\Delta_\mu^{\rm red}(\tilde{W}\tilde{H}\tilde{\nu})$ have opposite signs.
- For the equal mass case, $a_\mu^{\rm red}(\tilde{W}\tilde{H}\tilde{\nu})$ and $\Delta_\mu^{\rm red}(\tilde{W}\tilde{H}\tilde{\nu})$ dominate $\Longrightarrow$ negative $a_\mu^{\rm SUSY}$

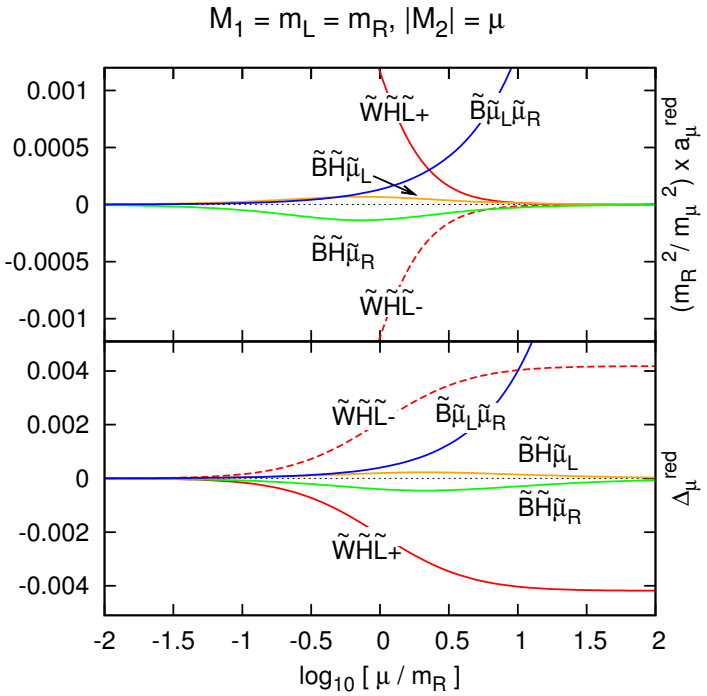
$$a_\mu^{\rm SUSY} = \frac{a_\mu^{\rm red}}{\Delta_\mu^{\rm red}}$$

$$a_\mu^{\rm SUSY} \approx -72 \times 10^{-10} \left(\frac{1\,\mathrm{TeV}}{M_{\rm SUSY}}\right)^2$$

**However! $a_\mu^{\rm SUSY}$ can be positive, if any of $\tilde{B}\tilde{H}\tilde{\mu}_L$, $\tilde{B}\tilde{H}\tilde{\mu}_R$, and $\tilde{B}\tilde{\mu}_L\tilde{\mu}_R$ dominates.**

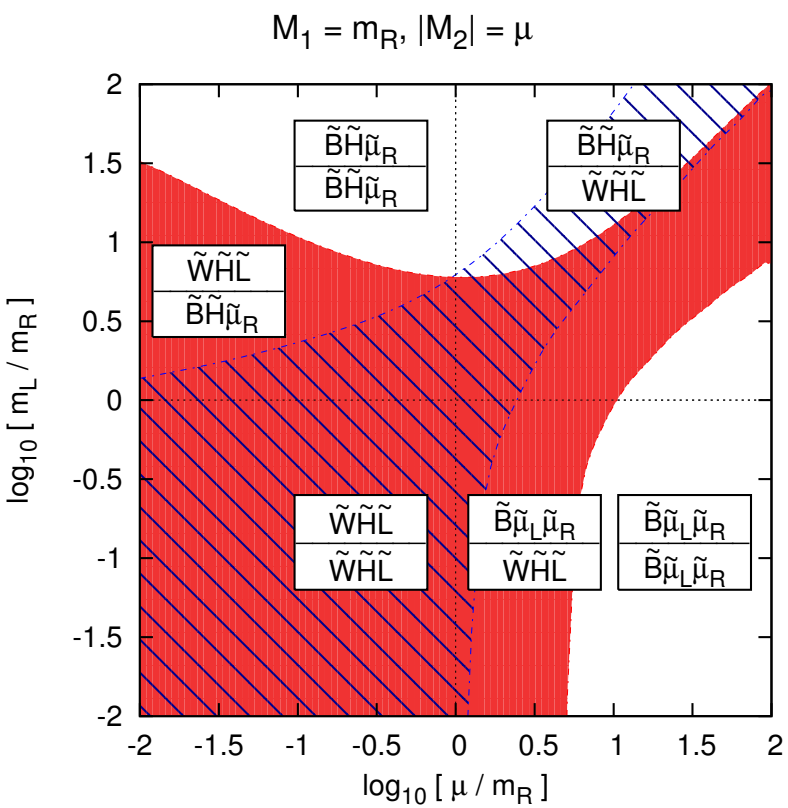



"$\mu_R$-dominance"




"large $\mu$-limit"

# The sign of $a_\mu^{\rm SUSY}$




At the equal mass limit,

$$a_\mu^{\rm SUSY} \approx -72\times 10^{-10}\left(\frac{1\,{\rm TeV}}{M_{\rm SUSY}}\right)^2$$

Generally,

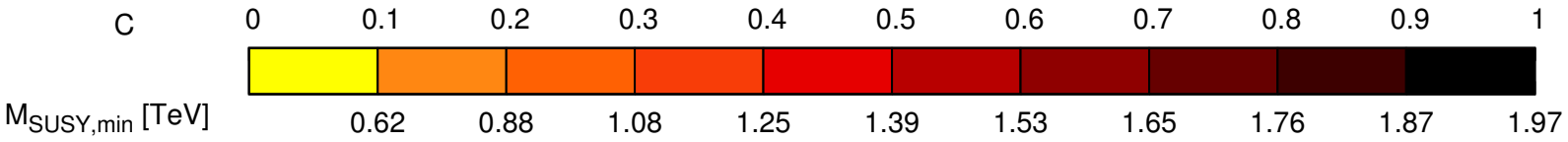
$$a_\mu^{\rm NP} = C_{\rm NP}\frac{m_\mu^2}{M_{\rm NP}^2},$$

$$C_{\rm NP} = \mathcal{O}(\delta m_\mu^{\rm NP}/m_\mu)$$

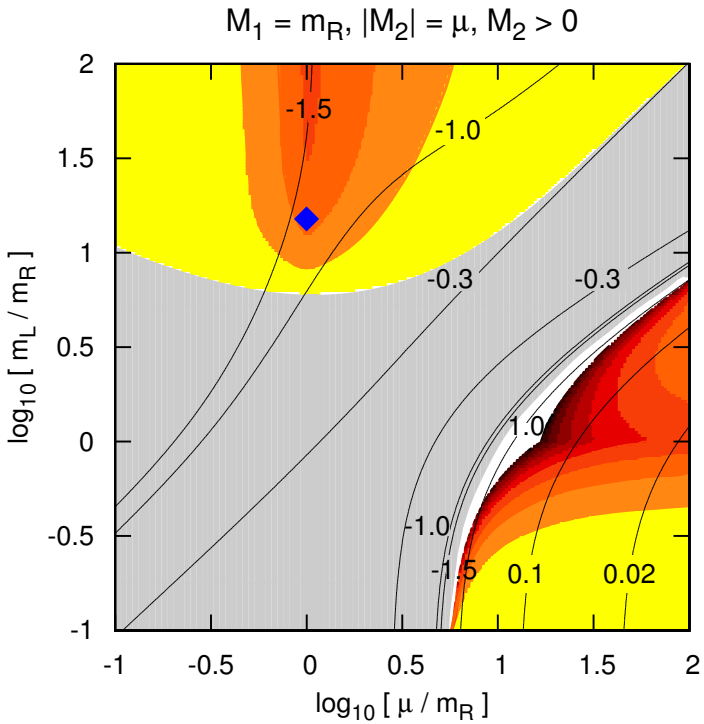
(model dependent)

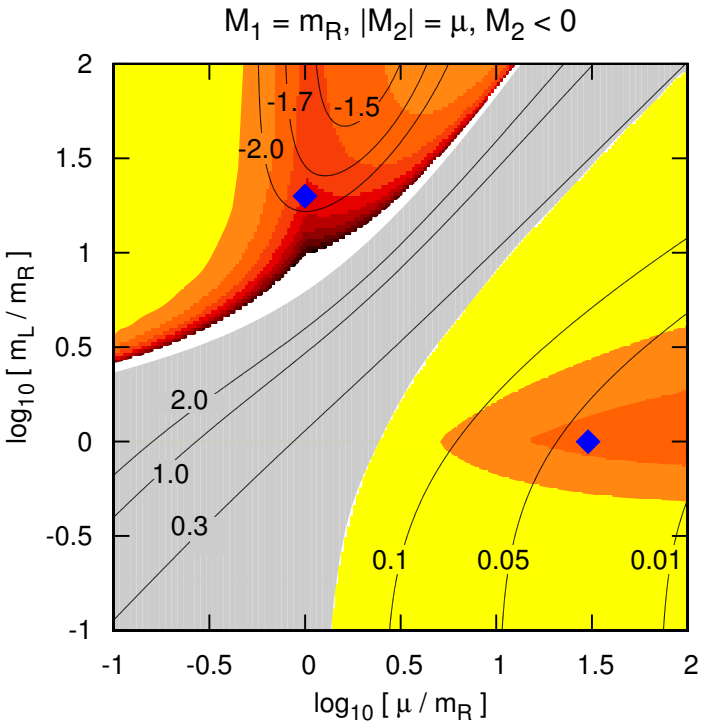
What can be the $C$-value/$a_\mu^{\rm SUSY}$ for the given parameter ratio space?

# $\tan\beta \to \infty$: $C$-value and $y_\mu$

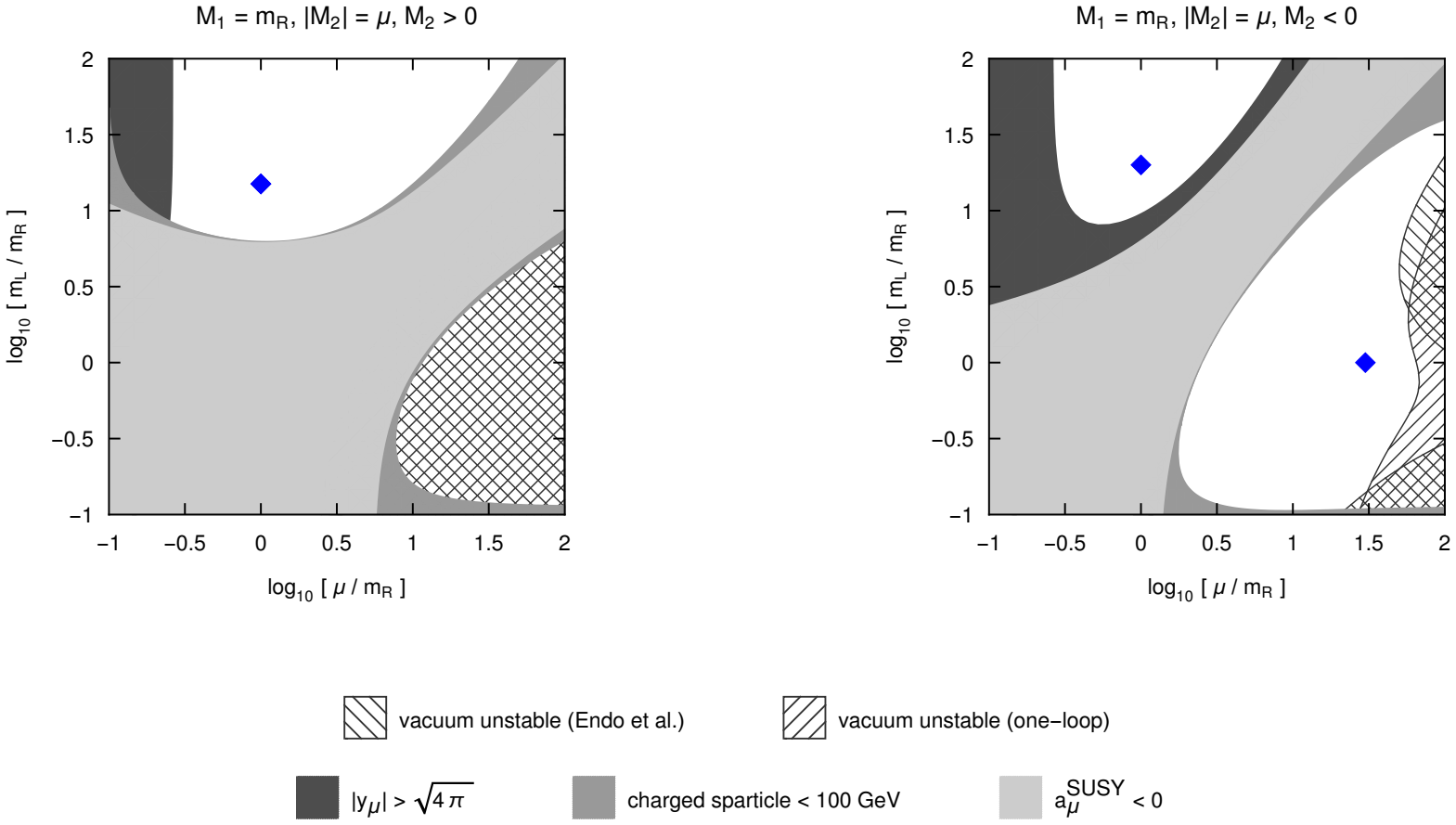



$M_{\rm SUSY,min}$ evaluated using $a_\mu^{\rm SUSY} = 28.7 \times 10^{-10}$.

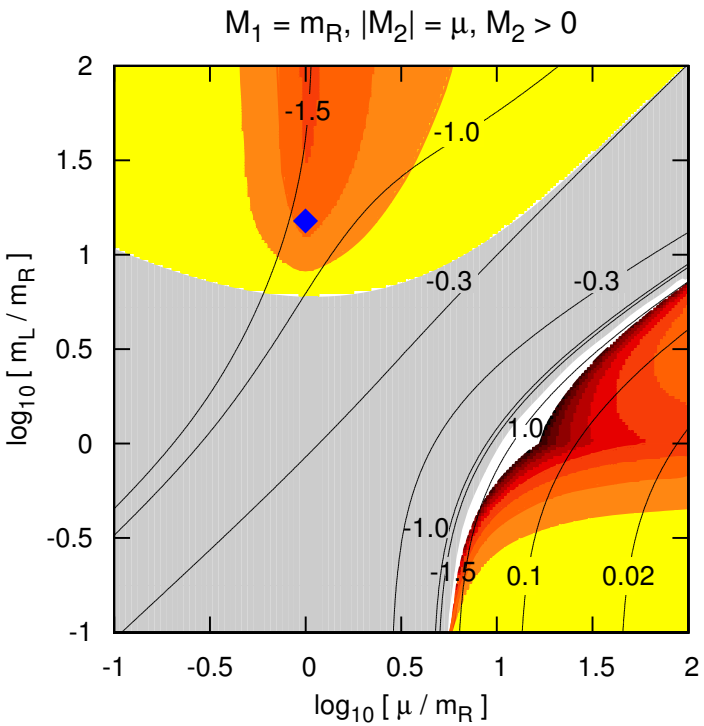


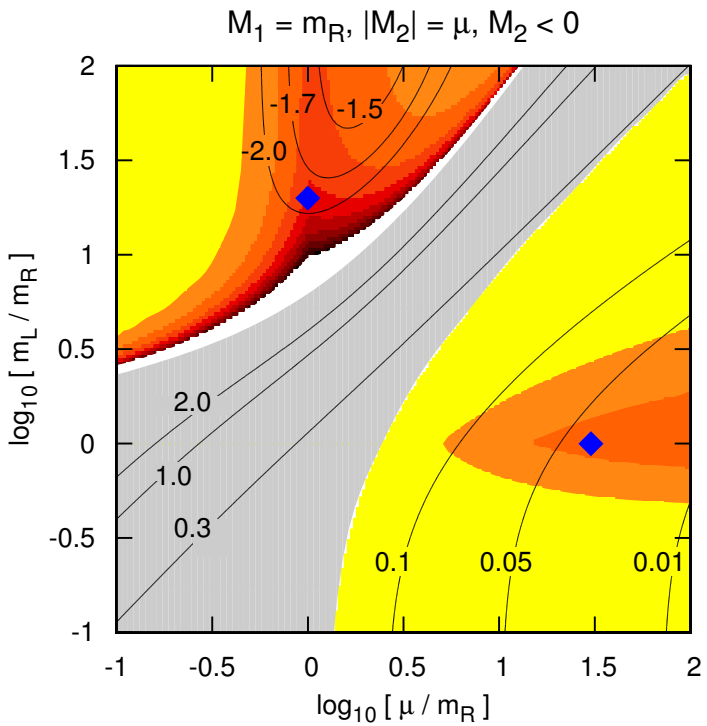



# Further constraints



# $\tan\beta \to \infty$: Benchmark points




| [TeV] | $\mu$ | $M_1$ | $M_2$ | $m_L$ | $m_R$ | $a_\mu/10^{-10}$ |
|---|---|---|---|---|---|---|
| | 1 | 1 | 1 | 15 | 1 | 26.4 |
| | 1.3 | 1.3 | -1.3 | 26 | 1.3 | 29.0 |
| | 30 | 1 | -30 | 1 | 1 | 28.0 |

# $a_\mu^{\rm SUSY}$ simple approximation

MSSM one-loop approximation with finite $\tan\beta$:

$$a_\mu^{\rm SUSY,1L} \approx 13 \times 10^{-10} {\rm sgn}(\mu) \tan\beta \left(\frac{100\,{\rm GeV}}{M_{\rm SUSY}}\right)^2$$

MSSM in the limit of $\tan\beta \to \infty$:

1. The equal mass case,

$$a_\mu^{\rm SUSY} \approx -72 \times 10^{-10} \left(\frac{1\,{\rm TeV}}{M_{\rm SUSY}}\right)^2$$

2. $|\mu| \gg |M_1| = m_L = m_R \equiv M_{\rm SUSY}$, $m_L \gg |\mu| = |M_1| = m_R \equiv M_{\rm SUSY}$

$$a_\mu^{\rm SUSY} \approx 37 \times 10^{-10} \left(\frac{1\,{\rm TeV}}{M_{\rm SUSY}}\right)^2$$

# Summary

- $3\sigma$ deviation in $a_\mu$ motivates New Physics.
- Precision improvement in ${a_\mu}^{\rm EW}$: Uncertainty in ${a_\mu}^{\rm EW}$ caused by $M_{\rm H}$ eliminated.
- MSSM $f\tilde{f}$ two-loop corrections solve the $\alpha$ ambiguity in one-loop result and show non-decoupling behaviour $\mathcal{O}(10\ldots30\%)$.
- Radiative muon mass generation in the limit of $\tan\beta \to \infty$
  - Relevant parameter regions different from the usual MSSM
  - TeV-scale SUSY masses can explain $a_\mu$.
  - "large $\mu$-limit" and "$\mu_R$-dominance" regions:

$$a_\mu^{\rm SUSY} \approx 37 \times 10^{-10} \left(\frac{1\,{\rm TeV}}{M_{\rm SUSY}}\right)^2$$