Potentially interesting aspects of the effective potential

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Ref.

- JhP, 1902.04559

Effective potential

Essential object, e.g. for studying:

- symmetry breaking
- vacuum stability
- phase transitions at zero and finite temperatures
- cosmic inflation
- radiative corrections to Higgs mass

Generating functionals

"Partition function (Zustandssumme)"

$$Z[J] = \int \mathcal{D}\phi \exp\left[i \int d^4x \left(\mathcal{L}(x) + J(x)\phi(x) \right) \right]$$
$$= Z[0] \langle \Omega | \Omega \rangle_J = \exp\left(i W[J] \right)$$

= generating functional for correlation functions

Phase of vacuum to vacuum amplitude

$$W[J] = -i \ln Z[J] = -T E_\Omega[J]$$

= generating functional for connected correlation functions

1PI effective action

Legendre transform of vacuum energy functional

Jona-Lasinio, Nuovo Cim.34(1964)1790

$$\Gamma[\phi_{\rm cl}] = W[J] - \int d^4x\, J(x)\phi_{\rm cl}(x)$$

= generating functional for 1PI correlation functions

$$\frac{\delta W[J]}{\delta J(x)} = \frac{-i}{Z[J]}\frac{\delta Z[J]}{\delta J(x)} = \frac{\int \mathcal{D}\phi\, e^{i\int d^4x\,(\mathcal{L}+J\phi)}\phi(x)}{\int \mathcal{D}\phi\, e^{i\int d^4x\,(\mathcal{L}+J\phi)}}$$

$$= \frac{\langle\Omega|\phi(x)|\Omega\rangle_J}{\langle\Omega|\Omega\rangle_J} = \phi_{\rm cl}(x)$$

= Expectation value of $\phi(x)$ in the presence of $J(x)$

$$\frac{\delta\Gamma[\phi_{\rm cl}]}{\delta\phi_{\rm cl}(x)} = -J(x) \qquad = 0 \text{ if } \phi_{\rm cl} = \langle\phi\rangle$$

Effective potential or “free energy”

Effective action for x-independent $\phi_{\rm cl}$

$$V_{\rm np}(\phi_{\rm cl}) = -\frac{1}{VT}\Gamma[\phi_{\rm cl}]\Big|_{\phi_{\rm cl}=\text{const.}}$$

Jackiw, PRD9(1974)1686

- real-valued by definition
- depends on gauge and renormalization scale

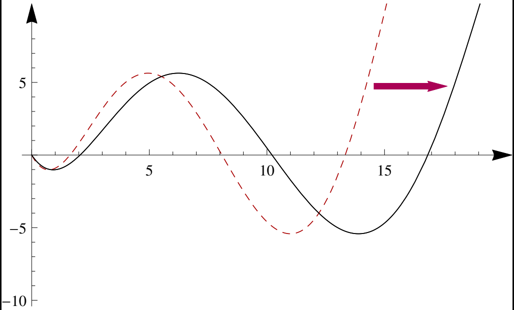


Fig. from Andreassen, Frost, Schwartz, PRD(2015)

Loop expansion of effective potential

$$V_{\rm np} \simeq V_{\rm pert} = V_0 + V_1 + V_2 + \cdots$$

$$V_0(\phi_{\rm cl}) = \text{tree-level potential in } \mathcal{L}$$

$$V_1(\phi_{\rm cl}) = \frac{M^4(\phi_{\rm cl})}{64\pi^2}\left(\ln\frac{M^2(\phi_{\rm cl})}{\mu^2} - \frac{3}{2}\right) \quad \text{in } \overline{\rm MS} \text{ scheme}$$

- develops imaginary part where V_0 is concave
- In this talk, allow $V_{\rm pert}$ to include nonperturbative contributions as in axion or meson potential

Energy interpretation

Kurt Symanzik, CMP16(1970)48

$$V_{\mathrm{np}}(\phi) = \frac{1}{V} \min_{\Omega} \frac{\langle \Omega | H | \Omega \rangle}{\langle \Omega | \Omega \rangle} \quad \text{s.t.} \quad \frac{\langle \Omega | \Phi | \Omega \rangle}{\langle \Omega | \Omega \rangle} = \phi$$

$V_{\mathrm{np}}(\phi)$ is the minimum of the expectation value of the energy density for all states constrained by the condition that the scalar fields Φ have expectation values ϕ

Convexity of effective potential

Kurt Symanzik, CMP16(1970)48

$$V_{\mathrm{np}}\big(x\phi_1 + (1-x)\phi_2\big) \leq xV_{\mathrm{np}}(\phi_1) + (1-x)V_{\mathrm{np}}(\phi_2)$$

for $0 \leq x \leq 1$

- Can be understood by taking linear combination of states

$$\sqrt{x}\,|\Omega_1\rangle + \sqrt{1-x}\,|\Omega_2\rangle \quad \text{with} \quad \langle\Omega_{1,2}|\Omega_{1,2}\rangle = 1$$

$$\langle\Omega_{1,2}|\Phi|\Omega_{1,2}\rangle = \phi_{1,2}, \quad \langle\Omega_{1,2}|H|\Omega_{1,2}\rangle/V = V_{\mathrm{np}}(\phi_{1,2})$$

or by imagining a state with volumes in different phases

- Convexity of Gibbs free energy is also known in statistical mechanics

"Maxwell construction"

- $V_{\rm np}(\phi)$ is linear between two local minima of $V_{\rm pert}(\phi)$

Fujimoto, O'Raifeartaigh, Parravicini, NPB212(1983)268

- Approximation by linear interpolation

$$V_{\rm np}\big(x\phi_1 + (1-x)\phi_2\big) \simeq xV_{\rm pert}(\phi_1) + (1-x)V_{\rm pert}(\phi_2)$$

for $0 < x < 1$

guaranteed to be real

- QFT analogue of Maxwell construction for free energies in thermodynamics

dS swampland conjecture

- A low energy effective theory belongs to landscape if it has string theory as its UV completion
- Otherwise it belongs to swampland
- Scalar potential of a low energy effective theory in landscape satisfies

Obied, Ooguri, Spodyneiko, Vafa (2018)

$$M_{\mathrm{Pl}} |\nabla V| > c\, V, \quad 0 < c \sim \mathcal{O}(1)$$

- Excludes de Sitter extrema
- Refined to include alternative condition

Ooguri, Palti, Shiu, Vafa (2018)

$$M_{\mathrm{Pl}}^2 \min(\nabla_i \nabla_j V) \leq -c'\, V, \quad 0 < c' \sim \mathcal{O}(1)$$

Old dS swampland conjecture vs HEP models

- Quintessence could accommodate observed cosmological constant and local maximum of Higgs potential

  Denef, Hebecker, Wrase (2018)

- Severely constrained by a long-range force and time dependence of proton-to-electron mass ratio

  Hamaguchi, Ibe, Moroi (2018)

- Quintessence + pion extremum requires $c < 1.4 \times 10^{-2}$

  K. Choi, D. Chway, C. S. Shin (2018)

- QCD axion becomes difficult

  Murayama, Yamazaki, Yanagida (2018)

- Root of problems is exclusion of positive local maxima of V

QFT questions about dS conjectures

- Which V? $V_{\rm np}$? V_0? $V_{\rm pert}$?
- How to interpret inequalities

$$M_{\rm Pl}\,|\nabla V| > c\,V, \quad 0 < c \sim \mathcal{O}(1)$$
$$M_{\rm Pl}^2 \min(\nabla_i\nabla_j V) \leq -c'\,V, \quad 0 < c' \sim \mathcal{O}(1)$$

if $\quad \mathrm{Im}\,V \neq 0$?

- Gauge and scale dependence of V

Could free energy interpretation save old dS conjecture?

Kobakhidze, 1901.08137; JhP, 1902.04559

Suppose V is V_{np} in dS swampland conjecture

$$M_{\text{Pl}} \left| \nabla V_{\text{np}} \right| > c\, V_{\text{np}}, \quad 0 < c \sim \mathcal{O}(1) \tag{1}$$

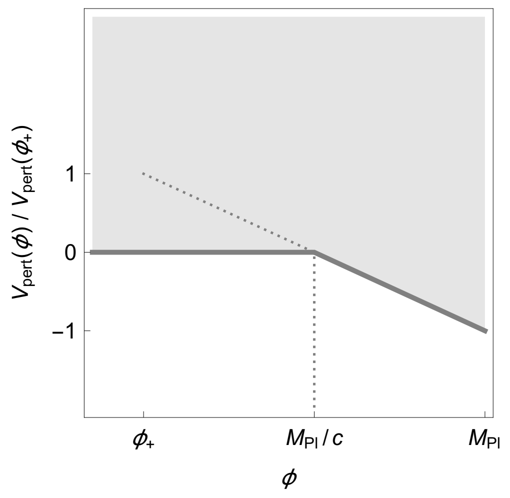
- V_{np} is real-valued, as quantum as possible
- More permissive than old and refined dS criteria as local maxima of V_{pert} are flattened in V_{np}
- Global minima still need to be AdS
- We are not living in a true vacuum, i.e. we are living in

```
!(true && vacuum) == (false || !vacuum)
```

Option !vacuum: living on a slope

- Quintessence solution to cosmological constant with $|\nabla V_{\mathrm{pert}}| > 0$
- Enough to add quintessence terms to Higgs potential as V_{np} has no local maximum
- No problem with a long-range force or time dependence of proton-to-electron mass ratio

Option `false vacuum`: $V_{\rm pert}$ with only two minima



Use Maxwell construction to represent condition (1) by

$$V_{\rm pert}(\phi_+) < \frac{M_{\rm Pl}}{c}\left|\frac{V_{\rm pert}(\phi_+) - V_{\rm pert}(\phi_-)}{\phi_+ - \phi_-}\right|, \qquad V_{\rm pert}(\phi_-) < 0$$

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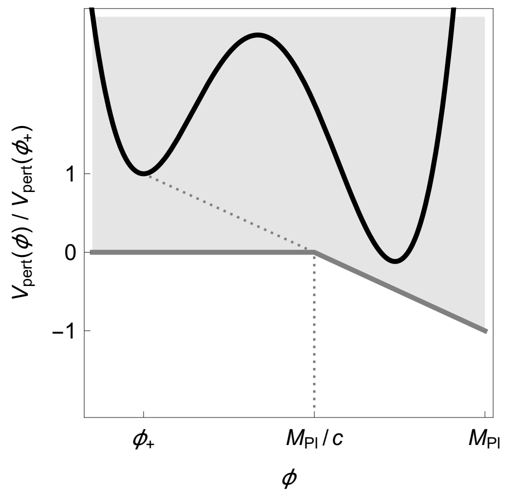

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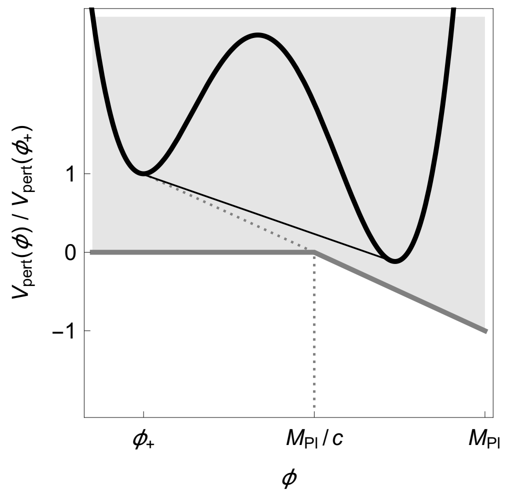

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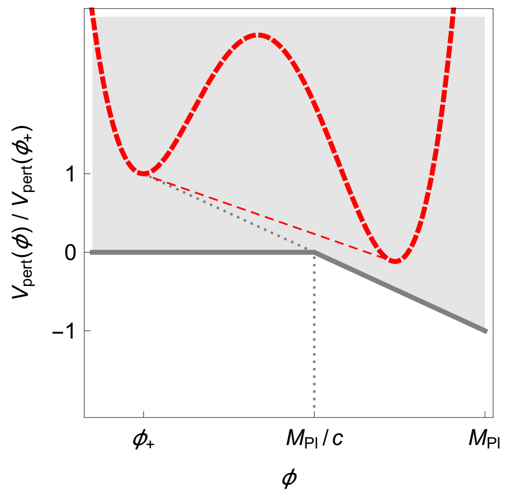

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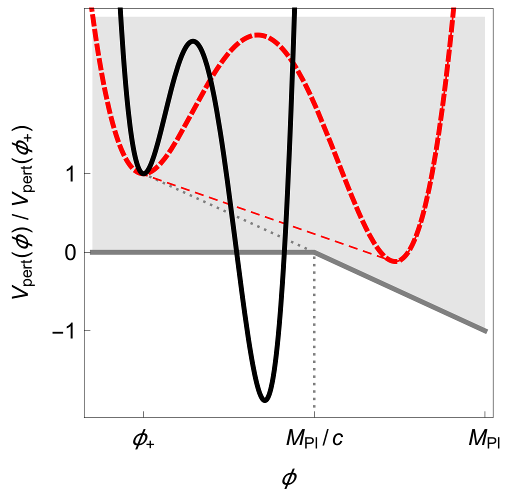

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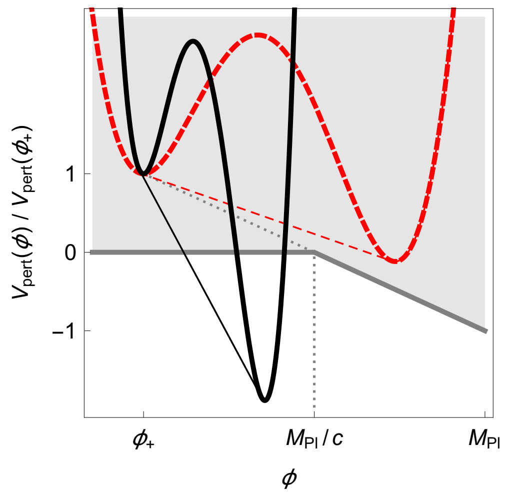

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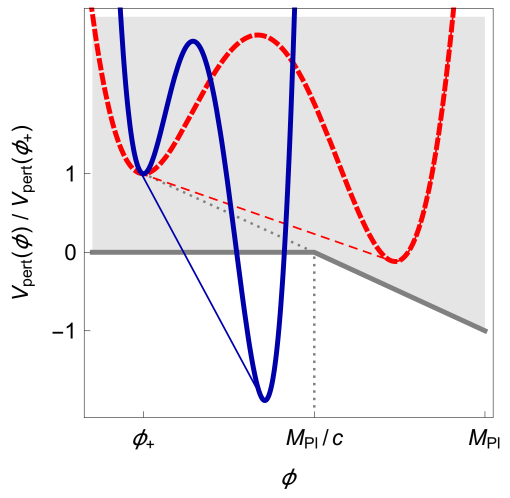

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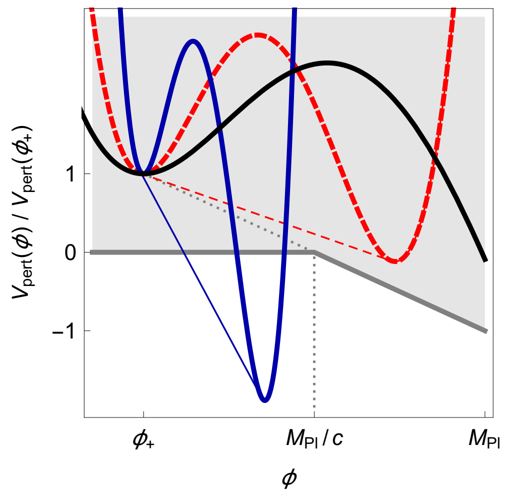

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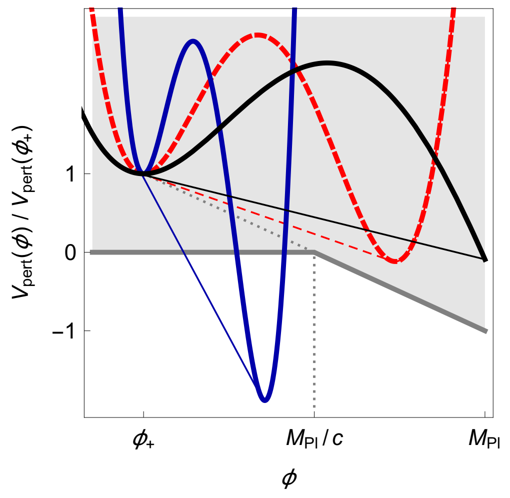

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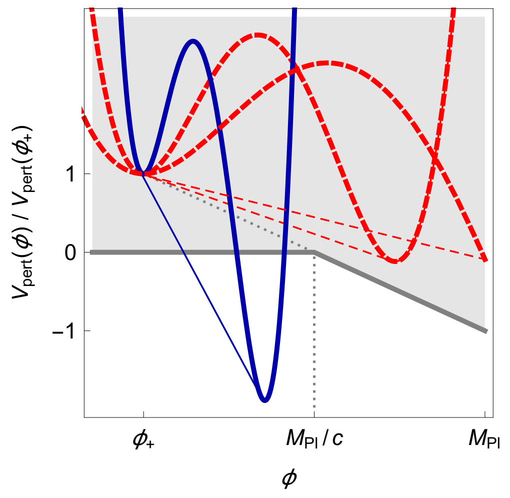

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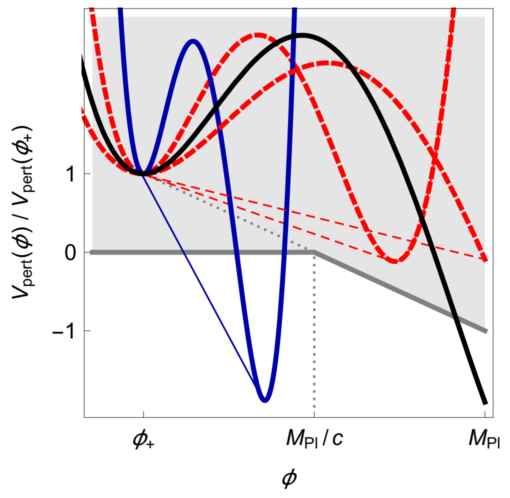

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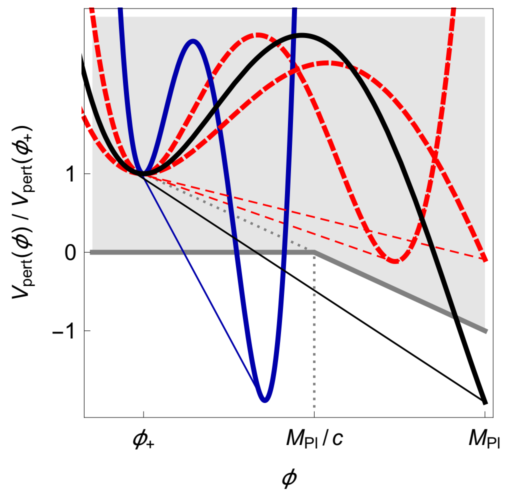

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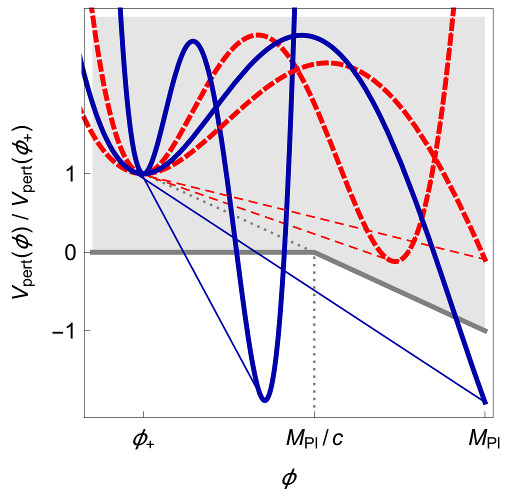

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Option `false vacuum`: symmetric V_{pert}

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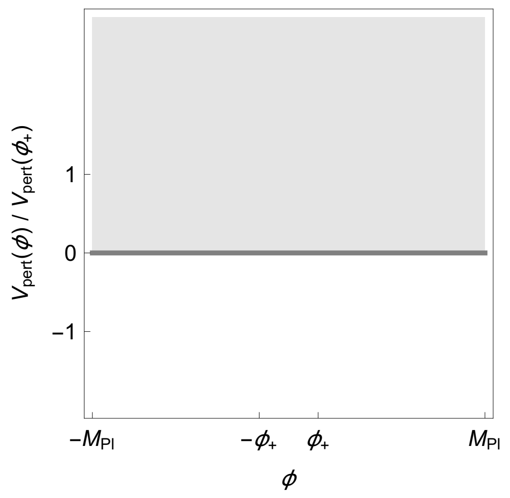

Maxwell construction is a constant between global minima

Option `false vacuum`: symmetric V_{pert}

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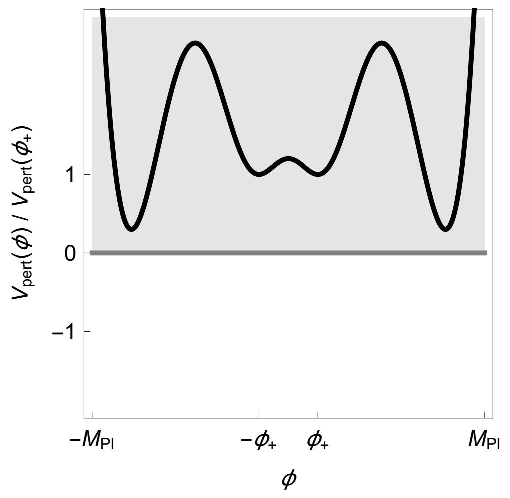

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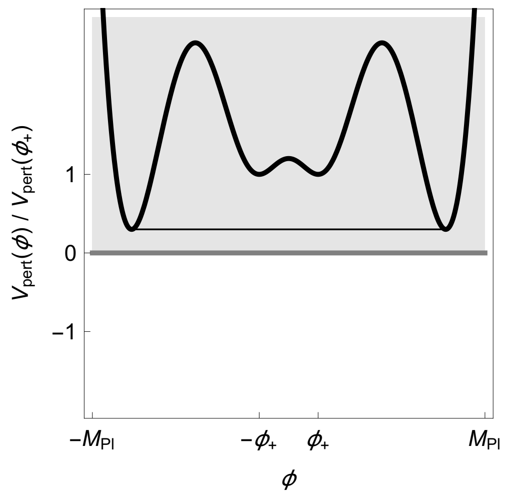

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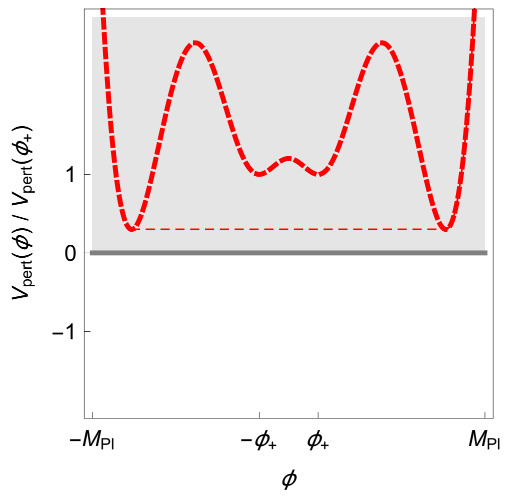

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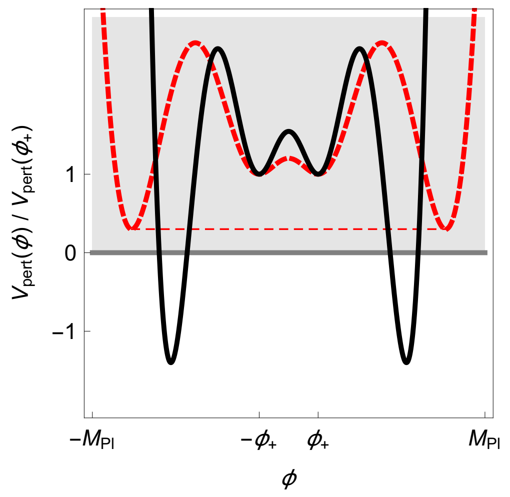

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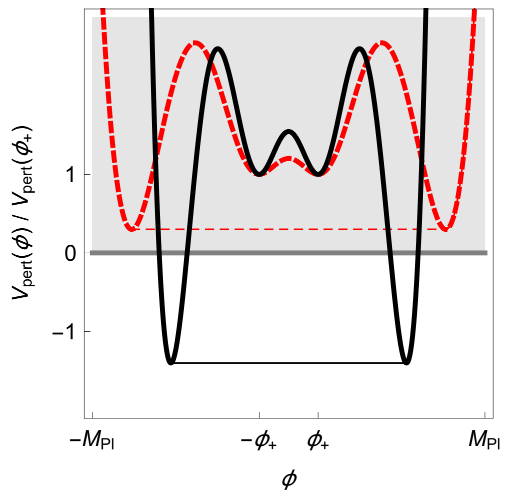

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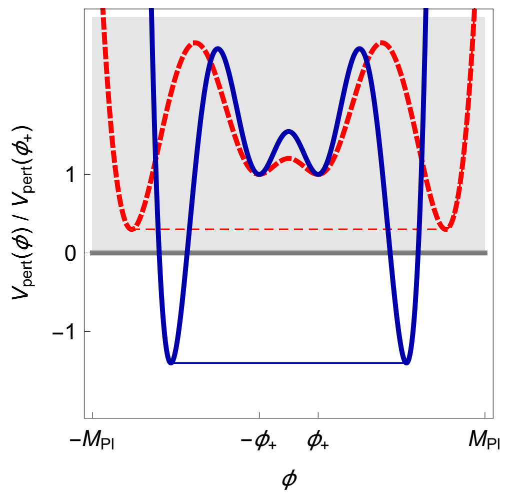

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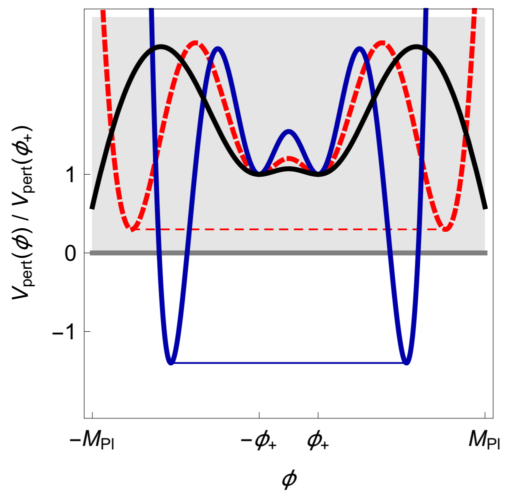

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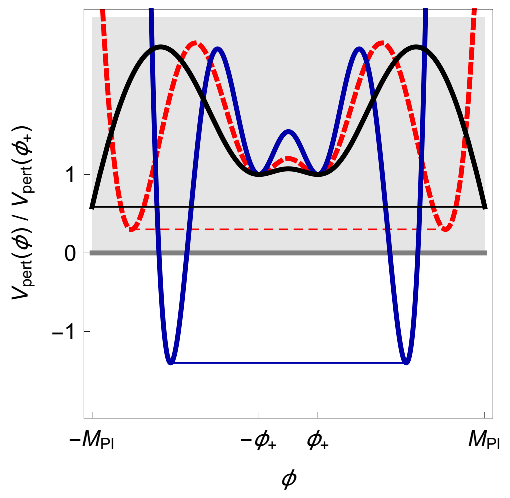

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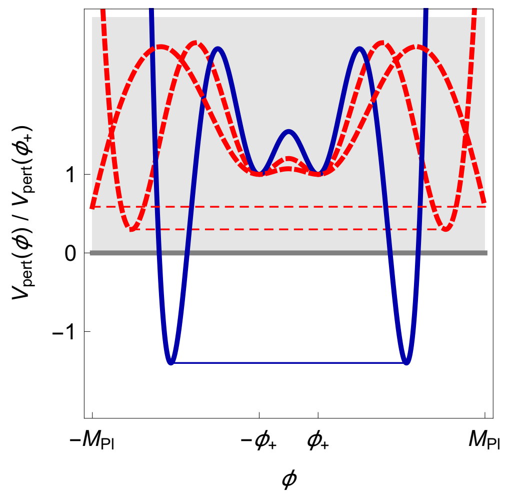

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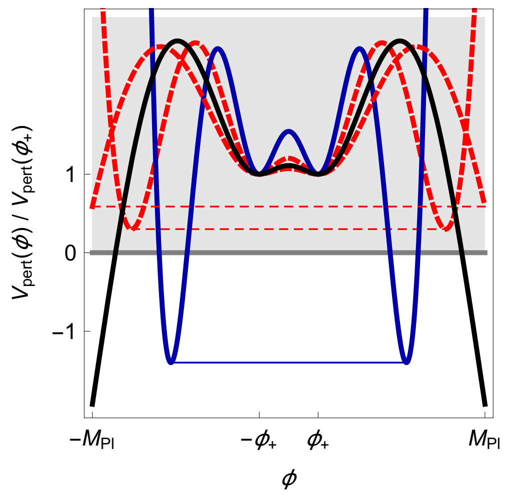

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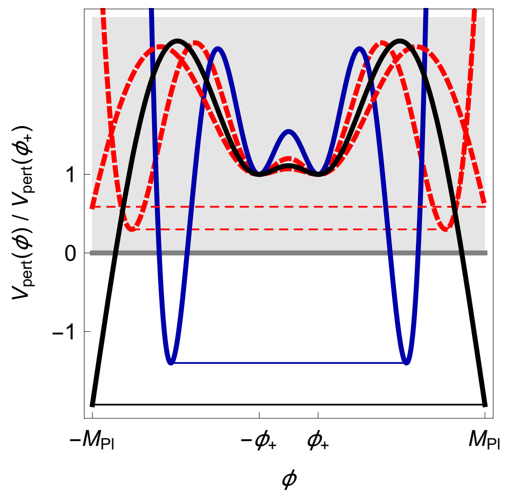

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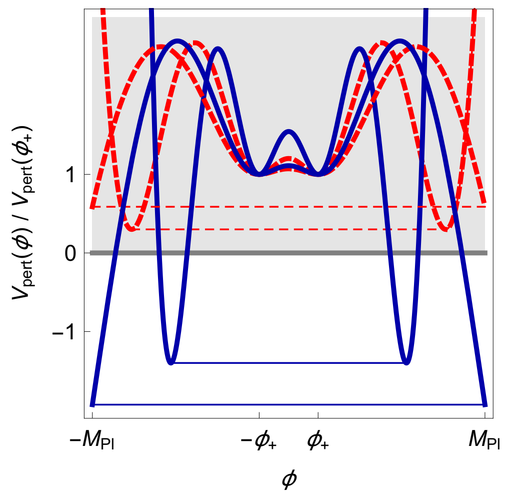

Maxwell construction is a constant between global minima

Implications for V_{pert}

- Our EW vacuum is required to be metastable
- Maybe due to an extra field such as in:
  - supersymmetry with CCB minima

    Claudson, Hall, Hinchliffe (1983)
  - metastable supersymmetry breaking sectors

    Intriligator, Seiberg, Shih (2006)
  - relaxion mechanism

    Graham, Kaplan, Rajendran (2015)
  - scalar extensions of the Higgs sector
- Otherwise metastability is due to SM Higgs

Near-criticality of Higgs potential

- RG-improved effective potential

$$V_{\text{pert}}(h) = \frac{\lambda(\mu = |h|)}{4} h^4$$

- Condition (1) + distance conjecture + UV cutoff of SM suggest $\lambda(\mu)$ should turn negative at $\mu \lesssim M_{\text{Pl}}$

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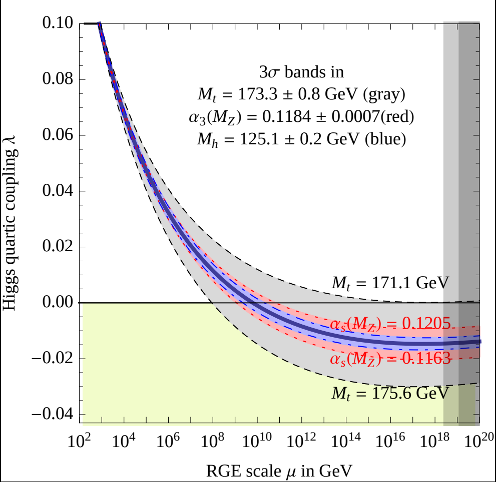

Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia (2013)

Connection to string theory?

- Hard to judge if condition (1) has something to do with string theory
- Admits false dS vacua contrary to original motivation for dS criteria
- Conjectures have not been proved

Summary

Yet another refinement of dS swampland conjecture

- Form is identical to original but effective potential integrates all (non)perturbative quantum effects
- V_{np} is real $\rightsquigarrow$ no inequality on complex numbers
- V_{np} is convex $\rightsquigarrow$ allows local maxima and false dS vacua in V_{pert} if slope of V_{np} is everywhere steep enough $\rightsquigarrow$ compatible with HEP models
- True vacua must still be AdS $\rightsquigarrow$ quintessence or metastable EW vacuum
- Reason for near-criticality of SM Higgs potential?