Searching for New Physics with precision calculations of Higgs properties

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based on

arXiv:1810.09388 with Mark Goodsell and Pietro Slavich

and

arXiv:1903.05417 with Shinya Kanemura

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Outline

1. A few words of general introduction: the Higgs as a probe of New Physics

2. Matching renormalisable couplings between generic theories
   *based on* [J.B., Goodsell, Slavich arXiv:1810.09388]

3. Two-loop corrections to the Higgs trilinear coupling in the 2HDM and IDM
   *based on* [J.B., Kanemura arXiv:1903:05417]
Going beyond the Standard Model

- Discovery of Higgs boson at the CERN LHC in 2012 completes the particle spectrum of the SM...
  ... but many questions remain about the Higgs boson!
  ▶ what is the nature of the Higgs? (elementary or composite)
  ▶ what is the form of the Higgs potential? (at the moment we only know the Higgs VEV and mass)
  ▶ what is the structure of the Higgs sector? (there is not really any motivation for the scalar sector to be minimal as in the SM)

- Many more reasons – independent of the Higgs discovery – motivate the existence of BSM Physics, e.g. to mention only a few:

  **Theory motivated**
  - the hierarchy problem
  - the strong-CP problem
  - the realisation of inflation
  - quantum gravity
  - etc.

  **Driven by experimental data**
  - evidence for dark matter and dark energy (from astrophysics/cosmology)
  - neutrino oscillations and masses
  - baryon asymmetry
  - exp. results for muon \((g - 2)\mu\)
  - possible anomalies in flavour Physics
  - etc.
Using the Higgs boson to search for BSM Physics

- Several of the problems of the SM are directly related to the Higgs sector
- Many BSM theories involve
  - extended Higgs sectors, e.g. 2nd Higgs doublet in MSSM, 2HDM, etc.
  - states that couple to the Higgs(es), e.g. stops in Supersymmetry (SUSY)
- Higgs mass $m_h$ measured with an astonishing accuracy

$$m_h^{\text{exp.}} = 125.09 \pm 0.21(\text{stat.}) \pm 0.11(\text{syst.}) \, \text{GeV}$$

(Atlas & CMS combined, Moriond 2015)

- $m_h$ sensitive to New Physics via
  - tree-level value, predicted in some BSM models (e.g. SUSY)
  - radiative corrections from new particles, with large couplings to the Higgs
- While their measurement is not yet as precise, the couplings of the Higgs to other particles ($h hh$, $hVV$, $hf\bar{f}$, etc.) are also sensitive to New Physics
  - at tree level, via mixing effects
  - at loop level, via radiative corrections from new particles
After the Higgs boson discovery, no clear sign of BSM Physics has been found

How can one explain this?

- new states are heavy, beyond the reach of the LHC
  $\rightarrow$ requires EFT calculations: see Part 1

- new states are made difficult to observe by some mechanism, \textit{e.g.}
  - alignment (without decoupling) $\rightarrow \text{i.e.}$ Higgs VEV is colinear to one of the (CP-even) mass eigenstates $\rightarrow$ to hide additional scalars
    $\rightarrow$ radiative corrections to Higgs coupling, \textit{e.g.} $\lambda_{hhh}$, may still be very large: see Part 2
  - suppression of decays by phase space (decays with compressed spectra)
  - suppression of production/decays by new symmetry or new property (\textit{e.g.} suppression of stop production in SUSY models with Dirac gauginos)
  - etc.
Part 1:
Matching renormalisable couplings between generic theories
Effective Field Theories

▶ Scale of New Physics $M_{NP}$ driven higher by experimental searches

⇒ in fixed-order calculations, large logarithmic terms $\propto \log \frac{M_{NP}}{m_{EW}}$ can spoil the accuracy, or even the validity, of the perturbative expansion, e.g.

$$\mathcal{O} = \alpha^0 a_0 + \alpha (b_1 L + a_1) + \alpha^2 (c_2 L^2 + b_2 L + a_2) + \cdots$$

$\alpha \equiv (g/4\pi)^2$, $L \equiv \log \frac{M_{NP}}{m_{EW}}$, $a_i, b_i, c_i \in \mathbb{C}$.

**Loss of perturbativity if**

$$\alpha L \gtrsim 1 \quad \cdots$$

The perturbative expansion must be **reorganised** $\rightarrow$ **EFT calculation**
Effective Field Theory calculations

- **Integrate out heavy fields** at some scale $\Lambda \sim M_{NP}$ and work in a low energy EFT below $\Lambda$

- Couplings in the EFT receive **threshold corrections** at the matching scale $\Lambda$

\[ \Lambda \sim M_{NP} \]

\[ \text{UV theory} \]
(light & heavy particles)

\[ \text{EFT} \]
(light particles only)

→ Use **RGEs** to run the couplings from the high input scale, to the low scale ($\ll M_{NP}$) at which the calculation is performed

\[ g = \tilde{g} + \Delta g \]

⇒ **large logs are resummed!**
Matching renormalisable couplings

- Many studies consider the effect of new heavy states via higher-dimensional operators...

- ... but it is often also interesting to study the threshold corrections to renormalisable couplings

- *Example*: in the context of Higgs mass calculations in SUSY models, heavy SUSY scenarios have been extensively investigated
  - Bounds on SUSY particles, especially coloured ones, are increasingly stringent (e.g. $m_{\tilde{t}} \gtrsim 1$ TeV in MSSM w/o compressed spectra)
  - Once masses of particles coupled to the Higgs are above a few TeV, fixed-order Higgs mass calculations lose accuracy
  - EFT calculation becomes necessary, integrating out heavy SUSY states and computing $m_h$ in EFT

→ Important matching conditions: scalar quartic and Yukawa couplings needed to compute $m_h$ in the EFT
Scalar couplings and Effective Field Theories

- Until recently, usually UV theory $\rightarrow$ MSSM, and EFT $\rightarrow$ SM

  see e.g. [Bernal, Djouadi, Slavich '07], [Draper, Lee, Wagner '13], [Bagnaschi, Giudice, Slavich, Strumia '14], [Pardo Vega, Villadoro '15], [Bagnaschi, Pardo Vega, Slavich '17], [Athron et al. '17], [Harlander, Klappert, Ochoa Franco, Voigt '18]

\[
\lambda_{SM}(M_S) = \frac{1}{4} (g^2(M_S) + g'^2(M_S)) \cos^2 2\beta + \Delta\lambda^{1\ell}(M_S) + \Delta\lambda^{2\ell}(M_S) + \Delta\lambda^{3\ell}(M_S)
\]

\[
m_h = f(\lambda_{SM}(m_t))
\]

- In many cases $M_S \gg v \Rightarrow$ effect of higher-dimensional operators $\propto v/M_S$ can be disregarded
Until recently, usually UV theory $\rightarrow$ MSSM, and EFT $\rightarrow$ SM

see e.g. [Bernal, Djouadi, Slavich '07], [Draper, Lee, Wagner '13], [Bagnaschi, Giudice, Slavich, Strumia '14], [Pardo Vega, Villadoro '15], [Bagnaschi, Pardo Vega, Slavich '17], [Athron et al. '17], [Harlander, Klappert, Ochoa Franco, Voigt '18]

but more and more scenarios are now being investigated!

see e.g. [Benakli, Darmé, Goodsell, Slavich '13], [Bagnaschi, Giudice, Slavich, Strumia '14], [Lee, Wagner '15], [Benakli, Goodsell, Williamson '18], [Bahl, Hollik '18], etc.
Many possible scenarios → huge amount of work to compute all RGEs and matching conditions for each scenario!

⇒ Automation

i.e. compute the relevant contributions/corrections for a generic model, then apply the result to the models one wants to study!

Increasingly many examples of automation:

- SARAH/SPheno \rightarrow spectrum generators, decays, etc.
- FlexibleSUSY
- MadGraph \rightarrow amplitude calculations, event generation, etc.
- H-COUP \rightarrow Higgs couplings and decay widths
- MicrOMEGAS \rightarrow DM properties (relic densities, direct/indirect detection rates, etc.)
- and many more...
Matching of scalar couplings between generic theories

- Two-loop RGEs are known for general QFTs ✓
  
  [Machacek, Vaughn '83,'84,'85], [Luo, Wang, Xiao '02], [Schienbein, Staub, Steudner, Svirina '18], [Sperling, Stöckinger, Voigt '13].

- ... but for the thresholds, generic results have been obtained only at one-loop:
  
  * for the case of matching a generic model onto the SM, in FlexibleSUSY [Athron et al. '17] and in SARAH [Staub, Porod '17], via **pole mass matching** i.e. extracting the threshold corrections to $\lambda_{\text{SM}}$ from
    
    \[ \frac{2\lambda_{\text{SM}}v_{\text{SM}}^2 + \Delta m_{\text{SM}}^2(m_h^2)}{\text{Higgs pole mass in the SM}} = \left( \frac{m_{\text{HET}}^2}{\text{tree}} + \Delta m_{\text{HET}}^2(m_h^2) \right) \]
    
  * in SMEFT studies where one computes $S_{\text{EFT}}$ from $S$ using functional methods
    
    see e.g. [Henning, Lu, Murayama '14,'16], [Drozd, Ellis, Quevillon, You '15], [Ellis, Quevillon, You, Zhang '16,'17], [Fuentes-Martin, Portoles, Ruiz-Femenia '16], [Zhang '16], [Bumm, Voigt '18]

\[
\begin{pmatrix}
\Delta_H & X_{HL} \\
X_{LH} & \Delta_L
\end{pmatrix} \equiv \begin{pmatrix}
\frac{\delta^2 S}{\delta \Phi_H^2} & \frac{\delta^2 S}{\delta \Phi_L \delta \Phi_H} \\
\frac{\delta^2 S}{\delta \Phi_L \delta \Phi_H} & \frac{\delta^2 S}{\delta \Phi_L^2}
\end{pmatrix}, \quad \text{with } \Phi_L: \text{light states, } \Phi_H: \text{heavy states}
\]

\[
S_{\text{EFT}}^{1\text{-loop}} = \left. \frac{i}{2} \text{Tr} \log(\Delta_H - X_{HL} \Delta_L^{-1} X_{LH}) \right|_{\text{hard}}
\]

but such results are difficult to implement in automated codes!
Matching of scalar couplings between generic theories

Our objective:

- All results for threshold corrections to scalar quartic and Yukawa couplings, when matching any high-energy model $A$ onto any low-energy model $B$

- Expressions that can be used in automated codes

- Method(s) that can be extended beyond one-loop

  → however there are challenges to address already from one-loop order!

[JB, Goodsell, Slavich 1810.09388]
MS/DR' matching of scalar quartic couplings and infrared divergences
Matching of scalar couplings at tree-level

- We consider a general theory of scalars, fermions, and gauge bosons\(^*\), with two mass scales: one light \(m_L\) and one heavy \(m_H \gg m_L\)
  
  \(^*\) however without heavy gauge bosons

- Integrating out heavy fields (\(i.e.\) of mass \(\gtrsim m_H\)), one finds at tree-level

\[
\begin{align*}
\text{Low-Energy Theory} & \quad \text{High-Energy Theory} \\
= & \quad + \\
\end{align*}
\]

\[a_{HLL}\]

\( \text{thin line: light state; thick line: heavy state} \)

- Trilinear couplings between light states – \(a_{LLL}\) – receive no threshold correction at tree-level

- In any case, we will consider the limit \(m_L \to 0\) in the following and then we must also take \(a_{LLL} \to 0\) (\textit{more on this in a few slides...})
Matching of scalar couplings in a toy model at one loop

- Considering now the **one-loop** matching → many diagrams contribute!

Low-Energy Model

\[ \begin{array}{c}
\begin{array}{c}
\text{grey blob}
\end{array}
\end{array} + \begin{array}{c}
\text{grey blob}
\end{array} = \begin{array}{c}
\text{grey blob}
\end{array} + \begin{array}{c}
\text{grey blob}
\end{array} \]

High-Energy Model

\[ \begin{array}{c}
\begin{array}{c}
\text{light state}
\end{array}
\end{array} + \begin{array}{c}
\text{heavy state}
\end{array} = \begin{array}{c}
\text{light state}
\end{array} + \begin{array}{c}
\text{heavy state}
\end{array} \]

Grey blob: 1PI one-loop subdiagrams with scalars, fermions, and (light) gauge bosons

*thin line:* light state; *thick line:* heavy state

difference of diagonal WFR between HET and EFT

Johannes Braathen (Osaka University) Seminar at IBS-CTPU
March 15, 2019
Matching of scalar couplings in a toy model at one loop

- Considering now the **one-loop** matching → many diagrams contribute!

  - **Low-Energy Model**
  - **High-Energy Model**

- Several diagrams are **IR divergent** in limit $m_L \to 0$, because of terms $\propto \log \frac{m_H}{m_L}$
IR parts in low and high energy theory must **exactly** cancel out, but because of $a_{HLL}$, divergent scalar diagrams are not in 1 to 1 correspondence → **automation impossible** as is!

We have derived complete expressions, **eliminating the IR divergent logs**
Eliminating IR divergences from the matching quartic couplings between generic theories

The matching condition in the general case is

\[
\text{Low-Energy Model} + \text{High-Energy Model} = \text{difference of WFR between HET and EFT}
\]

\(1\ell \) corrections (div. for \(m_L \to 0\))

\[B_0(0, 0) \to 0, \quad C_0(0, 0, X) \to -\frac{1}{X} B_0(0, X) = \frac{1}{X^2} A(X), \quad D_0(0, 0, X, Y) \to -\frac{1}{X - Y} \left(\frac{1}{X^2} A(X) - \frac{1}{Y^2} A(Y)\right)\]

where \(A(x) \equiv x(\log x/Q^2 - 1)\).
Eliminating IR divergences from the matching quartic couplings between generic theories

The matching condition in the general case is

\[ \text{Low-Energy Model} \quad + \quad \text{High-Energy Model} \quad = \quad \left( \begin{array}{c} + \\ + \end{array} \right) + \left( \begin{array}{c} + \\ + \end{array} \right) + \left( \begin{array}{c} + \\ + \end{array} \right) \]

\[ + \left( \begin{array}{c} + \\ + \\ + \end{array} \right) \]

\[ \text{difference of WFR} \quad \text{between HET and EFT} \]

\[ 1\ell \text{ corrections (IR safe!)} \]

\[ B_0(0, 0) \to 0, \quad C_0(0, 0, X) \to -\frac{1}{X} B_0(0, X) = \frac{1}{X^2} A(X), \quad D_0(0, 0, X, Y) \to -\frac{1}{X - Y} \left( \frac{1}{X^2} A(X) - \frac{1}{Y^2} A(Y) \right) \]

where \( A(x) \equiv x(\log x/Q^2 - 1) \).
More on the matching quartic couplings between generic theories

- In the absence of heavy gauge bosons, threshold corrections can be shown to be independent of the gauge couplings.

- If one considers $a_{LLL} \neq 0$, new IR divergent diagrams appear in the high-energy part of the matching.

These new IR divergences must be cancelled by additional terms in the low-energy part of the matching → from higher-dimensional operators.
Matching of quartic couplings and higher-dimensional operators

Dimension-5 operator

A 5-scalar interaction

\[ \mathcal{L}_{\text{EFT}} \supset - \frac{1}{5!} c_{pqrs}^{\phi_p \phi_q \phi_r \phi_s \phi_t} \]

is generated at tree-level in the low-energy theory

**Low-Energy Theory**

[Diagram showing the matching of low-energy and high-energy theories with thin lines for light states and thick lines for heavy states.]
Matching of quartic couplings and higher-dimensional operators

Dimension-5 operator

Tree-level matching of $c_5$:

High-energy Theory

In the one-loop matching of $\lambda$:

Low-energy Theory
Matching of quartic couplings and higher-dimensional operators

Dimension-6 operator

A dimension-6 correction to the scalar kinetic term

$$\mathcal{L}_{\text{EFT}} \supset -\frac{1}{4} k_{pqrs}^6 \phi_p \phi_q \partial_\mu \phi_r \partial^\mu \phi_s$$

is generated at tree-level by the second term of the expansion of

\[ \frac{i}{p^2 - m_H^2} = - \frac{i}{m_H^2} \left( 1 + \frac{p^2}{m_H^2} + \cdots \right) \]
Matching of quartic couplings and higher-dimensional operators

Dimension-6 operator

Tree-level matching of $k_6$:

In the one-loop matching of $\lambda$:

**High-energy Theory**

**Low-energy Theory**
Simplifying the matching condition
Simplifying the matching of quartic couplings between generic theories

LOW-ENERGY MODEL

HIGH-ENERGY MODEL

difference of WFR between HET and EFT

$1\ell$ corrections (IR safe!)
Simplifying the matching of quartic couplings between generic theories

Low-Energy Model

\[ + \quad + \quad = \quad + \quad + \quad + \\]

High-Energy Model

\[ + \quad + \quad + \quad + \quad + \quad + \\]

1\(\ell\) corrections (IR safe!)

Red boxes \(\rightarrow\) heavy-heavy and light-heavy mass corrections!
Simplifying the matching of quartic couplings between generic theories

$\delta m^2_{HH}$ (left): correction to heavy masses

$\delta m^2_{LH}$ (right): correction to the mixing between light and heavy states

Parameters of UV theory: (usually) arbitrary inputs $\rightarrow$ we are free to choose the finite part of mass counterterms to simplify the matching!

$\rightarrow$ eliminate mixing between light and heavy scalars + eliminate corrections to heavy masses

Generalisation of scheme devised in [Bagnaschi, Giudice, Slavich, Strumia ’14] for models with 2 Higgs doublets $\rightarrow$ choose $\tan \beta$ counterterm to remove off-diagonal WFR
Simplifying the matching of quartic couplings between generic theories

\[ \text{Low-Energy Model} \quad + \quad = \quad + \quad \text{High-Energy Model} \]

\[ 1\ell \text{ corrections (IR safe!)} \]

\[ \text{difference of WFR between HET and EFT} \]
Simplifying the matching of quartic couplings between generic theories

\[ + + = + + + + + + \]

1\(\ell\) corrections (IR safe!)

\[ 1\ell \text{ corrections (IR safe!)} \]

Redefinition of finite part of light-heavy mass counter-terms allows eliminating \(\delta m^2_{LH}\)

\[ \rightarrow \text{mixing between heavy and light states eliminated from the matching condition!} \]
Simplifying the matching of quartic couplings between generic theories

Redefinition of finite part of light-heavy mass counter-terms allows eliminating $\delta m^2_{LH}$

$\longrightarrow$ mixing between heavy and light states eliminated from the matching condition!

Redefinition of finite part of heavy-heavy mass counter-terms allows eliminating $\delta m^2_{HH}$

($numerical\ comparison\ in\ backup$)
A simple approach to matching using two-point functions

Pole-mass matching (see e.g. [Athron et al. '16])

- Extracting the threshold corrections to $\lambda_{SM}$ from

$$2\lambda_{SM}v_{SM}^2 + \Delta m_{SM}^2(p^2 = m_h^2) = (m_{HET}^2)^{\text{tree}} + \Delta m_{HET}^2(p^2 = m_h^2)$$

$$\Rightarrow \lambda_{SM} = \frac{2}{v_{HET}^2} \left[ m_{HET}^2 \left( 1 + [\Pi_{hh}'_{HET}(0) - \Pi_{hh}'_{SM}(0)] \right) - \frac{m_{HET}^2}{m_{Z}^2} (\Pi_{ZZ}^{HET}(0) - \Pi_{ZZ}^{SM}(0)) + (\Delta m_{HET}^2(0) - \Delta m_{SM}^2(0)) \right]$$

$\Pi_{hh}(0), \Pi_{ZZ}(0)$: Higgs and $Z$-boson self-energies at $p^2 = 0$, $\Delta m^2$: corrections to the Higgs mass

- only really tractable when EFT model does not have mixing in Higgs sector
- easier to extend beyond one-loop (as 2-point functions are easier to deal with)
- as is, requires cancellation of large (IR div.) logs, as was our problem earlier

- Formally equivalent to using the modified mass counterterms (c.f. previous slides)

- We obtain an efficient way to compute the threshold corrections to $\lambda_{SM}$ as

$$\lambda_{SM} = \frac{2}{v_{HET}^2} \left[ m_{HET}^2 \left( 1 + 2 \left[ \Pi_{hh}'_{HET}(0) - \Pi_{hh}'_{SM}(0) \right] \right) + \hat{\Delta}m_{HET}^2(0) \right]$$

w. light masses $\to 0$ terms w. light masses only $\to 0$
logs of light masses $\to 0$
gauge contributions $\to 0$
Matching of Yukawa couplings between generic theories
Matching of Yukawa couplings between generic theories

- Fewer diagrams contribute to the matching...
- ... but one must consider fermion mixing

\[
\begin{align*}
\text{off-diag. scalar WFR} & \quad \text{difference of diag. scalar WFR between HET and EFT} \\
\text{off-diag. fermion WFR} & \quad \text{difference of diag. fermion WFR between HET and EFT}
\end{align*}
\]
Matching of Yukawa couplings between generic theories

- Again, one can eliminate the mixing between light and heavy states (scalars or fermions) by modifying the finite part of mass counterterms → simpler matching!
Use of Effective Field Theories becomes increasingly necessary as $M_{NP}$ is driven higher by experimental searches.

When considering the calculation of a given observable in a wide range of scenarios or models, Automation can provide fast and accurate predictions.

Modified loop functions and renormalisation scheme choices now allow simple matching of scalar quartic and Yukawa couplings between generic theories (similar results implemented in SARAH in [Gabelmann, Mühlleitner, Staub 1810.12326]).

Efficient approach for pole mass matching, that will be easier to extend beyond one-loop.

Next: going beyond one-loop $\rightarrow$ use of modified scheme expected to become more important, consider pole-mass matching, ...
Part 2: Two-loop corrections to the Higgs trilinear coupling
The Higgs trilinear coupling $\lambda_{hhh}$
Investigating the Higgs trilinear coupling $\lambda_{hhh}$

**Theoretical motivations**

- Since the Higgs discovery, the existence of the Higgs potential is confirmed, but at the moment we only know:
  - the location of the EW minimum: $v \simeq 246$ GeV
  - the curvature of the potential around the EW minimum: $m_h \simeq 125$ GeV

However what we still don’t know is **shape** of the Higgs potential, which depends on $\lambda_{hhh}$.

- $\lambda_{hhh}$ determines whether the EWPT can be of strong first order or not ($\rightarrow$ necessary for EW baryogenesis)

  $\Rightarrow$ large deviation of $\lambda_{hhh}$ from SM prediction is necessary to have a strongly first-order EWPT
  [Grojean, Servant, Wells ’04], [Kanemura, Okada, Senaha ’04]

- In models where the Higgs mass(es) can be calculated, a computation of $\lambda_{hhh}$ with the same accuracy is needed to consistently interpret results with experimental data

- Higgs couplings such as $\lambda_{hhh}$ can in principle exhibit (large) effects from BSM Physics
Investigating the Higgs trilinear coupling $\lambda_{hhh}$

Current experimental limits

- Current limits on $\kappa_\lambda \equiv \lambda_{hhh}/\lambda_{hhh}^{SM}$ are (at 95% CL)
  
  $-5.0 < \kappa_\lambda < 12.1$ (ATLAS) and $-11 < \kappa_\lambda < 17$ (CMS)

Future prospects

- HL-LHC with 3 $ab^{-1}$ could reach $0.1 < \kappa_\lambda < 2.3$, and a 27-TeV HE-LHC with 15 $ab^{-1}$ $0.58 < \kappa_\lambda < 1.45$

- ILC-250 cannot measure $\lambda_{hhh}$, but 500-GeV and 1-TeV extensions could obtain measurements with precisions of 27% and 10% respectively

- CLIC 1.4 TeV + 3 TeV $\rightarrow$ 20% accuracy

- 100-TeV hadron collider with 30 $ab^{-1}$ $\rightarrow$ 5-7% accuracy

Double Higgs production
Radiative corrections to the Higgs trilinear coupling

- Higgs three-point function, $\Gamma_{hhh}(p_1^2, p_2^2, p_3^2)$, requires a diagrammatic calculation, with non-zero external momentum.

- Instead it is much more convenient to work with an effective Higgs trilinear coupling $\lambda_{hhh}$

\[
\mathcal{L} \supset -\frac{1}{6} \lambda_{hhh} h^3 \rightarrow \lambda_{hhh} = \frac{\partial^3 V_{\text{eff}}}{\partial h^3} \bigg|_{\text{min.}}
\]

$V_{\text{eff}} = V^{(0)} + \Delta V_{\text{eff}}$: effective potential (calculated in $\overline{\text{MS}}$ scheme).

- In effective-potential calculations, one should usually fix conditions for the lower derivatives of $V_{\text{eff}}$

\[
\left. \frac{\partial V_{\text{eff}}}{\partial h} \right|_{\text{min.}} = 0,
\]

\[
\left[ M_h^2 \right]_{V_{\text{eff}}} = \left. \frac{\partial^2 V_{\text{eff}}}{\partial h^2} \right|_{\text{min.}} - \frac{1}{v} \left. \frac{\partial V_{\text{eff}}}{\partial h} \right|_{\text{min.}}
\]

- Using these, we obtain

\[
\lambda_{hhh} = 3 \left[ \frac{M_h^2}{v} \right]_{V_{\text{eff}}} + D_3 \Delta V_{\text{eff}} \bigg|_{\text{min.}}, \quad \text{with} \quad D_3 \equiv \frac{\partial^3}{\partial h^3} - \frac{3}{v} \left[ -\frac{1}{v} \frac{\partial}{\partial h} + \frac{\partial^2}{\partial h^2} \right]
\]
Radiative corrections to the Higgs trilinear coupling

\( \Gamma_{hhh} \) and \( \lambda_{hhh} \) can be related as

\[
-\Gamma_{hhh}(0,0,0) = \hat{\lambda}_{hhh} = \left( \frac{Z_{h}^{\text{OS}}}{Z_{h}^{\text{MS}}} \right)^{3/2} \lambda_{hhh} = \left( 1 + \frac{3}{2} \frac{d}{dp^{2}} \Pi_{hh}(p^{2}) \bigg|_{p^{2} = M_{h}^{2}} \right) \lambda_{hhh}
\]

\( \delta Z_{h}^{\text{OS,MS}} = Z_{h}^{\text{OS,MS}} - 1 \): wave-function renormalisation counterterm in OS/MS scheme,
\( \Pi_{hh}(p^{2}) \): finite part of Higgs self-energy at ext. momentum \( p^{2} \)

Taking \( \Gamma_{hhh}(p_{1}^{2}, p_{2}^{2}, p_{3}^{2}) \approx \Gamma_{hhh}(0,0,0) \) is a good approximation

\( \rightarrow \) shown for \( \lambda_{hhh} \) at one loop in [Kanemura, Okada, Senaha, Yuan '04] (difference is only a few %)

\( \rightarrow \) no study including external momentum exists at two loops, but in the case of two-loop Higgs mass calculations, momentum effects are known to be subleading
Radiative corrections to the Higgs trilinear coupling and non-decoupling effects
The Two-Higgs-Doublet Model (2HDM)

► CP-conserving 2HDM, with softly-broken $\mathbb{Z}_2$ symmetry ($\Phi_1 \rightarrow \Phi_1, \Phi_2 \rightarrow -\Phi_2$) to avoid FCNCs

► 2 $SU(2)_L$ doublets $\Phi_{1,2} = \begin{pmatrix} \Phi_{1,2}^+ \\ \Phi_{1,2}^0 \end{pmatrix}$ of hypercharge 1/2

$$V_{2\text{HDM}}^{(0)} = m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - m_3^2(\Phi_2^\dagger \Phi_1 + \Phi_1^\dagger \Phi_2) + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_2^\dagger \Phi_1|^2 + \frac{\lambda_5}{2}(\Phi_2^\dagger \Phi_1)^2 + \text{h.c.}$$

► 7 free parameters in scalar sector: $m_3^2, \lambda_i (i = 1 \cdots 5)$, $\tan \beta \equiv \langle \Phi_2^0 \rangle / \langle \Phi_1^0 \rangle$ ($m_1^2, m_2^2$ eliminated with tadpole equations)

► Doublets expanded in terms of mass eigenstates as

$$\begin{pmatrix} \phi_{1}^+ \\ \phi_{2}^+ \end{pmatrix} = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} \quad \begin{pmatrix} \phi_{1}^0 \\ \phi_{2}^0 \end{pmatrix} = v \begin{pmatrix} c_\beta & -s_\beta \\ s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix} + i \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} A \end{pmatrix}$$

$h, H$: CP-even Higgses, $A$: CP-odd Higgs, $H^\pm$: charged Higgs, $\alpha$: CP-even Higgs mixing angle

► $\lambda_i (i = 1 \cdots 5)$ traded for mass eigenvalues $m_h, m_H, m_A, m_{H^\pm}$ and angle $\alpha$

► $m_3^2$ replaced by a soft-breaking mass scale $M^2 = 2m_3^2/s_2\beta$
Non-decoupling effects in $\lambda_{hhh}$ at one loop

First studies of the one-loop corrections to $\lambda_{hhh}$ in the 2HDM in [Kanemura, Kiyoura, Okada, Senaha, Yuan ’02] and [Kanemura, Okada, Senaha, Yuan ’04]

- Leading one-loop correction to $\lambda_{hhh}$

$$\delta^{(1)} \lambda_{hhh} = -\left. \frac{48 m_t^4}{v^3} \right\}_{\text{SM-like}} + \sum_{\Phi=H,A,H^\pm} \left( \frac{4 m_\Phi m^4_\Phi}{v^3} \left( 1 - \frac{M^2}{m^2_\Phi} \right)^3 \right)$$  (recall $\lambda_{hhh}^{(0)} = 3 m^2_h/v$)

- Masses of additional scalars $\Phi = H, A, H^\pm$ in 2HDM can be written as $m^2_\Phi = M^2 + \tilde{\lambda}_\Phi v^2$ ($\tilde{\lambda}_\Phi$: some combination of $\lambda_i$)

- Power-like dependence of BSM terms $\propto m^4_\Phi$, and

$$\left( 1 - \frac{M^2}{m^2_\Phi} \right)^3 \rightarrow \begin{cases} 0, & \text{for } M^2 \gg \tilde{\lambda}_\Phi v^2 \\ 1, & \text{for } M^2 \ll \tilde{\lambda}_\Phi v^2 \end{cases}$$

- Huge deviations possible, without violating unitarity!

![Diagram](image-url)  

figure from [Kanemura, Okada, Senaha, Yuan ’04]
State-of-the-art calculations of $\lambda_{hhh}$

At one loop

- Complete diagrammatic, OS-scheme, calculations been performed for a number of BSM models with extended sectors (with singlets, doublets, triplets)
- One-loop calculations available for 2HDMs, HSM, IDM in program $\mathbb{H}$-COUP [Kanemura, Kikuchi, Sakurai, Yagyu '17]

  Non-decoupling effects found for a range of BSM models at one loop
  $\Rightarrow$ what happens at two loops?

At two loops

- leading $O(\alpha_s\alpha_t)$ corrections in MSSM [Brucherseifer, Gavin, Spira '14] and NMSSM [Mühlleitner, Nhung, Ziesche '15], in effective-potential approximation
  $\Rightarrow$ two-loop effects can be up to $O(10\%)$; significant reduction of scale dependence in $\overline{\text{DR}}'$ calculations
- leading scalar corrections calculated in the IDM [Senaha, 1811.00336], also with the effective potential
  $\Rightarrow$ two-loop effects found to be a few percent ($\sim 2\%$), and can weaken the strength of the first-order electroweak phase transition in the model

We also want to investigate the fate of non-decoupling effects at two loops
$\Rightarrow$ [J.B., Kanemura 1903.05417]
OUR TWO-LOOP CALCULATION
Setup of our effective-potential calculation

- We obtain corrections to $\lambda_{hhh}$ as
  \[ \lambda_{hhh} = \left. \frac{\partial^3 V_{\text{eff}}}{\partial h^3} \right|_{\text{min.}} = \frac{3[M_h^2]}{v} V_{\text{eff}} + \left( \frac{\partial^3}{\partial h^3} - \frac{3}{v} \left[ \frac{1}{v} \frac{\partial}{\partial h} + \frac{\partial^2}{\partial h^2} \right] \right) \Delta V_{\text{eff}} \left|_{\text{min.}} \right. \]

- $\overline{\text{MS}}$-renormalised two-loop effective potential is
  \[ V_{\text{eff}} = V^{(0)} + \Delta V_{\text{eff}} = V^{(0)} + \kappa V^{(1)} + \kappa^2 V^{(2)} \quad (\kappa \equiv \frac{1}{16\pi^2}) \]

- $V^{(2)}$: 1PI vacuum bubble diags., and for the leading two-loop BSM corrections we need

- Here we only want the leading corrections from additional scalars and top quark $\Rightarrow$ we neglect subleading contributions from $h, G, G^\pm$, and light fermions ($\rightarrow$ no need to specify type of 2HDM)

- We consider scenarios without mixing: alignment in the 2HDM; and the IDM
Setup of our effective-potential calculation

- OS result is obtained as

\[
\hat{\lambda}_{hhh} = \left( \frac{Z_h^{OS}}{Z_h^{MS}} \right)^{3/2} \times \lambda_{hhh}
\]

\text{inclusion of WFR}

\text{MS parameters translated to OS ones}

- Let's suppose (for simplicity) that \( \lambda_{hhh} \) only depends on one parameter \( x \), as

\[
\lambda_{hhh} = f^{(0)}(x^{MS}) + \kappa f^{(1)}(x^{MS}) + \kappa^2 f^{(2)}(x^{MS})
\]

\( (\kappa = \frac{1}{16\pi^2}) \)

and

\[
x^{MS} = X^{OS} + \kappa \delta^{(1)} x + \kappa^2 \delta^{(2)} x
\]

then in terms of OS parameters

\[
\lambda_{hhh} = f^{(0)}(X^{OS}) + \kappa \left[ f^{(1)}(X^{OS}) + \frac{\partial f^{(0)}}{\partial x} (X^{OS}) \delta^{(1)} x \right]
\]

\[
+ \kappa^2 \left[ f^{(2)}(X^{OS}) + \frac{\partial f^{(1)}}{\partial x} (X^{OS}) \delta^{(1)} x + \frac{\partial f^{(0)}}{\partial x} (X^{OS}) \delta^{(2)} x + \frac{\partial^2 f^{(0)}}{\partial x^2} (X^{OS}) (\delta^{(1)} x)^2 \right]
\]
Setup of our effective-potential calculation

► OS result is obtained as

\[
\hat{\lambda}_{hhh} = \left( \frac{Z_{OS}^h}{Z_{MS}^h} \right)^{3/2} \times \lambda_{hhh}
\]

including of WFR

► Let’s suppose (for simplicity) that \( \lambda_{hhh} \) only depends on one parameter \( x \), as

\[
\lambda_{hhh} = f^{(0)}(\bar{x}^{MS}) + \kappa f^{(1)}(\bar{x}^{MS}) + \kappa^2 f^{(2)}(\bar{x}^{MS}) \quad (\kappa = \frac{1}{16\pi^2})
\]

and

\[
\bar{x}^{MS} = X^{OS} + \kappa \delta^{(1)} x + \kappa^2 \delta^{(2)} x
\]

then in terms of OS parameters

\[
\lambda_{hhh} = f^{(0)}(X^{OS}) + \kappa \left[ f^{(1)}(X^{OS}) + \frac{\partial f^{(0)}}{\partial x}(X^{OS}) \delta^{(1)} x \right]
\]

\[
+ \kappa^2 \left[ f^{(2)}(X^{OS}) + \frac{\partial f^{(1)}}{\partial x}(X^{OS}) \delta^{(1)} x + \frac{\partial f^{(0)}}{\partial x}(X^{OS}) \delta^{(2)} x + \frac{\partial^2 f^{(0)}}{\partial x^2}(X^{OS}) (\delta^{(1)} x)^2 \right]
\]

because we neglect \( m_h \) in the loop corrections and \( \lambda_{hhh}^{(0)} = 3m_h^2/v \) (in absence of mixing)
In the SM, 4 diagrams contribute to $V_{\text{eff}}$ at order $O(g_3^2 m_t^4)$ and $O(m_t^6/v^2)$.

In the limit $m_t \gg m_h, m_G, \cdots$, their expression reads

$$V^{(2)} = -4g_3^2 m_t^2 \left[ 4A(m_t^2) - 8m_t^2 - \frac{6A(m_t^2)^2}{m_t^2} \right] + 3y_t^2 \left[ 2m_t^2 I(m_t^2, m_t^2, 0) + m_t^2 I(m_t^2, 0, 0) + A(m_t^2)^2 \right]$$

$A(x) \equiv x(\log(x/Q^2) - 1)$, $I$: two-loop sunrise integral

Then we find in the $\overline{\text{MS}}$ scheme

$$\delta^{(2)} \lambda_{hhh} = \frac{128g_3^2 m_t^4 (1 + 6 \log m_t^2)}{v^3} - \frac{24m_t^4 y_t^2 (-7 + 6 \log m_t^2)}{v^3}$$

$(\log x \equiv \log x/Q^2)$
Two-loop Standard Model result

\[ \delta^{(2)} \lambda_{hhh} = \frac{128 g_3^2 m_t^4 (1 + 6 \log m_t^2)}{v^3} - \frac{24 m_t^4 y_t^2 (-7 + 6 \log m_t^2)}{v^3} \quad (\log x \equiv \log x/Q^2) \]

\[ m_t^2 \to M_t^2 - \Pi_{tt}(p^2 = M_t^2) \quad v \to \frac{1}{\sqrt{\sqrt{2} G_F}} + \delta v = v_{\text{phys}} + \delta v \]

+ include wave-function renormalisation

\[ \delta^{(2)} \hat{\lambda}_{hhh} = \frac{72 M_t^4}{v_{\text{phys}}^3} \left( 16 g_3^2 - \frac{13 M_t^2}{v_{\text{phys}}^2} \right) \]
Two-loop Standard Model result

\[ \Delta \lambda_{hhh}^{SM} = \lambda_{hhh}^{SM} - \lambda_{hhh}^{SM,(0)} \]
$\lambda_{hhh}$ in the Two-Higgs-Doublet Model
$\lambda_{hhh}$ at two loops in the 2HDM

15 new BSM diagrams appearing in $V^{(2)}$ in the 2HDM w.r.t. the SM case.
\[ \lambda_{hhh} \text{ at two loops in the 2HDM} \]

- We assume the additional scalars of the 2HDM – \( H, A, H^{\pm} \) – to have a degenerate mass \( m_{\Phi} \) → 3 mass scales in the calculation: \( m_t, m_{\Phi}, M \) (→ simpler analytical expressions)
- We take the alignment limit \( s_{\beta-\alpha} = 1 \) and neglect loop-induced deviations from this condition
- In the \( \overline{\text{MS}} \) scheme

\[
\delta^{(2)} \lambda_{hhh} = \frac{16 m_{\Phi}^4}{v^5} \left( 4 + 9 \cot^2 2\beta \right) \left( 1 - \frac{M^2}{m_{\Phi}^2} \right)^4 \left[ - 2 M^2 - m_{\Phi}^2 + (M^2 + 2 m_{\Phi}^2) \log m_{\Phi}^2 \right]
\]
\[
+ \frac{192 m_{\Phi}^6 \cot^2 2\beta}{v^5} \left( 1 - \frac{M^2}{m_{\Phi}^2} \right)^4 \left[ 1 + 2 \log m_{\Phi}^2 \right]
\]
\[
+ \frac{96 m_{\Phi}^4 m_t^2 \cot^2 \beta}{v^5} \left( 1 - \frac{M^2}{m_{\Phi}^2} \right)^3 \left[ - 1 + 2 \log m_{\Phi}^2 \right] + \mathcal{O} \left( \frac{m_{\Phi}^2 m_t^4}{v^5} \right)
\]
Decoupling behaviour

- Seeing whether corrections from additional BSM states decouple if said state is taken to be very massive is a good way to check the consistency of the calculation

\[ \delta^{(2)} \lambda_{hhh} = \frac{16 m_\Phi^4}{v^5} \left( 4 + 9 \cot^2 2\beta \right) \left( 1 - \frac{M^2}{m_\Phi^2} \right)^4 \left[ -2M^2 - m_\Phi^2 + (M^2 + 2m_\Phi^2) \log m_\Phi^2 \right] \]

\[ \delta^{(1)} \lambda_{hhh} = \frac{16 m_\Phi^4}{v^3} \left( 1 - \frac{M^2}{m_\Phi^2} \right)^3 + \frac{192 m_\Phi^6}{v^5} \cot^2 2\beta \left( 1 - \frac{M^2}{m_\Phi^2} \right)^4 \left[ 1 + 2 \log m_\Phi^2 \right] \]

\[ + \frac{96 m_\Phi^4 m_\phi^2}{v^5} \cot^2 \beta \left( 1 - \frac{M^2}{m_\Phi^2} \right)^3 \left[ -1 + 2 \log m_\Phi^2 \right] + \mathcal{O} \left( \frac{m_\Phi^2 m_t^4}{v^5} \right) \]

where \( m_\Phi^2 = M^2 + \tilde{\lambda} \Phi v^2 \)

- To have \( m_\Phi \to \infty \), then we must take \( M \to \infty \), otherwise the quartic couplings grow out of control

- Fortunately all of these terms go like

\[ \left( \frac{m_\Phi^2}{m_\Phi^2} \right)^{n-1} \left( 1 - \frac{M^2}{m_\Phi^2} \right)^n = \frac{(\tilde{\lambda} \Phi v^2)^n}{M^2 + \tilde{\lambda} \Phi v^2} \xrightarrow{M \to \infty} 0 \]
Decoupling behaviour and $\overline{\text{MS}}$ to OS scheme conversion

To express $\delta^{(2)} \hat{\lambda}_{hhh}$ in terms of physical parameters $(v_{\text{phys}}, M_t, M_A = M_H = M_{H^\pm} = M_{\Phi})$, we replace

$$
m_A^2 \rightarrow M_A^2 - \Pi_{AA}(M_A^2), \quad m_H^2 \rightarrow M_H^2 - \Pi_{HH}(M_H^2), \quad m_{H^\pm}^2 \rightarrow M_{H^\pm}^2 - \Pi_{H+H^-}(M_{H^\pm}^2),$$

$$v \rightarrow v_{\text{phys}} - \delta v, \quad m_t^2 \rightarrow M_t^2 - \Pi_{tt}(M_t^2)$$

A priori, $M$ is still renormalised in $\overline{\text{MS}}$ scheme, because it is difficult to relate to physical observable ... but then, expressions do not decouple for $M_{\Phi}^2 = M^2 + \tilde{\lambda}_{\Phi} v^2$ and $M \rightarrow \infty$!

This is because we should relate $M_{\Phi}$, renormalised in OS scheme, and $M$, renormalised in $\overline{\text{MS}}$ scheme, with a one-loop relation → then the two-loop corrections decouple properly.

We give a new “OS” prescription for the finite part of the counterterm for $M$ be requiring that

1. the decoupling of $\delta^{(2)} \hat{\lambda}_{hhh}$ (in OS scheme) is apparent using a relation $M_{\Phi}^2 = \tilde{M}^2 + \tilde{\lambda}_{\Phi} v^2$

2. all the log terms in $\delta^{(2)} \hat{\lambda}_{hhh}$ are absorbed in $\delta M^2$

$$\delta^{(2)} \hat{\lambda}_{hhh} = \frac{48 M_{\Phi}^6}{v_{\text{phys}}^5} \left(1 - \frac{\tilde{M}^2}{M_{\Phi}^2}\right)^4 \left\{4 + 3 \cot^2 2\beta \left[3 - \frac{\pi}{\sqrt{3}} \left(\frac{\tilde{M}^2}{M_{\Phi}^2} + 2\right)\right]\right\} + \frac{576 M_{\Phi}^6 \cot^2 2\beta}{v_{\text{phys}}^5} \left(1 - \frac{\tilde{M}^2}{M_{\Phi}^2}\right)^4$$

$$+ \frac{288 M_{\Phi}^4 M_t^2 \cot^2 2\beta}{v_{\text{phys}}^5} \left(1 - \frac{\tilde{M}^2}{M_{\Phi}^2}\right)^3 + \frac{168 M_{\Phi}^4 M_t^2}{v_{\text{phys}}^5} \left(1 - \frac{\tilde{M}^2}{M_{\Phi}^2}\right)^3 - \frac{48 M_{\Phi}^6}{v_{\text{phys}}^5} \left(1 - \frac{\tilde{M}^2}{M_{\Phi}^2}\right)^5 + O \left(\frac{M_{\Phi}^2 M_t^4}{v_{\text{phys}}^5}\right)$$
Decoupling behaviour

\[ M_H = M_A = M_{H^\pm} = M_\Phi \]

\[ s_\beta - \alpha = 1 \]

\[ t_\beta = 1.5 \]

\[ \delta R \equiv \frac{\lambda_{2HDM}^{hh}}{\lambda_{SM}^{hh}} - 1 \]

\[ \delta R \] size of BSM contributions to $\lambda_{hhh}$:

Radiative corrections from additional scalars + top quark indeed decouple properly for $\tilde{M} \to \infty$
\[ M_H = M_A = M_{H^\pm} = M_\Phi \]
\[ \tilde{M} = 0 \]
\[ s_{\beta - \alpha} = 1 \]
\[ t_\beta = 1.1 \]

\[ \tilde{M} = 0 \rightarrow \text{maximal non-decoupling effects} \]

\[ \delta R \equiv \left( \frac{\lambda_{2HDM}^{hhh}}{\lambda_{SM}^{hhh}} \right) - 1 \]

- Tree level unitarity is lost around \( M_\Phi \approx 600 \text{ GeV} \) [Kanemura, Kubota, Takasugi '93]
\( \lambda_{hhh} \) in the Inert Doublet Model
The Inert Doublet Model

- Model of 2 $SU(2)_L$ doublets, with $\mathbb{Z}_2$ symmetry under which $\Phi_1 \rightarrow \Phi_1$, $\Phi_2 \rightarrow -\Phi_2$ unbroken after EWSB

$$V^{(0)}_{IDM} = \mu_1^2 |\Phi_1|^2 + \mu_2^2 |\Phi_2|^2 + \frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \frac{\lambda_5}{2} ((\Phi_1^\dagger \Phi_2)^2 + \text{h.c.})$$

- Expand the doublets as

$$\Phi_1 = \left( \begin{array}{c} G^+ \\ \frac{1}{\sqrt{2}}(v + h + iG) \end{array} \right), \quad \Phi_2 = \left( \begin{array}{c} H^+ \\ \frac{1}{\sqrt{2}}(H + iA) \end{array} \right)$$

$H, A, H^\pm$ : inert scalars (no couplings to fermions, no scalar mixing)

- Tree-level masses of scalars read

$$m_h^2(h) = \mu_1^2 + \frac{3}{2} \lambda_1 (v + h)^2, \quad m_G^2(h) = m_{G^\pm}^2(h) = \mu_1^2 + \frac{1}{2} \lambda_1 (v + h)^2$$

$$m_H^2 = \mu_2^2 + \frac{1}{2} \lambda_H (v + h)^2, \quad m_A^2 = \mu_2^2 + \frac{1}{2} \lambda_A (v + h)^2, \quad m_{H^\pm}^2 = \mu_2^2 + \frac{1}{2} \lambda_3 (v + h)^2$$

- We consider a DM-inspired scenario in which $H$ is light and is DM ($M_H \simeq M_h/2$), and to maximise the leading corrections to $\lambda_{hhh}$ we consider $\mu_2$ small, i.e.

$$M_h, M_H, \mu_2 \ll M_A, M_{H^\pm}$$

$NB$: studied first in [Senaha 1811.00336]
$\lambda_{hhh}$ at two loops in the IDM

- 8 new diagrams appearing in $V^{(2)}$ in the IDM w.r.t. the SM

- Only (i) and (ii) studied in [Senaha '18] → in particular (vi)-(viii) depend on inert scalar quartic $\lambda_2$

- After conversion to the OS scheme

$$
\delta^{(2)}\hat{\lambda}_{hhh} = \frac{6\lambda_2}{v_{\text{phys}}^3} \left(3M_A^4 + 4M_A^2M_{H\pm}^2 + 8M_{H\pm}^4\right) + \frac{60(M_A^6 + 2M_{H\pm}^6)}{v_{\text{phys}}^5} + \frac{24(M_A^2 - M_{H\pm}^2)^2(M_A^2 + M_{H\pm}^2)}{v_{\text{phys}}^5} \\
+ \frac{24M_t^4(M_A^2 + 2M_{H\pm}^2)}{v_{\text{phys}}^5} + \frac{42M_t^2(M_A^4 + 2M_{H\pm}^4)}{v_{\text{phys}}^5} - \frac{2(M_A^4 + 2M_{H\pm}^4)(M_A^2 + 2M_{H\pm}^2)}{v_{\text{phys}}^5}.
$$
Numerical results

\[ \delta R \equiv \frac{\lambda_{h\Phi\Phi}^{IDM}}{\lambda_{h\Phi\Phi}^{SM}} - 1 \]

\[ M_A = M_{H^\pm} = M_{\Phi} \] to keep \( \rho \) parameter close to 1, 
\( \lambda_2 \) as large as possible under criterion of tree-level unitarity [Kanemura, Kubota, Takasugi]
Summary of Part 2

► First two-loop calculation of $\lambda_{hhh}$ in 2HDM, in a scenario with alignment and degenerate masses

► Continue the work of [Senaha '18] in the IDM, including more diagrams → found effect of $\lambda_2$

► Two-loop corrections to $\lambda_{hhh}$ remain smaller than one-loop contributions, at least as long as perturbative unitarity is maintained → typical size 10 – 20% of one-loop contributions
  => non-decoupling effects found at one loop are not drastically changed
  => in the (future) perspective of a precise measurement of $\lambda_{hhh}$, computing corrections beyond one loop will be necessary

► Many avenues for more work:
  → more complete two-loop calculation (effective potential, diagrammatic), including mixing effects, momentum dependence
  → investigate more precisely implications of two-loop calculation on allowed parameter space
  → consider more models (general renormalisable models?)
  → study two-loop corrections to other Higgs couplings
Thank you for your attention!
Backup
A simple toy model

Consider a theory with 3 real scalars – $h_1$, $h_2$, $S$ – and 2 $\mathbb{Z}_2$ symmetries:

$$(h_1, h_2, S) \xrightarrow{\mathbb{Z}_2^A} (-h_1, -h_2, S) \quad \text{and} \quad (h_1, h_2, S) \xrightarrow{\mathbb{Z}_2^B} (h_1, h_2, -S)$$

$h_1$, $h_2$ mix $\rightarrow$ 2 mass eigenstates $h$, $H$

Mass spectrum: $h$ light; $H$ and $S$ heavy

Allowed Lagrangian terms are

$$\mathcal{L} = \frac{1}{2} (\partial^\mu h_i)^2 + \frac{1}{2} (\partial^\mu S)^2 - \frac{1}{2} m_{ij} h_i h_j - \frac{1}{2} m_S^2 S^2 - \frac{1}{24} \tilde{\lambda}^{ijkl} h_i h_j h_k h_l - \frac{1}{4} \tilde{\lambda}^{ijSS} h_i h_j S^2 - \frac{1}{24} \lambda^{SSSS} S^4$$

We need to diagonalise the mass matrix of $h_{1,2}$, but the rotation matrix ($\sim$ mixing angle) depends on the scheme
Matching of the light scalar quartic coupling in the toy model

\[ \mathcal{L} = \frac{1}{2} (\partial^\mu h_i)^2 + \frac{1}{2} (\partial^\mu S)^2 - \frac{1}{2} m_{ij}^2 h_i h_j - \frac{1}{2} m_S^2 S^2 - \frac{1}{24} \tilde{\lambda}^{ijkl} h_i h_j h_k h_l - \frac{1}{4} \tilde{\lambda}^{ijSS} h_i h_j S^2 - \frac{1}{24} \tilde{\lambda}^{SSSS} S^4 \]

\[ \overline{\text{MS}} \text{ scheme} \]

\[
\begin{pmatrix}
  m_{11}^2 & m_{12}^2 \\
  m_{12}^2 & m_{22}^2
\end{pmatrix}
\rightarrow
\begin{pmatrix}
  m_h^2 & 0 \\
  0 & m_H^2
\end{pmatrix}
\]

\[ \tilde{\lambda}^{ijkl}, \tilde{\lambda}^{ijSS} \rightarrow \tilde{\lambda}^{hhhh}, \tilde{\lambda}^{hhHH}, \tilde{\lambda}^{hHHH}, \tilde{\lambda}^{HHHH}, \tilde{\lambda}^{hhSS}, \tilde{\lambda}^{hHSS}, \tilde{\lambda}^{HHSS} \]

The \( \overline{\text{MS}} \) matching of \( \lambda^{hhhh} \) gives

Blue: \( h \); Red: \( H \); Green: \( S \)
Matching of the light scalar quartic coupling in the toy model

\[ \text{MS scheme} \]

\[ \lambda_{ijkl}, \lambda_{ijSS} \xrightarrow{R} \tilde{\lambda}_{c.t.} \]

\[ \tilde{\lambda}_{ijkl}, \tilde{\lambda}_{ijSS} \xrightarrow{R'} \tilde{\lambda}_{c.t.} \]

\[ \tilde{\lambda}_{ijkl} + \tilde{\lambda}_{ijSS} \xrightarrow{R} \tilde{\lambda}_{c.t.} \]

\[ \tilde{\lambda}_{ijkl} + \tilde{\lambda}_{ijSS} \xrightarrow{R'} \tilde{\lambda}_{c.t.} \]

\[ = + + + + + \]

\[ \text{Modified counterterm scheme} \]

\[ R^T \left( \begin{array}{cc} m_h^2 & \bar{d} m_h^2 H \\ \bar{d} m_h^2 H & m_H^2 \end{array} \right) R \xrightarrow{R'} \left( \begin{array}{cc} (m_h^2)_{c.t.} & 0 \\ 0 & (m_H^2)_{c.t.} \end{array} \right) \]

\[ \tilde{\lambda}_{ijkl} + \tilde{\lambda}_{ijSS} \xrightarrow{R} \tilde{\lambda}_{c.t.} \]

\[ \tilde{\lambda}_{ijkl} + \tilde{\lambda}_{ijSS} \xrightarrow{R'} \tilde{\lambda}_{c.t.} \]

\[ = + + + + + \]

Blue: \( h \); Red: \( H \); Green: \( S \)
Dashed: tree-level; solid: one-loop

Red: $\overline{\text{MS}}$ scheme; Blue: modified counterterms

In the original basis:

$m^2_{11} = (100 \text{ GeV})^2$, $m^2_{12} = (400 \text{ GeV})^2$,
$m^2_{22} = (2000 \text{ GeV})^2$, $m^2_S = (5000 \text{ GeV})^2$,
$\tilde{\lambda}^{1111}_{1111} = 1$, $\tilde{\lambda}^{1112}_{1112} = 2$, $\tilde{\lambda}^{1122}_{1122} \in [-4, 4]$,
$\tilde{\lambda}^{1222}_{1222} = 1.5$, $\tilde{\lambda}^{2222}_{2222} = 0.5$,
$\tilde{\lambda}^{11SS}_{11SS} = 0$, $\tilde{\lambda}^{12SS}_{12SS} = 3.5$, $\tilde{\lambda}^{22SS}_{22SS} = 0$

$\Rightarrow m_h = 60 \text{ GeV}$, $m_H = 2002 \text{ GeV}$

This parameter point: small mixing between $h$ and $H$ at tree-level $\rightarrow$ loop-induced mixing is then large

$\Rightarrow$ large difference between schemes at tree-level

$\Rightarrow$ large loop corrections in $\overline{\text{MS}}$ calculation
In the original basis:

\[ m_{11}^2 = (100 \text{ GeV})^2, \quad m_{12}^2 = (400 \text{ GeV})^2, \]
\[ m_{22}^2 = (2000 \text{ GeV})^2, \quad m_S^2 = (5000 \text{ GeV})^2, \]
\[ \tilde{\lambda}_{1111} = 1, \quad \tilde{\lambda}_{1112} = 2, \quad \tilde{\lambda}_{1122} \in [-4, 4], \]
\[ \tilde{\lambda}_{1222} = 1.5, \quad \tilde{\lambda}_{2222} = 0.5, \]
\[ \tilde{\lambda}_{11SS} = 0, \quad \tilde{\lambda}_{12SS} = 3.5, \quad \tilde{\lambda}_{22SS} = 0 \]

\[ \Rightarrow m_h^2 = 60 \text{ GeV}, \quad m_H^2 = 2002 \text{ GeV} \]

*Blue dashed:* all couplings in modified scheme;

*Blue dot-dashed:* couplings in one-loop correction in $\overline{\text{MS}}$ scheme
An example of experimental limits on $\lambda_{hhh}$

Example of current limits on $\kappa_\lambda$ from the ATLAS search of $hh \rightarrow b\bar{b}\gamma\gamma$

(taken from [ATLAS collaboration 1807.04873])