Axion scales and couplings with Stückelberg mixing

Chang Sub Shin (IBS-CTPU)

based on 1909.11685
done with Kiwoon Choi (IBS), and Seokhoon Yun (KIAS)

at IBS-MultiDark-IPPP workshop
Oct 7 – Oct 11, 2019

Outline

Motivation

beyond the minimal Axion-Like Particle (ALP)

some confusion about couplings, form of the scalar potentials

General description of Stückelberg axion models

general kinetic terms

discrete gauge symmetries

manifest field basis

Examples

two Stückelberg axions with hierarchical decay constants

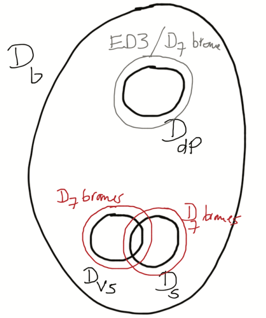
multiple Stückelberg axions (clockwork-type charge assignment)

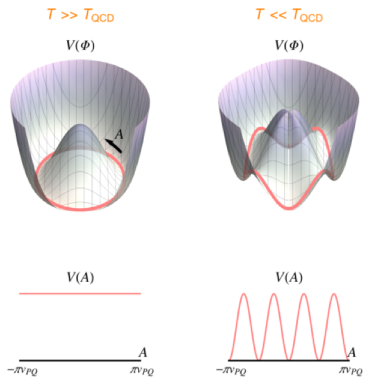
Motivation

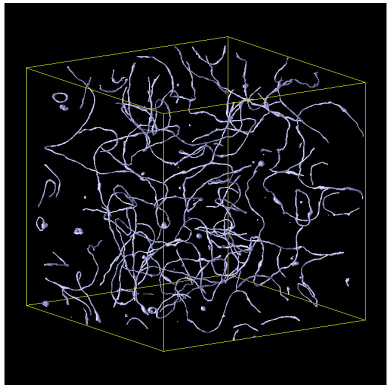
Axion, ALP

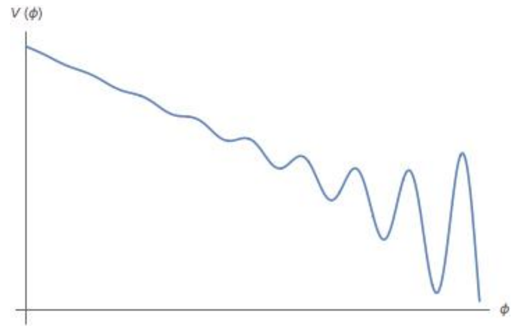
Well motivated from theory sides

string theory, inflation, dark matter candidates, quintessence, strong CP problem, hierarchy problem, etc.





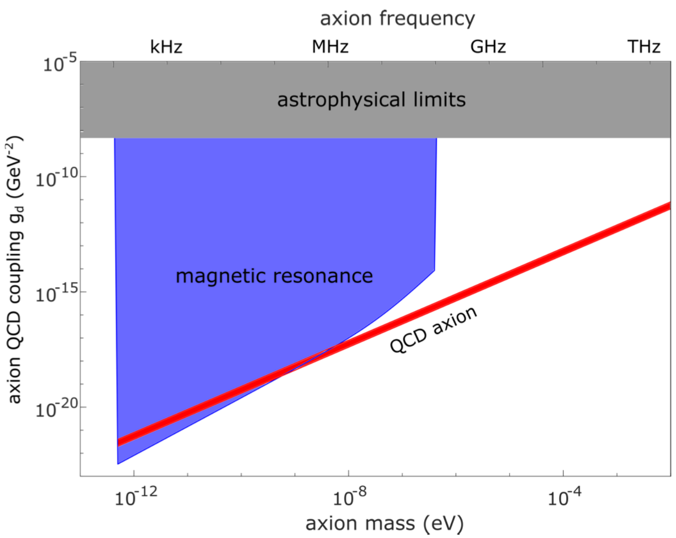




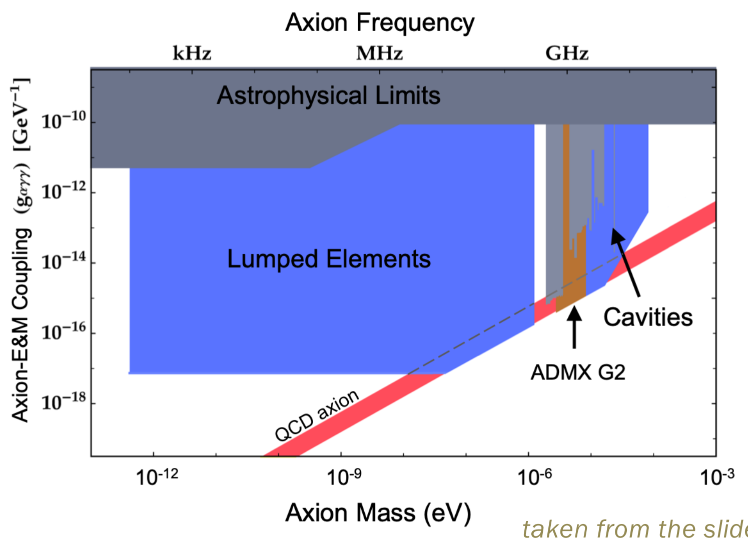
from experimental sides

CAPP, ADMX, HAYSTAC, ABRACADABRA, LC Circuit, CASPEr, etc.

nuclear coupling



electromagnetic coupling

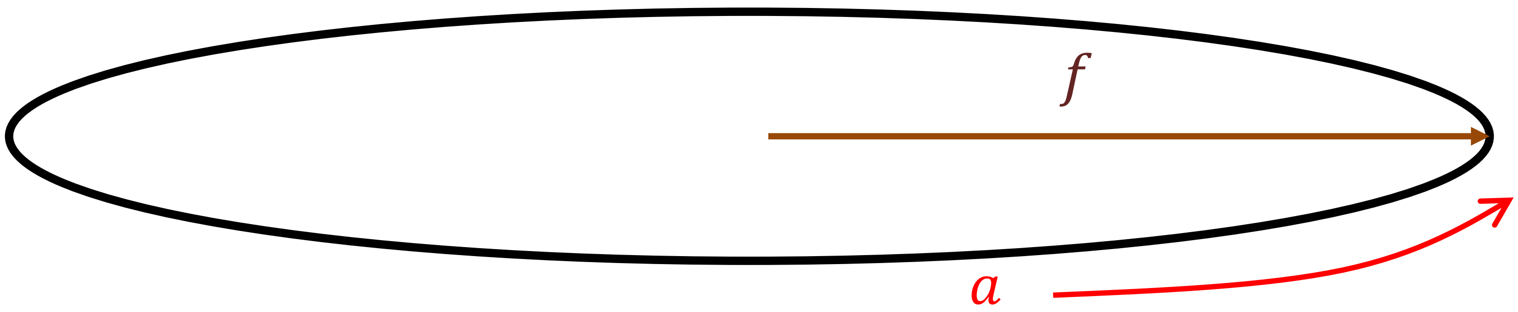


taken from the slides of P. Graham

Minimal ALP

ALP is the emergent, compact scalar field at low energies

1) discrete gauge symmetry: $a \to a + 2\pi f \mathbb{N}$



$$S[a] = S[a + 2\pi f \mathbb{N}]$$

It is natural to consider the ALP as the angular/phase field:

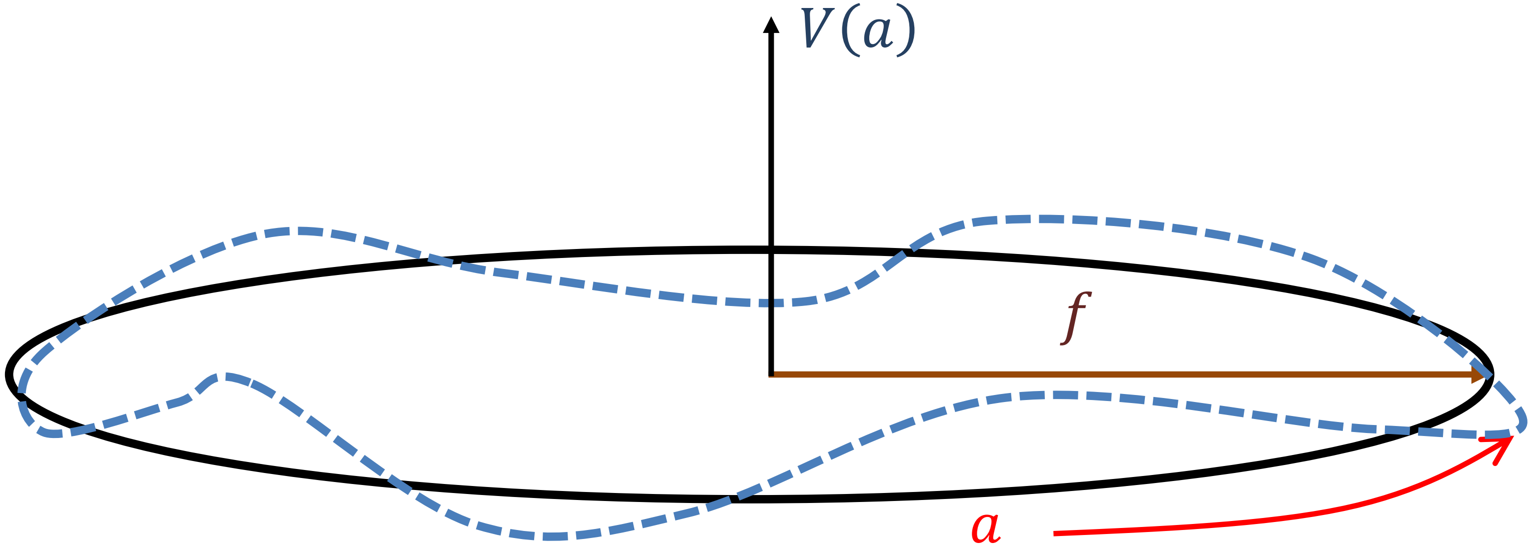
$$\theta(x) = a(x)/f \equiv \theta(x) + 2\pi$$

Minimal ALP

ALP is the emergent, compact scalar field at low energies

2) *(approximate) global symmetry:* $a \to a + 2\pi f c$, $(c \in \mathbb{R})$ *so called* $U(1)_{PQ}$

$U(1)_{PQ}$ *can be broken by various ways (for the QCD axion, from QCD/chiral anomaly)*



Nature of the angular variable is manifest in

$$\mathcal{L}_{eff}[\theta] = \frac{1}{2} f^2 (\partial_\mu \theta)(\partial^\mu \theta) - \sum_n V_n \cos(n\theta + \alpha_n)$$

$$+ \sum_i c_i (\partial_\mu \theta) J_i^\mu + \sum_a \frac{1}{32\pi^2} (n_a \theta) F_a \tilde{F}_a + \sum_b \left(e^{i n_b \theta} O_b(x) + h.c. \right)$$

$n, n_a, n_b \in \mathbb{Z}$

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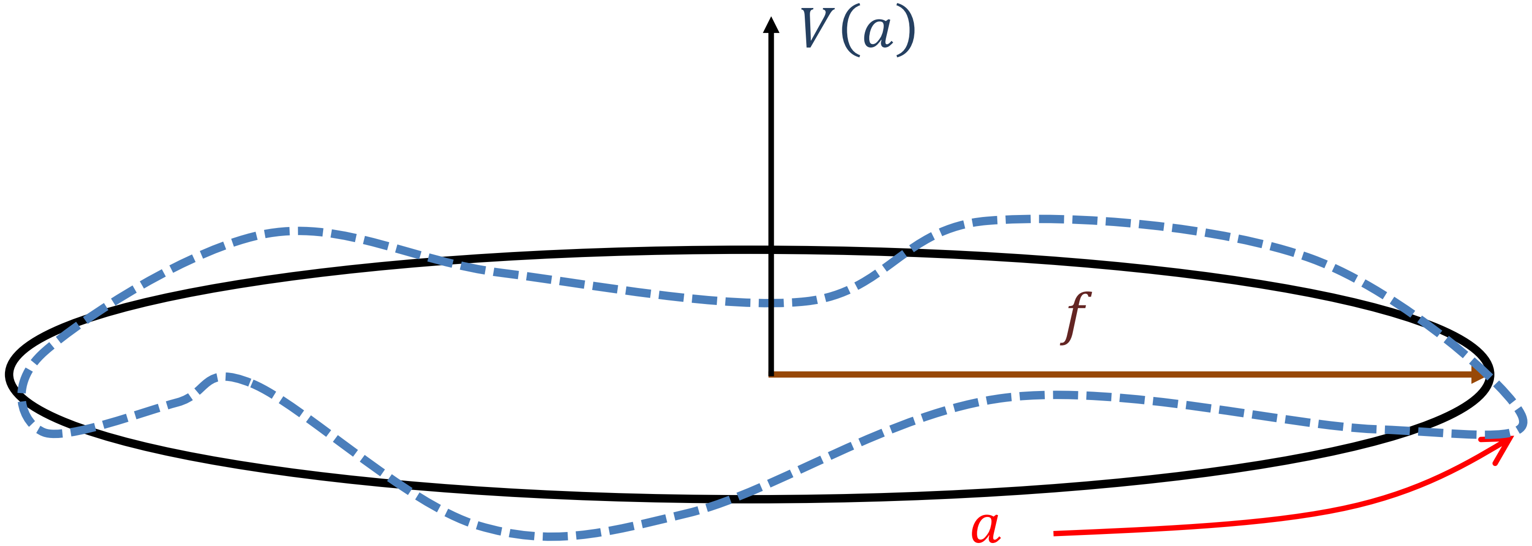


Interactions, dynamics of the ALP are manifest in

$$\mathcal{L}_{eff}[a] = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \sum_n V_n \cos\left(n\frac{a}{f} + \alpha_n\right)$$

$$+ \sum_i \frac{c_i}{f} \partial_\mu a \, J_i^\mu + \sum_a \frac{n_a}{32\pi^2 f} \, a \, F_a \tilde{F}_a + \sum_b \left(e^{i n_b a/f} O_b(x) + h.c.\right)$$

$a = f\theta$

Beyond the minimal ALP

Typically interactions of the ALP are governed by $1/f$.

However, for non-minimal ALP models, there are interesting variations.

Ex1) clockwork axion *[Choi Kim Yun 14, Choi Im 15, Kaplan Rattazzi 15]*

$$\mathcal{L} = \sum_{1,\cdots,N} \frac{1}{2} f^2 (\partial_\mu \theta^i)(\partial^\mu \theta^i) - \sum_{1,\cdots,N-1} V_i\left(q\,\theta^{i+1} - \theta^i\right) + \frac{1}{32\pi^2}\theta_1 F\tilde{F} + \frac{1}{32\pi^2}\theta_N G\tilde{G}$$

$$= \frac{1}{2}(\partial_\mu a)(\partial^\mu a) + \frac{a}{f_a}\left(\frac{q^{N-1}}{32\pi^2} F\tilde{F} + \frac{1}{32\pi^2} G\tilde{G}\right) + \text{heavy modes}$$

The axion field range ($a \equiv a + 2\pi f_a$):

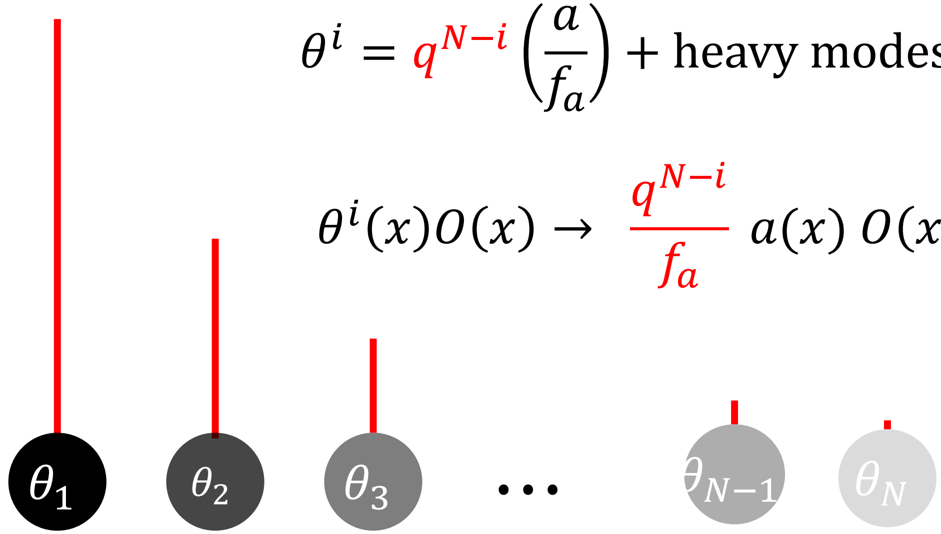
$$f_a = \left(\sum_{1,\cdots,N} q^{2(i-1)}\right)^{\frac{1}{2}} f \sim q^{N-1} f$$

Spectrum of effective decay constants:

$$\frac{g^2}{16\pi^2 f_a} \lesssim g_{aAA} \lesssim \frac{g^2\, q^{N-1}}{16\pi^2 f_a}$$

$$\theta^i = q^{N-i}\left(\frac{a}{f_a}\right) + \text{heavy modes}$$

$$\theta^i(x) O(x) \to \frac{q^{N-i}}{f_a}\, a(x)\, O(x)$$



Beyond the minimal ALP

Ex2) Stückelberg axions with the anomalous $U(1)_A$ gauge symmetry [Shui, Staessens, Ye 15]

$$\mathcal{L} = -\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} + \sum_{I=L,R}\bar{\psi}_I i\bar{\sigma}^\mu(D_\mu - iq_I A_\mu)\psi_I + \sum_{i=1,2}\frac{1}{2}f_i^2(\partial_\mu\theta^i - k^i A_\mu)^2 + \frac{1}{32\pi^2}\theta^1 G\tilde{G} + \cdots$$

Under

$$U(1)_A:\quad A_\mu \to A_\mu + \partial_\mu\Lambda,\qquad \theta^{1,2} \to \theta^{1,2} + k^{1,2}\Lambda,\qquad \psi_{L/R} \to e^{-i\,q_{L/R}\Lambda}\,\psi_{L/R}$$

mixed anomalous fermion loop contribution is canceled by gauge transformation of θ_1

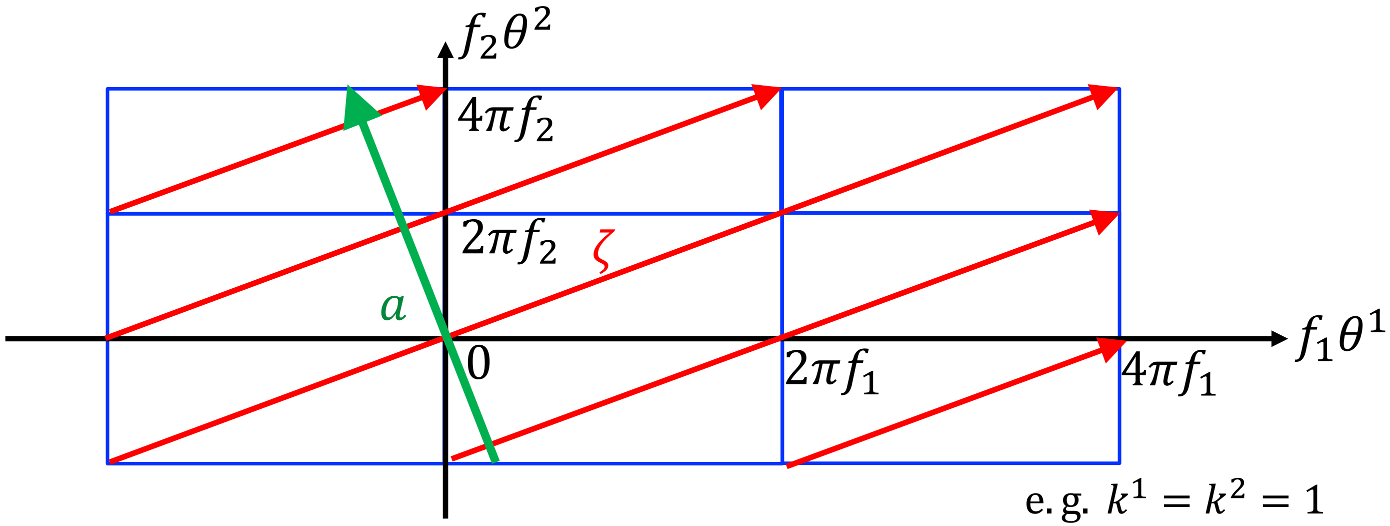
$$\delta_\Lambda\mathcal{L} = \frac{1}{32\pi^2}\big(k^1 - (q_L + q_R)\big)\Lambda(x)\,G_{\mu\nu}\tilde{G}^{\mu\nu} = 0$$

gauge invariance: $q_L + q_R = k^1$

One of combinations of θ^is (ζ) is absorbed by A_μ and becomes heavy, while the gauge invariant combination (a) remains light.

$$\zeta = \frac{f_1^2 k^1\theta^1 + f_2^2 k^2\theta^2}{(f_1 k^1)^2 + (f_2 k^2)^2}$$

$$a = \frac{f_1 f_2(k^1\theta^2 - k^2\theta^1)}{\sqrt{(f_1 k^1)^2 + (f_2 k^2)^2}}$$



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$$= -\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M_A^2(\partial_\mu\zeta - A_\mu)^2 + \frac{k^1}{32\pi^2}\zeta\, G\tilde{G} + \sum_{I=L,R}\bar{\psi}_I i\bar{\sigma}^\mu(D_\mu - iq_I A_\mu)\psi_I$$

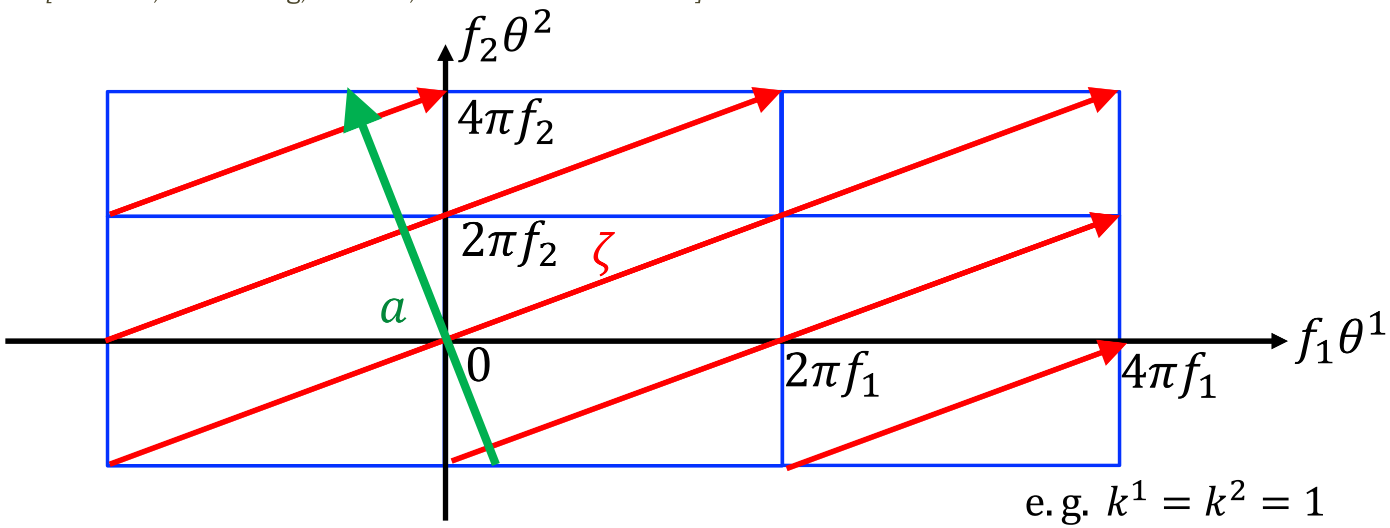
$$+ \frac{1}{2}(\partial_\mu a)^2 - \frac{1}{32\pi^2 f_*}\, a\, G\tilde{G} + \cdots$$

where $M_A^2 = (f_1k^1)^2 + (f_2k^2)^2$, $f_* = \frac{f_1}{f_2k^2}\sqrt{(f_1k^1)^2 + (f_2k^2)^2}$

In the limit of $f_1 \gg f_2$, $M_A \sim f_1$, $f_* \sim \frac{f_1}{f_2}M_A \gg f_1$, *the axion coupling to gauge bosons* $\left(\frac{g^2}{32\pi^2 f_*}\right)$ *is hierarchically suppressed.* [Fonseca, von Harling, de Lima, Machado 1906.10193]

$$\zeta \simeq \theta^1 + \frac{f_2^2k^2}{(f_1k^1)^2}\theta^2$$

$$a \simeq f_2\,(k^1\theta^2 - k^2\theta^1)$$

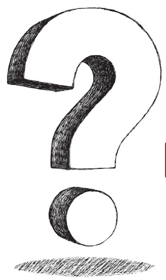


Beyond the minimal ALP

Ex2) Stückelberg axions with the anomalous $U(1)_A$ gauge symmetry

Can it give the axion potential with a period $2\pi f_* \simeq 2\pi f_2 (f_1^2/f_2^2) \gg 2\pi f_1, 2\pi f_2$?
→ *relevant for scalar dynamics: axion as the inflaton ($f_* \gg M_P$), dark matter etc.*

$$+\frac{1}{2}(\partial_\mu a)^2 + \boxed{\frac{1}{32\pi^2 f_*}\, a\, G\tilde{G}}$$



$$\boxed{V(a) = \Lambda^4 \cos\frac{a}{f_*}}$$

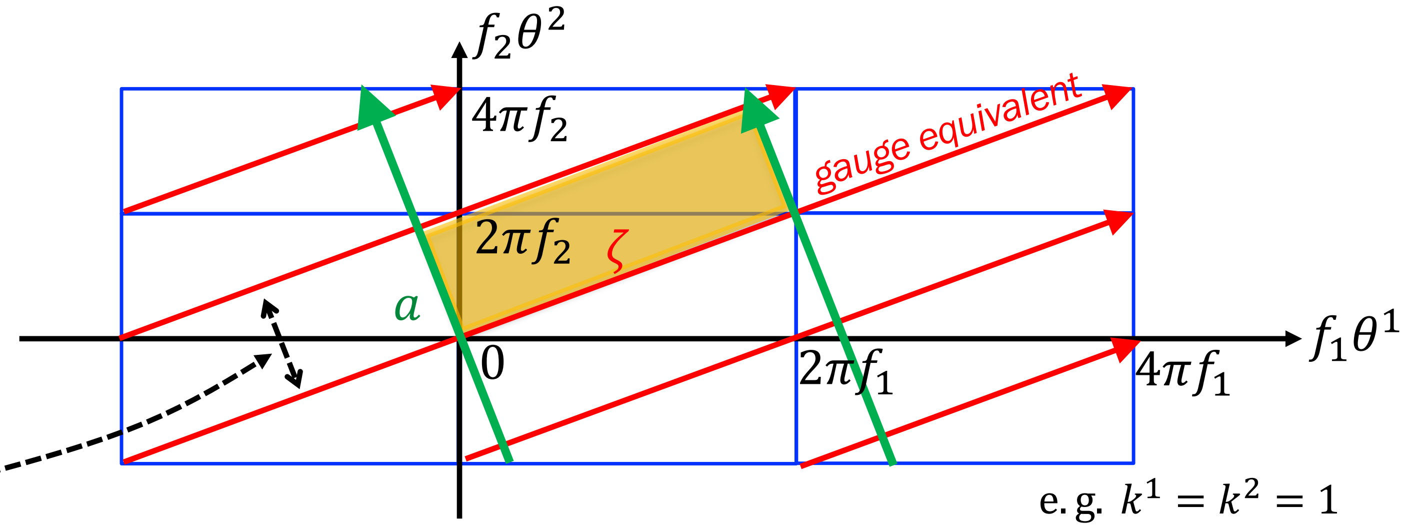
[Shui, Staessens 1807.00888]

f_a (a period, basis independent) vs f_* (a coupling, basis dependent)

$$\zeta \simeq \theta^1 + \frac{f_2^2 k^2}{(f_1 k^1)^2}\theta^2$$

$$a \simeq f_2\,(k^1\theta^2 - k^2\theta^1)$$

$$2\pi f_a \equiv \frac{2\pi\, f_1 f_2}{\sqrt{(f_1 k^1)^2 + (f_2 k^2)^2}}$$



General description of Stückelberg axions

[Choi, CSS, Yun 1909.11685]

General form

N Stückelberg axions θ^i, $N-1$ $U(1)_\alpha$ *gauge symmetries yield one light axion at low energies.* $i = 1, \cdots, N$ $\alpha = 1, \cdots, N-1$

$$\mathcal{L} = -\frac{1}{4e_\alpha^2} F^\alpha_{\mu\nu} F^{\alpha\,\mu\nu} + \frac{1}{2} G_{ij} (\partial_\mu \theta^i - k^i_\alpha A^\alpha_\mu)(\partial_\mu \theta^j - k^j_\alpha A^\alpha_\mu)$$
$$+ \frac{(r_i \theta^i)}{32\pi^2} G\tilde{G} + \left(\lambda_o \, e^{i n_i^o \theta^i} \prod_I \phi_I^{x_I^o}(x) + h.c. \right) + \kappa_{Ji} (\partial_\mu \theta^i - k^i_\alpha A^\alpha_\mu) J^\mu_\phi + \cdots$$

* *Stückelberg axions are the angular fields:*

$$\theta^i \equiv \theta^i + 2\pi \;\to k^i_\alpha, r_i, n_i^0 \in \mathbb{Z}$$

* *Gauge invariance:*

$$r_i k^i_\alpha = \sum_{\psi_I} 2 q_{I\alpha} \mathrm{Tr}[T(\psi_I)^2], \qquad n_i^o k^i_\alpha = \sum_I q_{I\alpha} x_I^o$$

* *Allow general form of the metric including the kinetic mixing:* G_{ij}

* *The previous example:* $N = 2,\ k_1^i = (1,1),\ r_i = (1,0),\ G_{ij} = \mathrm{diag}(f_1^2, f_2^2)$.

Useful relations

Gauge invariant combination: $a \propto \tilde{k}_i \theta^i$ *requires* $\tilde{k}_i k^i_\alpha = 0$. *The solution exists (with* $\tilde{k}_i \in \mathbb{Z}$ *)*
assuming $\mathrm{g.c.d.}(k^i_\alpha) = 1$ *for all* α

$$\tilde{k}_i = \det \begin{pmatrix} \delta_i^1 & \delta_i^2 & \cdots & \delta_i^N \\ k_1^1 & k_1^2 & \cdots & k_1^N \\ \vdots & \vdots & \ddots & \vdots \\ k_{N-1}^1 & k_{N-1}^2 & \cdots & k_{N-1}^N \end{pmatrix}$$

Since $\tilde{k}_i$ *is integer valued, there exists the integer valued vector* ℓ^i *such that* $\tilde{k}_i \ell^i = 1$. *Then we find for the* N *by* N *matrix* $[K]$,

$$[K] = \Big(\underbrace{\overbrace{k^i_\alpha}^{N-1}}_{} \quad \underbrace{\ell^i}_{1} \Big) \Big\} N$$

$\det[K] = 1$, *and it has the integer valued inverse matrix* $[K^{-1}]$ *as*

$$[K^{-1}] = \begin{pmatrix} \tilde{\ell}^\alpha_i \\ \tilde{k}_i \end{pmatrix}$$

Therefore $k^i_\alpha, \ell^i, \tilde{k}_i, \tilde{\ell}^\alpha_i \in \mathbb{Z}$ *and*

$$\tilde{\ell}^\alpha_i k^i_\beta = \delta^\alpha_\beta, \qquad \tilde{\ell}^\alpha_i \ell^i = 0, \qquad \sum_{k=1,\cdots,N} [K^{-1}]_i^k [K]_k^j = \tilde{\ell}^\alpha_i k^j_\alpha + \tilde{k}_i \ell^j = \delta_i^j$$

Identifying discrete gauge symmetry

Decomposing θ^is into the longitudinal modes of the gauge bosons (ζ^α: $\zeta^\alpha \to \zeta^\alpha + \Lambda^\alpha$) and the gauge invariant axion (a):

$$\theta^i = k^i_\alpha\, \zeta^\alpha + \frac{(G^{-1})^{ij}\,\tilde{k}_j}{(G^{-1})^{ij}\,\tilde{k}_i\tilde{k}_j}\,\frac{a}{f_a} = k^i_\alpha\left(\zeta^\alpha + \Gamma^\alpha(G,k)\frac{a}{f_a}\right) + \ell^i\,\frac{a}{f_a}$$

leads to

$$\frac{1}{2}G_{ij}(\partial_\mu\theta^i - k^i_\alpha A^\alpha_\mu)(\partial_\mu\theta^j - k^j_\alpha A^\alpha_\mu) = \frac{1}{2}(\partial_\mu a)^2 + \frac{1}{2}M^2_{\alpha\beta}(\partial_\mu\zeta^\alpha - A^\alpha_\mu)(\partial^\mu\zeta^\beta - A^{\beta\mu})$$

where

$$M^2_{\alpha\beta} = G_{ij}k^i_\alpha k^j_\beta, \qquad f_a = \frac{1}{\sqrt{(G^{-1})^{ij}\tilde{k}_i\tilde{k}_j}}$$

Then N discrete gauge symmetries

$$\mathbb{Z}_i:\ \theta^i \to \theta^i + 2\pi$$

Is equivalent to

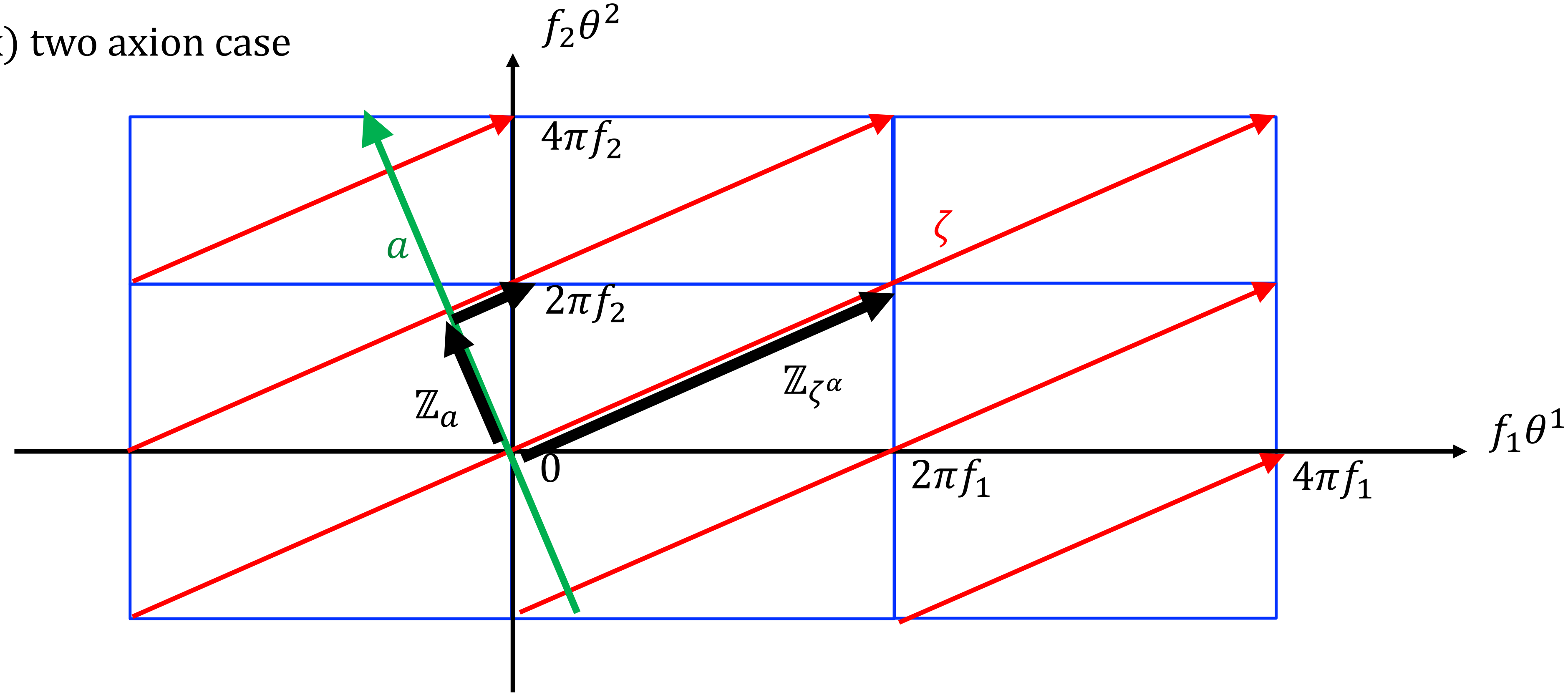
$$\mathbb{Z}_a:\ \frac{a}{f_a} \to \frac{a}{f_a} + 2\pi, \quad \zeta^\alpha \to \zeta^\alpha - 2\pi\,\Gamma^\alpha$$

and $N-1$ transformations of ζ^α

$$\mathbb{Z}_{\zeta^\alpha}:\ \zeta^\alpha \to \zeta^\alpha + 2\pi$$

Identifying discrete gauge symmetry

Ex) two axion case

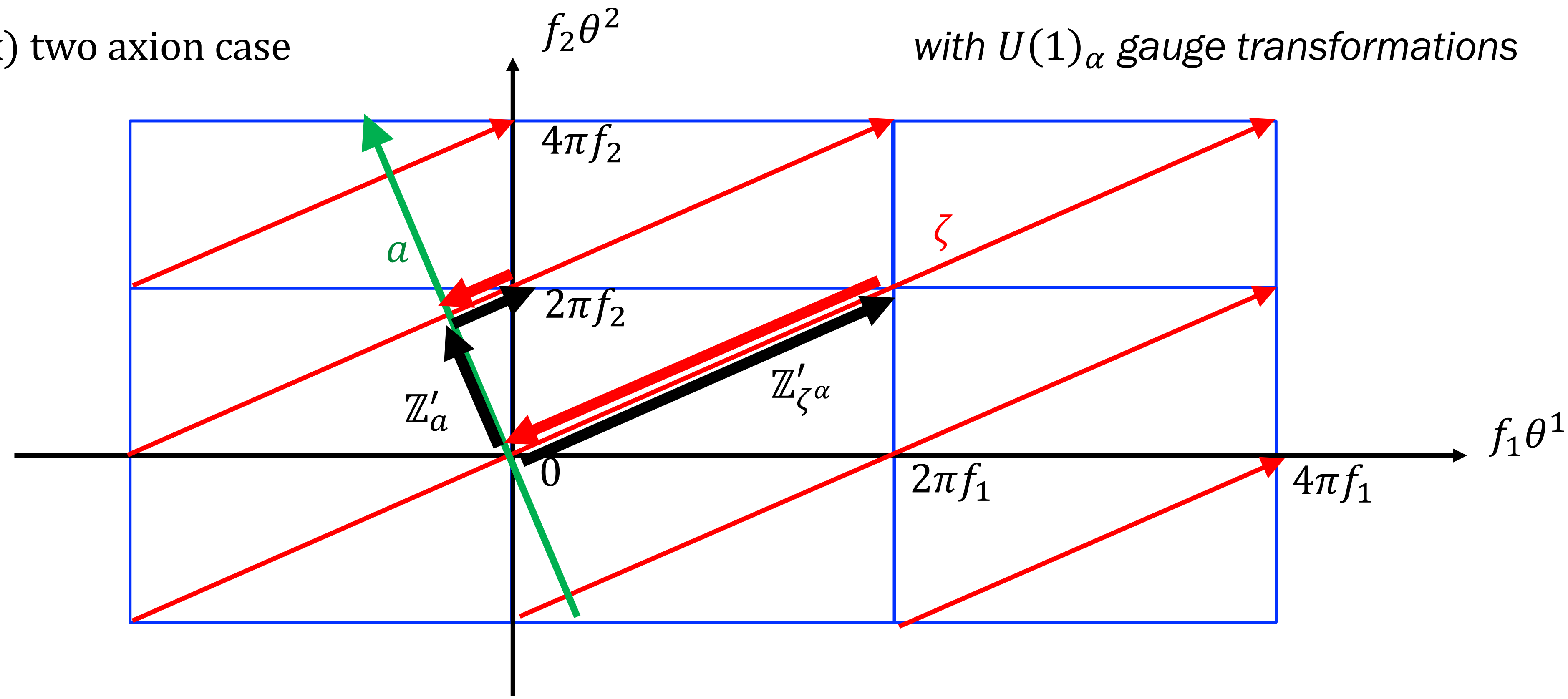


$\mathbb{Z}_a$: $\frac{a}{f_a} \to \frac{a}{f_a} + 2\pi$, $\quad \zeta^\alpha \to \zeta^\alpha - 2\pi\,\Gamma^\alpha$ $\qquad\qquad$ $\mathbb{Z}_{\zeta^\alpha}$: $\zeta^\alpha \to \zeta^\alpha + 2\pi$

Identifying discrete gauge symmetry

Ex) two axion case

with $U(1)_\alpha$ gauge transformations

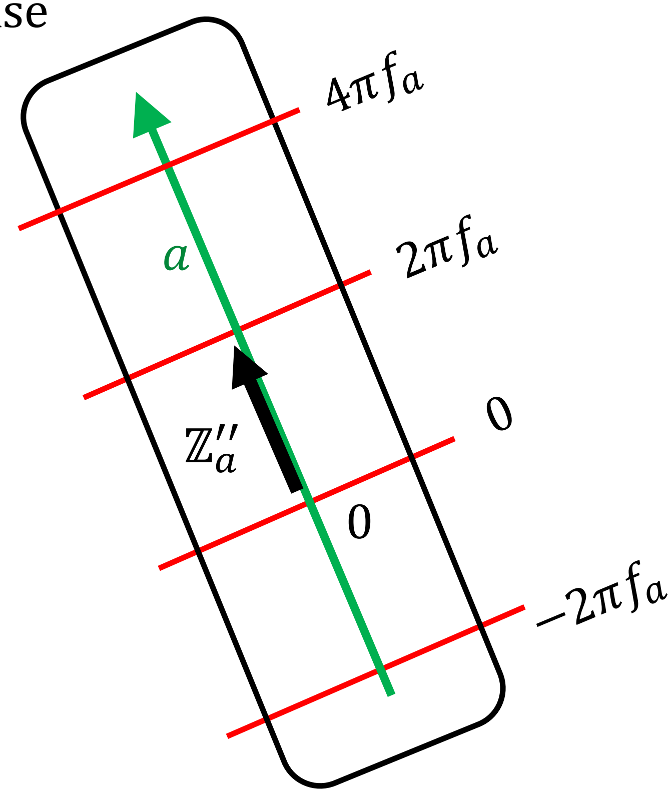


$\mathbb{Z}'_a$: $\frac{a}{f_a} \to \frac{a}{f_a} + 2\pi$, $\phi_I \to \phi_I \, e^{-i2\pi q_{I\alpha}\Gamma^\alpha}$ $\qquad$ $\mathbb{Z}'_{\zeta^\alpha}$: $\phi_I \to \phi_I$

Identifying discrete gauge symmetry

Ex) two axion case

with $U(1)_\alpha$ gauge transformations



with a field redefinition of matters $\phi_I \to \phi_I \exp(-q_{I\alpha}\Gamma^\alpha a/f_a)$

$$\mathbb{Z}_a'' : \frac{a}{f_a} \to \frac{a}{f_a} + 2\pi, \quad \phi_I \to \phi_I$$

The basis in which the periodicity of the axion is manifest!

General form in the new field basis

After integrating out gauge fields

$$\mathcal{L} = -\frac{1}{4e_\alpha^2} F^\alpha_{\mu\nu} F^{\alpha\,\mu\nu} + \frac{1}{2} G_{ij} (\partial_\mu \theta^i - k^i_\alpha A^\alpha_\mu)(\partial_\mu \theta^j - k^j_\alpha A^\alpha_\mu)$$
$$+ \frac{(r_i \theta^i)}{32\pi^2} G\tilde{G} + \left(\lambda_o\, e^{i n^o_i \theta^i} \prod_I \phi_I^{x_I^o}(x) + h.c. \right) + \kappa_{Ji} (\partial_\mu \theta^i - k^i_\alpha A^\alpha_\mu)\, J^\mu_\phi + \cdots$$

$$\mathcal{L}_{eff} = \frac{1}{2} (\partial_\mu a)^2 + \frac{r_i \ell^i}{32\pi^2} \frac{a}{f_a} G\tilde{G} + \left(\lambda_o\, e^{i\, n^o_i \ell^i\, a/f_a} \prod_I \phi_I^{x_I^o}(x) + h.c. \right)$$
$$+ \frac{1}{f_a} (\partial_\mu a) \left(\kappa_{Ji} (\ell^i + \Gamma^\alpha k^i_\alpha)\, J^\mu_\phi + \Gamma^\alpha J^\mu_\alpha(\phi) \right) \cdots$$

All non-derivative couplings of the axion are quantized in the unit of $1/f_a$:

$$r_i \ell^i,\; n^o_i \ell^i \in \mathbb{Z} \;\; \Rightarrow \;\; \mathcal{L}_{eff}(a) = \mathcal{L}_{eff}(a + 2\pi f_a)$$

** It is trivial that the scalar potentials of the axion should be* $V_{eff}(a) = V_{eff}(a + 2\pi f_a)$

** Derivative interactions (as the result of field redefinition) should be considered for the basis independent physical processes involving the axion.*

Lessons

The axion field range ($2\pi f_a$) is determined by the kinetic term of the axions regardless of detailed mechanism for generating scalar potentials and interactions.

We can always take the field basis in which all non-derivative couplings of the axion are quantized in the unit of $1/f_a$ so that the scalar potential of the axion is $2\pi f_a$ - periodic if there is no other mechanism to extend the period like the clockwork mechanism

This does not mean that all scattering amplitudes of the axion are suppressed by $O\left(\frac{1}{f_a}\right)$.

Examples

Two Stückelberg axions with a massive fermion

Again from the previous example with $(f_1 \gg f_2,\ q_L = 1,\ \ q_R = 0,\ n_1 = 0,\ n_2 = 1)$

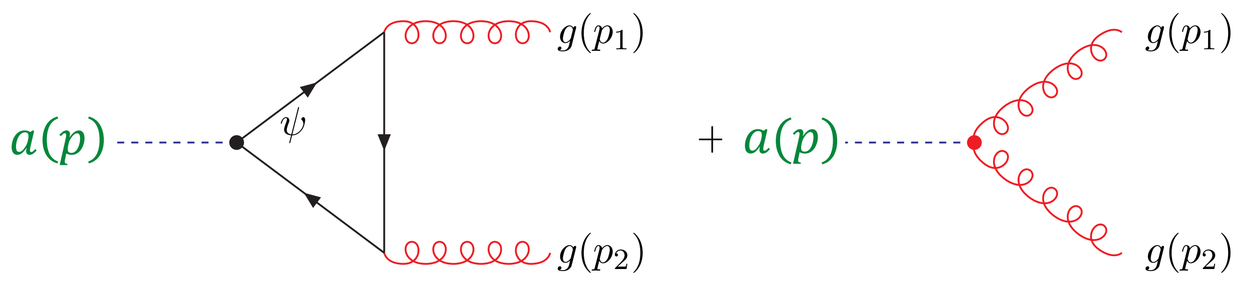
$$\mathcal{L}_{eff} = \frac{1}{2}(\partial_\mu a)^2 + \frac{1}{32\pi^2}\frac{f_2^2}{f_1^2+f_2^2}\frac{a}{f_a}\,G\tilde{G}$$
$$+\bar{\psi}_{L/R} i\bar{\sigma}^\mu D_\mu \psi_{L/R} - \left[\mu_\psi \exp\left(-i\frac{f_1^2}{f_1^2+f_2^2}\frac{a}{f_a}\right)\psi_L\psi_R + h.c.\right]$$

$$f_a = \frac{f_1 f_2}{\sqrt{f_1^2+f_2^2}}$$

Physically equivalent expression with the field redefinition: $\psi_L \to \psi_L \exp\left(i\frac{f_1^2}{f_1^2+f_2^2}\frac{a}{f_a}\right)$

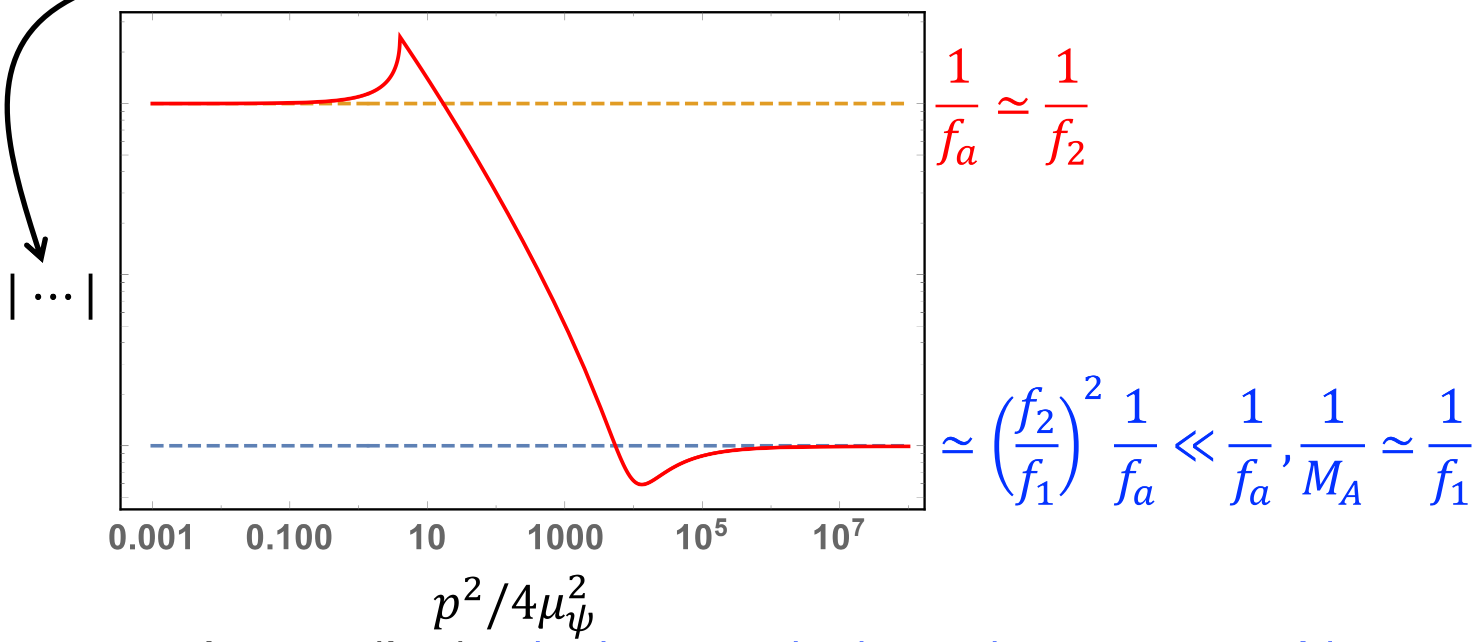
$$= \frac{1}{2}(\partial_\mu a)^2 + \frac{1}{32\pi^2}\frac{a}{f_a}\,G\tilde{G} + \bar{\psi}_{L/R} i\bar{\sigma}^\mu D_\mu \psi_{L/R} - (\mu_\psi\psi_L\psi_R + h.c.) - \frac{f_1^2}{f_1^2+f_2^2}\frac{\partial_\mu a}{f_a}\,\bar{\psi}_L\bar{\sigma}^\mu\psi_L$$

For the axion-gluon-gluon scattering amplitude

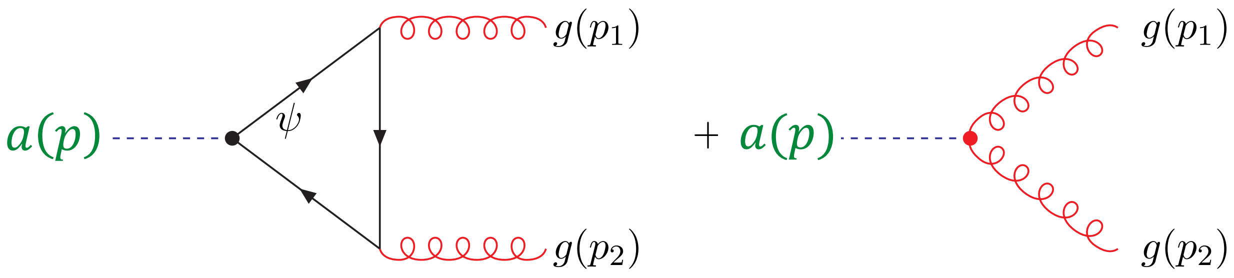


Two Stückelberg axions with a massive fermion

$$\mathcal{M}^{\mu\nu} = \frac{i\alpha_s}{2\pi}\epsilon^{\mu\nu\rho\sigma}p_{1\rho}p_{2\sigma}\left(\frac{1}{f_a} - \frac{1}{f_a}\frac{f_1^2}{f_1^2+f_2^2}F_{\text{Loop}}(p_1,p_2,\mu_\psi)\right)$$



For the axion-gluon-gluon scattering amplitude, the large scale dependence can provide quite different cosmological history (e.g. thermal production of the axion)



Multiple Stückelberg axions

With the min(max) eigenvalues of G as $f_{\min}^2$ $(f_{\max}^2)$

$$\frac{f_{\min}}{\|\tilde{k}\|} < f_a = \frac{1}{\sqrt{(G^{-1})^{ij}\tilde{k}_i\tilde{k}_j}} < \frac{f_{\max}}{\|\tilde{k}\|}$$

For $N \gg 1$ and generic $U(1)$ charges, $\|\tilde{k}\| \sim \sqrt{N!}\left\langle (k_\alpha^i)^2 \right\rangle^{\frac{N-1}{2}}$*, so that the axion field range $2\pi f_a$ is exponentially suppressed compared to the originally introduced one $2\pi f$*

$$f_a \sim \frac{f}{\sqrt{N!}\left\langle (k_\alpha^i)^2 \right\rangle^{\frac{N-1}{2}}} \ll f$$

Q: Is the coupling between the axion and matters also of $O\left(\frac{1}{f_a}\right)$?

A: NO! because the axion coupling through the mixing with θ^i is proportional to

$$\langle a|\theta^i\rangle = \frac{(G^{-1})^{ij}\,\tilde{k}_j}{(G^{-1})^{ij}\,\tilde{k}_i\tilde{k}_j}\frac{1}{f_a} \sim \frac{1}{\|\tilde{k}\|}\frac{1}{f_a} \ll \frac{1}{f_a} \qquad \text{(much suppressed)},$$

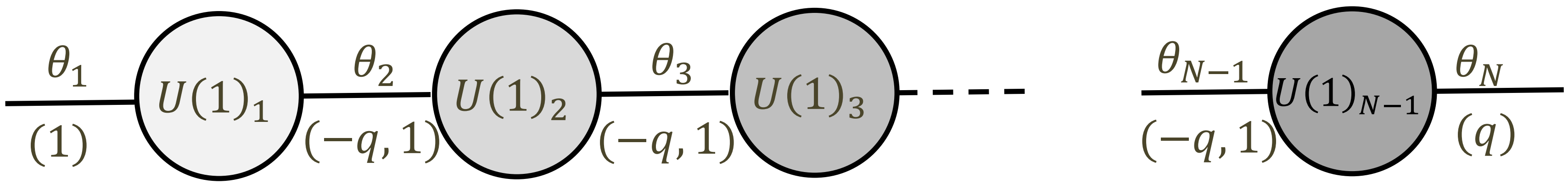
which means that the scalar potential $\left(e.g.\, \Lambda_*^4 \cos\frac{a}{f_a}\right)$ of the axion generically needs $O(\|\tilde{k}\|)$ insertions of the operators: "highly" protected PQ symmetry: $a/f_a \to a/f_a + c$, $(c \in \mathbb{R})$

Stückelberg Clockwork

The simple and clear example is the CW type charge assignment:

[Bonnefoy, Dudas, Pokorski 1804.01112]
[Choi, CSS, Yun 1909.11685]

$$\frac{1}{2}f^2\left(\partial_\mu\theta^1 - A_\mu^1\right)^2 + \frac{1}{2}f^2\left(\partial_\mu\theta^2 + q\,A_\mu^1 - A_\mu^2\right)^2 + \cdots \frac{1}{2}f^2\left(\partial_\mu\theta^N + q\,A_\mu^{N-1}\right)^2$$



$$a = f\,\frac{q^{N-1}\theta_1 + q^{N-2}\,\theta_2 \cdots + \theta_N}{\sqrt{1 + q^2 + \cdots + q^{2(N-1)}}}$$

Stückelberg Clockwork

The simple and clear example is the CW type charge assignment:

$$\frac{1}{2}f^2\left(\partial_\mu\theta^1 - A^1_\mu\right)^2 + \frac{1}{2}f^2\left(\partial_\mu\theta^2 + q\,A^1_\mu - A^2_\mu\right)^2 + \cdots\frac{1}{2}f^2\left(\partial_\mu\theta^N + q\,A^{N-1}_\mu\right)^2$$

The axion field range $2\pi f_a$ *and the mixing between the axion and original fields* θ^i *are*

$$f_a = \sqrt{\frac{q^2-1}{q^{2N}-1}}f \sim q^{-(N-1)}f, \qquad \langle a|\theta^i\rangle = \frac{q^2-1}{(q^N-q^{-N})q^i}\frac{1}{f_a} \sim \frac{1}{q^i f}$$

The axion field range is hidden in the individual axion couplings.

** An example of highly protecting the PQ symmetry* $(N=3, q=3)$*: Introducing* $N\ U(1)_\alpha$ *charged complex scalars* $(\phi_I \to \phi_I e^{i\delta^I_\alpha \Lambda^\alpha})$ *allows PQ breaking potentials at tree level*

$$\Delta V_{PQV} = \epsilon_1 e^{i4\theta^1}\phi_1^{*4} + \epsilon_2 e^{i\theta^2}\phi_1^3\phi_2^* + \epsilon_3 e^{i\theta^3}\phi_2^3\phi_3^* + \epsilon_4\phi_4^{*4} + h.c.$$

However the axion potential is only generated at 27-th loop as

$$V_{eff}(a) \sim \frac{\epsilon_1^9\epsilon_2^{12}\epsilon_3^4\epsilon_4}{(16\pi^2)^{27}}\Lambda^4\cos\frac{a}{f_a}$$

Stückelberg Clockwork

The simple an

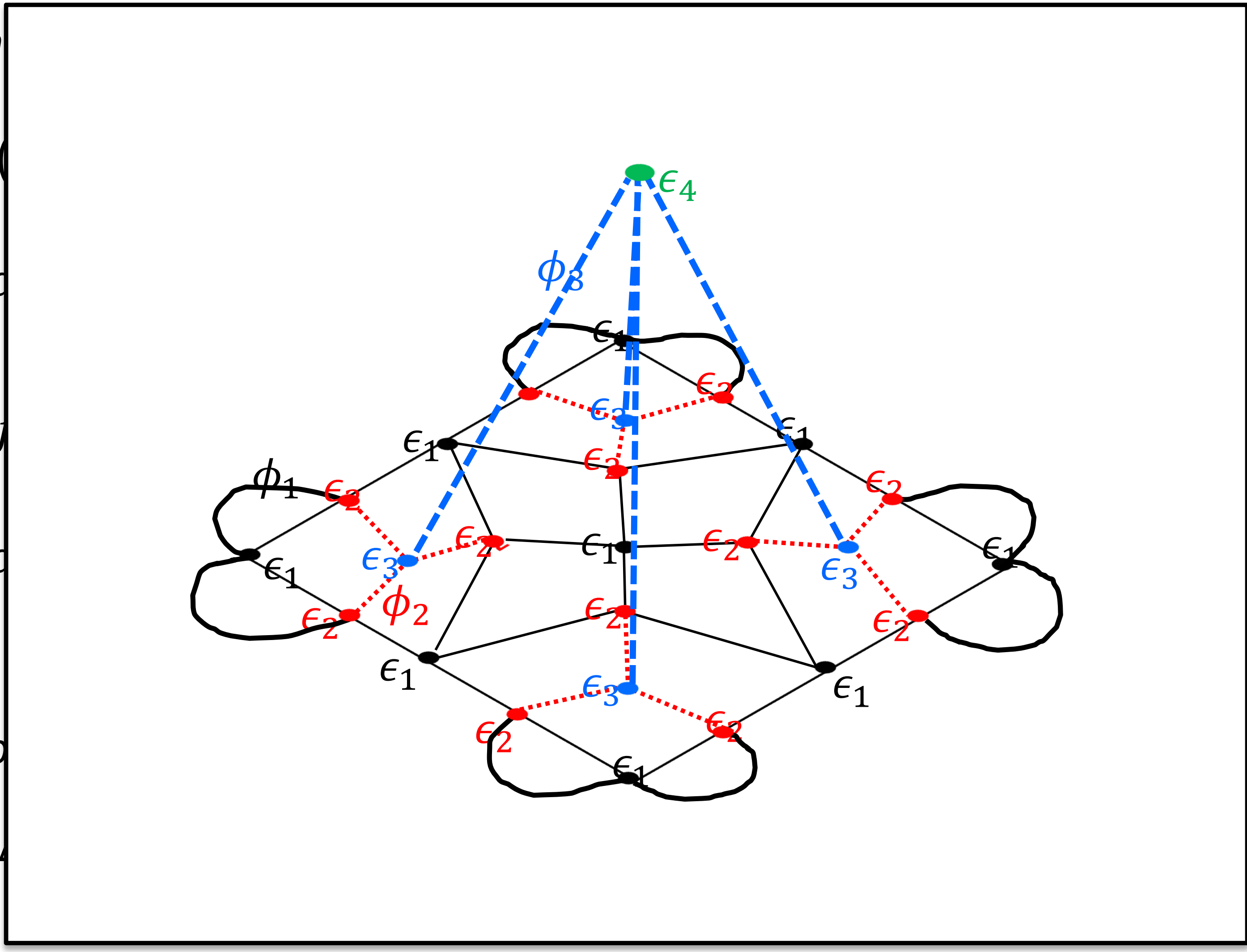
$\frac{1}{2}f^2($ … $^{1})^2$

The axion fiel … θ^i *are*

The axion fiel

** An example* … $N\ U(1)_\alpha$

charged comp … *level*



However the axion potential is only generated at 27-th loop as

$$V_{eff}(a) \sim \frac{\epsilon_1^9 \epsilon_2^{12} \epsilon_3^4 \epsilon_4}{(16\pi^2)^{27}} \Lambda^4 \cos\frac{a}{f_a}$$

Summary

The non-trivial light ALP can be generated by Stückelberg mixing: some of combinations become heavy by Stückelberg (Higgs) mechanism, and the gauge invariant combination becomes light.

We showed that the axion field range is unambiguously determined by the kinetic term of the Stückelberg axions: the large field excursion during cosmological evolution is (generically) forbidden. This can be manifest in a certain field basis

The counter-intuitive examples about the relation between the couplings (decay constant) and the axion period are studied.

For the case with the large number of Stückelberg axions, the axion field range is exponentially suppressed, which leads to the highly protected axion shift symmetry.