Higgs Physics

SONG

IBS. CTPU

1. Natural Unit

$$c = 1 = \hbar$$

$$[c] = \left[\frac{L}{T}\right] = 1 \qquad [L] = [T]$$

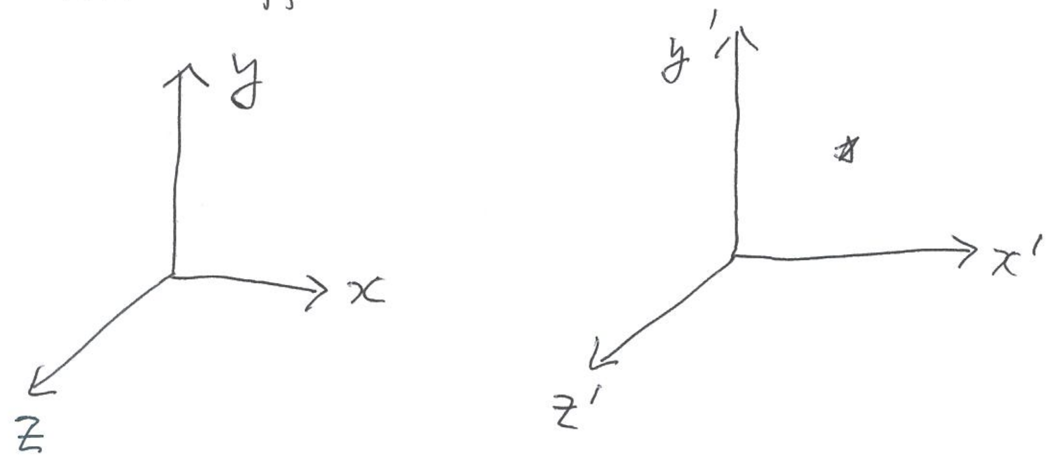
$$[\hbar] = \left[M\frac{L^2}{T}\right] = 1 \qquad \therefore\ [M] = \left[\frac{1}{L}\right]$$

$$\therefore [\text{mass}] = [\text{energy}] = [\text{momentum}]$$

$$= \left[\frac{1}{\text{length}}\right] = \left[\frac{1}{\text{time}}\right]$$

2. Lorentz Tf

- event : (t, x, y, z)
- Two different observers



$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$$

NOTE) $\gamma = \dfrac{1}{\sqrt{1-v^2}}$, $\beta = v$

$$\gamma^2 - \beta^2\gamma^2 = 1$$

$\therefore$ $\cosh\eta = \gamma$, $\sinh\eta = \gamma\beta$

where η : rapidity

Lorentz Tf matrix

$$\Lambda = \begin{pmatrix} \cosh\eta & -\sinh\eta & 0 & 0 \\ -\sinh\eta & \cosh\eta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

4-vector $x^\mu = (t, x, y, z)$

Under Lorentz Tf

$$x'^\mu = \Lambda^\mu{}_\nu x^\nu$$

We can consider this as a coordinate Tf

$$dx^\mu \longrightarrow dx'^\mu = \frac{\partial x'^\mu}{\partial x^\nu} dx^\nu \equiv \Lambda^\mu{}_\nu dx^\nu$$

$\exists$ another kind of 4-vector

$$\partial_\mu = \frac{\partial}{\partial x^\mu} \longrightarrow \partial'_\mu = \frac{\partial x^\nu}{\partial x'^\mu} \frac{\partial}{\partial x^\nu} = (\Lambda^{-1})_\mu{}^\nu \partial_\nu$$

contravariant vector

$$V^\mu \longrightarrow \Lambda^\mu{}_\nu V^\nu$$

covariant vector

$$V_\mu \longrightarrow (\Lambda^{-1})_\mu{}^\nu V_\nu$$

- Any product of a contravariant vector and a covariant vector is a Lorentz scalar

$$A'_\mu B'^\mu = (\Lambda^{-1})_\mu{}^\alpha A_\alpha (\Lambda)^\mu{}_\beta B^\beta = \delta^\alpha_\beta A_\alpha B^\beta = A_\mu B^\mu$$

$\boxed{Q}$ x_μ ?

Find a Lorentz scalar from x^μ !

$$(t')^2-(x')^2-(y')^2-(z')^2 = t^2-x^2-y^2-z^2$$
$$\equiv x_\mu x^\mu$$

$$\therefore\ x_\mu = (t, -x, -y, -z)$$
$$\equiv \eta_{\mu\nu} x^\nu$$

$$\eta_{\mu\nu} = \eta^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Minkowski metric

◎ Derivative

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = (\partial_t, \vec{\nabla})$$

$$\partial^\mu = \frac{\partial}{\partial x_\mu} = (\partial_t, -\vec{\nabla})$$

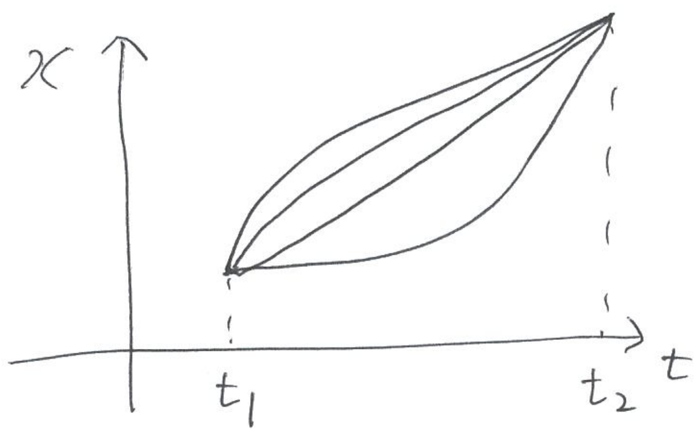
3. Eq. of motion $\Leftarrow$ Least action principle

In CM, orbit $x(t)$

t : Universal parameter

$L(x, \dot{x})$

$$S = \int_{t_1}^{t_2} dt\, L(x, \dot{x})$$



actual path : the path for $\delta S = 0$

$\Downarrow$

eq. of motion

$$\frac{\partial L}{\partial x} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)$$

SR : dt is NOT Lorent invariant.

x^μ : parameter $\longrightarrow$ field $\phi(x^\mu)$

Under $x^\mu \to x^\mu + \delta x^\mu$

$$\phi(x) \longrightarrow \phi'(x') \equiv \phi(x) + \Delta\phi$$

$$\begin{aligned}
\Delta\phi &= \phi'(x') - \phi(x) \\
&= \phi'(x') - \phi(x') + \phi(x') - \phi(x) \\
&= \delta\phi + \partial_\mu\phi\,\delta x^\mu
\end{aligned}$$

functional variation in ϕ

$$\delta S = \int d^4x'\, \mathcal{L}(\phi', \partial_\mu\phi'; x'^\mu) - \int d^4x\, \mathcal{L}(\phi, \partial_\mu\phi; x^\mu)$$

$$\boxed{d^4x' = \det\left(\frac{\partial x'^\mu}{\partial x^\nu}\right) = 1 + \partial_\mu(\delta x^\mu)}$$

$$= \int d^4x \left[\frac{\partial\mathcal{L}}{\partial\phi}\delta\phi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta(\partial_\mu\phi) + \underbrace{\frac{\partial\mathcal{L}}{\partial x^\mu}\delta x^\mu + \mathcal{L}\,\partial_\mu\delta x^\mu}_{\partial_\mu\left[\mathcal{L}\,\delta x^\mu\right]}\right]$$

$$\partial_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta\phi\right] - \partial_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right]\delta\phi$$

$$= \int_R d^4x \left[\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right)\right]\delta\phi + \int_{\partial R} d\sigma_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta\phi + \mathcal{L}\,\delta x^\mu\right]$$

Demand $\delta\phi = 0 = \delta x^\mu$ on ∂R

$$\boxed{\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right] = 0}$$ Euler-Lagrange

4. Klein-Gordon eq.

$$p^\mu p_\mu = E^2 - |\vec{p}|^2 = m^2$$

QM : $p^\mu \rightarrow i\partial^\mu$

$$-\partial_t^2 + \nabla^2 = m^2$$

$$\therefore \boxed{(\Box + m^2)\phi = 0} \quad \text{KG eq.}$$

$$\uparrow$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{m^2}{2}\phi^2$$

5. Energy-momentum tensor

↳ response to δx^μ

Consider the boundary term of δS

$$\delta S \supset \int_{\partial R} d\sigma_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \underbrace{\delta\phi}_{\Delta\phi - \partial_\nu \phi\, \delta x^\nu} + \mathcal{L}\, \delta x^\mu \right]$$

When focusing on a pure spacetime variation
$\Delta\phi = 0$

$$\delta S \supset -\int_{\partial R} d\sigma_\mu \Theta^\mu{}_\nu \delta x^\nu$$

$$\boxed{\Theta^\mu{}_\nu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial_\nu \phi - \delta^\mu_\nu \mathcal{L}}$$

↑
energy momentum tensor

$$\therefore \int_{\partial R} \Theta^\mu{}_\nu \, d\sigma_\mu = 0 = \int_R \partial_\mu \Theta^\mu{}_\nu \, d^4x$$

$$\therefore \boxed{\partial_\mu \Theta^\mu{}_\nu = 0}$$ continuity eq.

$\exists$ conserved charge

$$P_\nu = \int d^3x\, \Theta^0{}_\nu \longrightarrow \frac{dP_\nu}{dt} = 0$$

↑

6. Problems of KG eq.

① $E = \pm\sqrt{m^2 + |\vec{p}|^2}$: negative energy?

② Interpretation of ϕ

Try prob. density

Find $\boxed{\partial_\mu j^\mu = 0}$ from KG eq.

Then $\rho = i\left(\phi^* \frac{\partial\phi}{\partial t} - \phi\frac{\partial\phi^*}{\partial t}\right)$

Plane wave solution

$$\phi = N e^{-ip\cdot x}$$

Then $\rho = 2E|N|^2$

For negative E, $\rho < 0$

$\therefore$ ρ cannot be a prob. density

7. QFT

$$\phi(x) \longrightarrow \hat{\phi}(x)$$

quantum field

In QM, $[\hat{x}, \hat{p}] = i\hbar$

Fourier expansion of ϕ

$$\hat{\phi} = \sum_{k} \left[\hat{a}(k) e^{-ik\cdot x} + \hat{a}^{+}(k) e^{ik\cdot x} \right]$$

Conjugate field

$$\hat{\pi} \equiv \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$$

Impose $\quad [\phi(\vec{x},t), \pi(\vec{x}',t)] = i\delta^3(\vec{x}-\vec{x}')$

$$[\phi, \phi] = 0 = [\pi, \pi]$$

Then

$$[\hat{a}(\vec{k}), \hat{a}^{+}(\vec{k}')] = (2\pi)^3 2\omega_{\vec{k}} \delta^3(\vec{k}-\vec{k}')$$

where $\omega_{\vec{k}} = \sqrt{m^2 + |\vec{k}|^2}$

$$[a, a] = 0 = [a^{+}, a^{+}]$$

$$\downarrow$$

SHO

Number operator $N(\vec{k}) \propto a^{+}a$

Then $P^{\mu} = \sum_{k} k^{\mu}\left(N(k) + \frac{1}{2}\right)$

$a^{+}(k)$: creation operator

$a(k)$: annihilation operator

8. Dirac eq.

Dirac tried a linear expression for H

$$H = \vec{\alpha} \cdot \vec{p} + \beta m$$

Require $\quad (E^2 - |\vec{p}|^2 - m^2)\psi = 0$

Then

$$\{\alpha_i, \alpha_j\} = 2\delta_{ij}$$

$$\{\alpha_i, \beta\} = 0$$

$$\beta^2 = 1$$

only 4×4 matrices can satisfy them

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\therefore \quad \psi = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \quad \text{4-spinor}$$

Dirac spinor naturally includes spin

- Covariant Dirac eq.

$$\gamma^\mu = (\gamma^0, \vec{\gamma}) \equiv (\beta, \beta\vec{\alpha})$$

Then $$\boxed{\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}}$$

$$(i\gamma^\mu\partial_\mu - m)\psi = 0$$

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$$

where $\bar{\psi} = \psi^\dagger\gamma^0$

Several representations.

- Chiral representation (good for $v \sim 1$)

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$$

- Chirality = handedness

$$\gamma^5 = \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$

In the chiral reprentation $\gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\underbrace{\psi_L = P_L\psi = \begin{pmatrix} a \\ b \\ 0 \\ 0 \end{pmatrix}, \quad \psi_R = P_R\psi = \begin{pmatrix} 0 \\ 0 \\ c \\ d \end{pmatrix}}$$

Chirality ?

physical meaning ?

If $m = 0$,

$$E\gamma^0 \psi = \vec{\gamma}\cdot\vec{p}\,\psi$$

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

$$\psi_R = \frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}\psi_R \,, \quad -\psi_L = +\frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}\psi_L$$

In QM, spin operator $\vec{S} = \frac{1}{2}\vec{\sigma}$

Define the helicity op

$$h = \frac{\vec{S}\cdot\vec{p}}{|\vec{S}|\,|\vec{p}|} = \frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}$$

$\longrightarrow \vec{p}$

Spin $\Rightarrow$ right-handed

$\Leftarrow$ left-handed

$\therefore$ helicity : well-defined physical observable

9. Fermion mass

$$\mathcal{L} = -m \bar{\psi}_L \psi_R + h.c.$$

We need both ψ_L and ψ_R for mass.

BUT SM is a chiral theory

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad u_R, d_R$$

$$\begin{pmatrix} \nu_L \\ e^-_L \end{pmatrix}, \quad e^-_R, \cancel{\nu_R}$$

$\boxed{Q}$ How to write down the fermion mass?

10. U(1) gauge invariance

Maxwell eq. in Heaviside - Lorentz unit

$$\epsilon_0 = 1 = \mu_0$$

$$\vec{\nabla}\cdot\vec{B} = 0$$
$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

homogeneous eq

$$\vec{\nabla}\cdot\vec{E} = \rho$$
$$\vec{\nabla}\times\vec{B} = \vec{j} + \frac{\partial\vec{E}}{\partial t}$$

inhomogeneous eq.

Introduce $A^\mu = (\phi, \vec{A})$

$$\vec{B} = \vec{\nabla}\times\vec{A}$$
$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial\vec{A}}{\partial t}$$

Then two homogeneous equations are automatically satisfied.

$\boxed{Q}$ h ?

Field strength $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

$$= \begin{pmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -B^3 & B^2 \\ E^2 & B^3 & 0 & -B^1 \end{pmatrix}$$

Two inhomogeneous equations

$$\partial_\mu F^{\mu\nu} = j^\nu$$

where $j^\mu = (\rho, \vec{J})$

e.g. $\nu = 0$

$$\partial_1 F^{10} + \partial_2 F^{20} + \partial_3 F^{30} = \vec{\nabla} \cdot \vec{E} = \rho$$

OK.

$$\boxed{\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - A_\mu j^\mu}$$

☆

$$\frac{\partial \mathcal{L}}{\partial A_\mu} = -j^\mu$$

$$☆ = -\frac{1}{2}(\partial_\mu A_\nu \partial^\mu A^\nu - \partial_\mu A_\nu \partial^\nu A^\mu)$$

$$\therefore \frac{\partial \mathcal{L}}{\partial(\partial_\nu A_\mu)} = -(\partial^\nu A^\mu - \partial^\mu A^\nu) = -F^{\nu\mu}$$

$$\therefore \quad \partial_\nu F^{\nu\mu} = j^\mu$$

$\square$

11. Gauge invariance.

Observable are $\vec{E}$, $\vec{B}$

Under the gauge Tf of $A^\mu \to A^\mu + \partial^\mu \alpha$

$$\vec{E} \to \vec{E}, \quad \vec{B} \longrightarrow \vec{B}$$

- At the level of $\mathcal{L}$

$$F^{\mu\nu} \longrightarrow F^{\mu\nu}$$

$$\therefore \mathcal{L} \longrightarrow \mathcal{L}$$

Q $A_\mu j^\mu$?

NOT gauge-invariant ?

- Covariant derivative $\quad D_\mu = \partial_\mu + i q A_\mu$

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$$

$$= \mathcal{L}_0 - A_\mu \underbrace{q\bar{\psi}\gamma^\mu\psi}_{j^\mu}$$

⑥ Local gauge invariance

We require $\mathcal{L}' = \mathcal{L}$ under $\psi \to e^{i\alpha(x)}\psi$.

BUT $\quad e^{i\alpha}\left[\partial\psi + \boxed{i(\partial\alpha)\psi}\right]$

12. (collective) gauge Tf

$$\psi \to e^{-iq\alpha(x)}\psi$$

$$A_\mu \longrightarrow A_\mu + \partial_\mu \alpha$$

NOTE) Same $\alpha(x)$

For infinitesimal $q \ll 1$

$$\psi \to [1 - iq\alpha]\psi$$

$$D_\mu \psi \longrightarrow D_\mu' \psi'$$

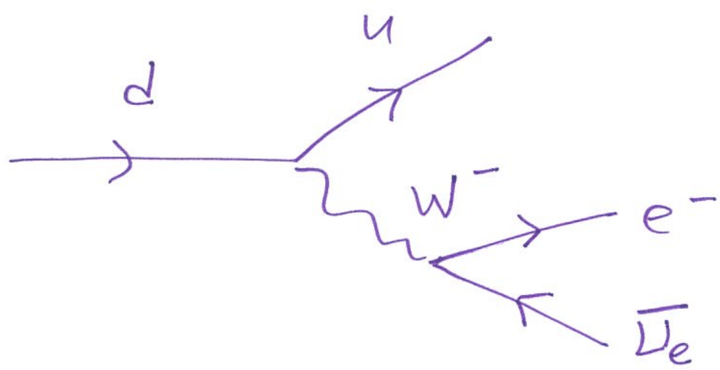
$$= \partial_\mu \psi - iq\alpha\,\partial_\mu \psi + iqA_\mu \psi$$

$$\bar{\psi} \longrightarrow \bar{\psi}(1 + iq\alpha)$$

Then $\bar{\psi} D_\mu \psi \longrightarrow \bar{\psi} D_\mu \psi$

13. Weak interaction & SU(2)

begin w/ β-decay



$$\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad \begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}$$

phase Tf. $\qquad e^{iX} \in SU(2)$

2x2 special unitary

$(\det = 1) \quad (U^\dagger U = 1)$

$\Rightarrow$ 3 real parameters

$\vec{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$

$$X = g\,\vec{\alpha}\cdot\frac{\vec{\sigma}}{2}$$

$\vec{\sigma}$: 3 Pauli matrices.

- SU(2) gauge Tf on $\quad Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$

$$Q \to e^{ig\vec{\alpha}\cdot\frac{\vec{\sigma}}{2}} Q$$

14. Strong interaction & SU(3)

$$e^{iX}\begin{pmatrix} u_R \\ u_G \\ u_B \end{pmatrix}$$

$X \in SU(3)$ generators.

8 indp parameters $\vec{\xi} = (\xi^1, \xi^2 \cdots \xi^8)$

$$X = g_s \vec{\xi} \circ \frac{\vec{\lambda}}{2}$$

$\vec{\lambda}$: 8개의 3x3 matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

15. Goldstone Thm

Consider a real scalar field ϕ

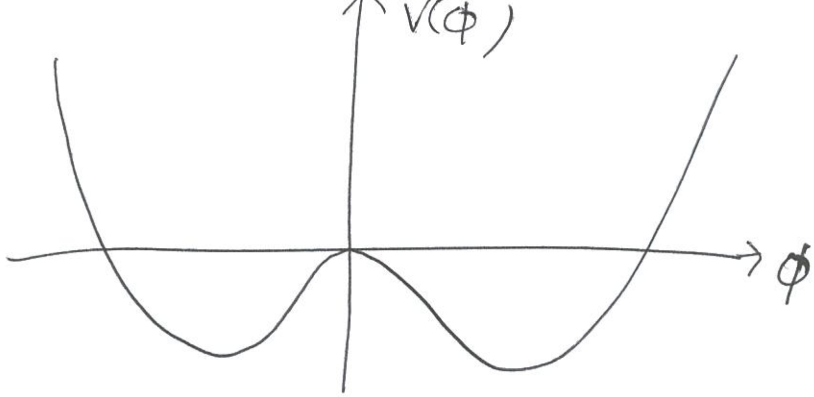
$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

$$V = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4$$

($\lambda>0$: bounded from below)

$\exists$ symmetry under $\phi \to -\phi$

If $\mu^2<0$



$V(\phi)$ has a minimum when $\frac{\partial V}{\partial \phi} = 0$.

$$\langle 0|\phi^2|0\rangle = -\frac{\mu^2}{\lambda} \equiv v^2$$

$\exists$ two equivalent vacua.

Let's choose $\boxed{\langle\phi\rangle_{VEV} = v}$

"Spontaneous"

$$\phi(x) = v + \sigma(x)$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}(-2\mu^2)\sigma^2 - \sqrt{-\mu^2\lambda}\,\boxed{\sigma^3} - \frac{\lambda}{4}\sigma^4 + \text{const}$$

Consider a complex scalar field.

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_i\,\partial^\mu\phi_i - \frac{1}{2}\mu^2(\phi_i\phi_i) - \frac{\lambda}{4}(\phi_i\phi_i)^2$$

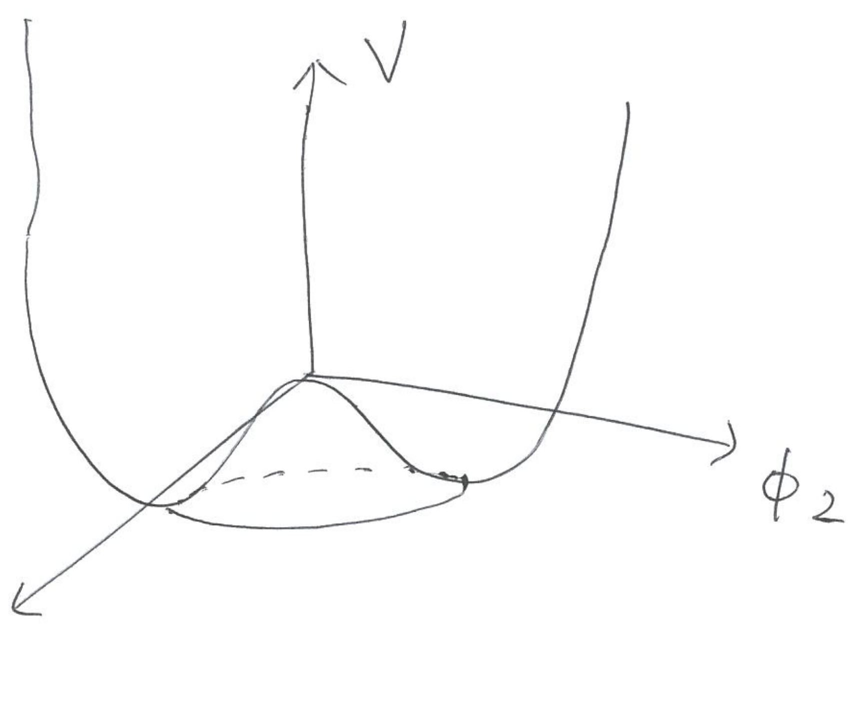
Sym. under $\phi_i \longrightarrow \phi_i' = R_{ij}\phi_j$

↳ $O(2)$ rotation

☆ $\mu^2 < 0$.

V has a minimum at

$$\langle 0|\phi_1^2 + \phi_2^2|0\rangle = -\frac{\mu^2}{\lambda} \equiv v^2$$



Choose $\phi_1 = v + \sigma(x)$, $\phi_2 = \pi(x)$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}(-2\mu^2)\sigma^2 - \lambda v\sigma^3 - \frac{\lambda}{4}\sigma^4$$
$$+ \frac{1}{2}(\partial_\mu\pi)(\partial^\mu\pi) - \frac{\lambda}{2}(\pi\pi)^2 - \lambda v\,\pi\pi\sigma - \frac{\lambda}{2}\pi\pi\sigma^2$$

$\therefore$ σ : massive

π : massless

Goldstone boson

16. Higgs mechanism

Goldstone Thm + U(1) gauge.

- Consider a complex scalar ϕ w/ U(1)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \phi|^2 - V(\phi)$$

where $D_\mu \phi = (\partial_\mu - ieA_\mu)\phi$

Then $\mathcal{L}' = \mathcal{L}$

under $\phi \longrightarrow e^{i\alpha(x)}\phi$

$A_\mu \longrightarrow A_\mu - \frac{1}{e}\partial_\mu \alpha(x)$

For $\mu^2 < 0$,

$$\phi(x) = \frac{1}{\sqrt{2}}\left[v + \phi_1(x) + i\phi_2(x)\right]$$

Then

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2}(\partial_\mu \phi_1)^2 + \frac{1}{2}(\partial_\mu \phi_2)^2$$
$$- v^2 \lambda \phi_1^2 + \frac{1}{2}(ev)^2 A_\mu A^\mu - \underline{e v A_\mu \partial^\mu \phi_2} + \cdots$$

$M_A = ev$: photon becomes massive

$M_\sigma = 2\lambda v$

$-evA_\mu \partial^\mu \phi_2$?

Define
$$\phi = \frac{1}{\sqrt{2}}\left(v + h(x)\right) e^{i\xi(x)/v}$$

Use the gauge freedom
$$A_\mu \to A_\mu - \frac{1}{ev}\partial_\mu \xi$$

All $\xi(x)$ terms disappear.

Unitary gauge !

Degrees of freedom

Before		d.o.f	After		
massless	A_μ	2	massive	A_μ	3
complex	ϕ	2		$h(x)$	1

$\xi(x)$: NOT physical

17. SM

gauge theory $\quad SU(3)_c \times SU(2)_L \times U(1)_Y$.

chiral theory

$$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad u_R, \; d_R$$

$$L = \begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}, \quad e_R^-$$

Hyperchange Y.

$$Y_Q = \frac{1}{3}, \quad Y_{u_R} = \frac{4}{3}, \quad Y_{d_R} = -\frac{2}{3}$$

$$Y_L = -1 \quad Y_{e_R} = -2$$

$$Y_\phi = +1$$

why? $\quad \mathcal{L} = -y \bar{Q} \Phi d_R$

$$-\frac{1}{3} + Y_\phi - \frac{2}{3} \stackrel{!}{=} 0$$

$$\therefore Y_\phi = +1$$

Lagrangian

$$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} \vec{W}_{\mu\nu} \cdot \vec{W}^{\mu\nu} - \frac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu}_{a}$$

$$+ \bar{Q}\, i\gamma^{\mu} D_{\mu} Q + \bar{u}_R\, i\gamma^{\mu} D_{\mu} u_R + \bar{d}_R\, i\gamma^{\mu} D_{\mu} d_R$$

$$+ (D^{\mu}\Phi)^{\dagger} D_{\mu}\Phi - V(\Phi)$$

- Covariant derivative of a fermion.

$$D_{\mu} Q = \left(\partial_{\mu} - i g_s \frac{\lambda^a}{2} G_{\mu}^{a} - i g_2 \frac{\sigma^a}{2} W_{\mu}^{a} - i g_1 \frac{1}{6} B_{\mu}\right) Q$$

$$D_{\mu} u_R = \left(\partial_{\mu} - i g_s \frac{\lambda^a}{2} G_{\mu}^{a} \qquad\qquad - i g_2 \frac{2}{3} B_{\mu}\right) u_R$$

$$\vdots$$

NOTE) At the Lagrangian level,
the gauge sym. is maintained!

Higgs mechanism in the SM

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda |\Phi^\dagger \Phi|^2$$

For $\mu^2 < 0$ in the unitary gauge,

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

Degrees of freedom

Before		After		
$\vec{W}$	6	massive	$W^\pm, Z$	9
B^μ	2	massless	A	2
Φ	4		H	1
total	12			12

Covariant derivative of Φ

$\Longmapsto$ masses for $W^{\pm}$, Z

$$D_\mu \Phi = \left(\partial_\mu - i g_2 \frac{\sigma^a}{2} W_\mu^a - i g_2 \frac{1}{2} B_\mu\right) \Phi$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} \partial_\mu - \frac{i}{2}(g_2 W_\mu^3 + g_1 B_\mu) & -\frac{i g_2}{2}(W_\mu^1 - i W_\mu^2) \\ -\frac{i g_2}{2}(W_\mu^1 + i W_\mu^2) & \partial_\mu + \frac{i}{2}(g_2 W_\mu^3 - g_1 B_\mu) \end{pmatrix} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

$$|D_\mu \Phi|^2$$

$$= \frac{1}{2}(\partial H)^2 + \frac{1}{8} g_2^2 (v+H)^2 \left| W_\mu^1 + i W_\mu^2 \right|^2 + \frac{1}{8}(v+H)^2 \left(g_2 W_\mu^3 - g_1 B_\mu \right)^2$$

Define

$$W_\mu^{\pm} = \frac{1}{\sqrt{2}} \left(W_\mu^1 \mp i W_\mu^2 \right)$$

$$Z_\mu = \frac{1}{\sqrt{g_1^2 + g_2^2}} \left(g_2 W_\mu^3 - g_1 B_\mu \right)$$

$$A_\mu = \frac{1}{\sqrt{g_1^2 + g_2^2}} \left(g_2 W_\mu^3 + g_1 B_\mu \right)$$

Then $\mathcal{L} \supset m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu$

NO $A_\mu A^\mu$ term $\Longmapsto$ massless A_μ

$$m_W = \frac{1}{2} g_2 v$$

$$m_Z = \frac{1}{2}\sqrt{g_1^2 + g_2^2}\ v$$

$$m_A = 0$$

Fermion mass

$$\mathcal{L}_{F} = -y_e \bar{L} \Phi e_R - y_d \bar{Q} \Phi d_R + h.c.$$

$$m_f = y_f \frac{v}{\sqrt{2}}$$

What about the up-type fermion mass ?

Hypercharge $\quad \bar{Q} \quad u_R$?

$$\qquad\qquad -\frac{1}{3} \quad \frac{4}{3}$$

We need $Y_{\Phi'} = -1$

In the SM, NO additional Higgs doublet !

$$\tilde{\Phi} = i\sigma_2 \Phi^* = \begin{pmatrix} \frac{v+H}{\sqrt{2}} \\ 0 \end{pmatrix}$$

$$\mathcal{L} \supset -y_u \bar{Q} \tilde{\Phi} u_R + h.c.$$

☆ Higgs self coupling

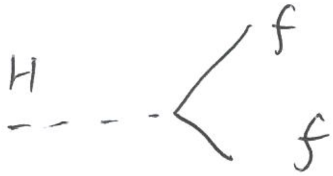
$$V(\Phi) = \lambda v^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4$$

$$m_H = 2\lambda v^2$$

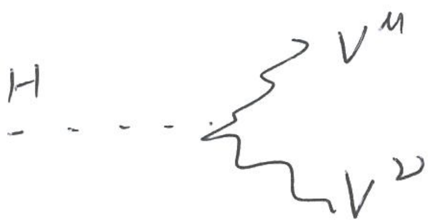
$$g_{H^3} = 3!\,\lambda v = 3\,\frac{m_H^2}{v}$$

$$g_{H^4} = 4!\,\frac{\lambda}{4} = 3\,\frac{M_H^2}{v^2}$$

☆ Vertex

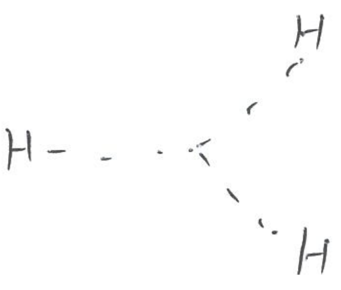


$$i\,\frac{m_f}{v}$$



$$\frac{2M_V^2}{v}(-i g_{\mu\nu})$$

$$\frac{2M_V^2}{v^2}(-i g_{\mu\nu})$$



$$i\,3\,\frac{M_H^2}{v}$$

$$i\,\frac{3 m_H^2}{v^2}$$