Searching for New Physics with precision calculations of Higgs properties

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based on
`arXiv:1810.09388` *with Mark Goodsell and Pietro Slavich*
and
`arXiv:1903.05417` *with Shinya Kanemura*

Seminar at the Institute for Basic Science
Center for Theoretical Physics of the Universe,
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Outline

1. A few words of general introduction: the Higgs as a probe of New Physics

2. Matching renormalisable couplings between generic theories
   based on [J.B., Goodsell, Slavich `arXiv:1810.09388`]

3. Two-loop corrections to the Higgs trilinear coupling in the 2HDM and IDM
   based on [J.B., Kanemura `arXiv:1903:05417`]

Going beyond the Standard Model

- Discovery of Higgs boson at the CERN LHC in 2012 completes the particle spectrum of the SM...
  ... but many questions remain about the Higgs boson!
  - what is the nature of the Higgs? (elementary or composite)
  - what is the form of the Higgs potential? (at the moment we only know the Higgs VEV and mass)
  - what is the structure of the Higgs sector? (there is not really any motivation for the scalar sector to be minimal as in the SM)

- Many more reasons – independent of the Higgs discovery – motivate the existence of BSM Physics, $e.g.$ to mention only a few:

Theory motivated

- the hierarchy problem
- the strong-CP problem
- the realisation of inflation
- quantum gravity
- etc.

Driven by experimental data

- evidence for dark matter and dark energy (from astrophysics/cosmology)
- neutrino oscillations and masses
- baryon asymmetry
- exp. results for muon $(g-2)_\mu$
- possible anomalies in flavour Physics
- etc.

Using the Higgs boson to search for BSM Physics

- Several of the problems of the SM are directly related to the Higgs sector
- Many BSM theories involve
  - **extended Higgs sectors**, $e.g.$ 2nd Higgs doublet in MSSM, 2HDM, etc.
    and/or
  - states that **couple** to the Higgs(es), $e.g.$ stops in Supersymmetry (SUSY)
- Higgs mass m_h measured with an <u>astonishing accuracy</u>

$$m_h^{\text{exp.}} = 125.09 \pm 0.21(\text{stat.}) \pm 0.11(\text{syst.})\ \text{GeV}$$

(ATLAS & CMS combined, Moriond 2015)

- m_h **sensitive to New Physics** via
  - tree-level value, predicted in some BSM models ($e.g.$ SUSY)
  - radiative corrections from new particles, with large couplings to the Higgs
- While their measurement is not yet as precise, the couplings of the Higgs to other particles (hhh, hVV, $hf\bar{f}$, etc.) are also sensitive to New Physics
  - at tree level, via mixing effects
  - at loop level, via radiative corrections from new particles

Current status of New Physics

After the Higgs boson discovery, **no clear sign of BSM Physics has been found**

How can one explain this?

- new states are heavy, beyond the reach of the LHC
  $\rightarrow$ requires EFT calculations: see Part 1

- new states are made difficult to observe by some mechanism, $e.g.$
  - alignment (without decoupling) – $i.e.$ Higgs VEV is colinear to one of the (CP-even) mass eigenstates – to hide additional scalars
    $\rightarrow$ radiative corrections to Higgs coupling, $e.g.$ λ_{hhh}, may still be very large: see Part 2
  - suppression of decays by phase space (decays with compressed spectra)
  - suppression of production/decays by new symmetry or new property ($e.g.$ suppression of stop production in SUSY models with Dirac gauginos)
  - etc.

Part 1:
Matching renormalisable couplings between generic theories

Effective Field Theories

- Scale of New Physics M_{NP} driven higher by experimental searches

$\Rightarrow$ in fixed-order calculations, **large logarithmic terms** $\propto \log \frac{M_{\text{NP}}}{m_{\text{EW}}}$ can spoil the accuracy, or even the validity, of the perturbative expansion, $e.g.$

$$\mathcal{O} = \underbrace{\alpha^0 a_0}_{\text{tree-level}} + \underbrace{\alpha(b_1 L + a_1)}_{\text{one-loop}} + \underbrace{\alpha^2(c_2 L^2 + b_2 L + a_2)}_{\text{two-loop}} + \cdots$$

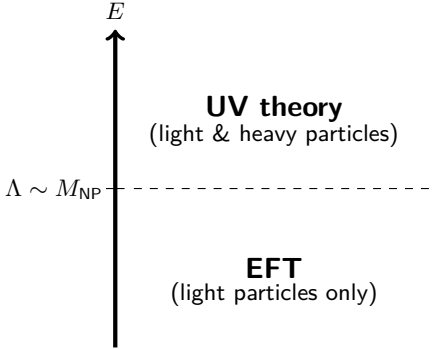
$\alpha \equiv (g/4\pi)^2$, $L \equiv \log M_{\text{NP}}/m_{\text{EW}}$, $a_i, b_i, c_i \in \mathbb{C}$.
Loss of perturbativity if

$$\alpha L \gtrsim 1 \ \cdots$$

The perturbative expansion must be reorganised $\rightarrow$ **EFT calculation**

Effective Field Theory calculations

- **Integrate out heavy fields** at some scale $\Lambda \sim M_{\text{NP}}$ and work in a low energy EFT below Λ
- Couplings in the EFT receive **threshold corrections** at the matching scale Λ



Coupling of the UV theory $\tilde{g}$

$\Downarrow$

Match effective actions computed in UV th. and EFT, at ℓ loops:

$$\tilde{\Gamma}^{\text{UV}}(Q=\Lambda) = \Gamma^{\text{EFT}}(Q=\Lambda)$$

$\Downarrow$

ℓ-loop threshold correction to coupling of the EFT g

$$g = \tilde{g} + \Delta g$$

- Use **RGEs** to run the couplings from the high input scale, to the low scale ($\ll M_{\text{NP}}$) at which the calculation is performed

$\Rightarrow$ **large logs are resummed!**

Matching renormalisable couplings

- Many studies consider the effect of new heavy states via higher-dimensional operators...
- ... but it is often also interesting to study the threshold corrections to **renormalisable couplings**
- *Example*: in the context of Higgs mass calculations in SUSY models, **heavy SUSY scenarios** have been extensively investigated
  - ▷ Bounds on SUSY particles, especially coloured ones, are increasingly stringent ($e.g.$ $m_{\tilde{t}} \gtrsim 1$ TeV in MSSM w/o compressed spectra)
  - ▷ Once masses of particles coupled to the Higgs are above a few TeV, fixed-order Higgs mass calculations lose accuracy
  - ▷ EFT calculation becomes necessary, integrating out heavy SUSY states and computing m_h in EFT

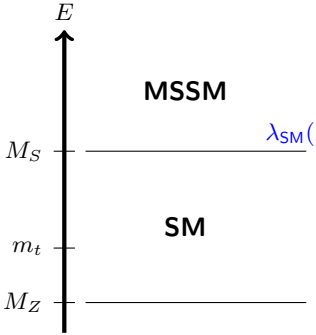
  → Important matching conditions: **scalar quartic and Yukawa couplings** needed to compute m_h in the EFT

Scalar couplings and Effective Field Theories

- Until recently, usually **UV theory** $\rightarrow$ **MSSM**, and **EFT** $\rightarrow$ **SM**

  see $e.g$ [Bernal, Djouadi, Slavich '07], [Draper, Lee, Wagner '13], [Bagnaschi, Giudice, Slavich, Strumia '14], [Pardo Vega, Villadoro '15], [Bagnaschi, Pardo Vega, Slavich '17], [Athron et al. '17], [Harlander, Klappert, Ochoa Franco, Voigt '18]

$e.g.$



MSSM inputs

$\Downarrow$

$$\lambda_{\text{SM}}(M_S) = \frac{1}{4}(g^2(M_S) + g'^2(M_S))\cos^2 2\beta + \Delta\lambda^{1\ell}(M_S) + \Delta\lambda^{2\ell}(M_S) + \Delta\lambda^{3\ell}(M_S)$$

$\Downarrow$

$$m_h = f(\lambda_{\text{SM}}(m_t))$$

fixed-order calculation in the SM

$\Uparrow$

SM inputs

[Full 2ℓ RGEs in MSSM]

[Full 2ℓ + leading 3ℓ and 4ℓ RGE for λ_{SM} in SM]

- In many cases $M_S \gg v \Rightarrow$ effect of higher-dimensional operators $\propto v/M_S$ can be disregarded

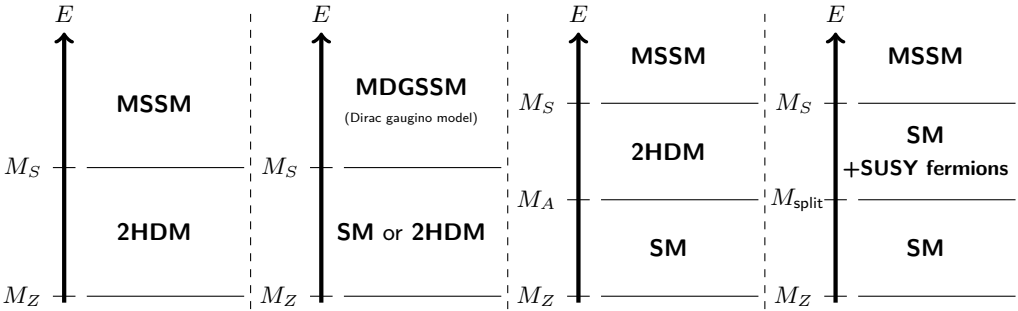
Scalar couplings and Effective Field Theories

- Until recently, usually **UV theory** $\rightarrow$ **MSSM**, and **EFT** $\rightarrow$ **SM**

  see $e.g$ [Bernal, Djouadi, Slavich '07], [Draper, Lee, Wagner '13], [Bagnaschi, Giudice, Slavich, Strumia '14], [Pardo Vega, Villadoro '15], [Bagnaschi, Pardo Vega, Slavich '17], [Athron et al. '17], [Harlander, Klappert, Ochoa Franco, Voigt '18]

  but more and more scenarios are now being investigated!

  see $e.g$ [Benakli, Darmé, Goodsell, Slavich '13], [Bagnaschi, Giudice, Slavich, Strumia '14], [Lee, Wagner '15], [Benakli, Goodsell, Williamson '18], [Bahl, Hollik '18], etc.



Automation

- Many possible scenarios $\rightarrow$ huge amount of work to compute all RGEs and matching conditions for each scenario!

  $\Rightarrow$ **Automation**

  i.e. compute the relevant contributions/corrections for a generic model, then apply the result to the models one wants to study!

- Increasingly many examples of automation:
  - `SARAH`/`SPheno` } $\rightarrow$ spectrum generators, decays, etc.
  - `FlexibleSUSY` }
  - `MadGraph` $\rightarrow$ amplitude calculations, event generation, etc.
  - `H-COUP` $\rightarrow$ Higgs couplings and decay widths
  - `MicrOMEGAS` $\rightarrow$ DM properties (relic densities, direct/indirect detection rates, etc.)
  - and many more...

Matching of scalar couplings between generic theories

- Two-loop RGEs are known for general QFTs ✓
  [Machacek, Vaughn '83,'84,'85], [Luo, Wang, Xiao '02], [Schienbein, Staub, Steudner, Svirina '18], [Sperling, Stöckinger, Voigt '13].

- ... but for the thresholds, generic results have been obtained only at one-loop:
  - for the case of matching a generic model onto the SM, in `FlexibleSUSY` [Athron et al. '17] and in `SARAH` [Staub, Porod '17], via ***pole mass matching*** $i.e.$ extracting the threshold corrections to λ_{SM} from

$$\underbrace{2\lambda_{\text{SM}} v_{\text{SM}}^2 + \Delta m_{\text{SM}}^2(m_h^2)}_{\text{Higgs pole mass in the SM}} = \underbrace{(m_{\text{HET}}^2)^{\text{tree}} + \Delta m_{\text{HET}}^2(m_h^2)}_{\text{Higgs pole mass in the UV theory}}$$

  - in SMEFT studies where one computes S_{EFT} from S using functional methods
    see $e.g.$ [Henning, Lu, Murayama '14,'16], [Drozd, Ellis, Quevillon, You '15], [Ellis, Quevillon, You, Zhang '16,'17], [Fuentes-Martin, Portoles, Ruiz-Femenia '16], [Zhang '16], [Bumm, Voigt '18]

$$\begin{pmatrix} \Delta_H & X_{HL} \\ X_{LH} & \Delta_L \end{pmatrix} \equiv \begin{pmatrix} \frac{\delta^2 S}{\delta \Phi_H^2} & \frac{\delta^2 S}{\delta \phi_L \delta \Phi_H} \\ \frac{\delta^2 S}{\delta \phi_L \delta \Phi_H} & \frac{\delta^2 S}{\delta \phi_L^2} \end{pmatrix}, \quad \text{with } \phi_L\text{: light states, } \Phi_H\text{: heavy states}$$

$$\Rightarrow S_{\text{EFT}}^{1-\text{loop}} = \frac{i}{2} \operatorname{Tr} \log(\Delta_H - X_{HL} \Delta_L^{-1} X_{LH}) \Big|_{\text{hard}}$$

but such results are difficult to implement in automated codes!

Our objective:

- All results for **threshold corrections** to scalar quartic and Yukawa couplings, when matching any high-energy model A onto any low-energy model B
- Expressions that can be used in automated codes
- Method(s) that can be extended beyond one-loop
  $\rightarrow$ however there are challenges to address already from **one-loop order**!

[JB, Goodsell, Slavich 1810.09388]

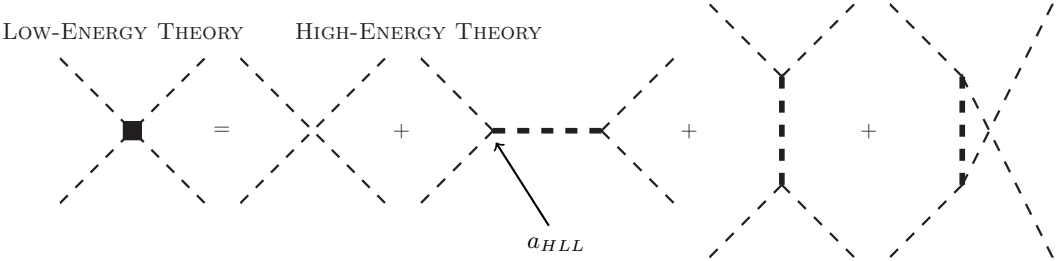
$\overline{\text{MS}}/\overline{\text{DR}}'$ matching of scalar quartic couplings and infrared divergences

Matching of scalar couplings at tree-level

- We consider a general theory of scalars, fermions, and gauge bosons*, with two mass scales: one light m_L and one heavy $m_H \gg m_L$

  * *however without heavy gauge bosons*

- Integrating out heavy fields ($i.e.$ of mass $\gtrsim m_H$), one finds at **tree-level**

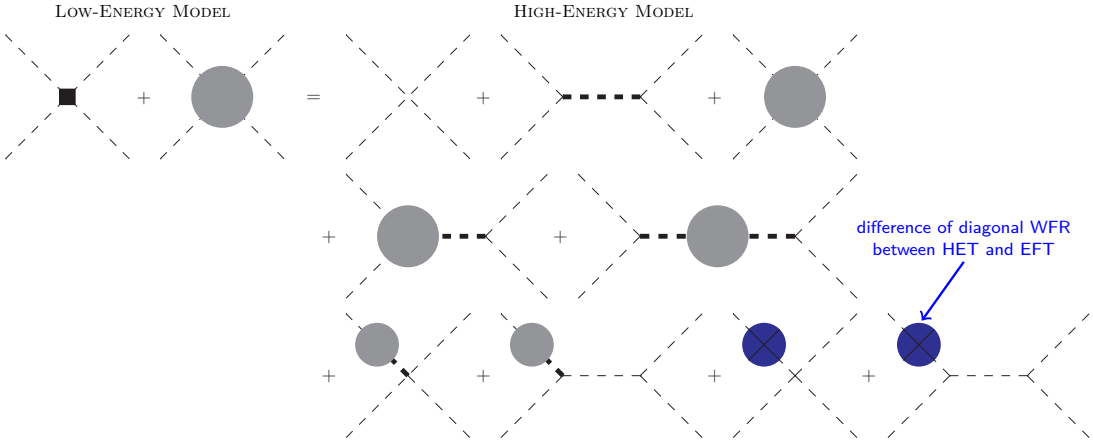


thin line: light state; *thick line*: heavy state

- Trilinear couplings between light states – a_{LLL} – receive **no threshold correction at tree-level**
- In any case, we will consider the limit $m_L \to 0$ in the following and then we must also take $a_{LLL} \to 0$ *(more on this in a few slides...)*

Matching of scalar couplings in a toy model at one loop

- Considering now the **one-loop** matching → many diagrams contribute!

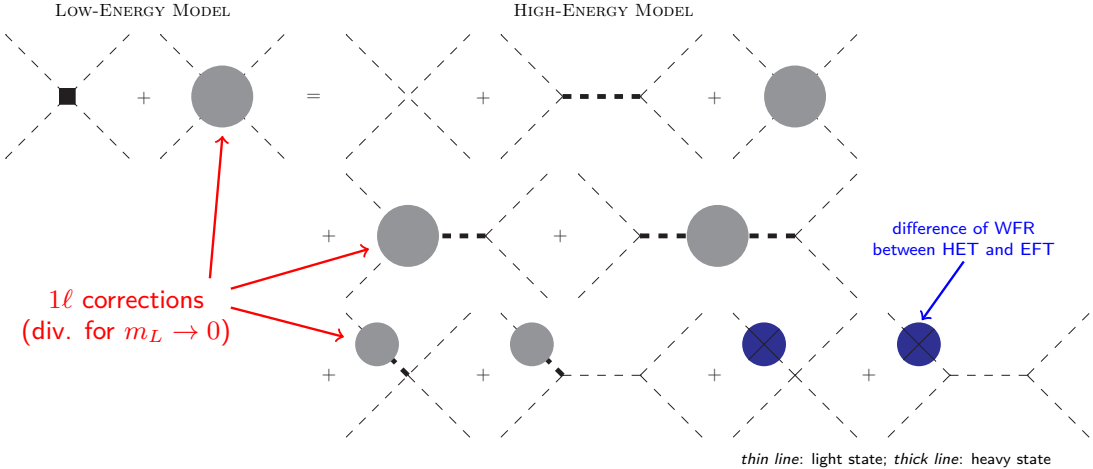


Grey blob: 1PI one-loop subdiagrams with scalars, fermions, and (light) gauge bosons

thin line: light state; *thick line*: heavy state

Matching of scalar couplings in a toy model at one loop

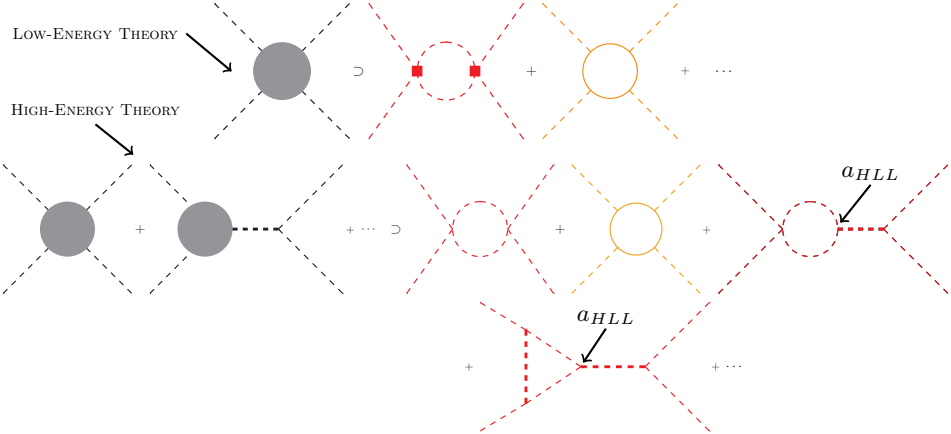
- Considering now the **one-loop** matching → many diagrams contribute!



- Several diagrams are **IR divergent** in limit $m_L \to 0$, because of terms $\propto \log m_H/m_L$

Matching of scalar couplings at one loop

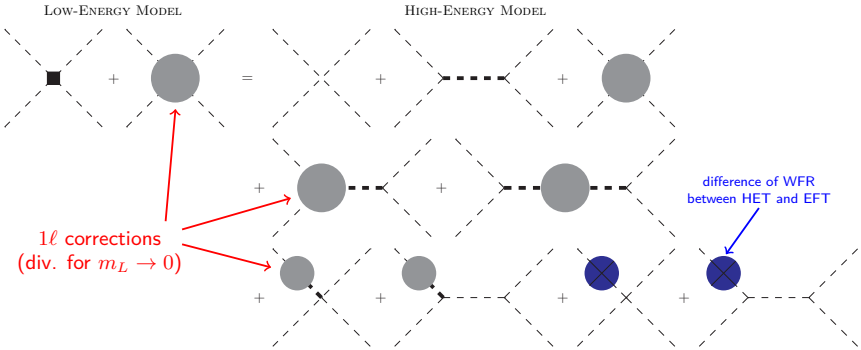
- IR parts in low and high energy theory must **exactly** cancel out, but because of a_{HLL}, divergent scalar diagrams are not in 1 to 1 correspondence $\rightarrow$ **automation impossible** as is!



$\Rightarrow$ We have derived complete expressions, **eliminating the IR divergent logs**

Eliminating IR divergences from the matching quartic couplings between generic theories

The matching condition in the general case is



- Expressions can be regularised by using modified (Passarino-Veltmann) loop functions

$$B_0(0,0) \to 0, \quad C_0(0,0,X) \to -\frac{1}{X} B_0(0,X) = \frac{1}{X^2} A(X), \quad D_0(0,0,X,Y) \to -\frac{1}{X-Y}\left(\frac{1}{X^2} A(X) - \frac{1}{Y^2} A(Y)\right)$$

where $A(x) \equiv x(\log x/Q^2 - 1)$.

Eliminating IR divergences from the matching quartic couplings between generic theories

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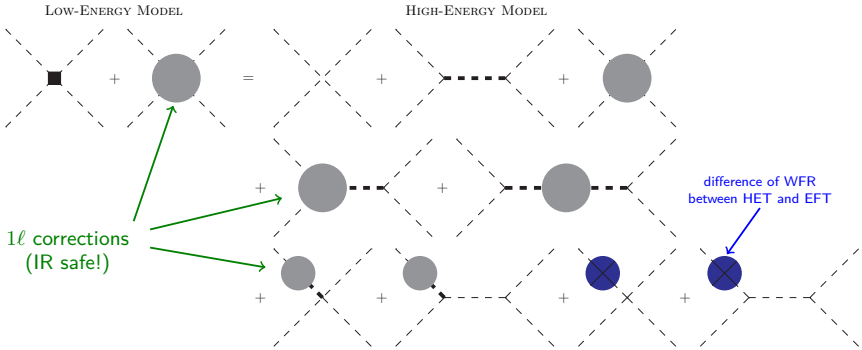

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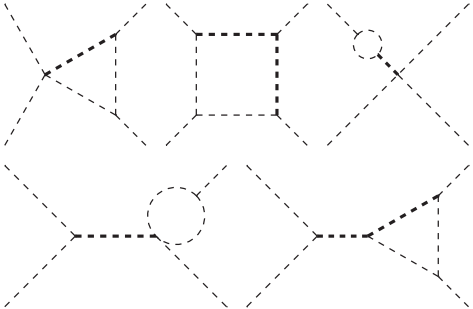
where $A(x) \equiv x(\log x/Q^2 - 1)$.

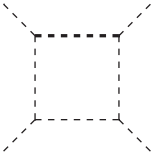
More on the matching quartic couplings between generic theories

- In the absence of heavy gauge bosons, threshold corrections can be shown to be independent of the gauge couplings

- If one considers $a_{LLL} \neq 0$, **new IR divergent diagrams** appear in the high-energy part of the matching



and



thin line: light state; *thick line*: heavy state

- These new IR divergences must be cancelled by additional terms in the low-energy part of the matching $\rightarrow$ from **higher-dimensional operators**

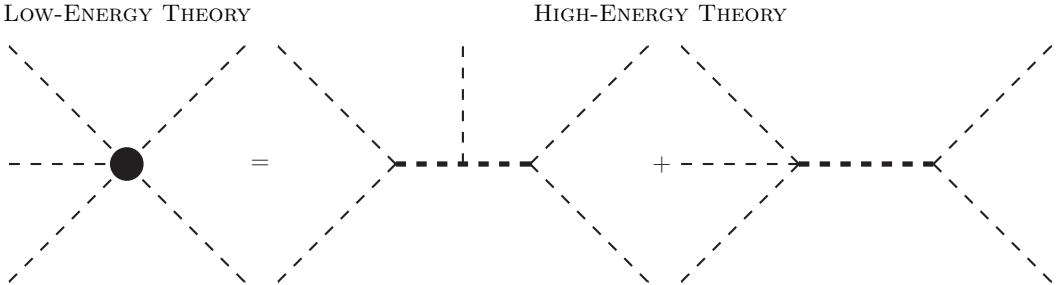
Matching of quartic couplings and higher-dimensional operators

Dimension-5 operator

A 5-scalar interaction

$$\mathcal{L}_{\text{EFT}} \supset -\frac{1}{5!} c_5^{pqrst} \phi_p \phi_q \phi_r \phi_s \phi_t$$

is generated at tree-level in the low-energy theory



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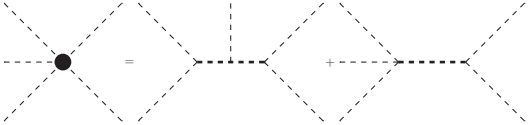
Matching of quartic couplings and higher-dimensional operators

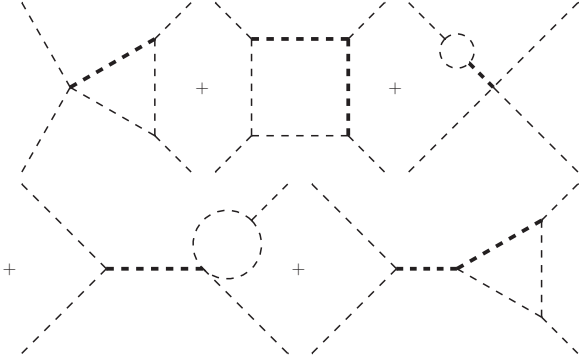
Dimension-5 operator

Tree-level matching of c_5:



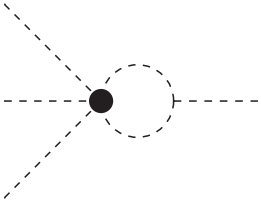
High-energy Theory

In the one-loop matching of λ:



$\longleftrightarrow$

Low-energy Theory



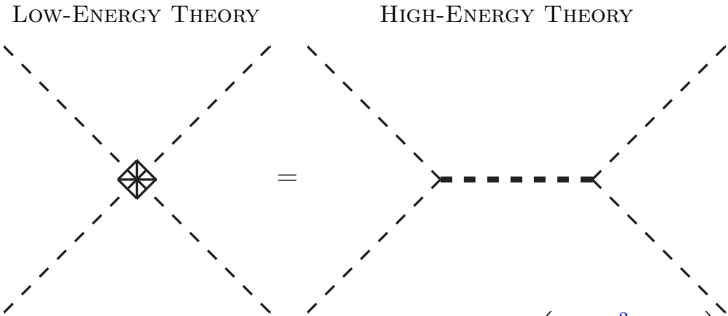
Matching of quartic couplings and higher-dimensional operators

Dimension-6 operator

A dimension-6 correction to the scalar kinetic term

$$\mathcal{L}_{\text{EFT}} \supset -\frac{1}{4} k_6^{pqrs} \phi_p \phi_q \partial_\mu \phi_r \partial^\mu \phi_s$$

is generated at tree-level by the second term of the expansion of

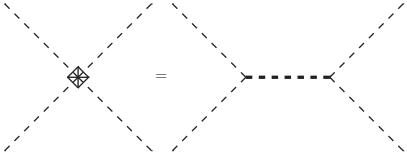


$$\frac{i}{p^2 - m_H^2} = -\frac{i}{m_H^2}\left(1 + \frac{p^2}{\underline{\underline{m_H^2}}} + \cdots\right)$$

Matching of quartic couplings and higher-dimensional operators

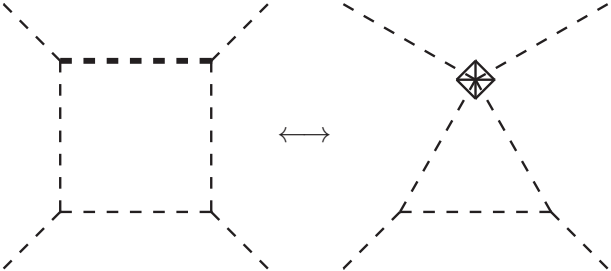
Dimension-6 operator

Tree-level matching of k_6:



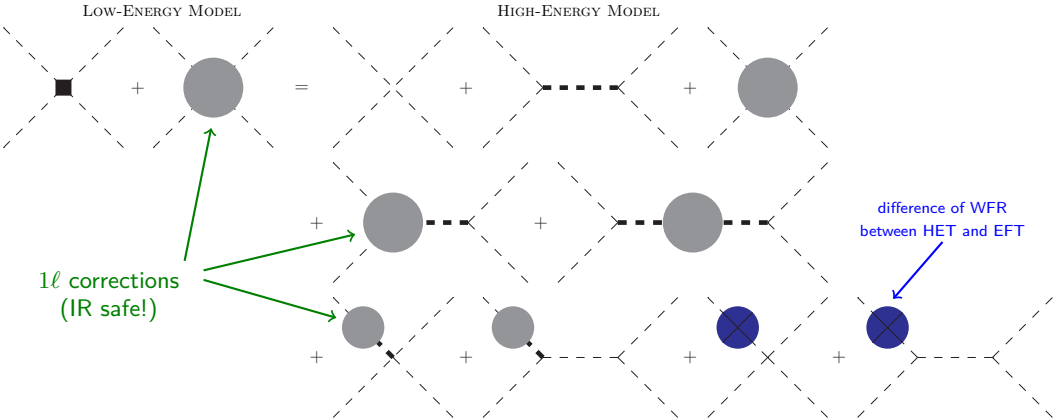
High-energy Theory Low-energy Theory

In the one-loop matching of λ:

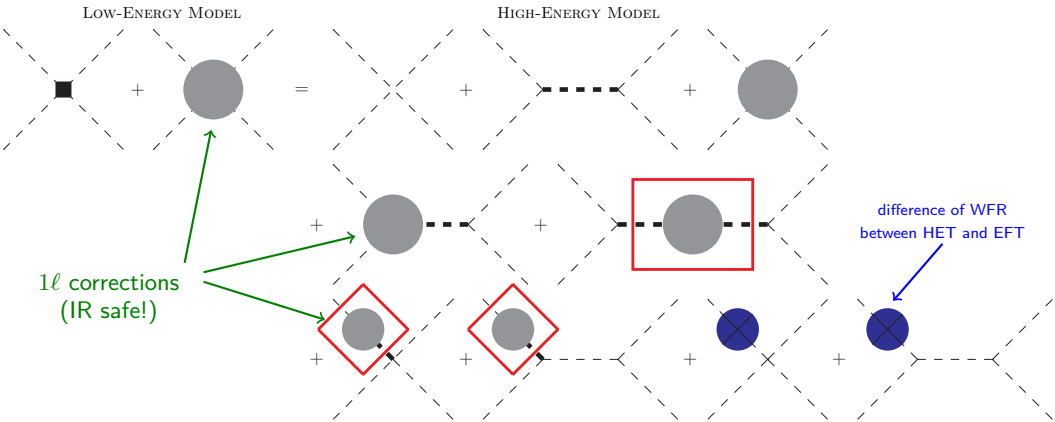


Simplifying the matching condition

Simplifying the matching of quartic couplings between generic theories



Simplifying the matching of quartic couplings between generic theories



▷ Red boxes → heavy-heavy and light-heavy **mass corrections**!

Simplifying the matching of quartic couplings between generic theories

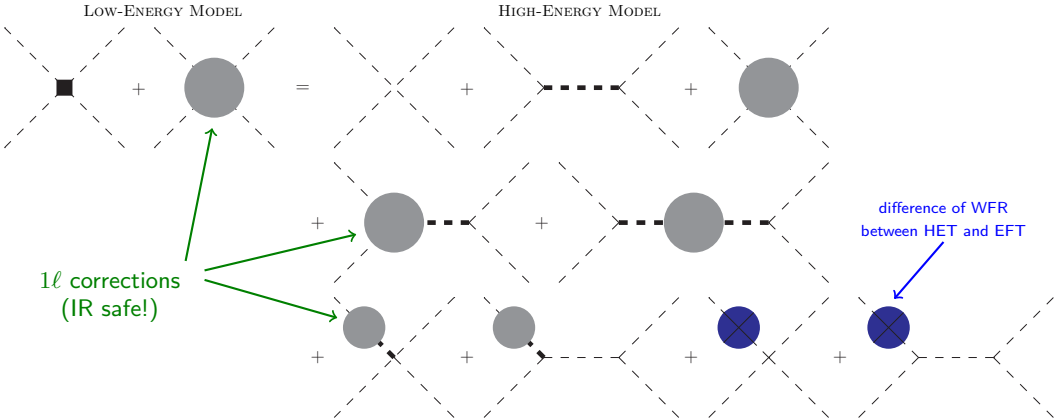
δm^2_{HH} δm^2_{LH}

- δm^2_{HH} *(left)*: correction to heavy masses
- δm^2_{LH} *(right)*: correction to the mixing between light and heavy states

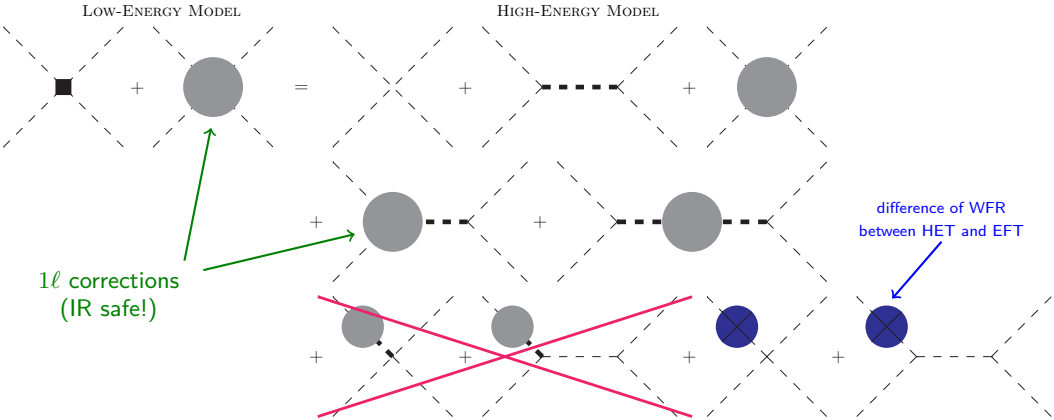
$\rightarrow$ parameters of high-energy theory

- Parameters of UV theory: (usually) arbitrary inputs $\rightarrow$ we are **free to choose the finite part of mass counterterms to simplify the matching!**
  $\longrightarrow$ eliminate mixing between light and heavy scalars + eliminate corrections to heavy masses
- Generalisation of scheme devised in [Bagnaschi, Giudice, Slavich, Strumia '14] for models with 2 Higgs doublets $\rightarrow$ choose $\tan\beta$ counterterm to remove off-diagonal WFR

Simplifying the matching of quartic couplings between generic theories

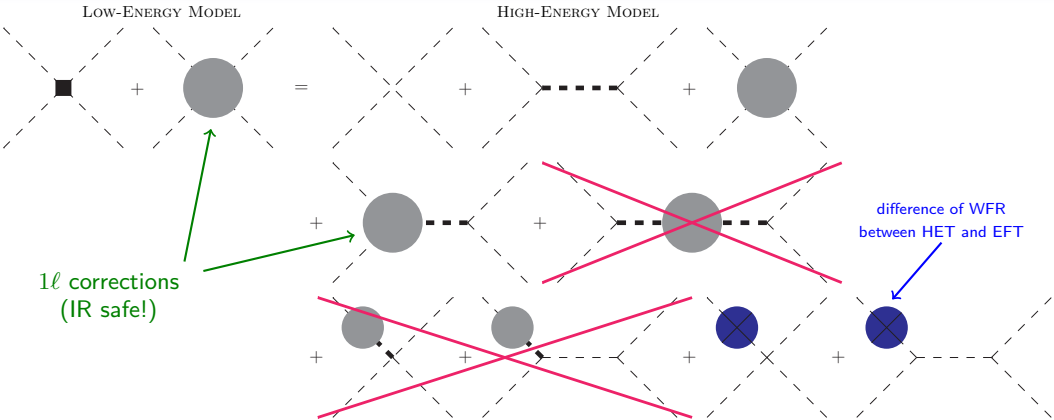


Simplifying the matching of quartic couplings between generic theories



- Redefinition of finite part of light-heavy mass counter-terms allows eliminating δm^2_{LH}
  $\longrightarrow$ **mixing between heavy and light states eliminated from the matching condition!**

Simplifying the matching of quartic couplings between generic theories



- Redefinition of finite part of light-heavy mass counter-terms allows eliminating δm^2_{LH}
  $\longrightarrow$ **mixing between heavy and light states eliminated from the matching condition!**
- Redefinition of finite part of heavy-heavy mass counter-terms allows eliminating δm^2_{HH}
  (numerical comparison in backup)

A simple approach to matching using two-point functions

Pole-mass matching (see $e.g.$ [Athron et al. '16])

- Extracting the threshold corrections to λ_{SM} from

$$\underbrace{2\lambda_{\text{SM}} v_{\text{SM}}^2 + \Delta m_{\text{SM}}^2(p^2 = m_h^2)}_{\text{Higgs pole mass in SM}} = \underbrace{(m_{\text{HET}}^2)^{\text{tree}} + \Delta m_{\text{HET}}^2(p^2 = m_h^2)}_{\text{Higgs pole mass in UV theory}}$$

$$\Rightarrow \lambda_{\text{SM}} = \frac{2}{v_{\text{HET}}^2}\left[m_{\text{HET}}^2\left(1 + [\Pi_{hh}^{\text{HET}\,\prime}(0) - \Pi_{hh}^{\text{SM}\,\prime}(0)]\right) - \frac{m_{\text{HET}}^2}{m_Z^2}\left(\Pi_{ZZ}^{\text{HET}}(0) - \Pi_{ZZ}^{\text{SM}}(0)\right) + \left(\Delta m_{\text{HET}}^2(0) - \Delta m_{\text{SM}}^2(0)\right)\right]$$

$\Pi_{hh}(0)$, $\Pi_{ZZ}(0)$: Higgs and Z-boson self-energies at $p^2 = 0$, Δm^2: corrections to the Higgs mass

- ▷ only really tractable when EFT model does not have mixing in Higgs sector
- ▷ easier to extend beyond one-loop (as 2-point functions are easier to deal with)
- ▷ as is, requires cancellation of large (IR div.) logs, as was our problem earlier

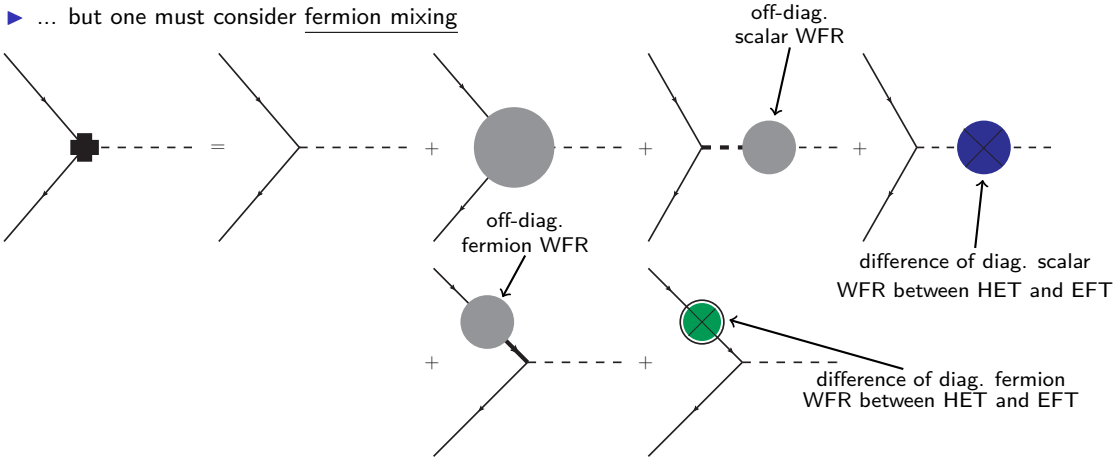
- Formally equivalent to using the modified mass counterterms (*c.f. previous slides*)
- We obtain an efficient way to compute the threshold corrections to λ_{SM} as

$$\lambda_{\text{SM}} = \frac{2}{v_{\text{HET}}^2}\left[m_{\text{HET}}^2\Big(1 + 2\underbrace{[\Pi_{hh}^{\text{HET}\,\prime}(0) - \Pi_{hh}^{\text{SM}\,\prime}(0)]}_{\text{w. light masses} \to 0}\Big) + \underbrace{\hat{\Delta} m_{HET}^2(0)}_{\substack{\text{terms w. light masses only} \to 0 \\ \text{logs of light masses} \to 0 \\ (\text{gauge contributions} \to 0)}}\right]$$

Matching of Yukawa couplings between generic theories

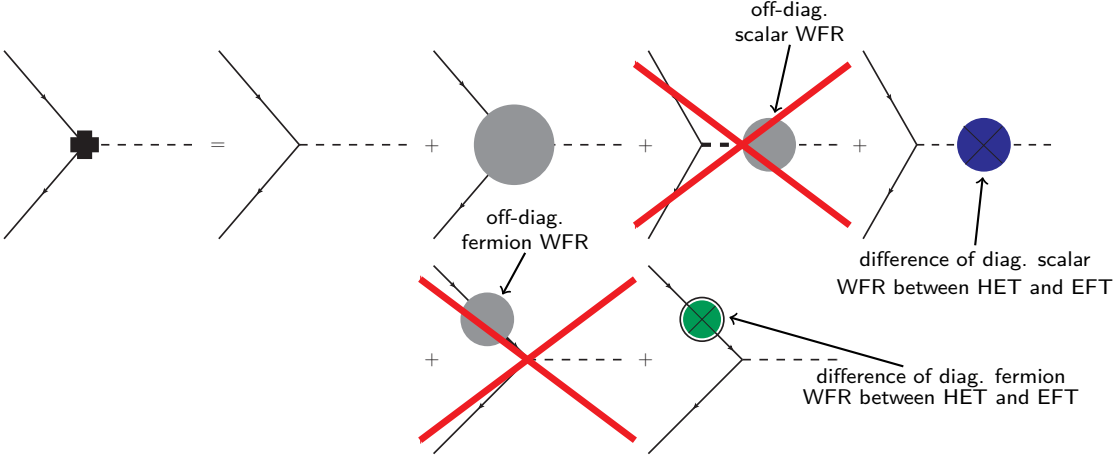
Matching of Yukawa couplings between generic theories

- Fewer diagrams contribute to the matching...
- ... but one must consider fermion mixing



Matching of Yukawa couplings between generic theories

- Again, one can eliminate the mixing between light and heavy states (scalars or fermions) by **modifying the finite part of mass counterterms** → **simpler matching!**



Summary of Part 1

- Use of **Effective Field Theories** becomes increasingly necessary as M_{NP} is driven higher by experimental searches
- When considering the calculation of a given observable in a wide range of scenarios or models
  $\longrightarrow$ **Automation** can provide fast and accurate predictions
- **Modified loop functions and renormalisation scheme choices** now allow simple matching of scalar quartic and Yukawa couplings between generic theories
  (similar results implemented in `SARAH` in [Gabelmann, Mühlleitner, Staub 1810.12326])
- **Efficient approach for pole mass matching**, that will be easier to extend beyond one-loop
- *Next*: going beyond one-loop $\longrightarrow$ use of modified scheme expected to become more important, consider pole-mass matching, ...

Part 2:
Two-loop corrections to the Higgs trilinear coupling

The Higgs trilinear coupling λ_{hhh}

Investigating the Higgs trilinear coupling λ_{hhh}

Theoretical motivations

- Since the Higgs discovery, the existence of the Higgs potential is confirmed, but at the moment we only know:
  - the location of the EW minimum: $v \simeq 246$ GeV
  - the curvature of the potential around the EW minimum: $m_h \simeq 125$ GeV

  However what we still don't know is **shape** of the Higgs potential, which depends on λ_{hhh}

- λ_{hhh} determines whether the EWPT can be of strong first order or not ($\rightarrow$ necessary for EW baryogenesis)

  $\Rightarrow$ large deviation of λ_{hhh} from SM prediction is necessary to have a strongly first-order EWPT [Grojean, Servant, Wells '04], [Kanemura, Okada, Senaha '04]

- In models where the Higgs mass(es) can be calculated, a computation of λ_{hhh} with the same accuracy is needed to consistently interpret results with experimental data

- Higgs couplings such as λ_{hhh} can in principle exhibit (large) effects from BSM Physics

Investigating the Higgs trilinear coupling λ_{hhh}

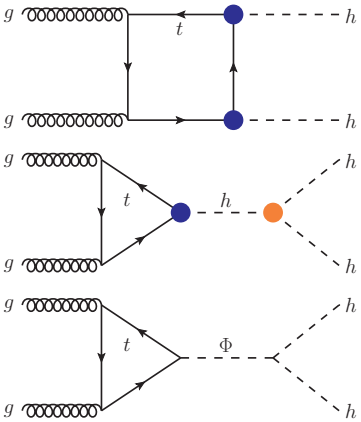
Current experimental limits

- Current limits on $\kappa_\lambda \equiv \lambda_{hhh}/\lambda_{hhh}^{\text{SM}}$ are (at 95% CL)

  $-5.0 < \kappa_\lambda < 12.1$ (ATLAS) and $-11 < \kappa_\lambda < 17$ (CMS)

Future prospects

- HL-LHC with 3 ab^{-1} could reach $0.1 < \kappa_\lambda < 2.3$, and a 27-TeV HE-LHC with 15 ab^{-1} $0.58 < \kappa_\lambda < 1.45$
- ILC-250 cannot measure λ_{hhh}, but 500-GeV and 1-TeV extensions could obtain measurements with precisions of 27% and 10% respectively
- CLIC 1.4 TeV + 3 TeV $\rightarrow$ 20% accuracy
- 100-TeV hadron collider with 30 ab^{-1} $\rightarrow$ 5-7% accuracy

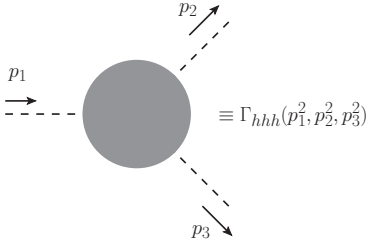


Double Higgs production

Radiative corrections to the Higgs trilinear coupling

- Higgs three-point function, $\Gamma_{hhh}(p_1^2, p_2^2, p_3^2)$, requires a diagrammatic calculation, with non-zero external momentum
- Instead it is much more convenient to work with an effective Higgs trilinear coupling λ_{hhh}

$$\mathcal{L} \supset -\frac{1}{6}\lambda_{hhh} h^3 \quad \rightarrow \quad \underbrace{\lambda_{hhh} = \left.\frac{\partial^3 V_{\text{eff}}}{\partial h^3}\right|_{\text{min.}}}_{\overline{\text{MS}}\text{ result}}$$



$V_{\text{eff}} = V^{(0)} + \Delta V_{\text{eff}}$: effective potential (calculated in $\overline{\text{MS}}$ scheme)

- In effective-potential calculations, one should usual fix conditions for the lower derivatives of V_{eff}

$$\underbrace{\left.\frac{\partial V_{\text{eff}}}{\partial h}\right|_{\text{min.}} = 0,}_{\text{tadpole condition}} \qquad \underbrace{[M_h^2]_{V_{\text{eff}}} = \left.\frac{\partial^2 V_{\text{eff}}}{\partial h^2}\right|_{\text{min.}} - \frac{1}{v}\left.\frac{\partial V_{\text{eff}}}{\partial h}\right|_{\text{min.}}}_{\text{curvature mass of the Higgs}}$$

- Using these, we obtain

$$\lambda_{hhh} = \frac{3[M_h^2]_{V_{\text{eff}}}}{v} + \mathcal{D}_3 \Delta V_{\text{eff}}\Big|_{\text{min.}}, \qquad \text{with } \mathcal{D}_3 \equiv \frac{\partial^3}{\partial h^3} - \frac{3}{v}\left[-\frac{1}{v}\frac{\partial}{\partial h} + \frac{\partial^2}{\partial h^2}\right]$$

Radiative corrections to the Higgs trilinear coupling

- Γ_{hhh} and λ_{hhh} can be related as

$$-\Gamma_{hhh}(0,0,0) = \underbrace{\hat{\lambda}_{hhh}}_{\text{OS result}} = \left(\frac{Z_h^{\text{OS}}}{Z_h^{\overline{\text{MS}}}}\right)^{3/2} \underbrace{\lambda_{hhh}}_{\overline{\text{MS}}\text{ result}} = \left(1 + \frac{3}{2}\frac{d}{dp^2}\Pi_{hh}(p^2)\Big|_{p^2=M_h^2}\right)\lambda_{hhh}$$

$\delta Z_h^{\text{OS},\overline{\text{MS}}} = Z_h^{\text{OS},\overline{\text{MS}}} - 1$: wave-function renormalisation counterterm in OS/$\overline{\text{MS}}$ scheme,
$\Pi_{hh}(p^2)$: finite part of Higgs self-energy at ext. momentum p^2

- Taking $\Gamma_{hhh}(p_1^2, p_2^2, p_3^2) \simeq \Gamma_{hhh}(0,0,0)$ is a good approximation
  - → shown for λ_{hhh} at one loop in [Kanemura, Okada, Senaha, Yuan '04] (difference is only a few %)
  - → no study including external momentum exists at two loops, but in the case of two-loop Higgs mass calculations, momentum effects are known to be subleading

Radiative corrections to the Higgs trilinear coupling and non-decoupling effects

The Two-Higgs-Doublet Model (2HDM)

- CP-conserving 2HDM, with softly-broken $\mathbb{Z}_2$ symmetry ($\Phi_1 \to \Phi_1, \Phi_2 \to -\Phi_2$) to avoid FCNCs
- 2 $SU(2)_L$ doublets $\Phi_{1,2} = \begin{pmatrix} \Phi^+_{1,2} \\ \Phi^0_{1,2} \end{pmatrix}$ of hypercharge $1/2$

$$V^{(0)}_{\text{2HDM}} = m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - m_3^2(\Phi_2^\dagger\Phi_1 + \Phi_1^\dagger\Phi_2) + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_2^\dagger\Phi_1|^2 + \frac{\lambda_5}{2}\left((\Phi_2^\dagger\Phi_1)^2 + \text{h.c.}\right)$$

- 7 free parameters in scalar sector: $m_3^2,\ \lambda_i\ (i = 1\cdots5),\ \tan\beta \equiv \langle\Phi_2^0\rangle/\langle\Phi_1^0\rangle$
  (m_1^2, m_2^2 eliminated with tadpole equations)
- Doublets expanded in terms of mass eigenstates as

$$\begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix} = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix}\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} \qquad \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix} = v\begin{pmatrix} c_\beta \\ s_\beta \end{pmatrix} + \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix}\begin{pmatrix} H \\ h \end{pmatrix} + i\begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix}\begin{pmatrix} G \\ A \end{pmatrix}$$

  h, H: CP-even Higgses, A: CP-odd Higgs, $H^\pm$: charged Higgs, α: CP-even Higgs mixing angle
- $\lambda_i\ (i = 1\cdots5)$ traded for mass eigenvalues m_h, m_H, m_A, $m_{H^\pm}$ and angle α
- m_3^2 replaced by a soft-breaking mass scale $M^2 = 2m_3^2/s_{2\beta}$

Non-decoupling effects in λ_{hhh} at one loop

First studies of the one-loop corrections to λ_{hhh} in the 2HDM in [Kanemura, Kiyoura, Okada, Senaha, Yuan '02] and [Kanemura, Okada, Senaha, Yuan '04]

- Leading one-loop correction to λ_{hhh}

$$\delta^{(1)}\lambda_{hhh} = \underbrace{-\frac{48 m_t^4}{v^3}}_{\text{SM-like}} + \sum_{\Phi=H,A,H^\pm} \underbrace{\frac{4 n_\Phi m_\Phi^4}{v^3}\left(1-\frac{M^2}{m_\Phi^2}\right)^3}_{\text{BSM}}$$

(recall $\lambda_{hhh}^{(0)} = 3m_h^2/v$)

- Masses of additional scalars $\Phi = H, A, H^\pm$ in 2HDM can be written as $m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$
($\tilde{\lambda}_\Phi$: some combination of λ_i)
- Power-like dependence of BSM terms $\propto m_\Phi^4$, and

$$\left(1-\frac{M^2}{m_\Phi^2}\right)^3 \to \begin{cases} 0, & \text{for } M^2 \gg \tilde{\lambda}_\Phi v^2 \\ 1, & \text{for } M^2 \ll \tilde{\lambda}_\Phi v^2 \end{cases}$$

- Huge deviations possible, without violating unitarity!

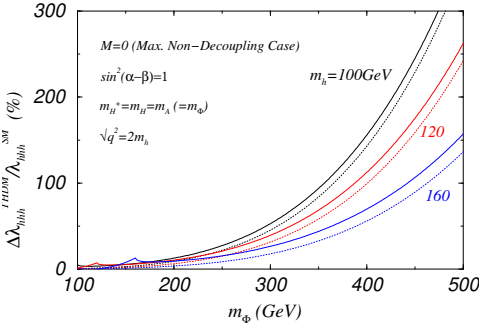


figure from [Kanemura, Okada, Senaha, Yuan '04]

State-of-the-art calculations of λ_{hhh}

At one loop

- Complete diagrammatic, OS-scheme, calculations been performed for a number of BSM models with extended sectors (with singlets, doublets, triplets)
- One-loop calculations available for 2HDMs, HSM, IDM in program `H-COUP` [Kanemura, Kikuchi, Sakurai, Yagyu '17]

Non-decoupling effects found for a range of BSM models at one loop

$\Rightarrow$ **what happens at two loops?**

At two loops

- leading $\mathcal{O}(\alpha_s\alpha_t)$ corrections in MSSM [Brucherseifer, Gavin, Spira '14] and NMSSM [Mühlleitner, Nhung, Ziesche '15], in effective-potential approximation
  $\rightarrow$ two-loop effects can be up to $\mathcal{O}(10\%)$; significant reduction of scale dependence in $\overline{\mathrm{DR}}'$ calculations
- leading scalar corrections calculated in the IDM [Senaha, 1811.00336], also with the effective potential
  $\rightarrow$ two-loop effects found to be a few percent ($\sim 2\%$), and can weaken the strength of the first-order electroweak phase transition in the model

We also want to investigate the fate of non-decoupling effects at two loops

$\Rightarrow$ [J.B., Kanemura 1903.05417]

Our two-loop calculation

Setup of our effective-potential calculation

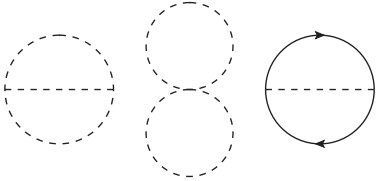
- We obtain corrections to λ_{hhh} as

$$\lambda_{hhh} = \frac{\partial^3 V_{\text{eff}}}{\partial h^3}\bigg|_{\text{min.}} = \frac{3[M_h^2]_{V_{\text{eff}}}}{v} + \left(\frac{\partial^3}{\partial h^3} - \frac{3}{v}\left[-\frac{1}{v}\frac{\partial}{\partial h} + \frac{\partial^2}{\partial h^2}\right]\right)\Delta V_{\text{eff}}\bigg|_{\text{min.}}$$

- $\overline{\text{MS}}$-renormalised two-loop effective potential is

$$V_{\text{eff}} = V^{(0)} + \Delta V_{\text{eff}} = V^{(0)} + \kappa V^{(1)} + \kappa^2 V^{(2)} \qquad \left(\kappa \equiv \frac{1}{16\pi^2}\right)$$

- $V^{(2)}$: 1PI vacuum bubble diags., and for the leading two-loop BSM corrections we need



$V^{(2)}_{SSS}$ $\quad$ $V^{(2)}_{SS}$ $\quad$ $V^{(2)}_{FFS}$

- Here we only want the leading corrections from **additional scalars** and **top quark** $\Rightarrow$ we neglect subleading contributions from h, $G, G^{\pm}$, and light fermions ($\rightarrow$ no need to specify type of 2HDM)
- We consider scenarios **without mixing**: alignment in the 2HDM; and the IDM

Setup of our effective-potential calculation

- OS result is obtained as

$$\hat{\lambda}_{hhh} = \underbrace{\left(\frac{Z_h^{\mathrm{OS}}}{Z_h^{\overline{\mathrm{MS}}}}\right)^{3/2}}_{\text{inclusion of WFR}} \times \underbrace{\lambda_{hhh}}_{\substack{\overline{\mathrm{MS}}\text{ parameters} \\ \text{translated to OS ones}}}$$

- Let's suppose (for simplicity) that λ_{hhh} only depends on one parameter x, as

$$\lambda_{hhh} = f^{(0)}(x^{\overline{\mathrm{MS}}}) + \kappa f^{(1)}(x^{\overline{\mathrm{MS}}}) + \kappa^2 f^{(2)}(x^{\overline{\mathrm{MS}}}) \qquad \left(\kappa = \frac{1}{16\pi^2}\right)$$

and

$$x^{\overline{\mathrm{MS}}} = X^{\mathrm{OS}} + \kappa\delta^{(1)}x + \kappa^2\delta^{(2)}x$$

then in terms of OS parameters

$$\begin{aligned}\lambda_{hhh} =& f^{(0)}(X^{\mathrm{OS}}) + \kappa\left[f^{(1)}(X^{\mathrm{OS}}) + \frac{\partial f^{(0)}}{\partial x}(X^{\mathrm{OS}})\delta^{(1)}x\right] \\ &+ \kappa^2\left[f^{(2)}(X^{\mathrm{OS}}) + \frac{\partial f^{(1)}}{\partial x}(X^{\mathrm{OS}})\delta^{(1)}x + \frac{\partial f^{(0)}}{\partial x}(X^{\mathrm{OS}})\delta^{(2)}x + \frac{\partial^2 f^{(0)}}{\partial x^2}(X^{\mathrm{OS}})(\delta^{(1)}x)^2\right]\end{aligned}$$

Setup of our effective-potential calculation

- OS result is obtained as

$$\hat{\lambda}_{hhh} = \underbrace{\left(\frac{Z_h^{\mathrm{OS}}}{Z_h^{\overline{\mathrm{MS}}}}\right)^{3/2}}_{\text{inclusion of WFR}} \times \underbrace{\lambda_{hhh}}_{\substack{\overline{\mathrm{MS}}\text{ parameters} \\ \text{replaced by OS ones}}}$$

- Let's suppose (for simplicity) that λ_{hhh} only depends on one parameter x, as

$$\lambda_{hhh} = f^{(0)}(x^{\overline{\mathrm{MS}}}) + \kappa f^{(1)}(x^{\overline{\mathrm{MS}}}) + \kappa^2 f^{(2)}(x^{\overline{\mathrm{MS}}}) \qquad \left(\kappa = \frac{1}{16\pi^2}\right)$$

and

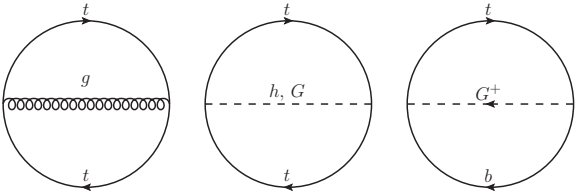
$$x^{\overline{\mathrm{MS}}} = X^{\mathrm{OS}} + \kappa\delta^{(1)}x + \kappa^2\delta^{(2)}x$$

then in terms of OS parameters

$$\lambda_{hhh} = f^{(0)}(X^{\mathrm{OS}}) + \kappa\left[f^{(1)}(X^{\mathrm{OS}}) + \cancel{\frac{\partial f^{(0)}}{\partial x}(X^{\mathrm{OS}})\delta^{(1)}x}\right]$$
$$+ \kappa^2\left[f^{(2)}(X^{\mathrm{OS}}) + \frac{\partial f^{(1)}}{\partial x}(X^{\mathrm{OS}})\delta^{(1)}x + \cancel{\frac{\partial f^{(0)}}{\partial x}(X^{\mathrm{OS}})\delta^{(2)}x} + \cancel{\frac{\partial^2 f^{(0)}}{\partial x^2}(X^{\mathrm{OS}})(\delta^{(1)}x)^2}\right]$$

because we neglect m_h in the loop corrections and $\lambda_{hhh}^{(0)} = 3m_h^2/v$ (in absence of mixing)

Two-loop Standard Model result



- In the SM, 4 diagrams contribute to V_{eff} at order $\mathcal{O}(g_3^2 m_t^4)$ and $\mathcal{O}(m_t^6/v^2)$
- In the limit $m_t \gg m_h, m_G, \cdots$, their expression reads

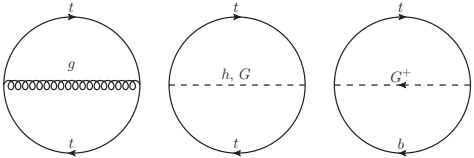
$$V^{(2)} = -4g_3^2 m_t^2 \left[4A(m_t^2) - 8m_t^2 - \frac{6A(m_t^2)^2}{m_t^2}\right] + 3y_t^2 \left[2m_t^2 I(m_t^2, m_t^2, 0) + m_t^2 I(m_t^2, 0, 0) + A(m_t^2)^2\right]$$

$A(x) \equiv x(\log(x/Q^2) - 1)$, I: two-loop sunrise integral

- Then we find in the $\overline{\text{MS}}$ scheme

$$\delta^{(2)}\lambda_{hhh} = \frac{128 g_3^2 m_t^4 (1 + 6\,\overline{\log}\, m_t^2)}{v^3} - \frac{24 m_t^4 y_t^2 (-7 + 6\,\overline{\log}\, m_t^2)}{v^3} \qquad (\overline{\log}\, x \equiv \log x/Q^2)$$

Two-loop Standard Model result



- $\overline{\mathrm{MS}}$ expression

$$\delta^{(2)}\lambda_{hhh} = \frac{128 g_3^2 m_t^4 (1 + 6\,\overline{\log}\, m_t^2)}{v^3} - \frac{24 m_t^4 y_t^2 (-7 + 6\,\overline{\log}\, m_t^2)}{v^3} \qquad (\overline{\log}\, x \equiv \log x/Q^2)$$

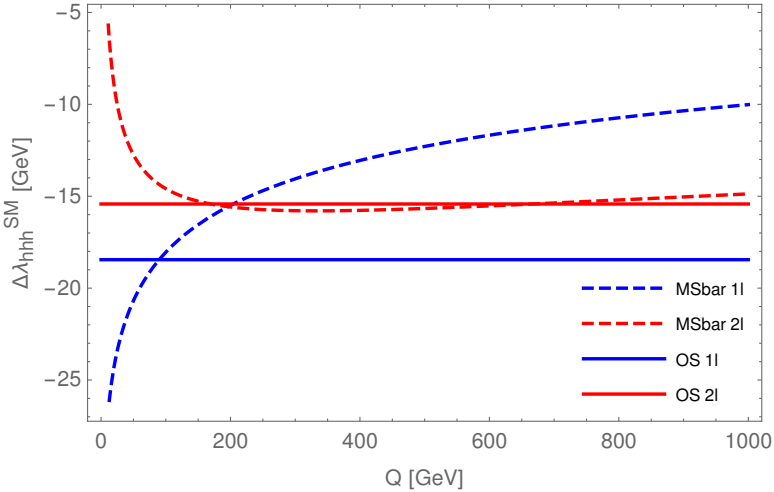
- Translate top quark mass and Higgs VEV from $\overline{\mathrm{MS}}$ to OS scheme in $\delta^{(1)}\lambda_{hhh} = -\frac{48 m_t^4}{v^3}$

$$m_t^2 \to M_t^2 - \Pi_{tt}(p^2 = M_t^2) \qquad v \to \frac{1}{\sqrt{\sqrt{2} G_F}} + \delta v = v_{\text{phys}} + \delta v$$

$+$ include wave-function renormalisation
$\to$ OS-scheme result

$$\delta^{(2)}\hat{\lambda}_{hhh} = \frac{72 M_t^4}{v_{\text{phys}}^3}\left(16 g_3^2 - \frac{13 M_t^2}{v_{\text{phys}}^2}\right)$$

Two-loop Standard Model result

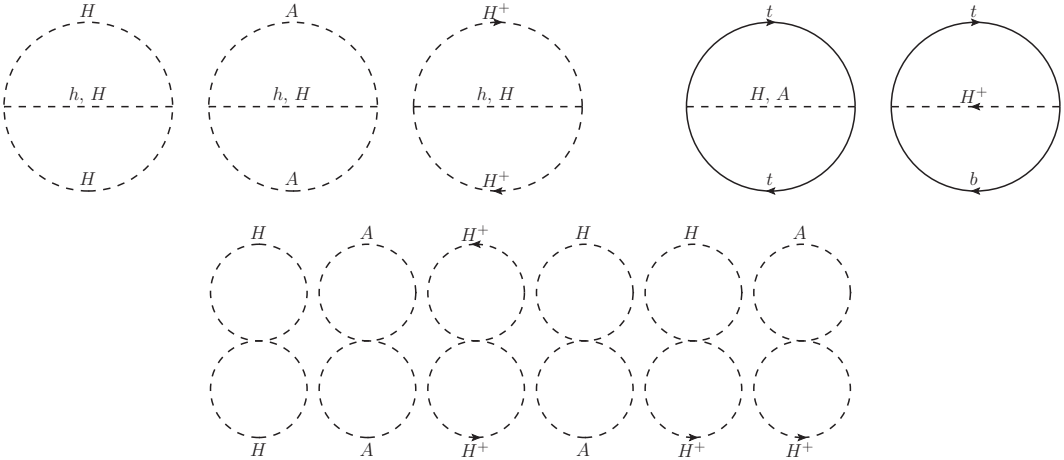


$$\Delta\lambda_{hhh}^{\text{SM}} = \lambda_{hhh}^{\text{SM}} - \lambda_{hhh}^{\text{SM},(0)}$$

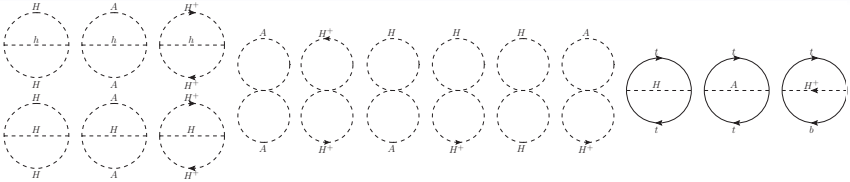
λ_{hhh} in the Two-Higgs-Doublet Model

λ_{hhh} at two loops in the 2HDM

15 new BSM diagrams appearing in $V^{(2)}$ in the 2HDM w.r.t. the SM case



λ_{hhh} at two loops in the 2HDM



- We assume the additional scalars of the 2HDM – H, A, $H^\pm$ – to have a degenerate mass m_Φ $\to$ 3 mass scales in the calculation: m_t, m_Φ, M ($\to$ simpler analytical expressions)
- We take the alignment limit $s_{\beta-\alpha} = 1$ and neglect loop-induced deviations from this condition
- In the $\overline{\text{MS}}$ scheme

$$\delta^{(2)}\lambda_{hhh} = \frac{16 m_\Phi^4}{v^5}\left(4 + 9\cot^2 2\beta\right)\left(1 - \frac{M^2}{m_\Phi^2}\right)^4 \left[-2M^2 - m_\Phi^2 + (M^2 + 2m_\Phi^2)\,\overline{\log}\, m_\Phi^2\right]$$
$$+ \frac{192 m_\Phi^6 \cot^2 2\beta}{v^5}\left(1 - \frac{M^2}{m_\Phi^2}\right)^4 \left[1 + 2\,\overline{\log}\, m_\Phi^2\right]$$
$$+ \frac{96 m_\Phi^4 m_t^2 \cot^2\beta}{v^5}\left(1 - \frac{M^2}{m_\Phi^2}\right)^3 \left[-1 + 2\,\overline{\log}\, m_\Phi^2\right] + \mathcal{O}\left(\frac{m_\Phi^2 m_t^4}{v^5}\right)$$

Decoupling behaviour

- Seeing whether corrections from additional BSM states decouple if said state is taken to be very massive is a good way to check the consistency of the calculation

$$\delta^{(2)}\lambda_{hhh} = \frac{16m_\Phi^4}{v^5}\left(4+9\cot^2 2\beta\right)\left(1-\frac{M^2}{m_\Phi^2}\right)^4\left[-2M^2-m_\Phi^2+(M^2+2m_\Phi^2)\,\overline{\log}\,m_\Phi^2\right]$$

$$\delta^{(1)}\lambda_{hhh} = \frac{16m_\Phi^4}{v^3}\left(1-\frac{M^2}{m_\Phi^2}\right)^3 + \frac{192m_\Phi^6\cot^2 2\beta}{v^5}\left(1-\frac{M^2}{m_\Phi^2}\right)^4\left[1+2\,\overline{\log}\,m_\Phi^2\right] + \frac{96m_\Phi^4 m_t^2\cot^2\beta}{v^5}\left(1-\frac{M^2}{m_\Phi^2}\right)^3\left[-1+2\,\overline{\log}\,m_\Phi^2\right] + \mathcal{O}\left(\frac{m_\Phi^2 m_t^4}{v^5}\right)$$

where $m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$

- To have $m_\Phi \to \infty$, then we must take $M \to \infty$, otherwise the quartic couplings grow out of control
- Fortunately all of these terms go like

$$(m_\Phi^2)^{n-1}\left(1-\frac{M^2}{m_\Phi^2}\right)^n \underset{m_\Phi^2 = M^2+\tilde{\lambda}_\Phi v^2}{=} \frac{(\tilde{\lambda}_\Phi v^2)^n}{M^2+\tilde{\lambda}_\Phi v^2} \xrightarrow[\tilde{\lambda}_\Phi v^2 \text{ fixed}]{M\to\infty} 0$$

Decoupling behaviour and $\overline{\text{MS}}$ to OS scheme conversion

- To express $\delta^{(2)}\lambda_{hhh}$ in terms of physical parameters (v_{phys}, M_t, $M_A = M_H = M_{H^\pm} = M_\Phi$), we replace

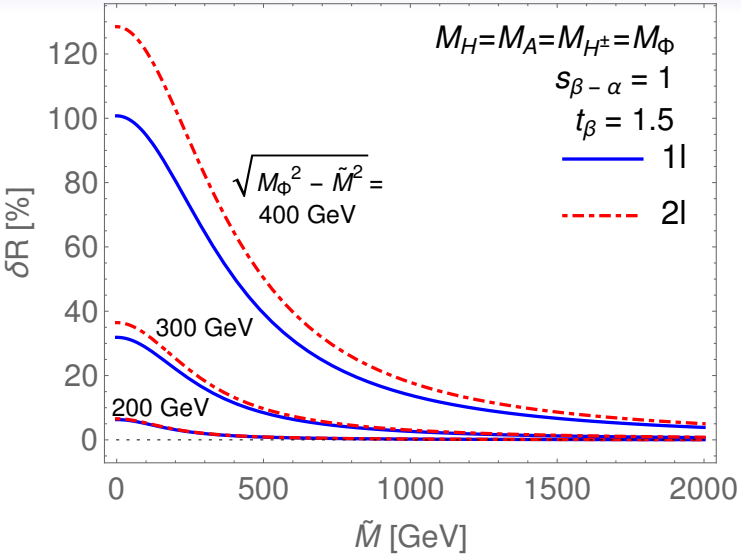
$$m_A^2 \to M_A^2 - \Pi_{AA}(M_A^2), \quad m_H^2 \to M_H^2 - \Pi_{HH}(M_H^2), \quad m_{H^\pm}^2 \to M_{H^\pm}^2 - \Pi_{H^+H^-}(M_{H^\pm}^2),$$

$$v \to v_{\text{phys}} - \delta v, \quad m_t^2 \to M_t^2 - \Pi_{tt}(M_t^2)$$

- A priori, M is still renormalised in $\overline{\text{MS}}$ scheme, because it is difficult to relate to physical observable
  ... but then, expressions do not decouple for $M_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$ and $M \to \infty$!
- This is because we should relate M_Φ, renormalised in OS scheme, and M, renormalised in $\overline{\text{MS}}$ scheme, with a **one-loop relation** $\to$ then the two-loop corrections decouple properly
- We give a new "OS" prescription for the finite part of the counterterm for M be requiring that
  1. the decoupling of $\delta^{(2)}\hat{\lambda}_{hhh}$ (in OS scheme) is apparent using a relation $M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$
  2. all the log terms in $\delta^{(2)}\hat{\lambda}_{hhh}$ are absorbed in δM^2

$$\delta^{(2)}\hat{\lambda}_{hhh} = \frac{48M_\Phi^6}{v_{\text{phys}}^5}\left(1 - \frac{\tilde{M}^2}{M_\Phi^2}\right)^4 \left\{4 + 3\cot^2 2\beta\left[3 - \frac{\pi}{\sqrt{3}}\left(\frac{\tilde{M}^2}{M_\Phi^2} + 2\right)\right]\right\} + \frac{576M_\Phi^6\cot^2 2\beta}{v_{\text{phys}}^5}\left(1 - \frac{\tilde{M}^2}{M_\Phi^2}\right)^4$$
$$+ \frac{288M_\Phi^4 M_t^2\cot^2\beta}{v_{\text{phys}}^5}\left(1 - \frac{\tilde{M}^2}{M_\Phi^2}\right)^3 + \frac{168M_\Phi^4 M_t^2}{v_{\text{phys}}^5}\left(1 - \frac{\tilde{M}^2}{M_\Phi^2}\right)^3 - \frac{48M_\Phi^6}{v_{\text{phys}}^5}\left(1 - \frac{\tilde{M}^2}{M_\Phi^2}\right)^5 + \mathcal{O}\left(\frac{M_\Phi^2 M_t^4}{v_{\text{phys}}^5}\right)$$

Decoupling behaviour

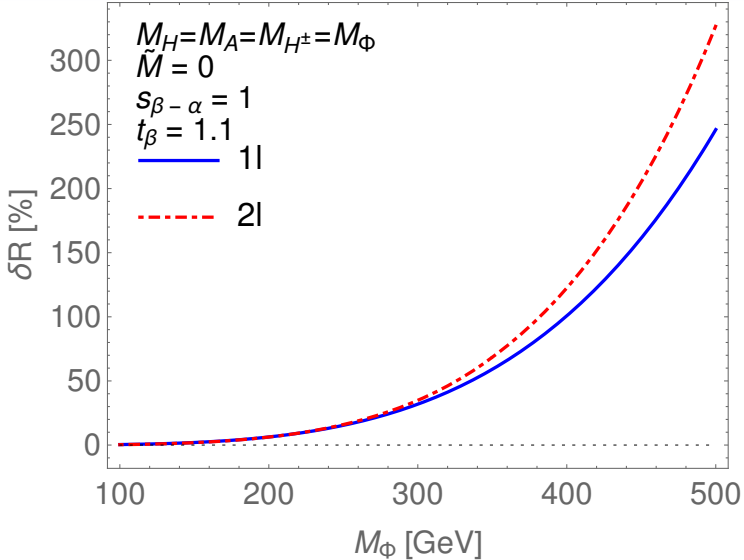


- δR size of BSM contributions to λ_{hhh}:

$$\delta R \equiv \frac{\lambda_{hhh}^{\text{2HDM}}}{\lambda_{hhh}^{\text{SM}}} - 1$$

- Radiative corrections from additional scalars + top quark indeed decouple properly for $\tilde{M} \to \infty$

Non-decoupling effects



$$\delta R \equiv \frac{\lambda_{hhh}^{\text{2HDM}}}{\lambda_{hhh}^{\text{SM}}} - 1$$

- Other limit of interest: $\tilde{M} = 0 \rightarrow$ maximal non-decoupling effects
- $\delta^{(1)}\hat{\lambda}_{hhh} \rightarrow \propto M_\Phi^4$
- $\delta^{(2)}\hat{\lambda}_{hhh} \rightarrow \propto M_\Phi^6$
- For $\tilde{M} = 0$, $\tan\beta = 1.1$, tree-level unitarity is lost around $M_\Phi \approx 600$ GeV [Kanemura, Kubota, Takasugi '93]

λ_{hhh} in the Inert Doublet Model

The Inert Doublet Model

- Model of 2 $SU(2)_L$ doublets, with $\mathbb{Z}_2$ symmetry under which $\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$ unbroken *after EWSB*

$$V_{\text{IDM}}^{(0)} = \mu_1^2|\Phi_1|^2 + \mu_2^2|\Phi_2|^2 + \frac{\lambda_1}{2}|\Phi_1^2|^4 + \frac{\lambda_2}{2}|\Phi_2^2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \frac{\lambda_5}{2}\left((\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right)$$

- Expand the doublets as

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h + iG) \end{pmatrix} \qquad \Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H + iA) \end{pmatrix}$$

$H, A, H^\pm$: inert scalars (no couplings to fermions, no scalar mixing)

- Tree-level masses of scalars read

$$m_h^2(h) = \mu_1^2 + \frac{3}{2}\lambda_1(v+h)^2, \qquad m_G^2(h) = m_{G^\pm}^2(h) = \mu_1^2 + \frac{1}{2}\lambda_1(v+h)^2$$

$$m_H^2 = \mu_2^2 + \frac{1}{2}\lambda_H(v+h)^2, \qquad m_A^2 = \mu_2^2 + \frac{1}{2}\lambda_A(v+h)^2, \qquad m_{H^\pm}^2 = \mu_2^2 + \frac{1}{2}\lambda_3(v+h)^2$$

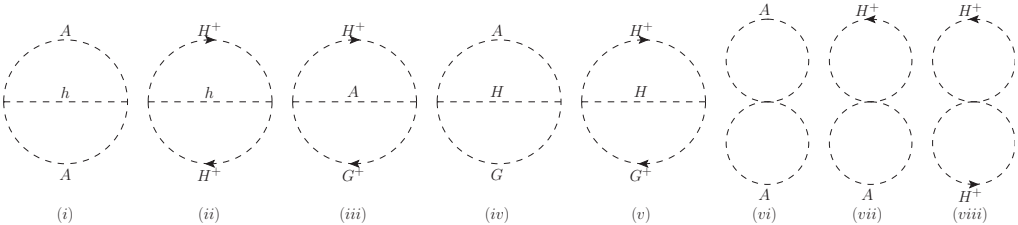
- We consider a DM-inspired scenario in which H is light and is DM ($M_H \simeq M_h/2$), and to maximise the leading corrections to λ_{hhh} we consider μ_2 small, *i.e.*

$$M_h, M_H, \mu_2 \ll M_A, M_{H^\pm}$$

NB: studied first in [Senaha 1811.00336]

λ_{hhh} at two loops in the IDM

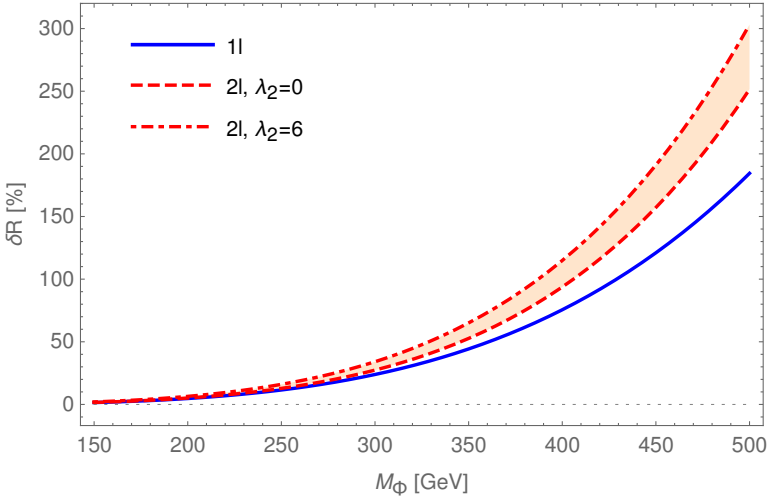
- 8 new diagrams appearing in $V^{(2)}$ in the IDM w.r.t. the SM



- Only (i) and (ii) studied in [Senaha '18] $\rightarrow$ in particular (vi)-$(viii)$ depend on inert scalar quartic λ_2
- After conversion to the OS scheme

$$\delta^{(2)}\hat{\lambda}_{hhh} = \frac{6\lambda_2}{v_{\text{phys}}^3}\left(3M_A^4 + 4M_A^2 M_{H^\pm}^2 + 8M_{H^\pm}^4\right) + \frac{60(M_A^6 + 2M_{H^\pm}^6)}{v_{\text{phys}}^5} + \frac{24(M_A^2 - M_{H^\pm}^2)^2(M_A^2 + M_{H^\pm}^2)}{v_{\text{phys}}^5}$$
$$+ \frac{24M_t^4(M_A^2 + 2M_{H^\pm}^2)}{v_{\text{phys}}^5} + \frac{42M_t^2(M_A^4 + 2M_{H^\pm}^4)}{v_{\text{phys}}^5} - \frac{2(M_A^4 + 2M_{H^\pm}^4)(M_A^2 + 2M_{H^\pm}^2)}{v_{\text{phys}}^5}\,.$$

Numerical results



$$\delta R \equiv \frac{\lambda_{hhh}^{\text{IDM}}}{\lambda_{hhh}^{\text{SM}}} - 1$$

$M_A = M_{H^\pm} = M_\Phi$ to keep ρ parameter close to 1,

λ_2 as large as possible under criterion of tree-level unitarity [Kanemura, Kubota, Takasugi]

Summary of Part 2

- **First two-loop calculation of λ_{hhh} in 2HDM**, in a scenario with alignment and degenerate masses

- Continue the work of [Senaha '18] in the IDM, including more diagrams $\rightarrow$ **found effect of λ_2**

- Two-loop corrections to λ_{hhh} remain smaller than one-loop contributions, at least as long as perturbative unitarity is maintained $\rightarrow$ **typical size $10 - 20\%$ of one-loop contributions**
  - $\Rightarrow$ non-decoupling effects found at one loop are **not drastically changed**
  - $\Rightarrow$ in the (future) perspective of a precise measurement of λ_{hhh}, computing corrections beyond one loop will be **necessary**

- Many avenues for more work:
  - $\rightarrow$ more complete two-loop calculation (effective potential, diagrammatic), including mixing effects, momentum dependence
  - $\rightarrow$ investigate more precisely implications of two-loop calculation on allowed parameter space
  - $\rightarrow$ consider more models (general renormalisable models?)
  - $\rightarrow$ study two-loop corrections to other Higgs couplings

Thank you for your attention!

Backup

A simple toy model

- Consider a theory with 3 real scalars – h_1, h_2, S – and 2 $\mathbb{Z}_2$ symmetries:

$$(h_1, h_2, S) \xrightarrow{\mathbb{Z}_2^A} (-h_1, -h_2, S) \quad \text{and} \quad (h_1, h_2, S) \xrightarrow{\mathbb{Z}_2^B} (h_1, h_2, -S)$$

- h_1, h_2 mix $\longrightarrow$ 2 mass eigenstates h, H

- **Mass spectrum:** h light; H and S heavy

- Allowed Lagrangian terms are

$$\mathcal{L} = \frac{1}{2}\left(\partial^\mu h_i\right)^2 + \frac{1}{2}\left(\partial^\mu S\right)^2 - \frac{1}{2}m_{ij}^2 h_i h_j - \frac{1}{2}m_S^2 S^2 - \frac{1}{24}\tilde{\lambda}^{ijkl} h_i h_j h_k h_l - \frac{1}{4}\tilde{\lambda}^{ijSS} h_i h_j S^2 - \frac{1}{24}\tilde{\lambda}^{SSSS} S^4$$

- We need to diagonalise the mass matrix of $h_{1,2}$, but the **rotation matrix** ($\sim$**mixing angle**) **depends on the scheme**

Matching of the light scalar quartic coupling in the toy model

$$\mathcal{L} = \frac{1}{2}\left(\partial^\mu h_i\right)^2 + \frac{1}{2}\left(\partial^\mu S\right)^2 - \frac{1}{2}m_{ij}^2 h_i h_j - \frac{1}{2}m_S^2 S^2 - \frac{1}{24}\tilde{\lambda}^{ijkl} h_i h_j h_k h_l - \frac{1}{4}\tilde{\lambda}^{ijSS} h_i h_j S^2 - \frac{1}{24}\tilde{\lambda}^{SSSS} S^4$$

$\overline{\text{MS}}$ scheme

$$\begin{pmatrix} m_{11}^2 & m_{12}^2 \\ m_{12}^2 & m_{22}^2 \end{pmatrix} \xrightarrow{R} \begin{pmatrix} m_h^2 & 0 \\ 0 & m_H^2 \end{pmatrix}$$

$$\tilde{\lambda}^{ijkl}, \tilde{\lambda}^{ijSS} \xrightarrow{R} \tilde{\lambda}^{hhhh}, \tilde{\lambda}^{hhhH}, \tilde{\lambda}^{hhHH}, \tilde{\lambda}^{hHHH}, \tilde{\lambda}^{HHHH}, \tilde{\lambda}^{hhSS}, \tilde{\lambda}^{hHSS}, \tilde{\lambda}^{HHSS}$$

The $\overline{\text{MS}}$ matching of λ^{hhhh} gives

Blue: h; Red: H; Green: S

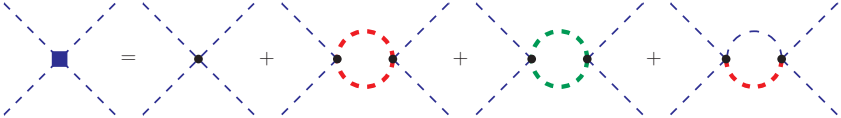
Matching of the light scalar quartic coupling in the toy model

$\overline{\text{MS}}$ scheme

Modified counterterm scheme

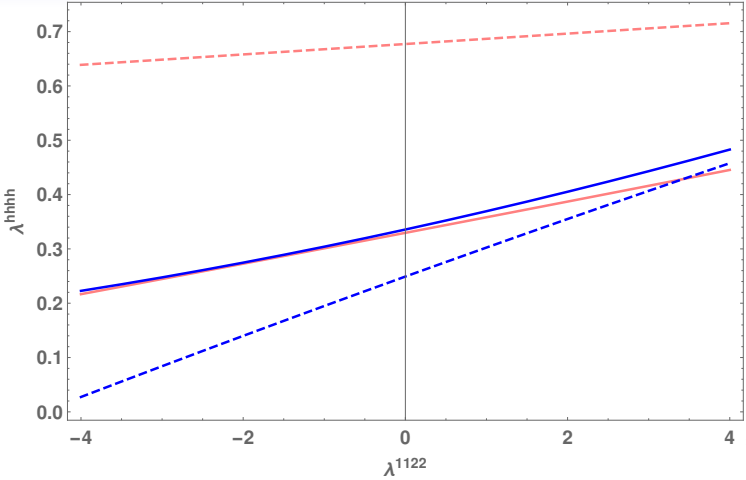
$$R^T \begin{pmatrix} m_h^2 & \overline{\delta} m_{hH}^2 \\ \overline{\delta} m_{hH}^2 & m_H^2 \end{pmatrix} R \xrightarrow{R'} \begin{pmatrix} (m_h^2)_{\text{c.t.}} & 0 \\ 0 & (m_H^2)_{\text{c.t.}} \end{pmatrix}$$

$$\tilde{\lambda}^{ijkl}, \tilde{\lambda}^{ijSS} \xrightarrow{R'} \tilde{\lambda}^{hhhh}_{\text{c.t.}}, \tilde{\lambda}^{hhhH}_{\text{c.t.}}, \tilde{\lambda}^{hhHH}_{\text{c.t.}}, \tilde{\lambda}^{hHHH}_{\text{c.t.}}, \tilde{\lambda}^{HHHH}_{\text{c.t.}}, \tilde{\lambda}^{hhSS}_{\text{c.t.}}, \tilde{\lambda}^{hHSS}_{\text{c.t.}}, \tilde{\lambda}^{HHSS}_{\text{c.t.}}$$



Blue: h; Red: H; Green: S

Numerical comparison



Dashed: tree-level; *solid:* one-loop
Red: $\overline{\text{MS}}$ scheme; *Blue:* modified counterterms

In the original basis:

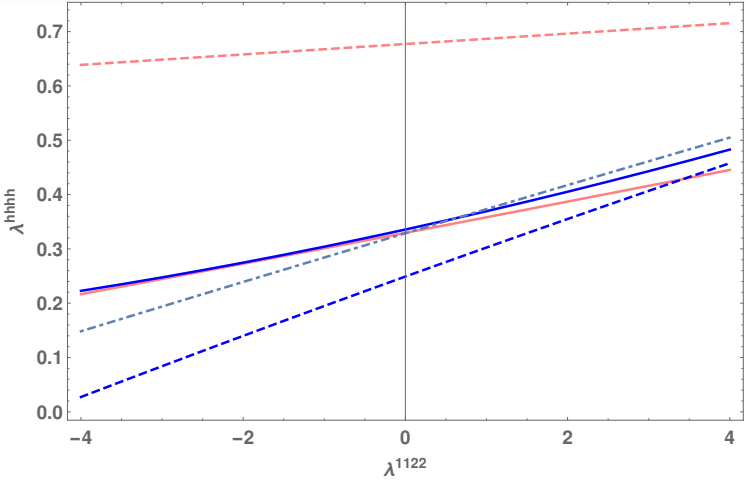
$m_{11}^2 = (100\text{ GeV})^2$, $m_{12}^2 = (400\text{ GeV})^2$,
$m_{22}^2 = (2000\text{ GeV})^2$, $m_S^2 = (5000\text{ GeV})^2$,
$\tilde{\lambda}^{1111} = 1$, $\tilde{\lambda}^{1112} = 2$, $\tilde{\lambda}^{1122} \in [-4, 4]$,
$\tilde{\lambda}^{1222} = 1.5$, $\tilde{\lambda}^{2222} = 0.5$,
$\tilde{\lambda}^{11SS} = 0$, $\tilde{\lambda}^{12SS} = 3.5$, $\tilde{\lambda}^{22SS} = 0$
$\Rightarrow m_h = 60$ GeV, $m_H = 2002$ GeV

- This parameter point: small mixing between h and H at tree-level $\rightarrow$ loop-induced mixing is then large

$\Rightarrow$ large difference between schemes at tree-level

$\Rightarrow$ large loop corrections in $\overline{\text{MS}}$ calculation

Numerical comparison



Dashed: tree-level; *solid & dot-dashed:* one-loop
Red: $\overline{\text{MS}}$ scheme; *Blue:* modified counterterms

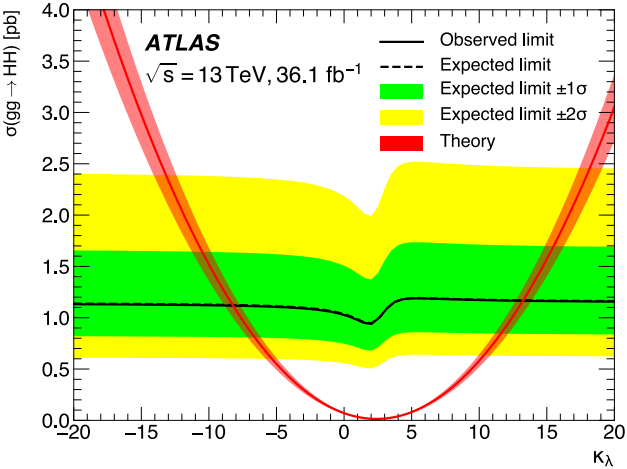
In the original basis:

$m_{11}^2 = (100\text{ GeV})^2$, $m_{12}^2 = (400\text{ GeV})^2$,
$m_{22}^2 = (2000\text{ GeV})^2$, $m_S^2 = (5000\text{ GeV})^2$,
$\tilde{\lambda}^{1111} = 1$, $\tilde{\lambda}^{1112} = 2$, $\tilde{\lambda}^{1122} \in [-4, 4]$,
$\tilde{\lambda}^{1222} = 1.5$, $\tilde{\lambda}^{2222} = 0.5$,
$\tilde{\lambda}^{11SS} = 0$, $\tilde{\lambda}^{12SS} = 3.5$, $\tilde{\lambda}^{22SS} = 0$

$$\Rightarrow m_h^2 = 60\text{ GeV},\ m_H^2 = 2002\text{ GeV}$$

Blue dashed: all couplings in modified scheme;
Blue dot-dashed: couplings in one-loop correction in $\overline{\text{MS}}$ scheme

An example of experimental limits on λ_{hhh}



Example of current limits on κ_λ from the ATLAS search of $hh \to b\bar{b}\gamma\gamma$
(*taken from* [ATLAS collaboration 1807.04873])