Formation of primordial black holes and their cosmological implications

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Summary

Introduction

BHs of a wide range of masses could have been formed in the very early Universe through a variety of mechanisms, well before structures such as galaxies or stars arise.

Collapse of primordial fluctuations during radiation domination is the most often discussed PBH formation mechanism.

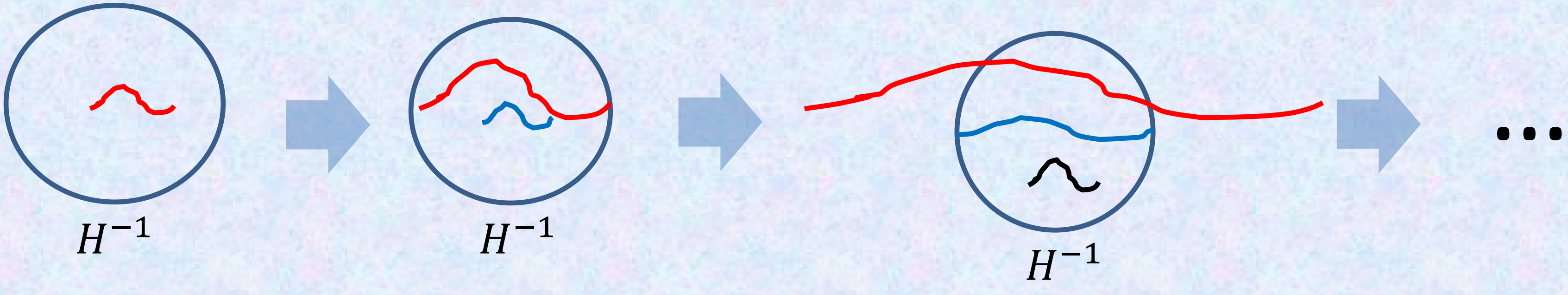
In most of this talk, I focus on this mechanism of PBH.

Primordial fluctuations with a vast range of scales

We first consider quantum fluctuations of e.g. inflaton well inside the Hubble radius ($H^{-1} = a/\dot{a}$), nearly constant during inflation.

They get stretched due to the exponential spatial expansion, exiting the Hubble radius.

This process repeats itself during inflation, resulting in fluctuations with a **vast** range of wavelengths.



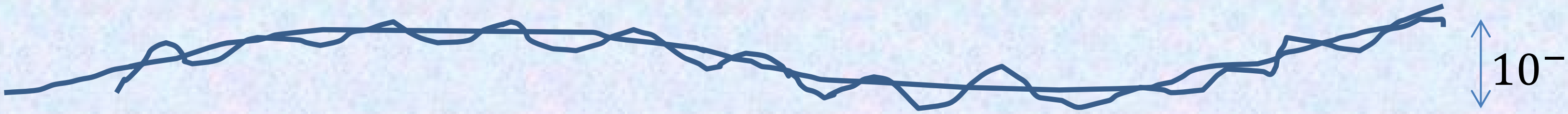
Primordial fluctuations with large comoving (current) wavelengths (Mpc-Gpc) eventually lead to the anisotropy in CMB and large-scale-structure (LSS) of the Universe.

By observing CMB and LSS, their properties such as the amplitude can be probed.

On very large comoving scales (Mpc-Gpc), the amplitude of primordial fluctuations is known to be $\sim 10^{-5}$ with only slight scale dependence.

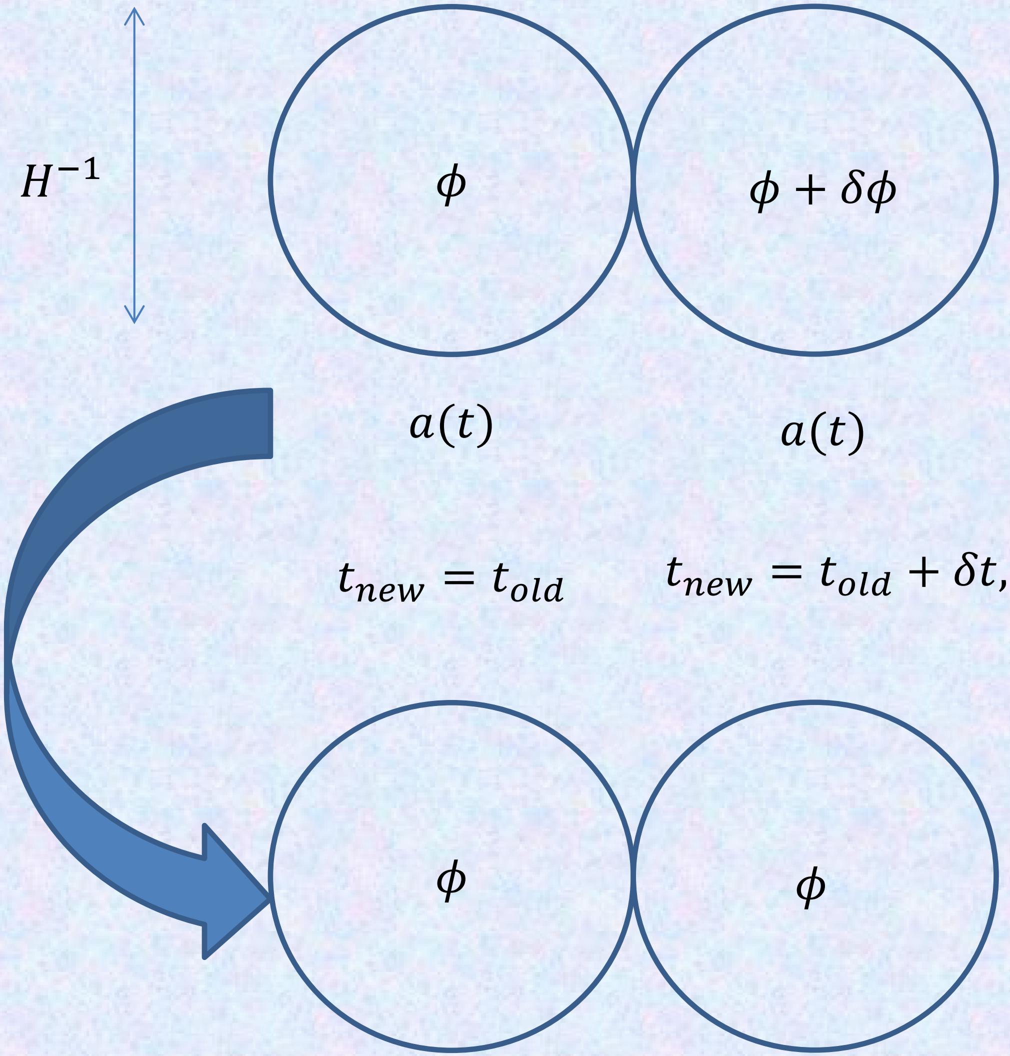
The amplitude on small scales is unknown, and could be substantially larger than 10^{-5}.

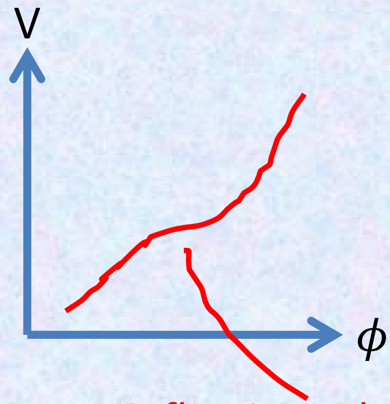
nearly-scale invariant



enhanced small-scale fluctuations?

How small-scale primordial fluctuations could be enhanced?





inflection, plateau

fluctuations generated around this feature are enhanced

$t_{new} = t_{old}$

$$t_{new} = t_{old} + \delta t, \delta t = \frac{\delta\phi}{\dot{\phi}}$$

$$\delta\phi \simeq H/2\pi$$

$$\zeta = -\frac{H\delta\phi}{\dot{\phi}} \simeq \frac{H^3}{6\pi V'}$$

$$\dot{\phi} \simeq -\frac{V'}{3H}$$

$a(t_{old}) = a(t_{new})$

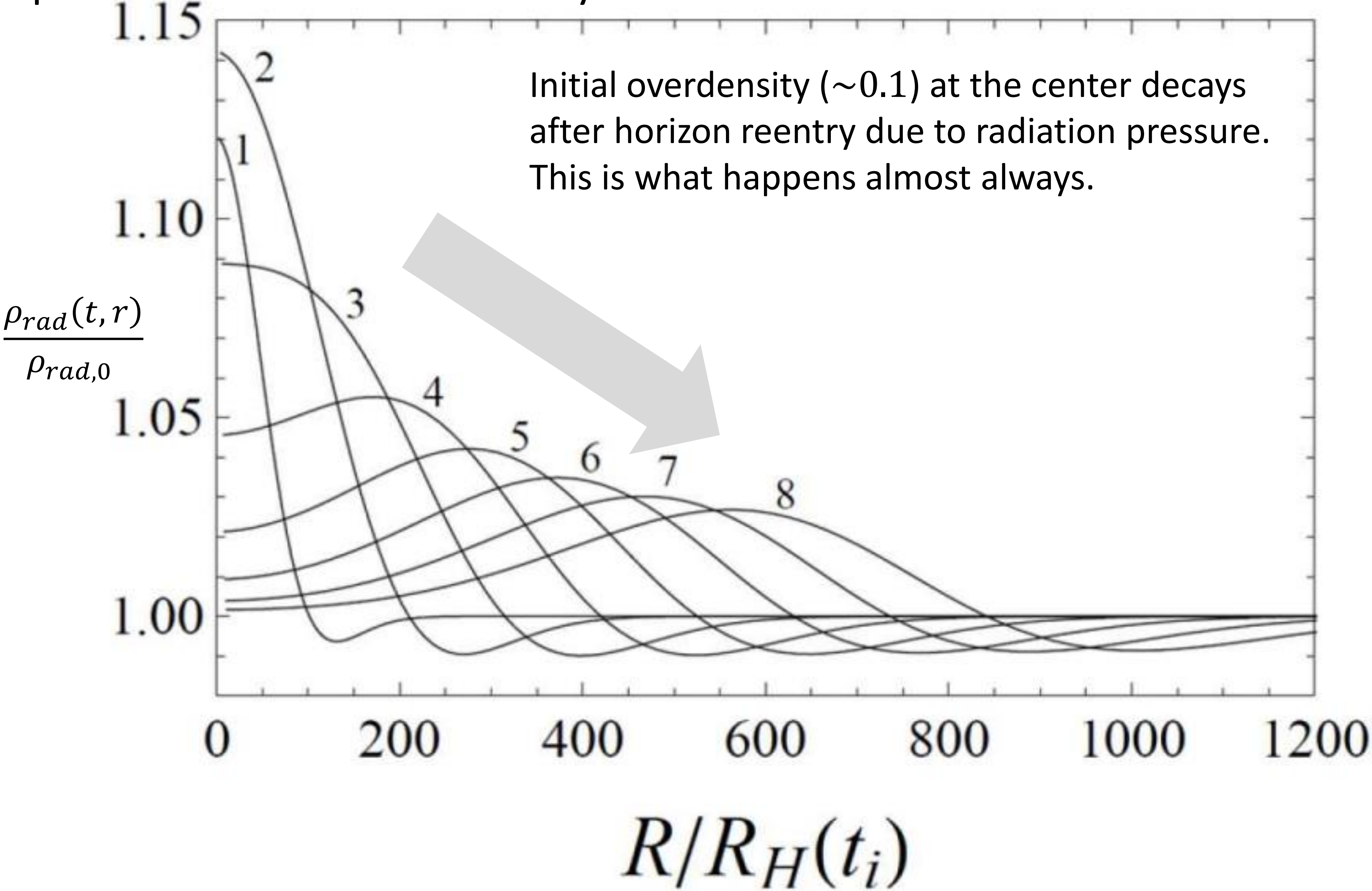
$$a(t_{old}) = a(t_{new} - \delta t) = a(t_{new}) - \dot{a}\delta t$$
$$= a(t_{new})(1 - \underbrace{H\delta\phi/\dot{\phi}}_{\zeta})$$

Let us focus on the fate of small-scale primordial fluctuations.

They result in over(under) densities in radiation after the Universe became dominated by radiation.

I show how a small-amplitude radiation overdensity evolves during radiation domination.

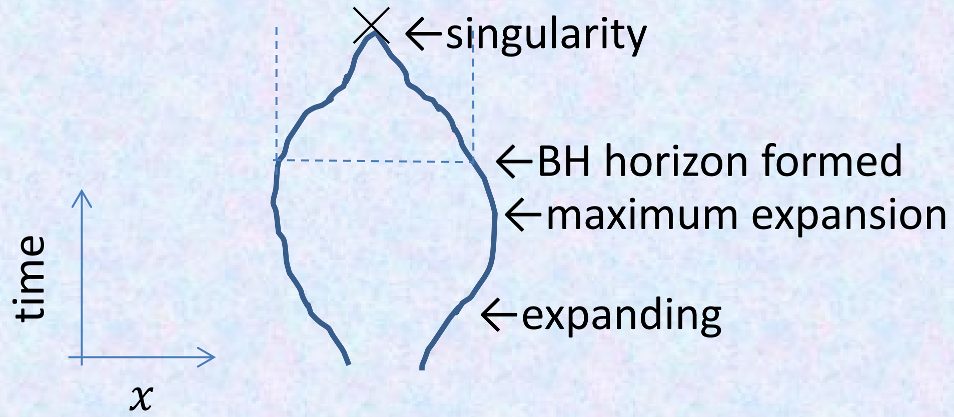
Radiation overdensities with **small amplitude** decay due to radiation pressure after horizon reentry



Initial overdensity (~ 0.1) at the center decays after horizon reentry due to radiation pressure. This is what happens almost always.

TN, Harada, Polnarev, Yokoyama, 2013

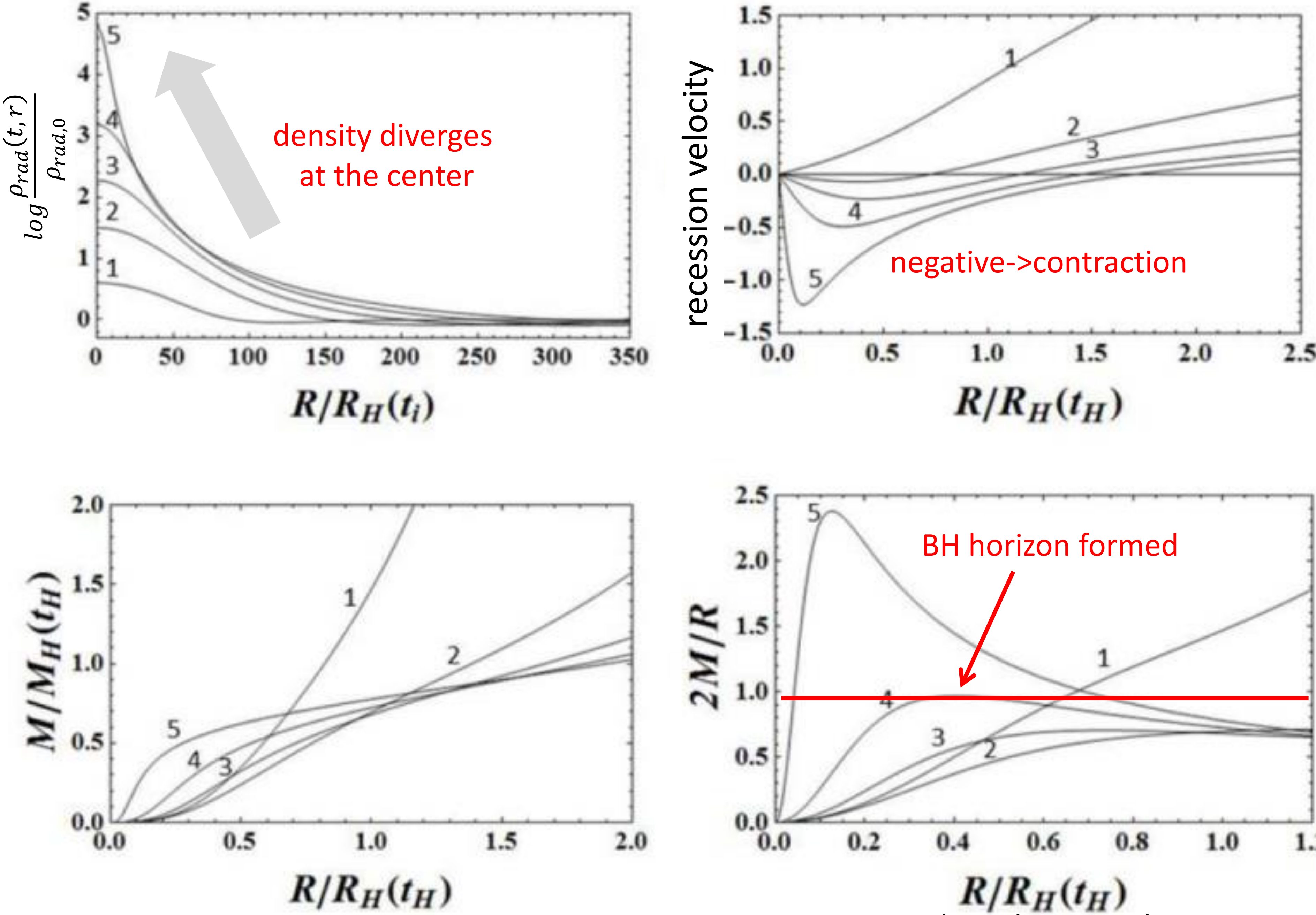
On the other hand when the initial amplitude is **sufficiently large**, the overdensity starts contraction at some point, and it soon falls within its gravitational radius, forming a PBH.



In this case gravity forces overcome pressure gradient forces.

This mostly happens shortly after horizon reentry.

PBH formation in numerical simulation



TN, Harada, Polnarev, Yokoyama, 2013

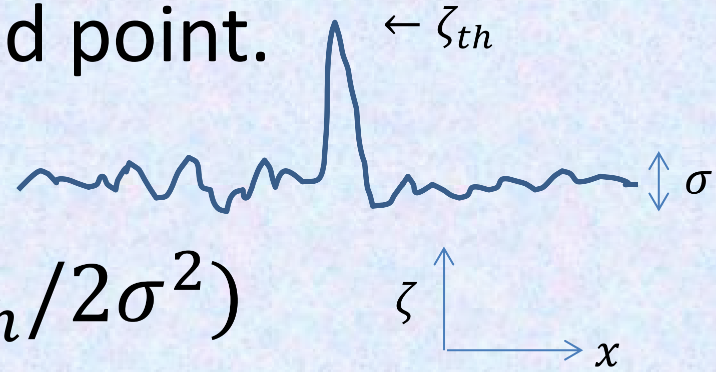
PBH is formed when the perturbation amplitude is larger than some large value (**threshold**).

PBH formation **threshold** is very crudely $\zeta_{th}\sim 0.5$, but it depends on perturbation profile. See TN, Harada, Polnarev, Yokoyama, 2013.

It also depends on the gauge and what quantities one adopts to characterize perturbations. See Harada, Yoo, TN, Koga, 2015.

How common is PBH formation?

Let $P(\zeta)d\zeta$ denote the probability of ζ lying in $(\zeta, \zeta + d\zeta)$ at a randomly selected point.



For Gaussian fluctuations,

$$P(\zeta_{th}) \propto \exp(-\zeta_{th}^2/2\sigma^2)$$

σ is the root-mean-square (or typical) amplitude.

When smoothed over large scales, $\sigma \sim 10^{-5}$, so if one assumes nearly-scale-invariant primordial fluctuations, PBH formation probability is vanishingly small and there would be no PBHs in our observable Universe.

If small-scale fluctuations are significantly enhanced, PBHs can play an important cosmological role.

Mostly, PBHs are formed shortly after horizon reentry $R = H^{-1}$, and their mass is roughly horizon mass $\rho_{rad} H^{-3}$ at reentry.

Smaller-scale primordial fluctuations reenter the horizon earlier, hence PBHs from them are formed earlier and their masses are smaller.

$$M \sim 10^5 M_\odot \left(\frac{t}{1s}\right) \sim 10^5 M_\odot \left(\frac{T}{MeV}\right)^{-2} \sim 10^{17} M_\odot \left(\frac{z}{z_{eq}}\right)^{-2}$$

$$\sim 10 M_\odot \left(\frac{k}{10^6 Mpc^{-1}}\right)^{-2}$$

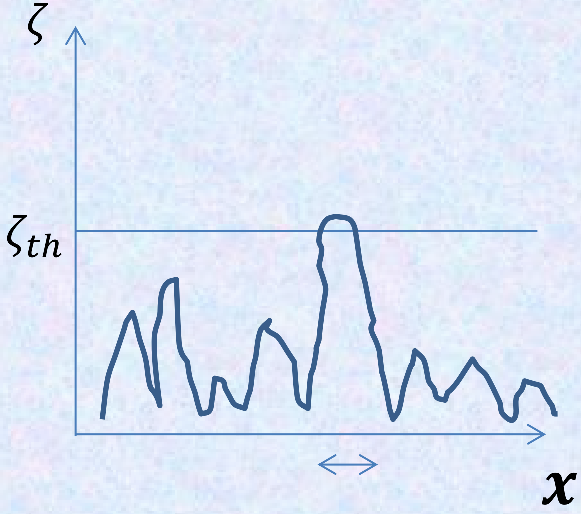
k: comoving wave number

Their abundance has been constrained through different observations, providing valuable information for cosmology.

They can also be used to e.g. provide cold dark matter, explain high-redshift supermassive black holes, gravitational-wave events.

For simplicity, in the following we assume all PBHs have a single mass, which is a good approximation when one considers a sharp spike in the primordial spectrum leading to PBH formation.

Crudely, the abundance of PBHs can be estimated by



$$\int_{\zeta_{th}}^{\infty} P(\zeta)d\zeta = \frac{V_{PBH}}{V_{tot}} = \frac{\rho_{rad}V_{PBH}}{\rho_{rad}V_{tot}} = \frac{\rho_{PBH}}{\rho_{rad}} \equiv \beta$$

↑evaluated at formation epoch

$P(\zeta)$: Probability density function of curvature perturbation ζ

ζ_{th}: threshold for PBH formation $\zeta_{th} \sim 0.5$

$$f = \frac{\rho_{PBH}}{\rho_{DM}} = \frac{\rho_{PBH,eq}}{\rho_{rad,eq}} = \beta \frac{a_{eq}}{a_*}$$

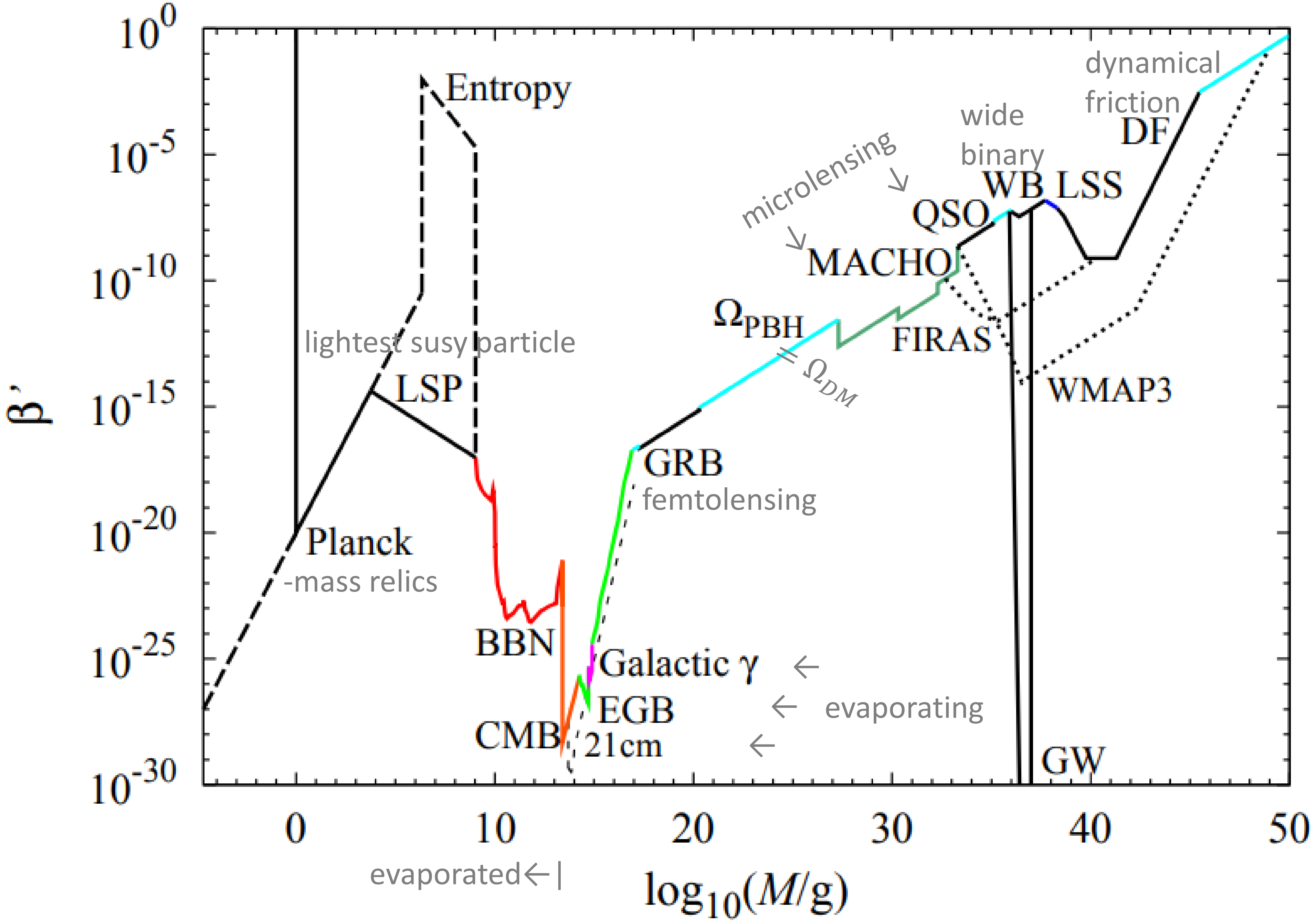
a_*: scale factor at PBH formation

$f \leq 1$, hence $\beta \ll 1$ for $a_* \ll a_{eq}$

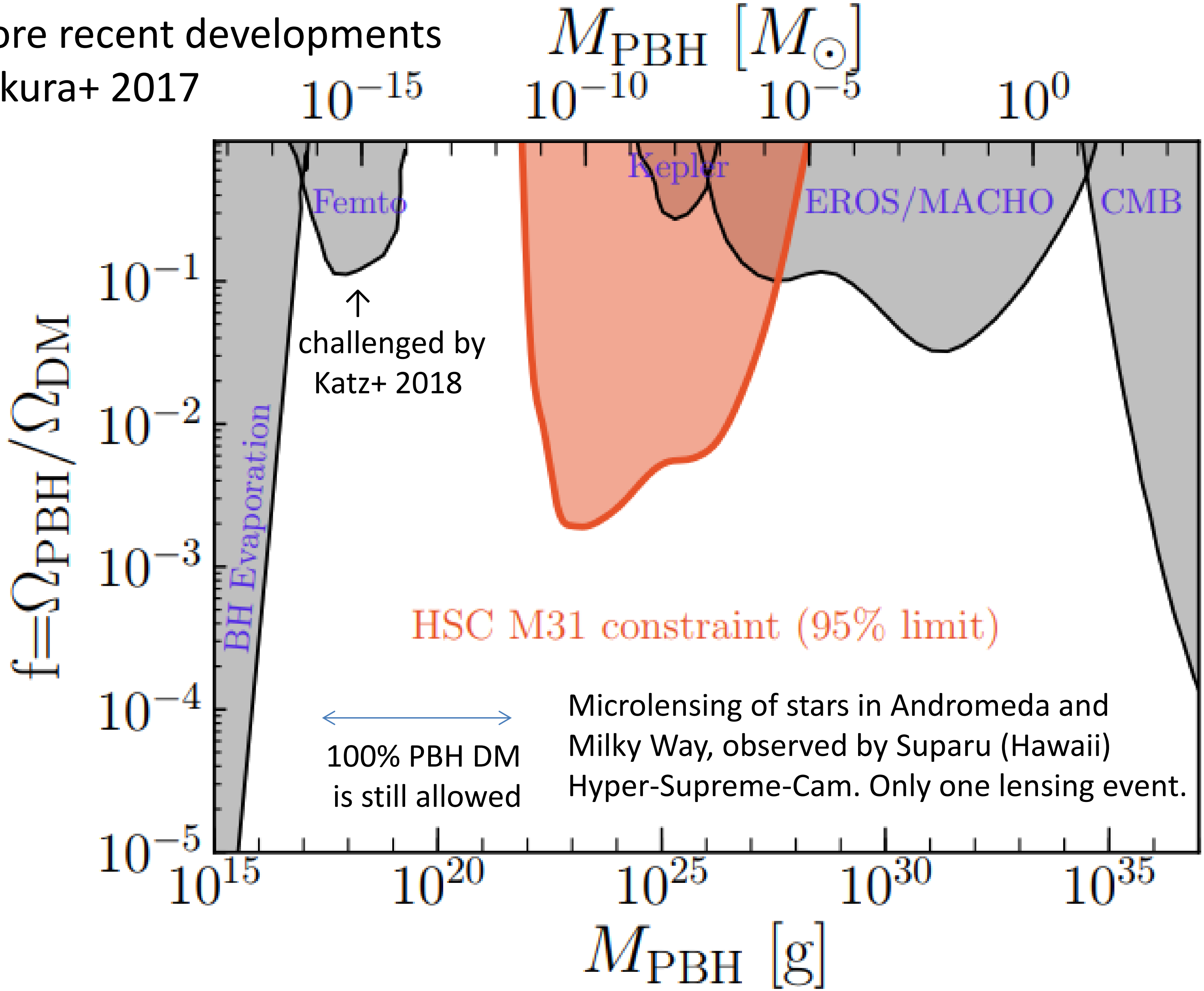
PBH formation had to be extremely rare, even when they play cosmologically interesting roles.

$f = 1$: all the cold dark matter is PBHs

Limits on PBHs, Carr, Kohri, Sendouda, Yokoyama, 2009



More recent developments
Niikura+ 2017



challenged by Katz+ 2018

100% PBH DM is still allowed

Microlensing of stars in Andromeda and Milky Way, observed by Suparu (Hawaii) Hyper-Supreme-Cam. Only one lensing event.

PBHs as probes of primordial fluctuations

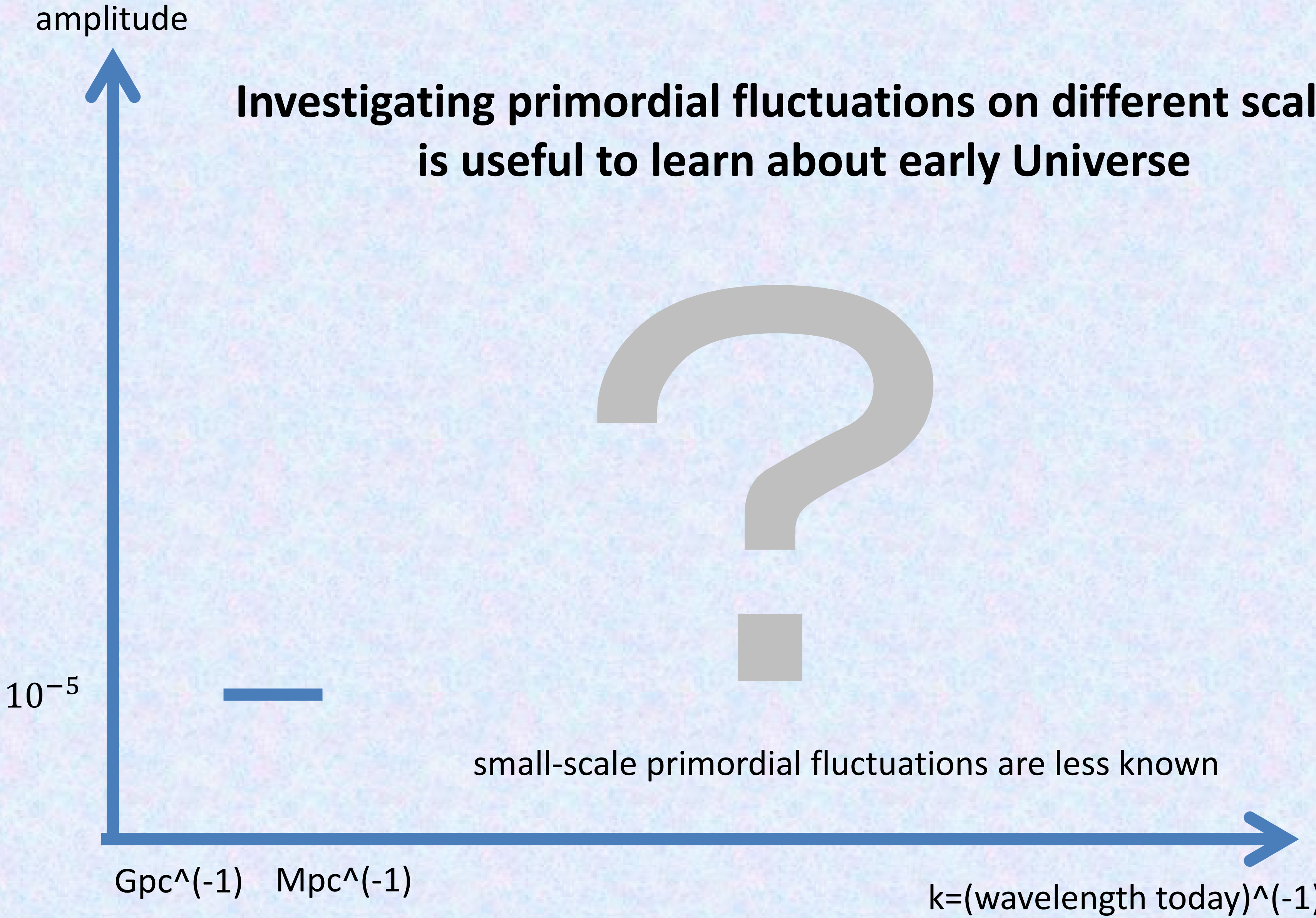
During inflation, primordial fluctuations of a wide range of scales were generated quantum mechanically.

Large-scale primordial fluctuations (Mpc-Gpc) lead to anisotropy in CMB, and galaxies etc.

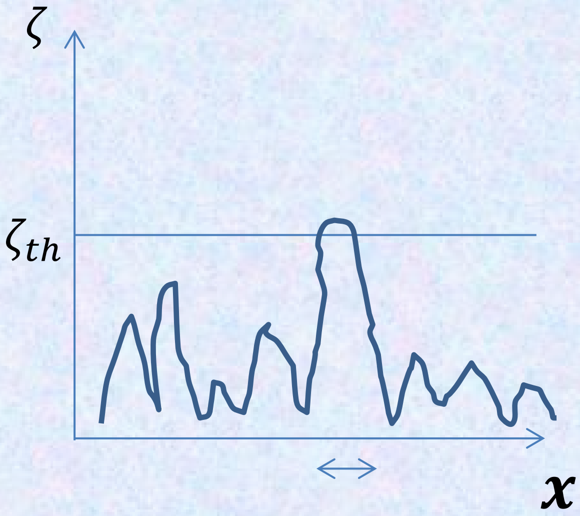
Their properties have been well determined by observing CMB, structures such as galaxies.

These are only a tiny fraction of total modes of primordial fluctuations.

Investigating primordial fluctuations on different scales is useful to learn about early Universe



Crudely, the abundance of PBHs can be estimated by



$$\beta = \frac{\rho_{PBH}}{\rho_{rad}} \sim \int_{\zeta_{th}}^{\infty} P(\zeta) d\zeta$$

↑evaluated at formation epoch

$P(\zeta)$: Probability density function of curvature perturbation ζ

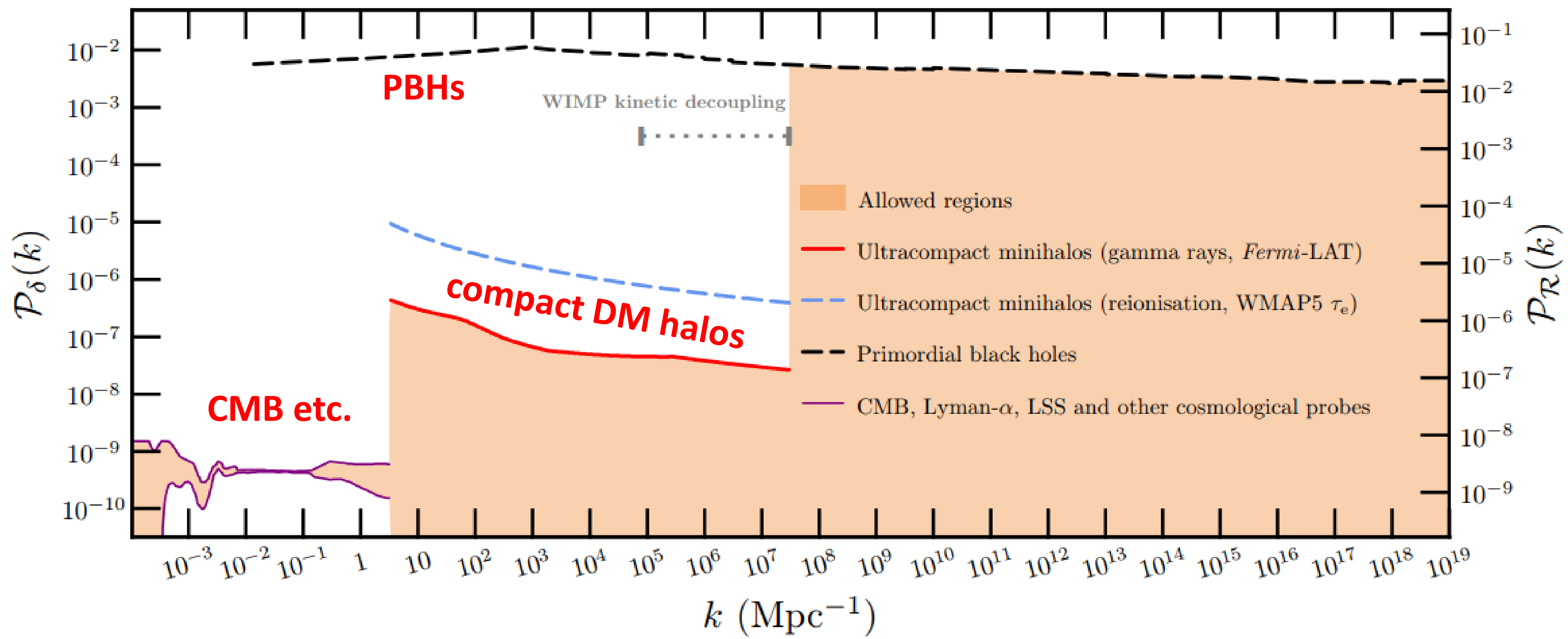
ζ_{th}: threshold for PBH formation $\zeta_{th} \sim 0.5$

If fluctuations follow a Gaussian statistics,

$$P(\zeta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\zeta^2}{2\sigma^2}\right), \quad \sigma^2 = \int \zeta^2 P(\zeta) d\zeta \sim \mathcal{P}_\zeta$$

Upper limits on $\boldsymbol{\beta}$ can be translated into upper limits on $\mathcal{P}_\zeta < \mathcal{O}(0.01)$.

Limits on primordial fluctuations on different scales



Bringmann+ 2011

Short summary for Introduction

PBHs of different masses could have been formed in the early Universe through a variety of mechanisms, the most popular one is collapse of primordial fluctuations during radiation domination.

Their abundance is constrained by observations on different masses.

PBHs can be used to explain the dark matter or probe primordial fluctuations on small scales.

PBHs and gravitational wave observations

Ding, TN, Silk, Wang, 2019

GWs from merging PBHs

If two PBHs are sufficiently close to each other during radiation domination, they would form a binary, taking into account the gravitational force due to a third PBH, closest to the two PBHs.

Ioka, Chiba, Tanaka, Nakamura, 1998

Some of them may merge today or at very low redshifts to be (have been) observed by gravitational-wave experiments.

Assuming all PBHs have the same mass, event rate is function of mass and $f = \Omega_{\mathrm{PBH}}/\Omega_{\mathrm{DM}}$.

For $M \sim 30 M_{\odot}$, $f \sim 10^{-3}$ is sufficient to account for the event rate inferred by the LIGO first detection.

Sasaki, Suyama, Tanaka, Yokoyama 2016

Could we also detect more massive PBH mergers in future space-based experiments?

Event rate increases as f increases for fixed M.

Event rate decreases as M increases for fixed f, mostly because of the smaller number density of PBHs.

Hence, higher f is required for higher M to realize a sufficiently large event rate, detectable by GW experiments.

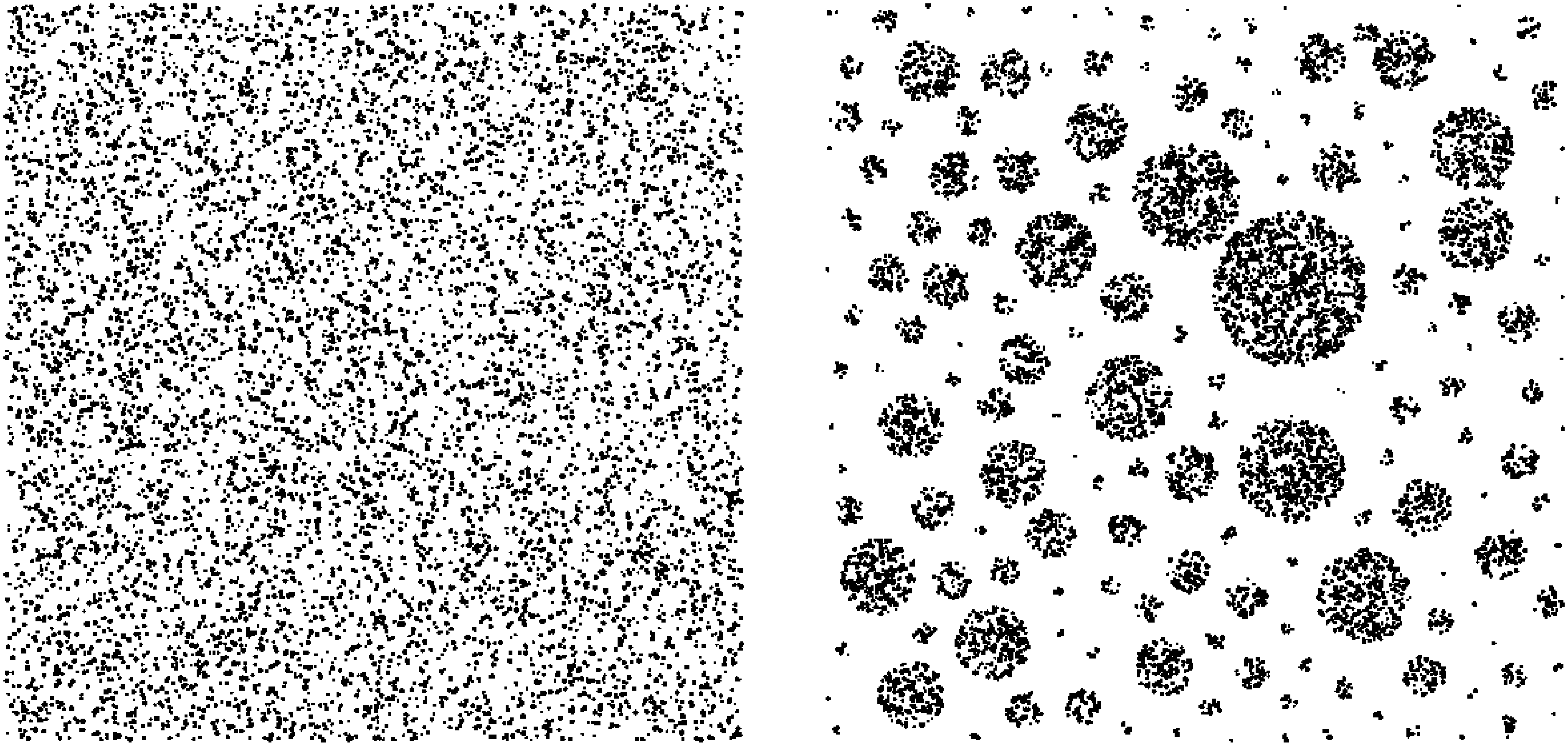
But large f is constrained by observations, so very massive PBH mergers are unlikely to be observed in future space-based experiments.

PBHs with initial clustering

This conclusion is altered if we introduce spatial clustering of PBHs at their formation epoch.

Ding, TN, Silk, Wang, arXiv:1903.07337

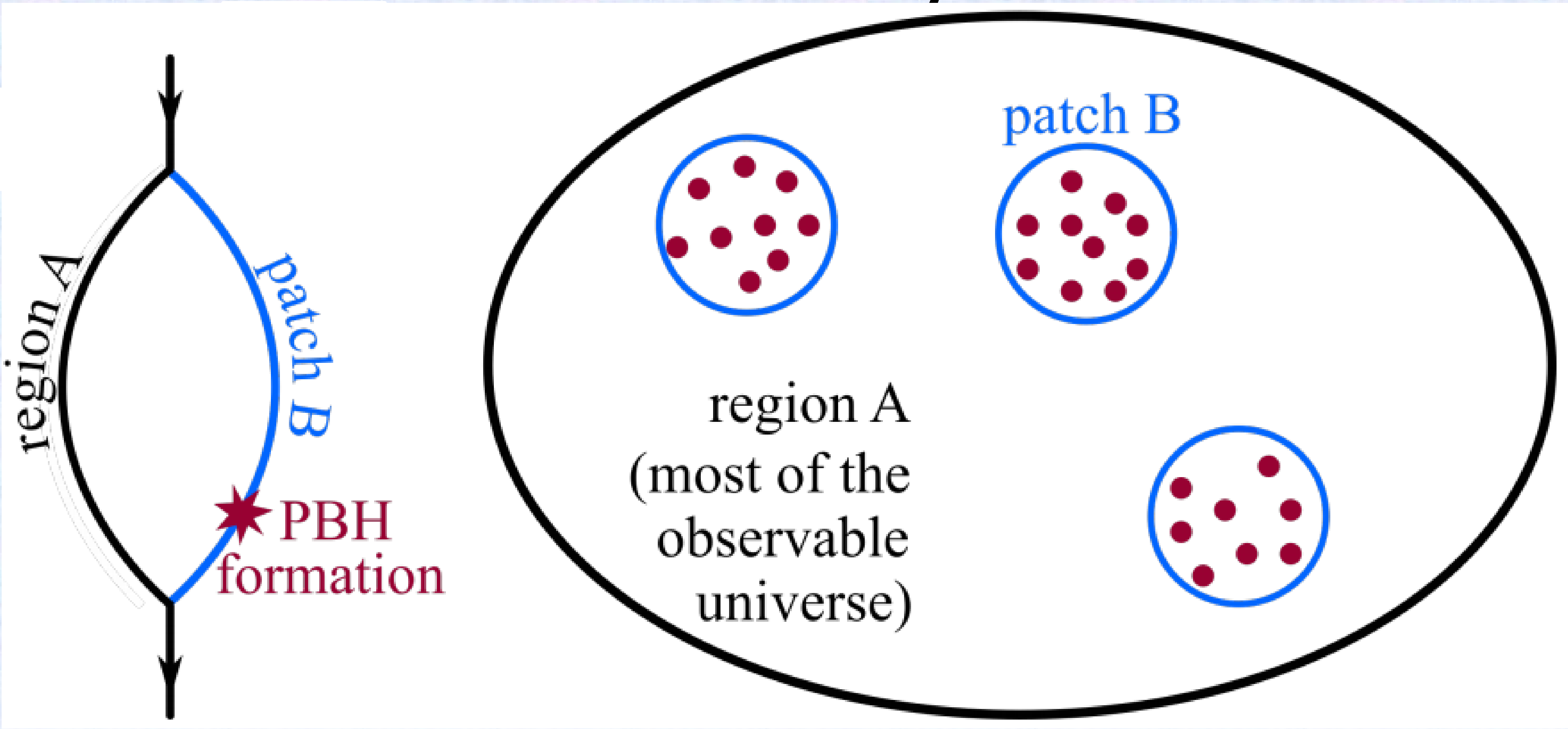
Spatial distribution of PBHs shortly after their formation



small clustering higher clustering

Total number of PBH is the same but event rate is higher for the right.
Sufficiently high event rate can be realized without increasing global number of PBHs too much by clustering, thus evading constraints on PBHs.

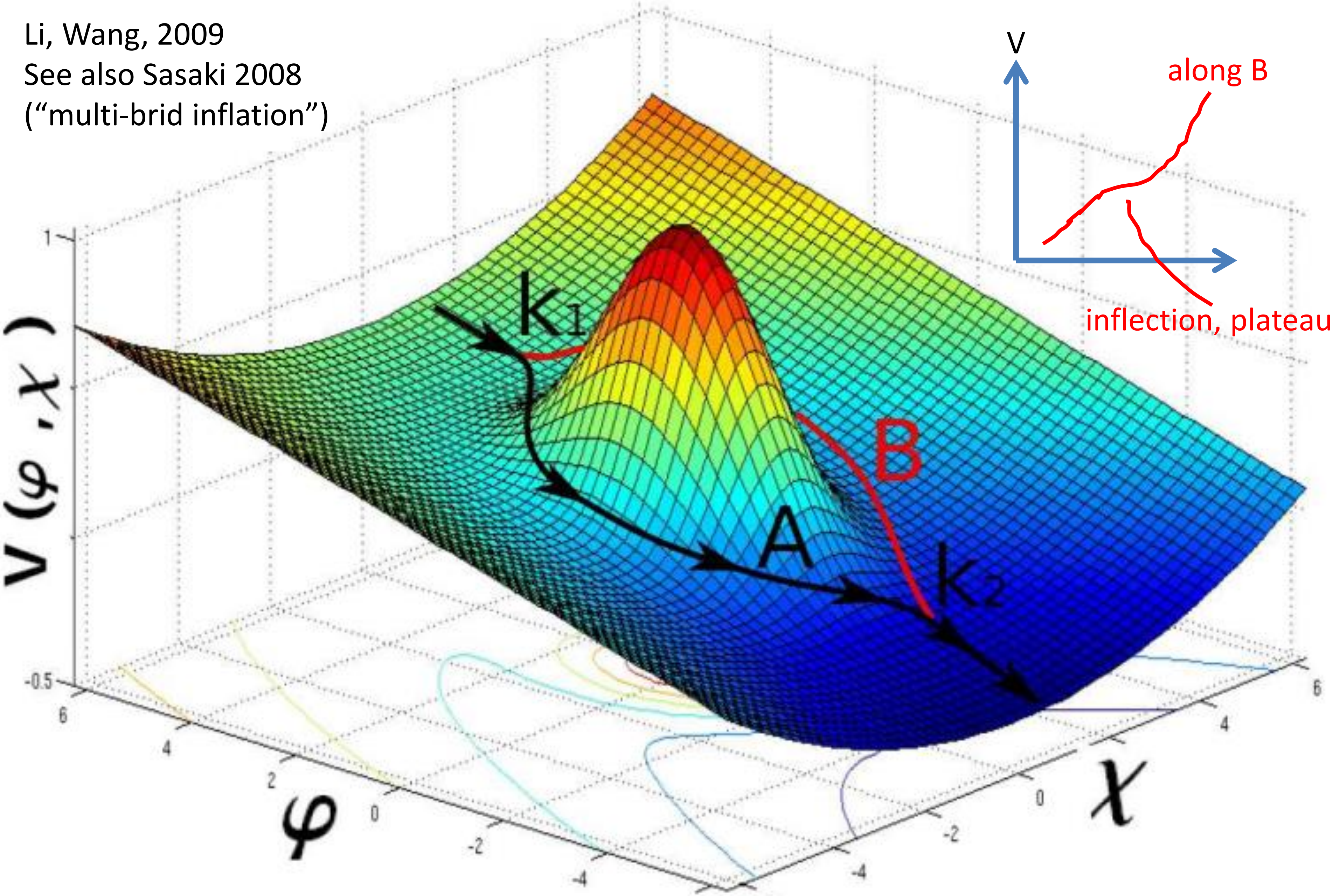
How such a situation could have been realized in the early Universe?



The inflaton trajectory may split or bifurcate in multi-field inflation (multi-stream inflation). Then one can assume any potential shape leading to PBH formation of single field inflation often discussed in the literature on trajectory B.

Li, Wang, 2009, TN, Suyama, Yokoyama, 2016

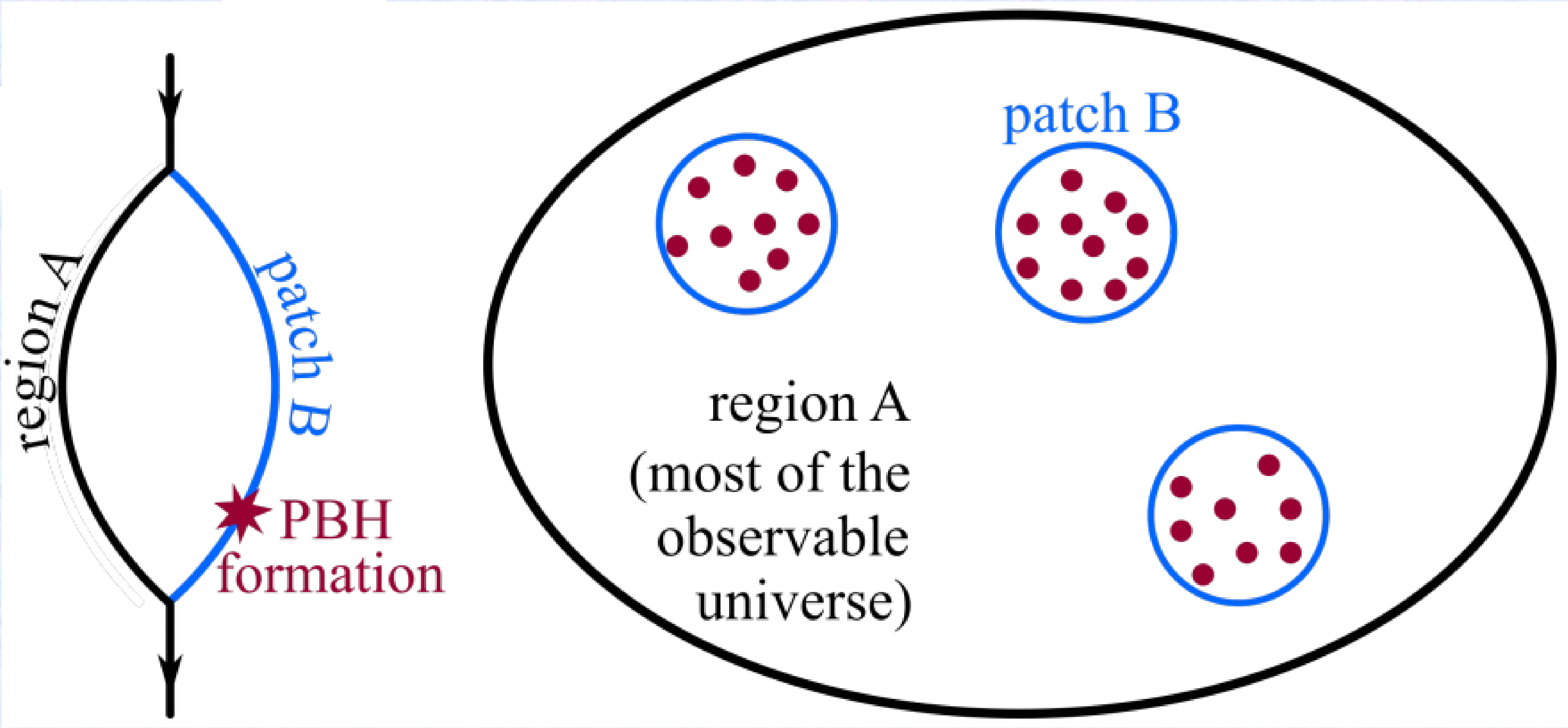
Li, Wang, 2009
See also Sasaki 2008
("multi-brid inflation")



To have more interesting physics, we assume the paths A and B have a slightly different potential energy $\delta V \equiv V(\varphi, \chi_A) - V(\varphi, \chi_B)$, with δV non-vanishing only if $\varphi_1 < \varphi < \varphi_2$.

Model parameters

Ding, TN, Silk, Wang, arXiv:1903.07337



$\beta_1 (< 1)$: volume fraction of B patches

k_1: comoving wave number specifying the radius of B patches

$\beta_2 (< 1)$: fraction of volume inside B patches collapsing to PBHs

k_2: comoving wave number of fluctuations collapsing to PBHs inside B patches

k_2 determines the mass of PBHs.

Global abundance of PBHs at formation is $\beta = \beta_1\beta_2 = \rho_{\mathrm{PBH}}/\rho_{\mathrm{rad}}$, where the ratio should be evaluated at PBH formation.

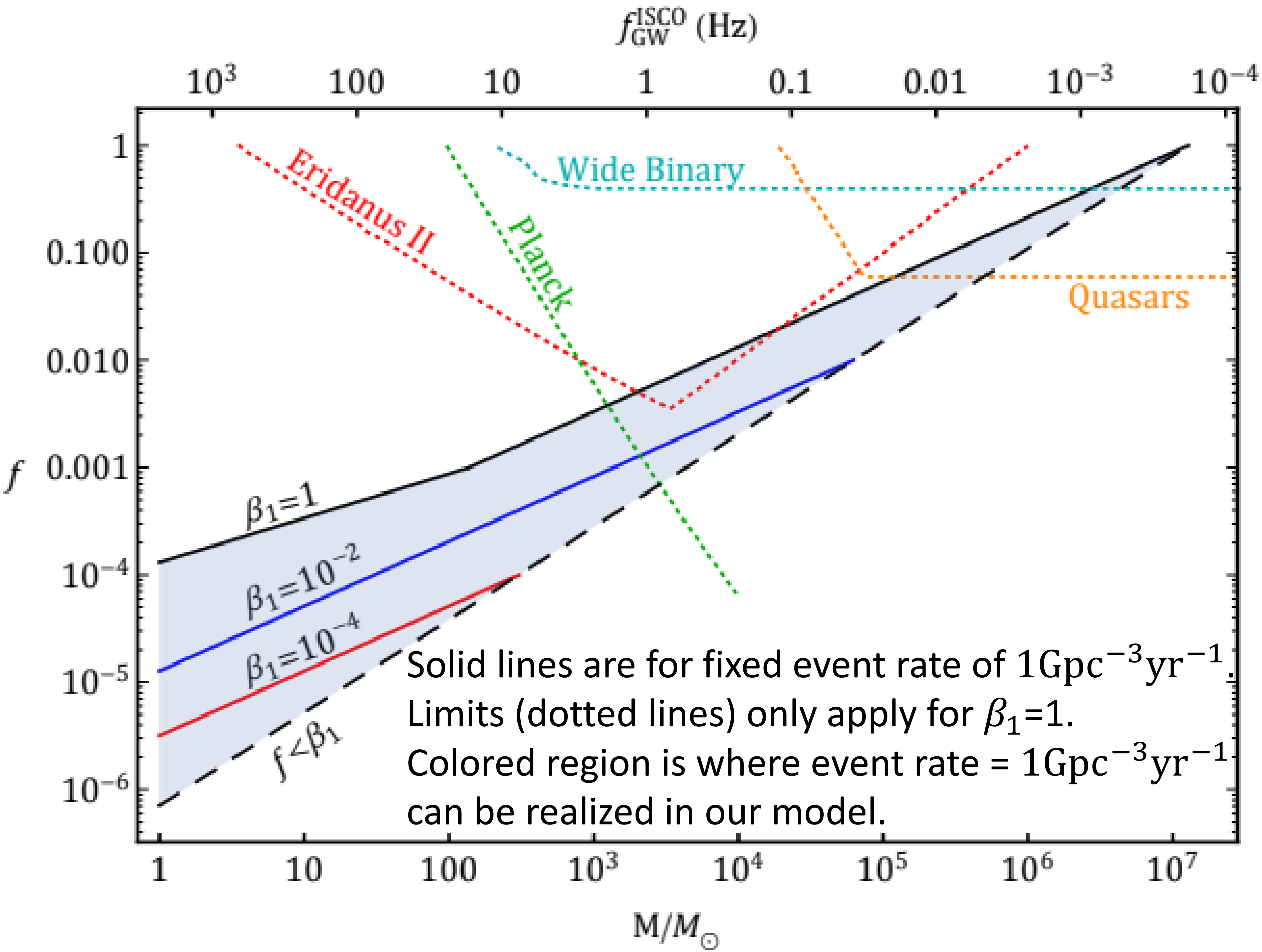
$$\beta_{eq} = \frac{a_{eq}}{a_*}\beta = \frac{\rho_{PBH,eq}}{\rho_{rad,eq}} = \frac{f\rho_{DM,eq}}{\rho_{rad,eq}} = f$$

Local abundance β_2 is enhanced by β_1^{-1} for fixed β, leading to enhanced merger rate in B patches.

Sasaki+ calculated the event rate by considering binary formation during radiation domination, without initial spatial clustering of PBHs.

We can apply Sasaki+ calculations to B patches to estimate the event rate from PBHs with initial spatial clustering.

$\beta_1(< 1)$: volume fraction of B patches
k_1: comoving wave number specifying the radius of B patches
$\beta_2(< 1)$: fraction of volume inside B patches collapsing to PBHs
k_2: comoving wave number of fluctuations collapsing to PBHs inside B patches



Solid lines are for fixed event rate of $1\mathrm{Gpc}^{-3}\mathrm{yr}^{-1}$.
Limits (dotted lines) only apply for β_1=1.
Colored region is where event rate = $1\mathrm{Gpc}^{-3}\mathrm{yr}^{-1}$ can be realized in our model.

It is possible to realize high event rate for massive PBH mergers.

$f < \beta_1$ is needed to ensure validity of our simple estimations of the event rate based on Sasaki+.

If $\beta_2 (< 1)$ is too large, B patches become locally matter dominant well before the global matter-radiation equality. If this happens, simple calculations based on Sasaki+ become invalid.

So we require B patches do not become locally matter dominant well before global matter-radiation equality.

★ $$\beta_2 \times \frac{a_{eq,B}}{a_{PBH}} = 1, a_{eq,B} > a_{eq}, \rightarrow a_{PBH}\beta_2^{-1} > a_{eq} \rightarrow \beta_1 > \beta \frac{a_{eq}}{a_{PBH}} = f$$

$\uparrow$

$\beta = \beta_1 \beta_2$

In the previous figure $\beta_1 < f$ was omitted to use simple analysis, but it would also be possible to explore that regime (work in prep.)

★

Limits on massive PBHs

Planck:

Accretion on PBHs should have affected Planck CMB temperature/polarization date if sufficiently many massive PBHs were present.

Ali-Haïmoud, Kamionkowski, 2016

Eridanus II:

A recently discovered star cluster near the center of the ultra-faint dwarf galaxy Eridanus II (a distant satellite of Milky Way?) should have been disrupted if too many massive PBHs were present.

Brandt 2016

Wide binary:

Observed wide binaries (separation is ~0.1 pc) should have been disrupted if too many massive PBHs were present.

Bahcall, Hut, Tremaine, 1985

Quasar:

Millilensing (image separation ~ milliarcsec) of quasars should have been observed if sufficiently many massive PBHs were present.

Wilkinson+ 2001

Short summary

If initial spatial distribution of PBHs is clustered, coalescence of very massive PBHs $(\mathrm{M} \gg 30 M_{\odot})$ could also be detected by future space-based gravitational wave detectors.

Ding, TN, Silk, Wang, 2019

Cosmological signatures associated with PBH formation from primordial fluctuations

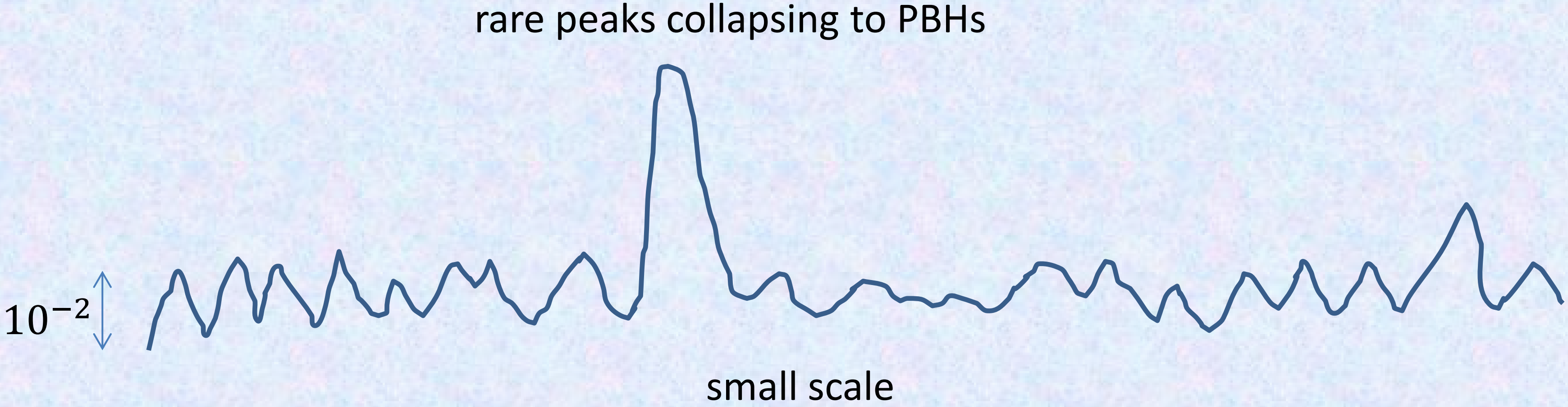
1. Stochastic gravitational waves
   TN, Silk, Kamionkowski, 2016

2. CMB spectral distortions
   Kohri, TN, Suyama, 2014, TN, Suyama, Yokoyama, 2016, TN, Carr, Silk, 2017

In order for PBHs to play some cosmological role (explain dark matter, quasars, or GW events), their formation probability had to be sufficiently large.

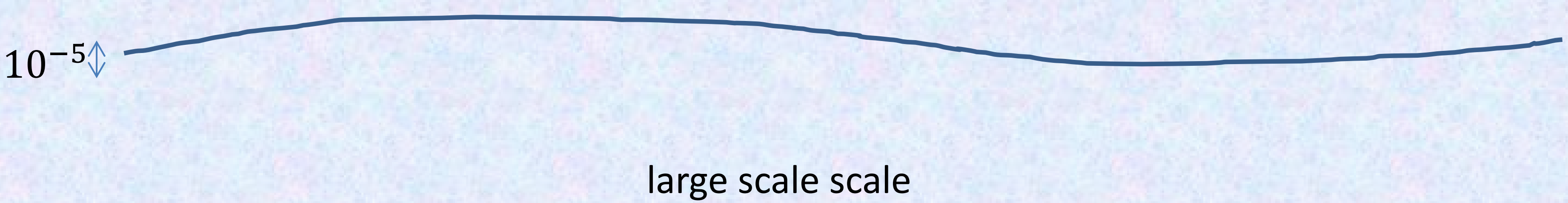
For Gaussian fluctuations, this means the root-mean-square amplitude of small-scale primordial fluctuations is $\sim\mathcal{O}(0.01)$ to realize a cosmologically interesting amount of PBHs.

This is much larger than the amplitude of primordial fluctuations on large scales 10^{-5}.



large-amplitudes of small-scale fluctuations

-> stochastic GWs, CMB distortions



Gravitational waves induced at second order in scalar perturbations

Cosmological perturbations can be decomposed in scalar, vector, tensor perturbations.

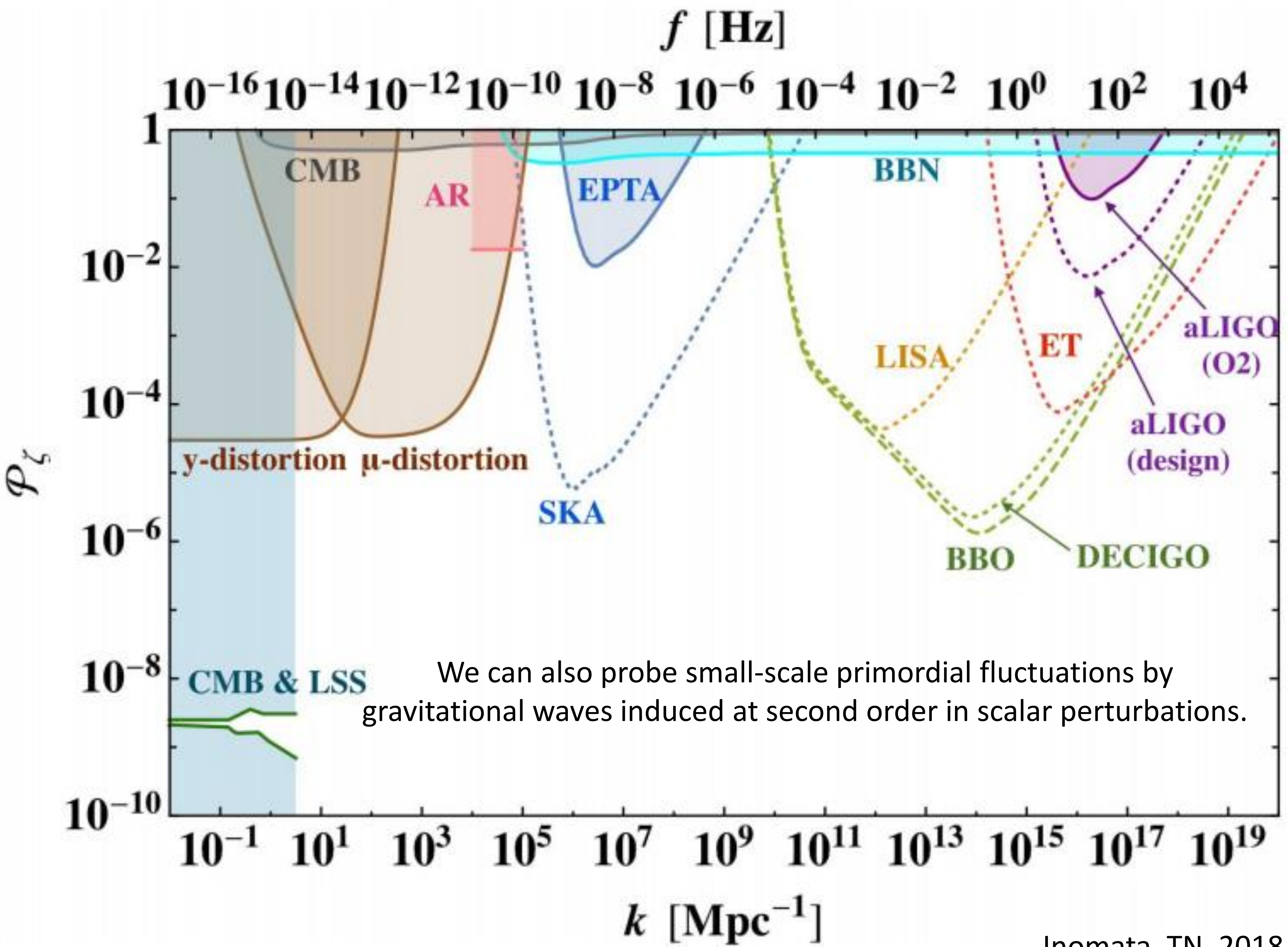
They evolve independently in linear pert. theory.

They are coupled at non-linear level.

For instance, tensor perturbations can be generated by scalar perturbations at second order (induced GWs).

If small-scale perturbations are enhanced, induced GWs become large.

Ongoing and future GW experiments can probe induced GWs or small-scale scalar perturbations.



We can also probe small-scale primordial fluctuations by gravitational waves induced at second order in scalar perturbations.

Inomata, TN, 2018

Induced GWs associated with PBH formation

The root mean square amplitude of small-scale fluctuations determines induced GWs. $\Omega_{GW} \sim \sigma^4 \Omega_{rad}$

To produce some interesting amount of PBHs, small-scale fluctuations need to be enhanced, leading to large induced GWs.

RMS required to produce some amount of PBHs depends on primordial non-Gaussianity.

Induced GWs associated with PBH formation depend on primordial non-Gaussianity.

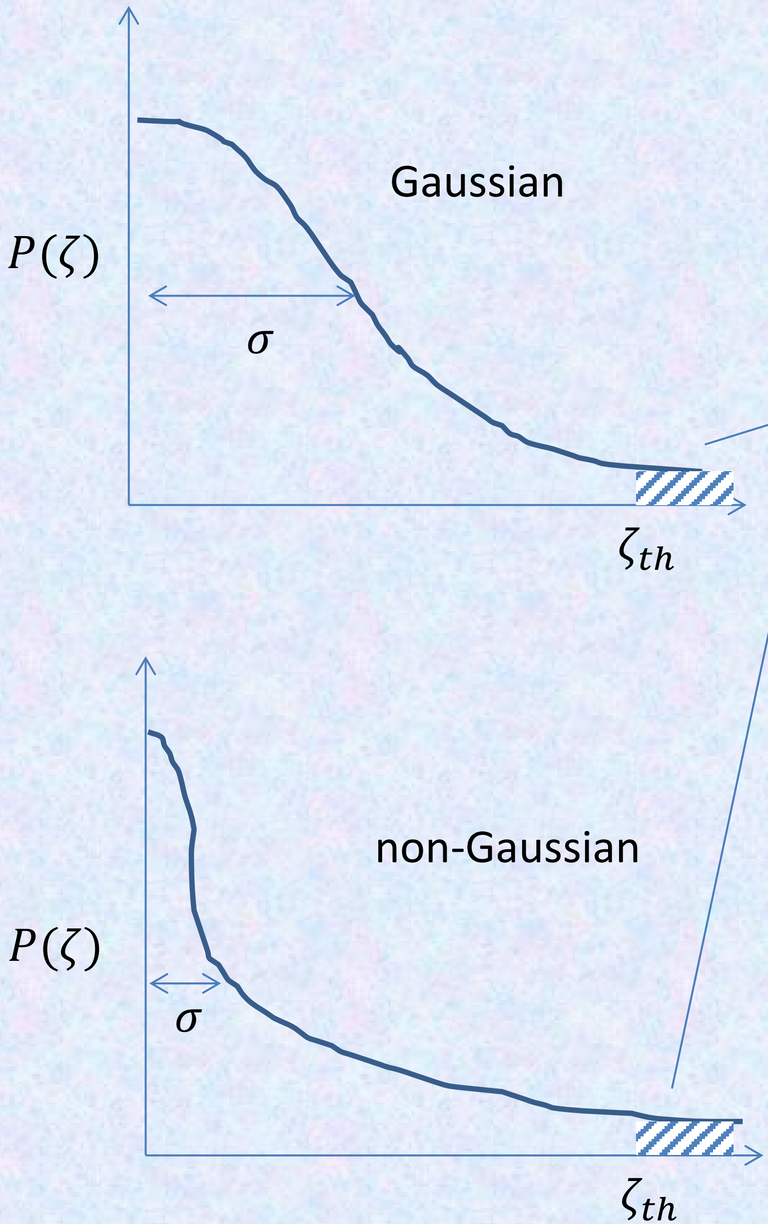
It is natural non-Gaussianity (NG) is large for small-scale perturbations, enhanced such that PBH formation probability is relatively large.

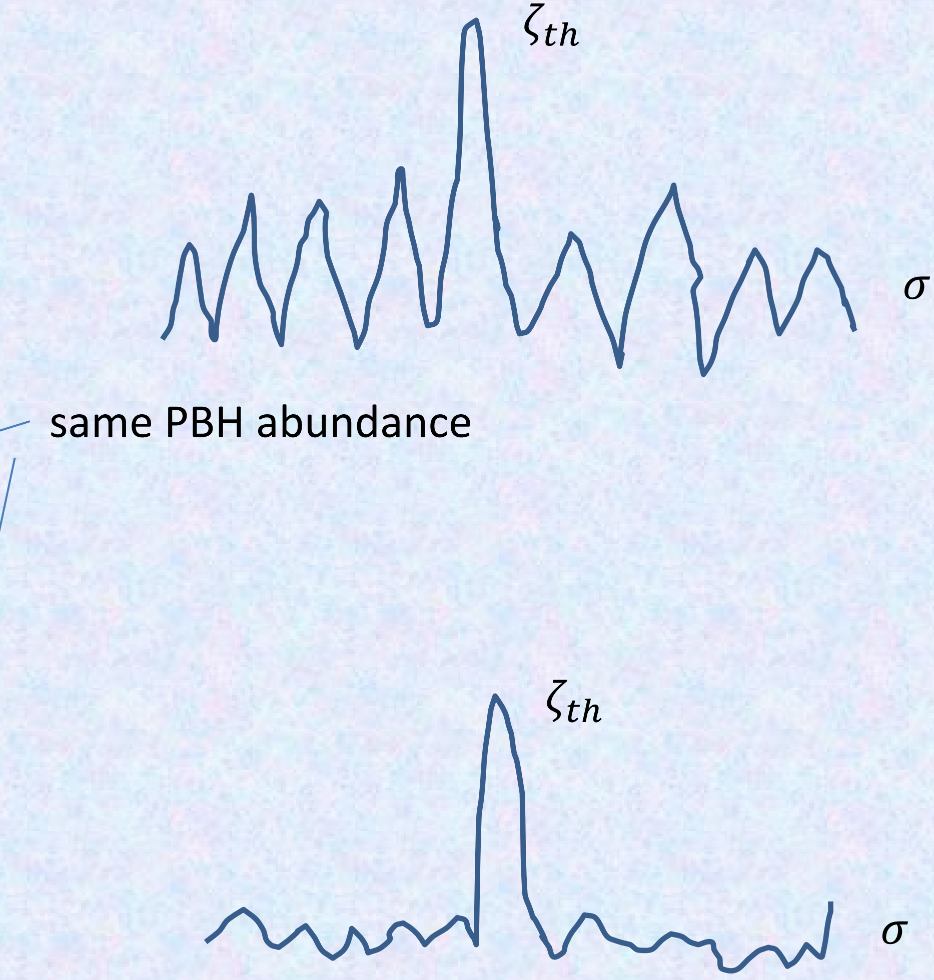
This is partly because only non-linear perturbations can collapse to form PBHs, and non-linearity implies NG.

(Note: On large scales fluctuation amplitudes are known to be small and Gaussianity to be a good approximation.)

It is mostly very challenging to calculate NG for each of many models predicting PBH formation.

Hence we use a few toy or phenomenological descriptions of NG for now to investigate the effects of NG.

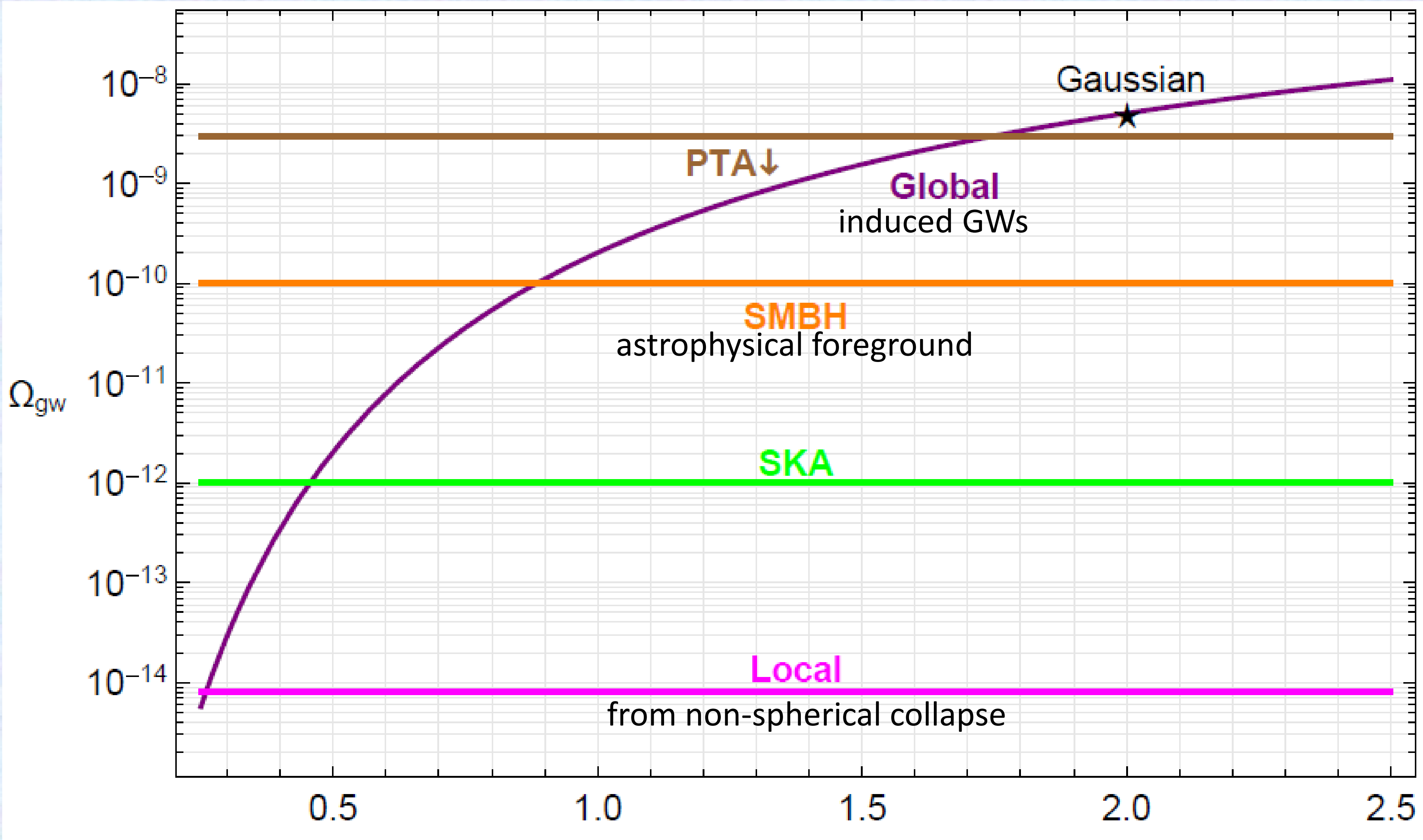




← σ is smaller for this non-Gaussian PDF, compared with σ of a Gaussian PDF leading to the same amount of PBHs.

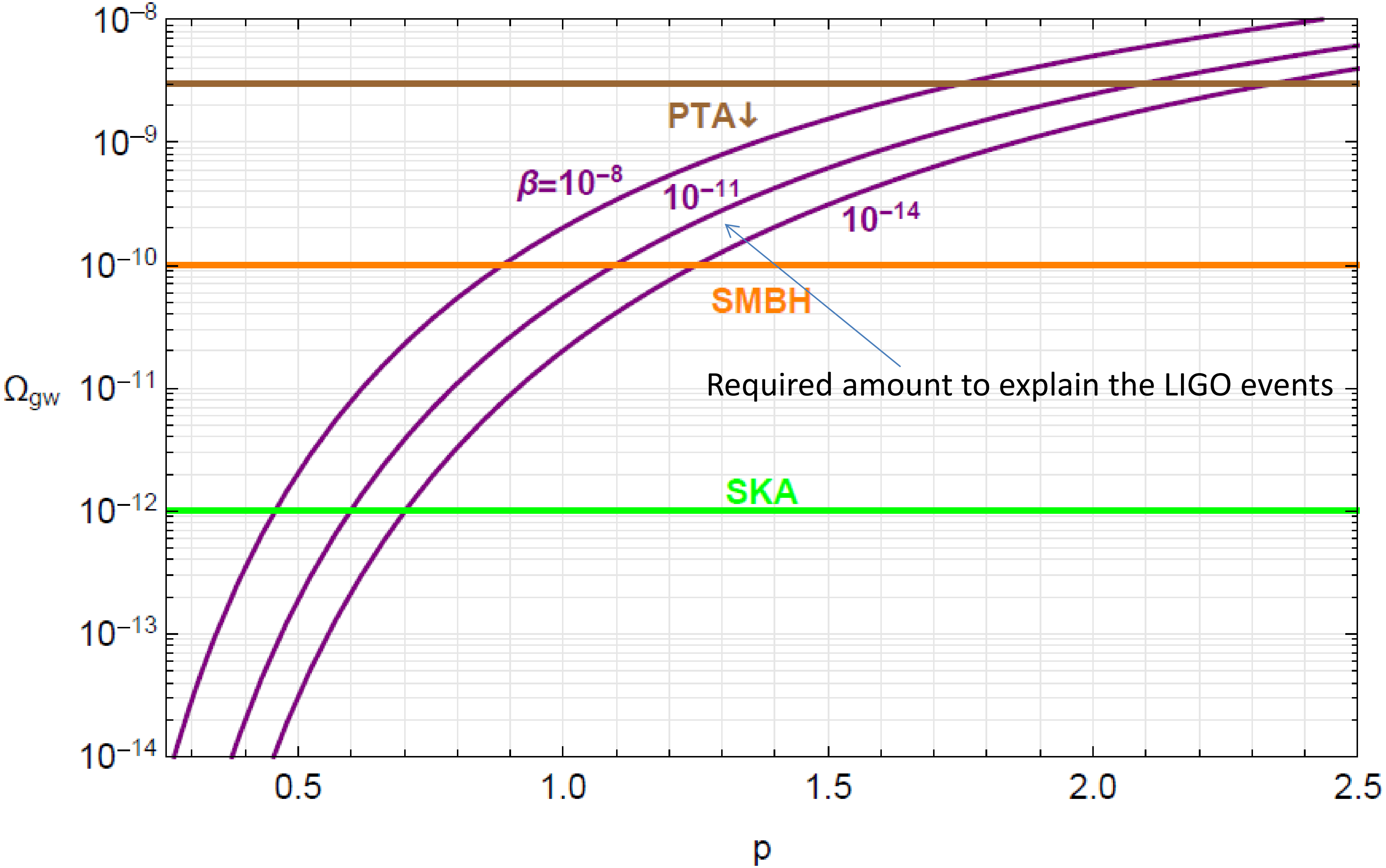
This non-Gaussianity leads to smaller stochastic gravitational wave backgrounds.

Induced GWs from formation of $30M_{\odot}$ with $\mathrm{f} = \frac{\Omega_{PBH}}{\Omega_{\mathrm{DM}}} = 1$ at ~nHz

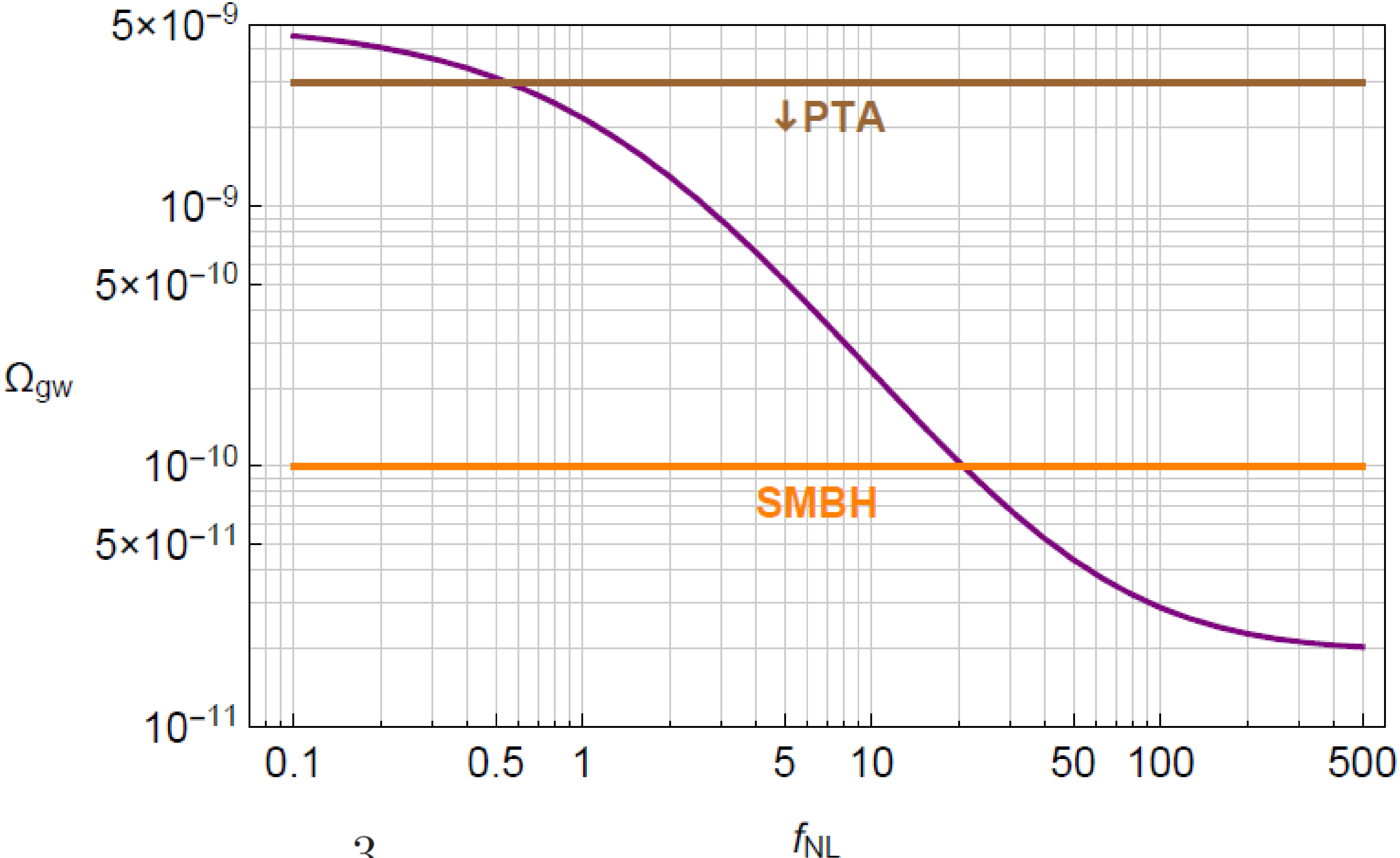


$$P(\zeta) = \frac{1}{2\sqrt{2}\tilde{\sigma}\Gamma\left(1+1/p\right)} \exp\left[-\left(\frac{|\zeta|}{\sqrt{2}\tilde{\sigma}}\right)^{p}\right]$$ p

Nakama, Silk, Kamionkowski, 2016

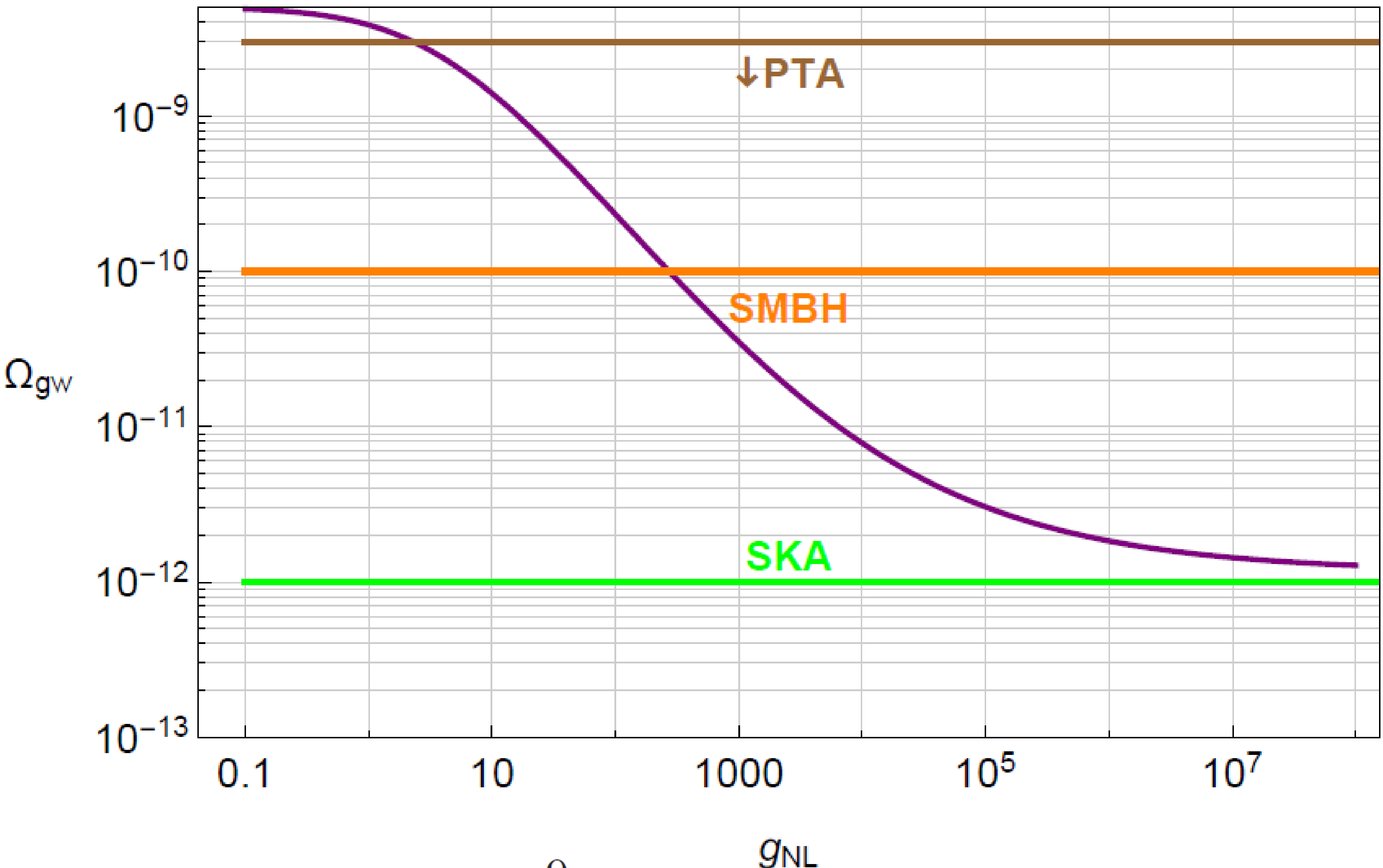


PTA↓
β=10^-8
10^-11
10^-14
SMBH
Required amount to explain the LIGO events
SKA
Ω_gw
10^-8
10^-9
10^-10
10^-11
10^-12
10^-13
10^-14
0.5
1.0
1.5
2.0
2.5
p



$$\zeta = \zeta_G + \frac{3}{5} f_{\rm NL}(\zeta_G^2 - \sigma_G^2)$$

See 1711.08956.



$$\zeta = \zeta_G + g\zeta_G^3, \quad g \equiv \frac{9}{25} g_{\rm NL}$$

Short summary

Gravitational waves are induced from scalar perturbations at second order.

Such induced GWs can be large when PBHs are formed from enhanced small-scale primordial fluctuations.

Induced GWs associated with a fixed amount of PBHs depend on primordial non-Gaussianity.

Cosmological signatures associated with PBH formation from primordial fluctuations

1. Stochastic gravitational waves

   TN, Silk, Kamionkowski, 2016

2. CMB spectral distortions

   Kohri, TN, Suyama, 2014, TN, Suyama, Yokoyama, 2016, TN, Carr, Silk, 2017

Small-scale acoustic waves are damped due to photon diffusion (Silk damping).

After damping, the energy stored in acoustic waves gets transferred to background Universe.

At $z < 2 \times 10^6$, reactions needed to maintain thermodynamic equilibrium are inefficient.

So this kind of energy injection leads to the photon energy spectrum slightly deviating from Planck spectrum (CMB distortions).

It has been tightly constrained by experiments.

★

At $z > 2 \times 10^6$, $\rho_\gamma \sim T^4$, $n_\gamma \sim T^3$, energy injection increases T, then n_γ increases, mostly by double Compton scattering.

At $z < 2 \times 10^6$, it becomes inefficient, n_γ can not change, but Compton scattering is still efficient until $z \sim 5 \times 10^4$.

In this case energy injection leads to Bose-Einstein distribution with μ parameter. $n^{-1} \propto \exp\left(\frac{h\nu}{kT} + \mu\right) - 1$

This additional parameter is needed to ensure number conservation.

$5 \times 10^4 < z < 2 \times 10^6$ is called μ era.

Later Compton scattering becomes inefficient as well, then energy injection leads to y-type distortions.

CMB distortions are determined by the RMS amplitude of small-scale primordial fluctuations.

To create PBHs, large RMS is needed, so large CMB distortions are created.

Existing/future limits on CMB distortions can place tight limits on PBHs.

But it again depends on primordial non-Gaussianity.

Let us consider the following dimensionless delta-function power spectrum:

$$\mathcal{P}_\zeta = \sigma^2 k \delta(k - k_*), \tag{8}$$

which leads to the μ distortion [70]

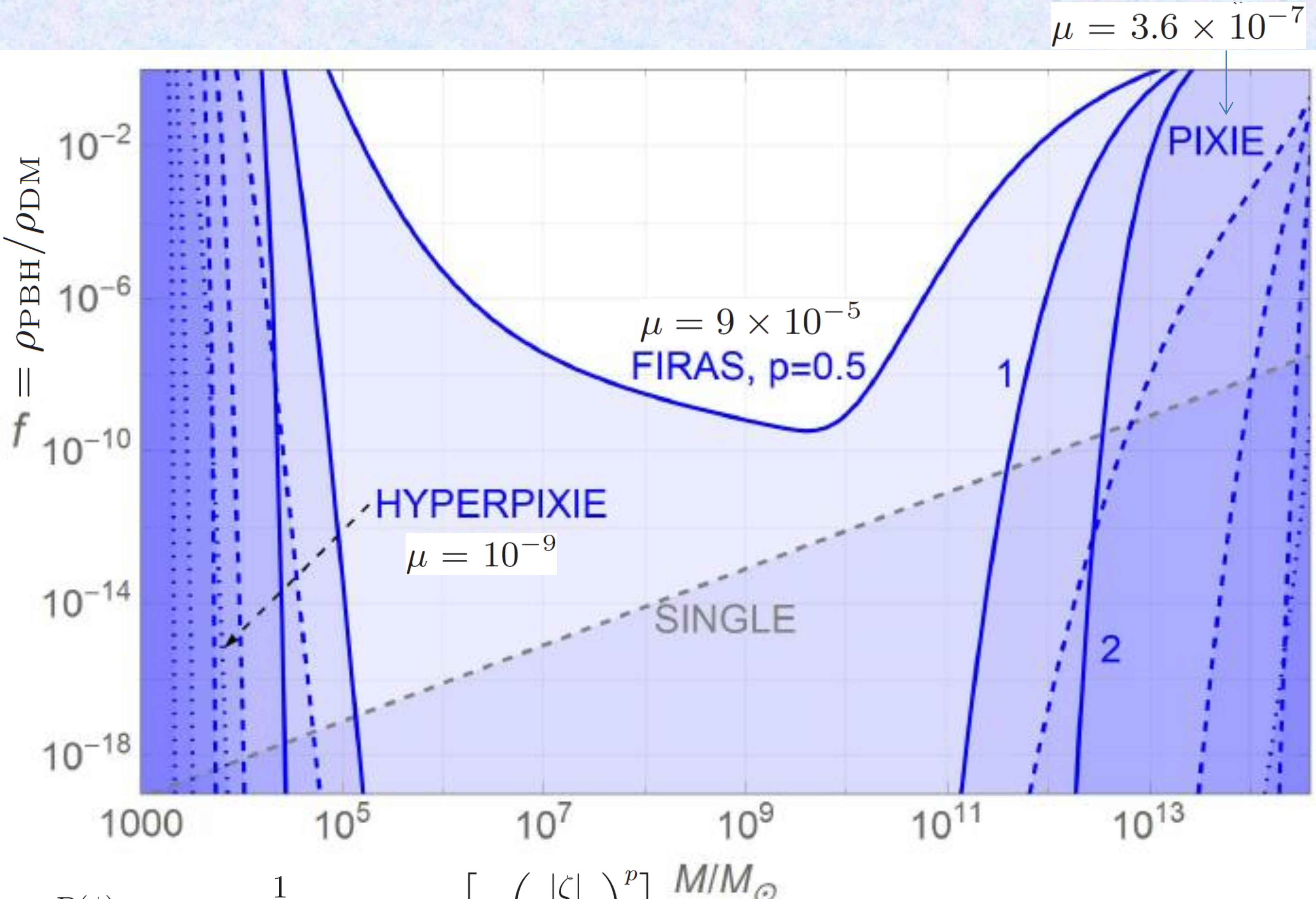
$$\mu \simeq 2.2\sigma^2 \left[\exp\left(-\frac{\hat{k}_*}{5400}\right) - \exp\left(-\left[\frac{\hat{k}_*}{31.6}\right]^2\right)\right], \tag{9}$$

where $\hat{k}_*$ is the wave number in units of Mpc^{-1}. The wave number and the PBH mass are related via [35]

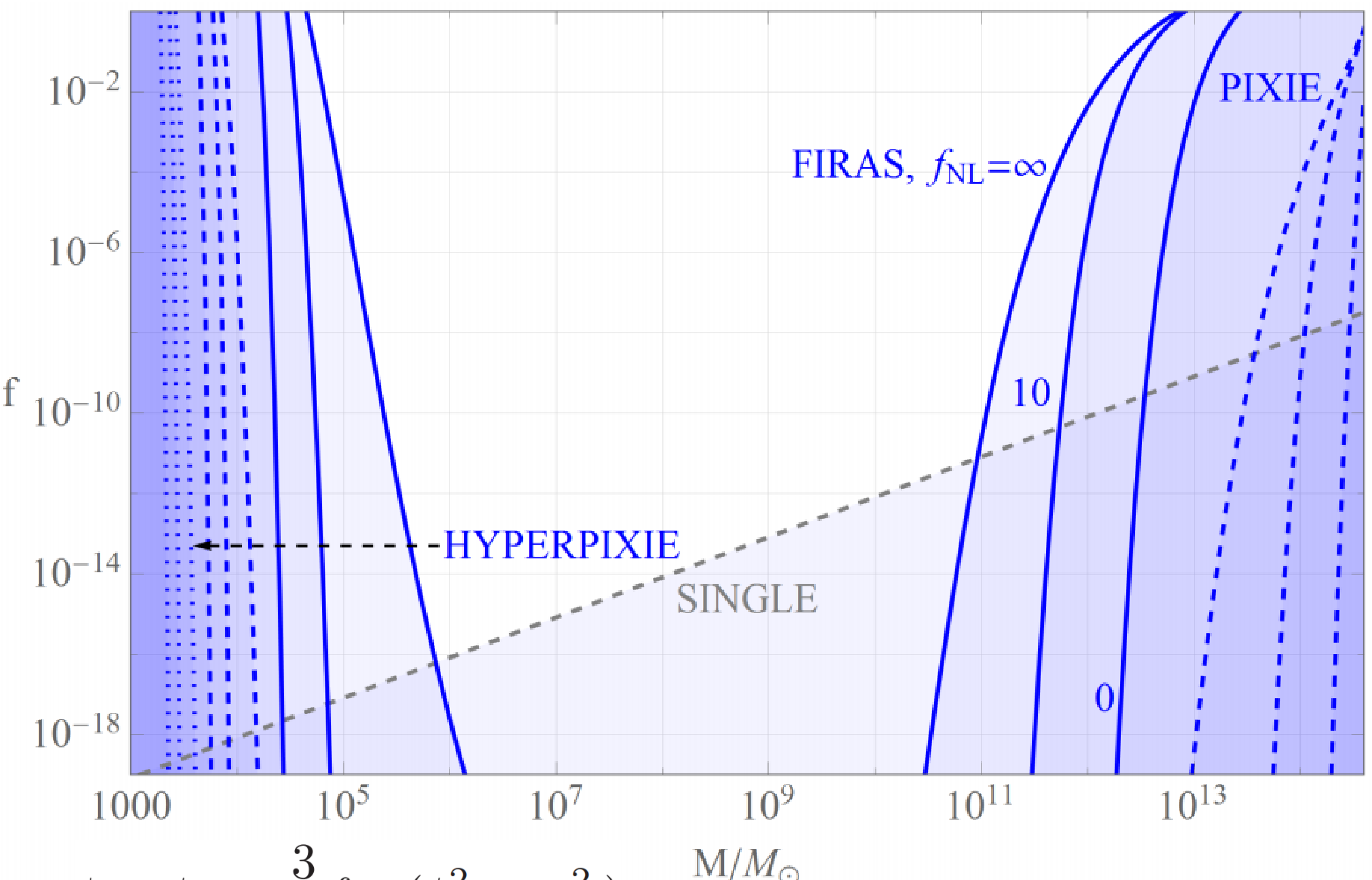
$$k \simeq 7.5 \times 10^5 \gamma^{1/2} \mathrm{Mpc}^{-1} \left(\frac{g}{10.75}\right)^{-1/12} \left(\frac{M}{30 M_\odot}\right)^{-1/2}, \tag{10}$$

$$\beta \sim \int_{\zeta_{th}}^{\infty} P_{NG}(\zeta, \sigma, p) d\zeta$$

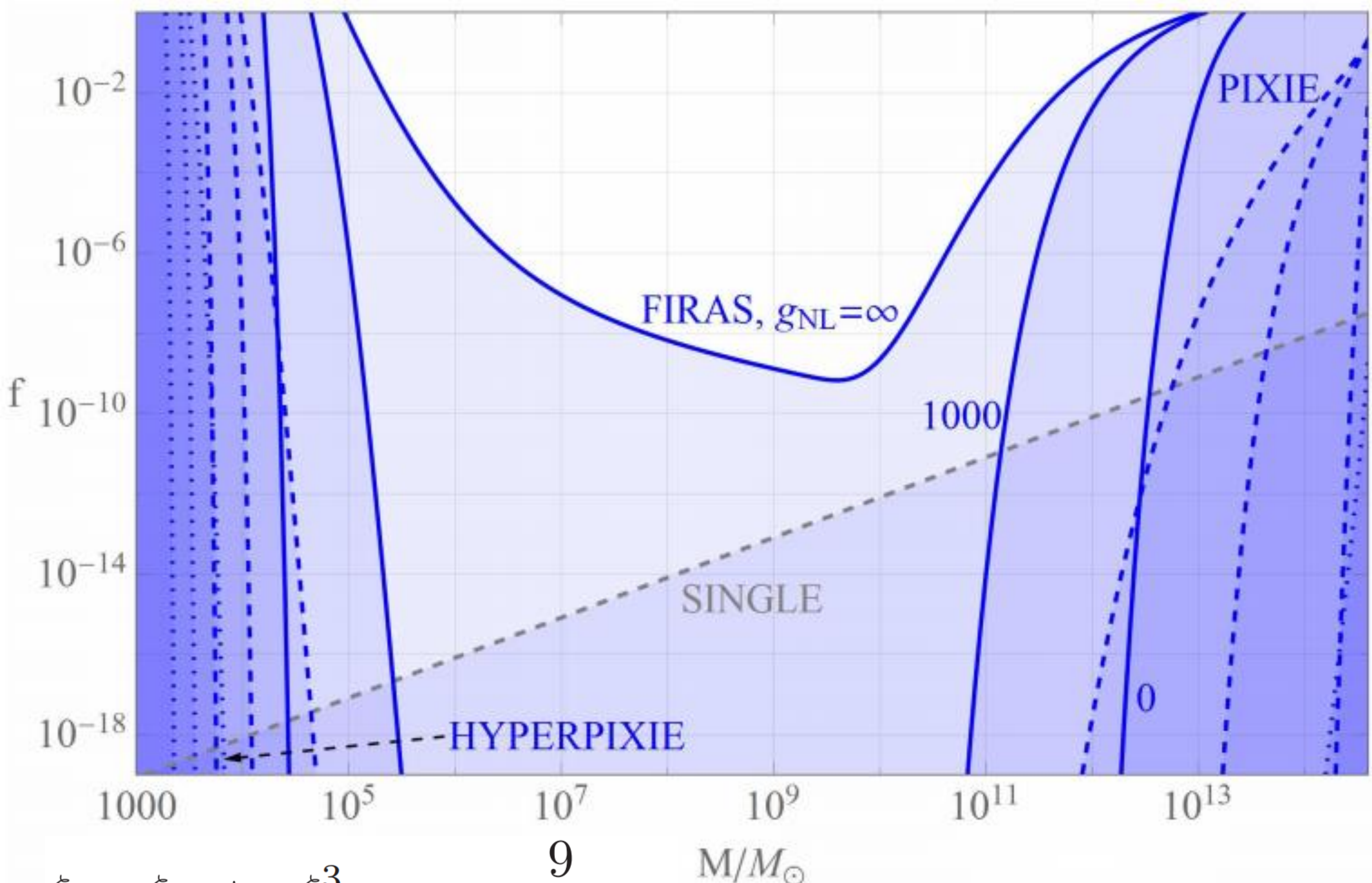
At each k_*, translated into PBH mass, upper limit on μ can be translated into upper limit on σ. This upper limit on σ can be translated into upper limit on β (or f), but that translation depends on a parameter controlling the strength of non-Gaussianity.



$$P(\zeta) = \frac{1}{2\sqrt{2}\tilde{\sigma}\Gamma\left(1+1/p\right)} \exp\left[-\left(\frac{|\zeta|}{\sqrt{2}\tilde{\sigma}}\right)^p\right]$$



$$\zeta = \zeta_G + \frac{3}{5} f_{\rm NL}(\zeta_G^2 - \sigma_G^2)$$



$$\zeta = \zeta_G + g\zeta_G^3, \quad g \equiv \frac{9}{25} g_{\rm NL}$$

Short summary

Diffusion damping of acoustic waves causes CMB spectral distortions, tiny deviations of the CMB energy spectrum from Planck spectrum.

Massive PBHs $(10^5 - 10^{11} M_\odot)$, relevant to high-redshift quasars, can be tightly constrained by existing/future limits on CMB spectral distortions.

Those constraints depend on primordial non-Gaussianity.

Existence of PBHs and fine-tuning

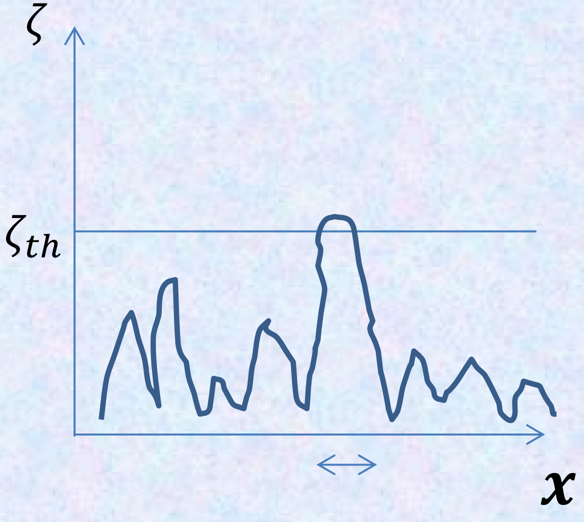
TN, Wang, 2018

PBHs may be used to provide all the cold dark matter, to explain the gravitational-wave events, or to explain high-redshift supermassive black holes.

Often, abundance of PBHs is extremely sensitive to a parameter controlling the abundance of PBHs.

This implies fine-tuning is required to realize some amount of PBHs for some purposes.

Crudely, the abundance of PBHs can be estimated by



$$\beta = \frac{\rho_{PBH}}{\rho_{rad}} \sim \int_{\zeta_{th}}^{\infty} P(\zeta) d\zeta$$

↑evaluated at formation epoch

$P(\zeta)$: Probability density function of curvature perturbation ζ

ζ_{th}: threshold for PBH formation $\zeta_{th} \sim 0.5$

If fluctuations follow a Gaussian statistics,

$$P(\zeta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\zeta^2}{2\sigma^2}\right), \quad \sigma^2 = \int \zeta^2 P(\zeta) d\zeta \sim \mathcal{P}_\zeta$$

Upper limits on β can be translated into upper limits on $\mathcal{P}_\zeta < \mathcal{O}(0.01)$.

PBH abundance is exponentially sensitive to σ.

This implies fine-tuning of σ is needed to realize some PBH abundance in range $\beta_m < \beta < \beta_M$ to explain dark matter (DM), GW events or quasars by PBHs formed by collapse of Gaussian (G) fluctuations during radiation domination (RD).

That is, σ_m and σ_M, corresponding to β_m and β_M are very close, or $\epsilon = \frac{\sigma_M - \sigma_m}{(\sigma_M + \sigma_m)/2}$ is very small.

Degree of fine tuning changes for PBHs formed from non-Gaussian fluctuations (NG), for those formed during an early matter era, which may be realized after inflation and before RD.

We quantified degree of fine tuning required for several cases.

Case Name	Role	$M/M_\odot$	$f=\Omega_{\rm PBH}/\Omega_{\rm DM}$	$\beta=\rho_{\rm PBH}/\rho_{\rm r}$	Era	Stat.	σ range	ϵ
DMRDG	All DM	10^{-12}	1	$(3.4\times10^{-16}, 1.0\times10^{-13})$	RD	G	$(0.0545, 0.0585)$	0.071
GWRDG	GW events	30	$(10^{-3}, 10^{-2})$	$(10^{-11}, 10^{-10})$	RD	G	$(0.0624, 0.0648)$	0.038
DMeMDG	All DM	10^{-12}	1	$(2.0\times10^{-14}, 2.8\times10^{-12})$	eMD	G	$(0.00324, 0.00874)$	0.92
DMRDNG	All DM	10^{-12}	1	$(3.4\times10^{-16}, 1.0\times10^{-13})$	RD	NG	$(0.00492, 0.00684)$	0.33
Quasars	Quasars	10^{9}	$(10^{-11}, 10^{-9})$	$(10^{-15}, 10^{-13})$	RD	NG	$(0.00263, 0.00365)$	0.32

$p=0.5$

$$P(\zeta)=\frac{1}{2\sqrt{2}\tilde{\sigma}\Gamma(1+1/p)}\exp\left[-\left(\frac{|\zeta|}{\sqrt{2}\tilde{\sigma}}\right)^p\right]$$

Dependence of abundance of PBHs formed during early matter era is power law, not exponential, implying less fine tuning:

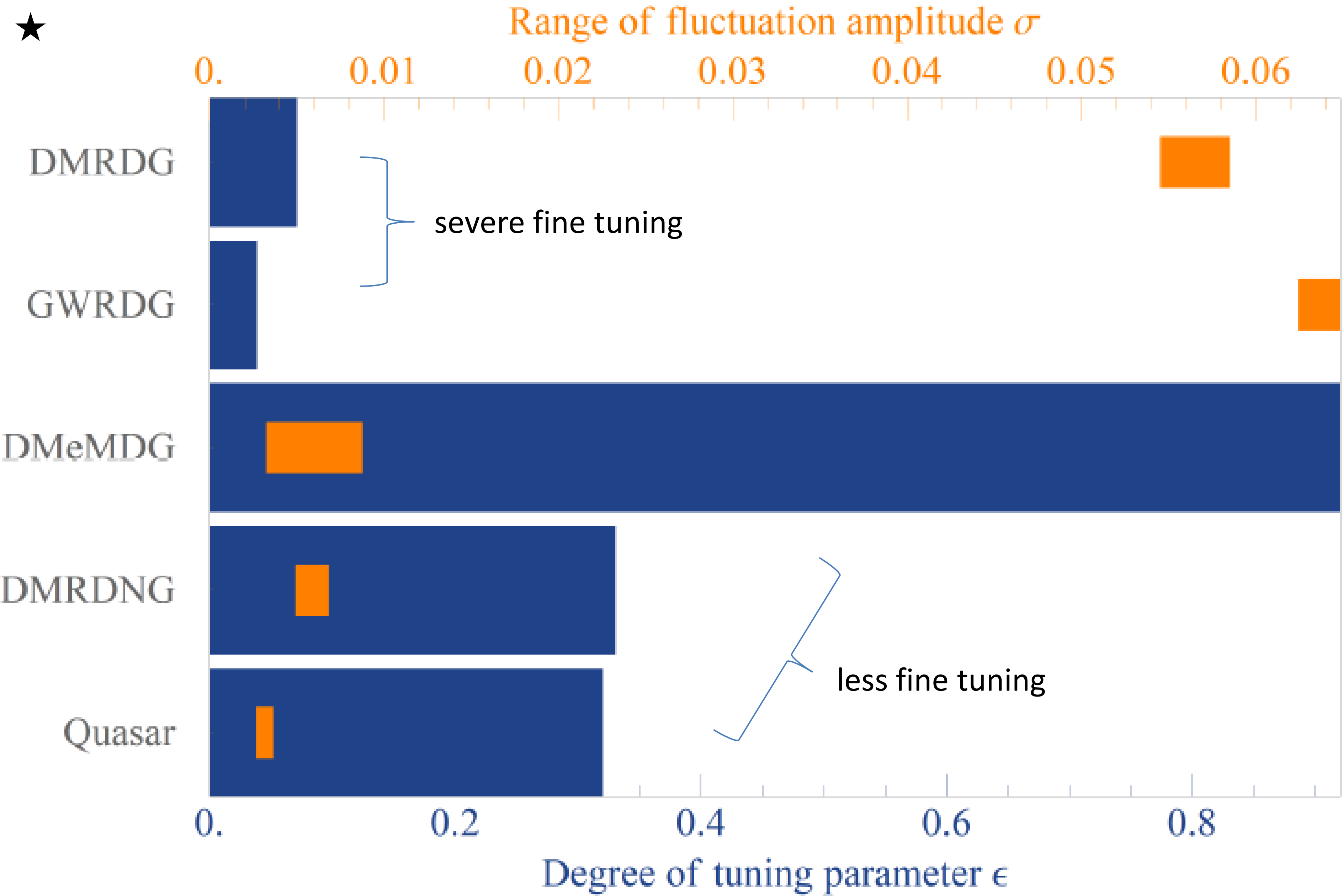
$$\beta\sim 0.05556\sigma^5$$

Harada+ 1609.01588

$$\epsilon=\frac{\sigma_M-\sigma_m}{(\sigma_M+\sigma_m)/2}$$

For PBHs formed from NG fluctuations and formed during early matter era, fine tuning can be significantly alleviated.

★



$$\epsilon = \frac{\sigma_M - \sigma_m}{(\sigma_M + \sigma_m)/2}$$

Another cosmological scenario

Tiny PBHs might have been formed abundantly in the early Universe, to the extent that they dominate the energy density of the Universe.

Universe may have been reheated by Hawking radiation of such PBHs.

Further, Planck mass relics may be left over, instead of their complete evaporating. These relics may account for cold dark matter.

No fine tuning is required for this scenario, since PBH abundance does not have to be tuned.

★

For DM scenarios we assumed a range of baryon-to-dark-matter ratio Rm=1/300<R<RM=1 obtained from anthropic arguments astro-ph/0511774.

This value of Rm was estimated from the instability of our galactic disk, so that the disk fragments and star formation takes place.

For the value of RM, one may expect RM ~ 1, since in a baryon-dominated Universe matter inhomogeneities on galactic scales are suppressed around the recombination due to Silk damping.

One may also find it useful to see the needed degree of tuning assuming some range for R obtained by recent experiments such as the Planck satellite, instead of using a range for R from an anthropic argument. Let us use $0.182 < R < 0.192$ [16], then we find $0.055563 < \sigma < 0.055597$, with $\epsilon \simeq 0.00061$.

Short summary

PBHs formed by collapse of Gaussian primordial perturbations during radiation domination require severe fine tuning.

It can be significantly alleviated by non-Gaussianity, or for PBHs formed during an early matter era.

No tuning is required for scenarios where reheating and dark matter are simultaneously accounted for by tiny PBHs.

Overall Summary

PBHs of a wide range of masses could have been produced by different mechanisms.

The most popular mechanism is gravitational collapse of overdensities during radiation domination.

PBHs might explain dark matter, gravitational-wave events or high-redshift supermassive BHs.

Low-frequency GW bursts from **massive** PBH binaries may be detected by future space-based GW experiments if PBHs are clustered at formation time.

Relatively large stochastic GWs are produced when PBH formation probability is interestingly large. The magnitude depends on primordial non-Gaussianity for fixed PBH abundance.

Massive PBHs $\sim 10^4 - 10^{10} M_\odot$ are tightly constrained by CMB distortions, but this also depends on primordial non-Gaussianity.

Often fine-tuning is required to create PBHs for some purpose, but fine-tuning can be significantly alleviated by primordial non-Gaussianity, or for PBHs formed during an early matter domination.