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Dark Matter Sommerfeld-enhanced annihilation and bound-state decay at finite temperature

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based on

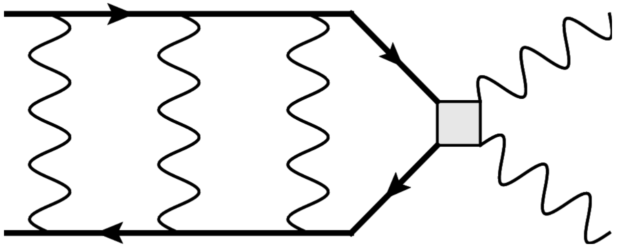
[arXiv:1808.06472] Phys.Rev. **D98** (2018) no.11, 115023

in collaboration with

Laura Covi (Göttingen) and **Kyohei Mukaida** (DESY).

Motivation

- **How heavy can (neutralino) dark matter be?**
  (title of [H. Fukuda *et al.* '19])



- **Sommerfeld enhancement + Bound states**

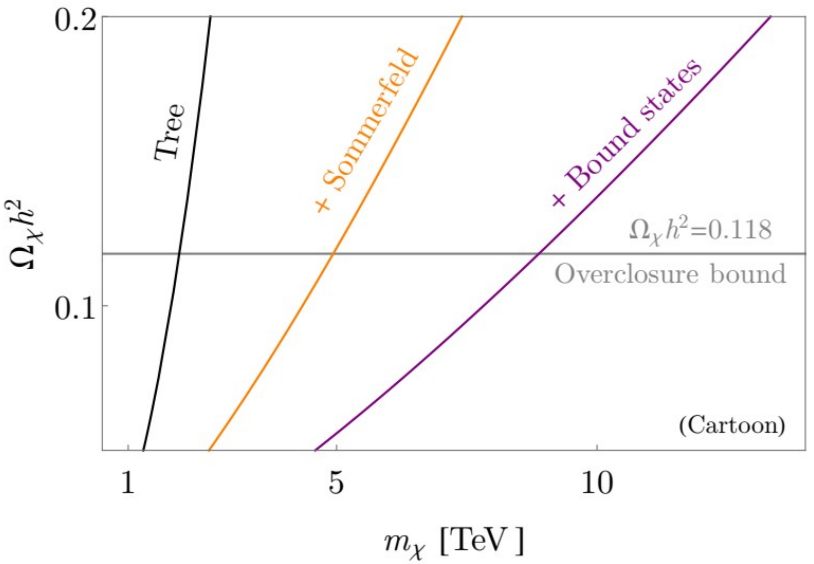
Criteria for single attractive Yukawa potential: $\frac{1}{\alpha m_\chi} \lesssim \frac{1}{m_\phi}$

Ex.: Wino, co-annihilation of colored charged particles, Higgs mediated bound states, hidden charged dark sectors, SIDM with light mediators, ...
[J. Hisano *et al.* '07, ...] [J. Feng *et al.* '09, Harling&Petraki '14, ..., Harz&Petraki '19]

$$(\chi\bar{\chi})_{\rm sc} \rightleftharpoons \text{SM SM}', \qquad \textbf{annihilation}$$

$$\left.\begin{aligned}(\chi\bar{\chi})_{\rm sc} &\rightleftharpoons \phi + (\chi\bar{\chi})_{B_i}, \\ (\chi\bar{\chi})_{B_i} &\rightleftharpoons \phi + (\chi\bar{\chi})_{B_j},\end{aligned}\right\} \quad \text{radiative processes}$$

$$(\chi\bar{\chi})_{B_i} \rightleftharpoons \text{SM SM}'. \qquad \textbf{decay}$$



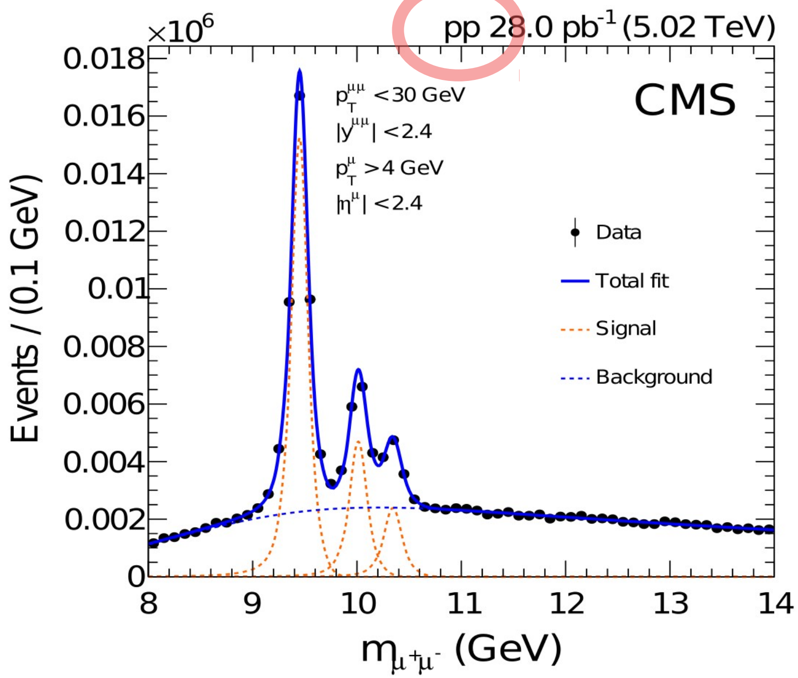
⇒ Effects generically allow for **larger DM masses**.

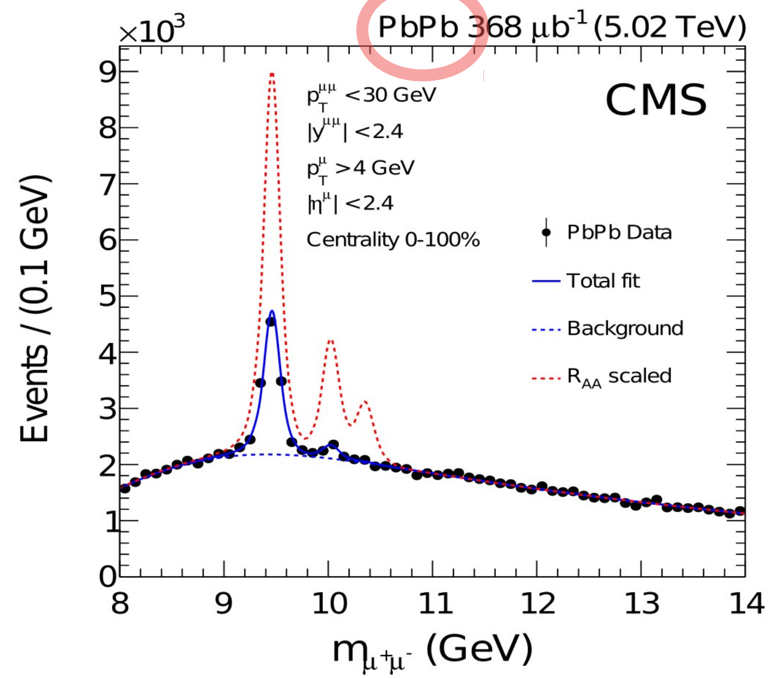
⇒ **Evading detection** at, e.g., the LHC.

⇒ Eventually consulting construction of future colliders.

Heavy quarkonium annihilation/decay in QGP

CMS collaboration, Phys.Lett. **B790** (2019) 270-293





- Sequential melting of [Υ(1S), Υ(2S), Υ(3S)] in QGP observed.

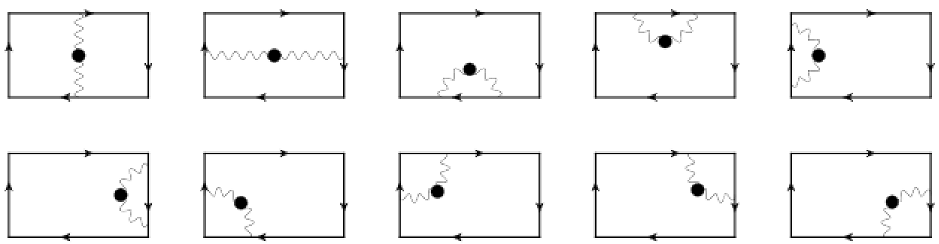
Effective in-medium potential

Equilibrium 4-Polyakov loop (Wilson lines) method [M. Laine *et al.* '07]

$$C_E(\tau, \mathbf{r}) \equiv \langle W[(0,\mathbf{r});(\tau,\mathbf{r})] W[(\tau,\mathbf{r});(\tau,\mathbf{0})] W[(\tau,\mathbf{0});(0,\mathbf{0})] W[(0,\mathbf{0}),(\tau,\mathbf{r})] \rangle,$$

$$W(\tilde{x},\tilde{y}) = \exp\left[ig \int_{\tilde{x}}^{\tilde{y}} A_\mu(\tilde{z}) \mathrm{d}\tilde{z}_\mu \right]. \qquad \mathrm{i}\partial_t C(t,r) \equiv V(t,r) C(t,r)$$

At leading order g_χ^2 :



Explicit cancellation of gauge parameter: ***Gauge invariant potential!***

$$\langle A_\mu(\tilde{x}) A_\nu(\tilde{y}) \rangle = \sum_{\tilde{q}_0} \int_{\mathbf{q}} e^{i\tilde{Q}(\tilde{x}-\tilde{y})} \left[\frac{P^T_{\mu\nu}(\tilde{Q})}{\tilde{Q}^2 + \Pi_T(\tilde{Q})} + \frac{P^L_{\mu\nu}(\tilde{Q})}{\tilde{Q}^2 + \Pi_L(\tilde{Q})} + \xi \frac{\tilde{q}_\mu \tilde{q}_\nu}{(\tilde{Q}^2)^2} \right].$$

In static limit and ***Hard-Thermal-Loop*** resummed approximation:

$$\lim_{t\to\infty} V(t,\mathbf{r}) = -\alpha m_D - \frac{\alpha}{r} e^{-m_D r} - i\alpha T \phi(m_D r)$$

Thermal width leads to melting of bound states, ill-defined principle quantum number.

Dynamics?

DM freeze-out is transition from chemical equilibrium to chemical non-equilibrium. Need dynamical formulation (**beyond quarkonia literature**). Difficult to find number density equations if principle quantum number of BS is ill-defined.

<u>"Linear Response theory" method</u> **[Bödeker&Laine '12]:**

$$\dot{\delta n} = -\Gamma_{\rm chem}\delta n(t) + \xi(t) \text{ (Langevin equation)}$$

$\Gamma_{\rm chem}$ depends on in-medium potential

matching $\Gamma_{\rm chem}$

$$\dot{n} + 3Hn = -\Gamma_{\rm chem}(n - n_{\rm eq}) + \mathcal{O}([n - n_{\rm eq}]^2) \text{ (linearized BE)}$$

$$\dot{n} + 3Hn = -\frac{\Gamma_{\rm chem}}{2n_{\rm eq}}(n^2 - n_{\rm eq}^2) \text{ (quadratic BE)}$$

How to interpret n? Why quadratic if we have also BS decay? Moreover, for $T \lesssim |E_B|$:

$\Gamma_{\rm chem.} \propto e^{\beta |E_B|}$ ("Late times become problematic")

Advantage of Non-equilibrium QFT approach

Previous literature:

1) Effective in-medium potential

System similar to heavy quarkonium decay in QGP.

⇒ **Equilibrium 4-Polyakov loop (Wilson lines) method** **[M. Laine *et al.* '07]**

2) DM number density equation (What is the "BE" to use?)

Langevin equation: $\delta \dot{n} = -\Gamma_{\rm chem} \delta n(t) + \xi(t)$ **[Bödeker&Laine '12]**

Our approach:
Non-equilibrium QFT ⇒ In-medium effects + full dynamics

In some limits, previous literature results should be (are) recovered.

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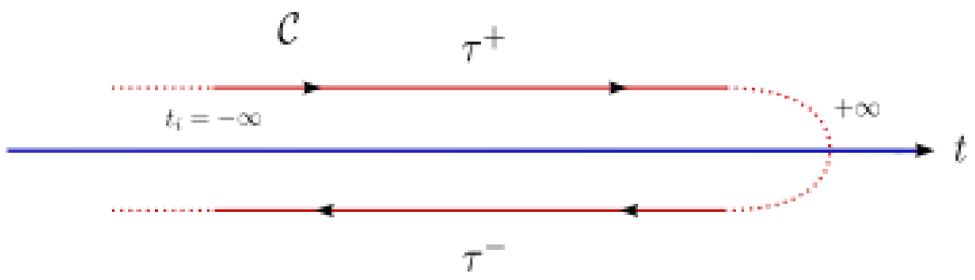
Scheme overview for CTP-formalism

$$G_O(x,y) \equiv \langle T_{\mathcal{C}} O(x) O^\dagger(y) \rangle$$
$$\langle ... \rangle \equiv \mathrm{Tr}[\rho ...]$$



I Non-relativistic effective action

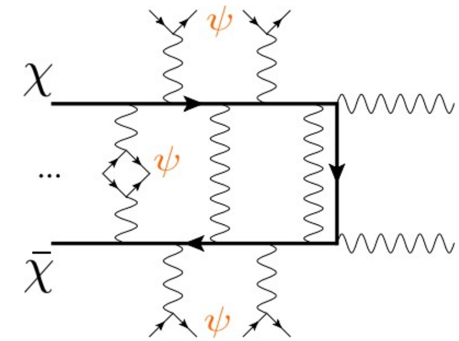
II Number density eq. from EoM of two-point fct.

III Resummation and effective potential

IV KMS condition and spectral function

Scheme

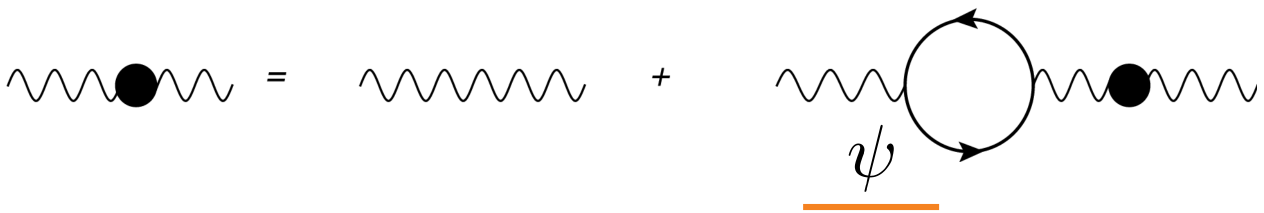
$$\mathcal{L} \supset g_\chi \bar{\chi}\gamma^\mu \chi A_\mu + \underline{g_\psi \bar{\psi}\gamma^\mu \psi A_\mu}$$



I <u>Non-relativistic effective action on CTP-contour</u>

$$S_{\rm NR}[\eta,\xi] = \int_{x\in\mathcal{C}} \eta^\dagger\left[i\partial_t + \frac{\Delta}{2M}\right]\eta + \xi^\dagger\left[i\partial_t - \frac{\Delta}{2M}\right]\xi + \int_{x,y\in\mathcal{C}} i\frac{g_\chi^2}{2}\underbrace{J(x)D(x,y)J(y)}_{\text{“potential scattering”}} + i\underbrace{O^\dagger(x)\Gamma(x,y)O(y)}_{\text{“annihilation”}},$$

- Separation of short- and long-range contributions, $J \equiv \eta^\dagger\eta + \xi^\dagger\xi, O \equiv \xi^\dagger\eta$
- Electric correlator: $D(x,y) \equiv \langle T_{\mathcal{C}} A_0(x) A_0(y)\rangle$



Scheme

II Number density equation from EoM of two-point function

$$\dot{n}_\eta + 3Hn_\eta = -2(\sigma v)_0 \left[G_{\eta\xi}^{++--}(x,x,x,x) - G_{\eta\xi}^{++--}(x,x,x,x)\Big|_{\rm eq} \right].$$

$$n_\eta(x) \equiv \langle \eta^\dagger(x)\eta(x)\rangle,\ G_{\eta\xi}(x,y,z,w) \equiv \langle T_{\mathcal{C}}\eta(x)\xi^\dagger(y)\xi(w)\eta^\dagger(z)\rangle$$

- Exact result! “Only” have to compute a 4-point function.
- Truncation of Martin-Schwinger hierarchy introduces approximations.
- To lowest order:

$$2G_{\eta\xi}^{++--}(x,x,x,x) \simeq n_\eta n_\xi$$

→ *Lee-Weinberg equation*

- For bound states to occur, need non-perturbative solution

Scheme

III Resummation and effective potential

The problem reduces after truncation of Martin-Schwinger hierarchy to the solution of:

$$G_\eta = G_\eta^0 + g_\chi^2 \int_{\mathcal{C}} G_\eta^0 D \left[G_{\eta\xi} + G_{\eta\eta}\right]$$

$$G_{\eta\xi} = G_\eta G_\xi + g_\chi^2 \int_{\mathcal{C}} G_\eta G_\xi D G_{\eta\xi}$$

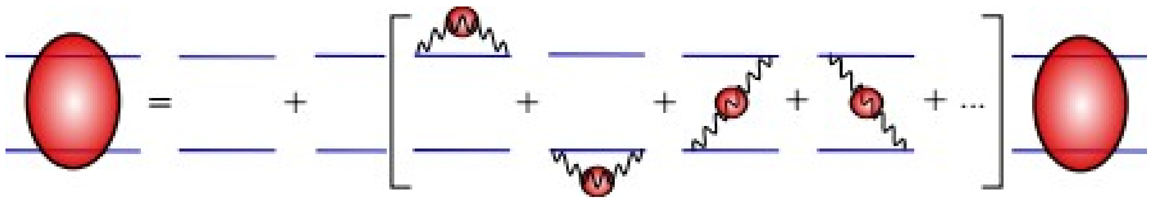
Still too complicated: 3 time problem, KMS condition ... further approximation needed.
Instead, we compute the two-point function to order g_χ^2

$$G_\eta = G_\eta^0 + g_\chi^2 \int_{\mathcal{C}} G_\eta^0 \Sigma_\eta G_\eta^0$$

and insert this into the four-point correlator

$$G_{\eta\xi} = G_\eta^0 G_\xi^0 + g_\chi^2 \int_{\mathcal{C}} G_\eta^0 G_\xi^0 \left[D G_\eta^0 G_\xi^0 + \Sigma_\eta G_\eta^0 + \Sigma_\xi G_\xi^0\right]$$

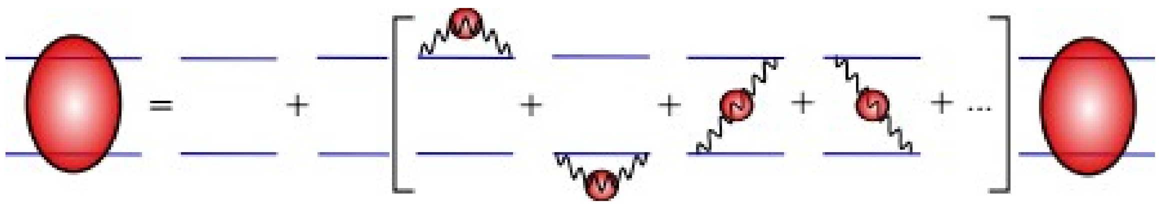
Then, (after some rearrangements) we **resum all contributions** to order g_χ^2



This **resummation scheme** has some nice features.
(respects KMS condition, reproduces 4-Polykov result, contains FT corrections,...)

Scheme

III Resummation and effective potential



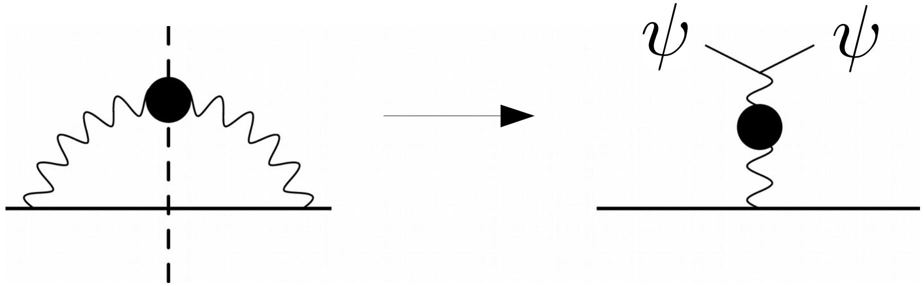
$$\left[\frac{\nabla_{\mathbf{r}}^2}{M} + E + i\epsilon - V_{\text{eff}}(\mathbf{r}, T)\right] G_{\eta\xi}^R(\mathbf{r}, \mathbf{r}'; E) = 2i\delta(\mathbf{r} - \mathbf{r}')$$

In static limit and Hard-Thermal-Loop approximation:

$$V_{\text{eff}}(\mathbf{r}, T) \equiv -ig_\chi^2 \int \frac{\mathrm{d}^3 p}{(2\pi)^3}(1 - e^{i\mathbf{p}\mathbf{r}})D^{++}(0, \mathbf{p}) = -\alpha_\chi m_D - \frac{\alpha_\chi}{r}e^{-m_D r} - i\alpha_\chi T\phi(m_D r)$$

consistent with **[M. Laine *et al.* '07]**

Interpretation of thermal width:



Soft bath particle scattering
aka "Landau damping"

Scheme

IV Kubo-Martin-Schwinger relation and two-particle spectral function

$$\dot{n}_\eta + 3Hn_\eta = -2(\sigma v)_0 \left[G^{++--}_{\eta\xi}(x,x,x,x) - G^{++--}_{\eta\xi}(x,x,x,x)\big|_{\text{eq}} \right].$$

- Treat annihilation as perturbation
- Assume grand canonical state $\rho \propto e^{-\beta(\hat{H}-\mu_\eta \hat{N}_\eta - \mu_\xi \hat{N}_\xi)}$ to relate:

$$G^{++--}_{\eta\xi}(x,x,x,x) = e^{-2\beta(M-\mu)} \int \frac{\mathrm{d}^3\mathbf{P}}{(2\pi)^3} e^{-\beta \mathbf{P}^2/(4M)} \int_{-\infty}^{\infty} \frac{\mathrm{d}E}{(2\pi)} e^{-\beta E} G^{\rho}_{\eta\xi}(\mathbf{0},\mathbf{0};E).$$

$$G^{\rho}_{\eta\xi} = 2\Im[iG^R]$$

- Positive and negative energy (bound state) solutions included.

Consistency check

$$\dot{n} + 3Hn = -2(\sigma v)_0 G^{++--}_{\eta\xi}(x,x,x,x)\big|_{\rm eq}\left[e^{\beta 2\mu} - 1\right],$$

$$G^{++--}_{\eta\xi}\big|_{\rm eq} = e^{-2\beta M}\int\frac{{\rm d}^3\mathbf{P}}{(2\pi)^3}e^{-\beta\mathbf{P}^2/(4M)}\int_{-\infty}^{\infty}\frac{{\rm d}E}{(2\pi)}e^{-\beta E}G^{\rho}_{\eta\xi}(\mathbf{0},\mathbf{0};E).$$

$$V_{\rm eff}(r,T) = 0$$

Free spectral function:

$$G^{\rho}_{\eta\xi} \propto \theta(E)E^{1/2}$$

+

Ideal dilute gas:

$$n = n_s^{\rm eq}e^{\beta\mu}$$

$$\Rightarrow \beta\mu = \ln[n/n_s^{\rm eq}]$$

$$\dot{n} + 3Hn = -(\sigma v)_0\left[n^2 - (n_s^{\rm eq})^2\right]$$

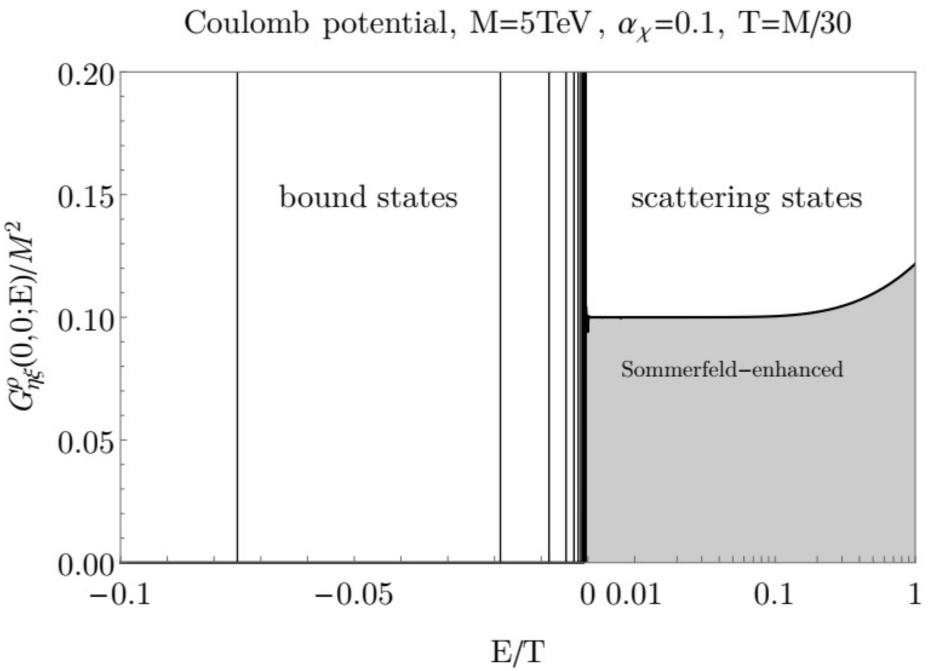
Lee-Weinberg equation ✓

Consistency check

$$\dot{n} + 3Hn = -2(\sigma v)_0 G^{++--}_{\eta\xi}(x,x,x,x)\big|_{\mathrm{eq}} \left[e^{\beta 2\mu} - 1\right],$$

$$G^{++--}_{\eta\xi}\big|_{\mathrm{eq}} = e^{-2\beta M} \int \frac{\mathrm{d}^3\mathbf{P}}{(2\pi)^3} e^{-\beta \mathbf{P}^2/(4M)} \int_{-\infty}^{\infty} \frac{\mathrm{d}E}{(2\pi)} e^{-\beta E} G^{\rho}_{\eta\xi}(\mathbf{0},\mathbf{0};E).$$

$$\lim_{T\to 0} V_{\mathrm{eff}}(r,T)$$



Consistency check

$$\dot{n} + 3Hn = -2(\sigma v)_0 G_{\eta\xi}^{++--}(x,x,x,x)\big|_{\rm eq}\left[e^{\beta 2\mu} - 1\right],$$

$$G_{\eta\xi}^{++--}\big|_{\rm eq} = e^{-2\beta M}\int \frac{{\rm d}^3\mathbf{P}}{(2\pi)^3} e^{-\beta\mathbf{P}^2/(4M)} \int_{-\infty}^{\infty}\frac{{\rm d}E}{(2\pi)} e^{-\beta E} G_{\eta\xi}^{\rho}(\mathbf{0},\mathbf{0};E).$$

$$\lim_{T\to 0} V_{\rm eff}(r,T)$$

Spectral function:

$$(\sigma v)_0 G_{\eta\xi}^{\rho}(E)|_{E>0} \propto E^{1/2}(\sigma v)_0|\psi(0)|^2$$

$$(\sigma v)_0 G_{\eta\xi}^{\rho}(E)|_{E<0} \propto \sum_n \delta(E - E_n)\Gamma_n$$

Chemical potential:

$$n = n_s^{\rm eq} e^{\beta\mu} + \sum_i n_i^{\rm eq} e^{2\beta\mu}$$

$$\Rightarrow \beta\mu = \ln[\alpha n / n_s^{\rm eq}]$$

$$\dot{n} + 3Hn = -\left(\langle(\sigma v)_0 S\rangle + \sum_i \Gamma_i \frac{n_i^{\rm eq}}{(n_s^{\rm eq})^2}\right)\left[(\alpha n)^2 - (n_s^{\rm eq})^2\right]$$

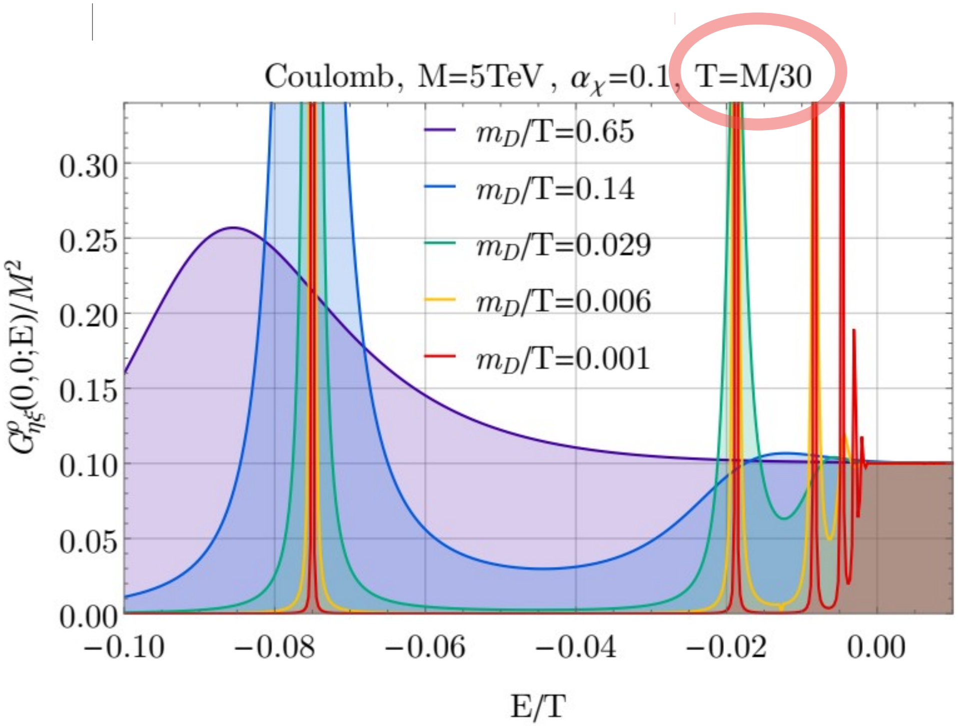
BEs in (Saha) ionization equilibrium ✓

Finite temperature corrections

$$\dot{n} + 3Hn = -2(\sigma v)_0 G^{++--}_{\eta\xi}(x,x,x,x)\big|_{\rm eq}\left[e^{\beta 2\mu} - 1\right],$$

$$G^{++--}_{\eta\xi}\big|_{\rm eq} = e^{-2\beta M}\int \frac{d^3\mathbf{P}}{(2\pi)^3} e^{-\beta\mathbf{P}^2/(4M)} \int_{-\infty}^{\infty} \frac{dE}{(2\pi)} e^{-\beta E} G^{\rho}_{\eta\xi}(\mathbf{0},\mathbf{0};E).$$

full $V_{\rm eff}(r,T)$



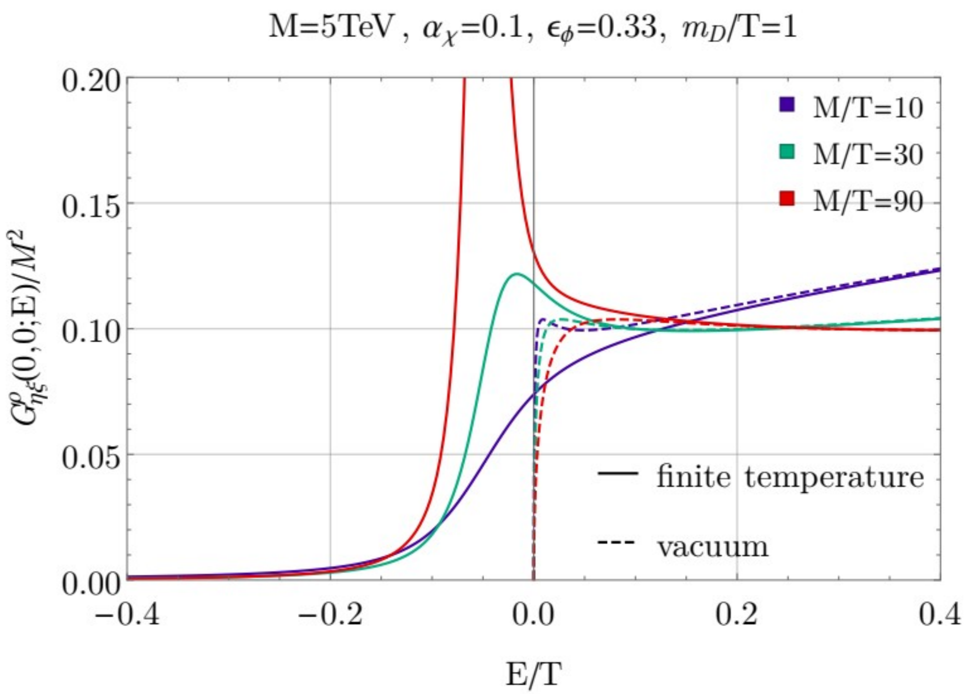
⇒ Thermal width can exceed binding energy

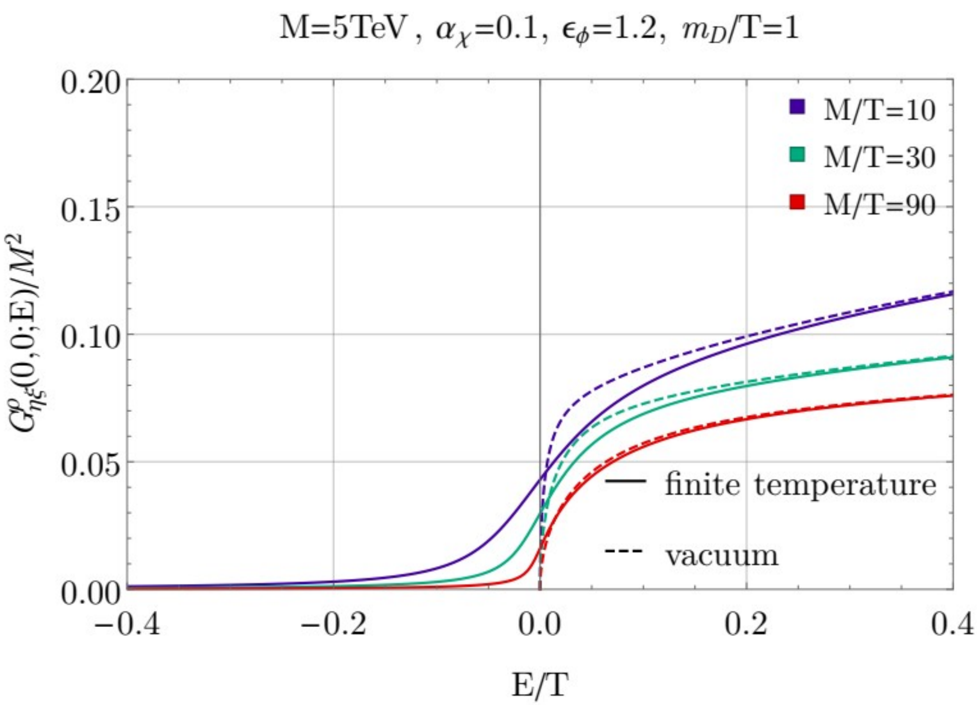
⇒ Spectrum is continuous (can not distinguish bound from scattering state)

⇒ Melting of ground-state pole at the time of DM freeze-out possible!

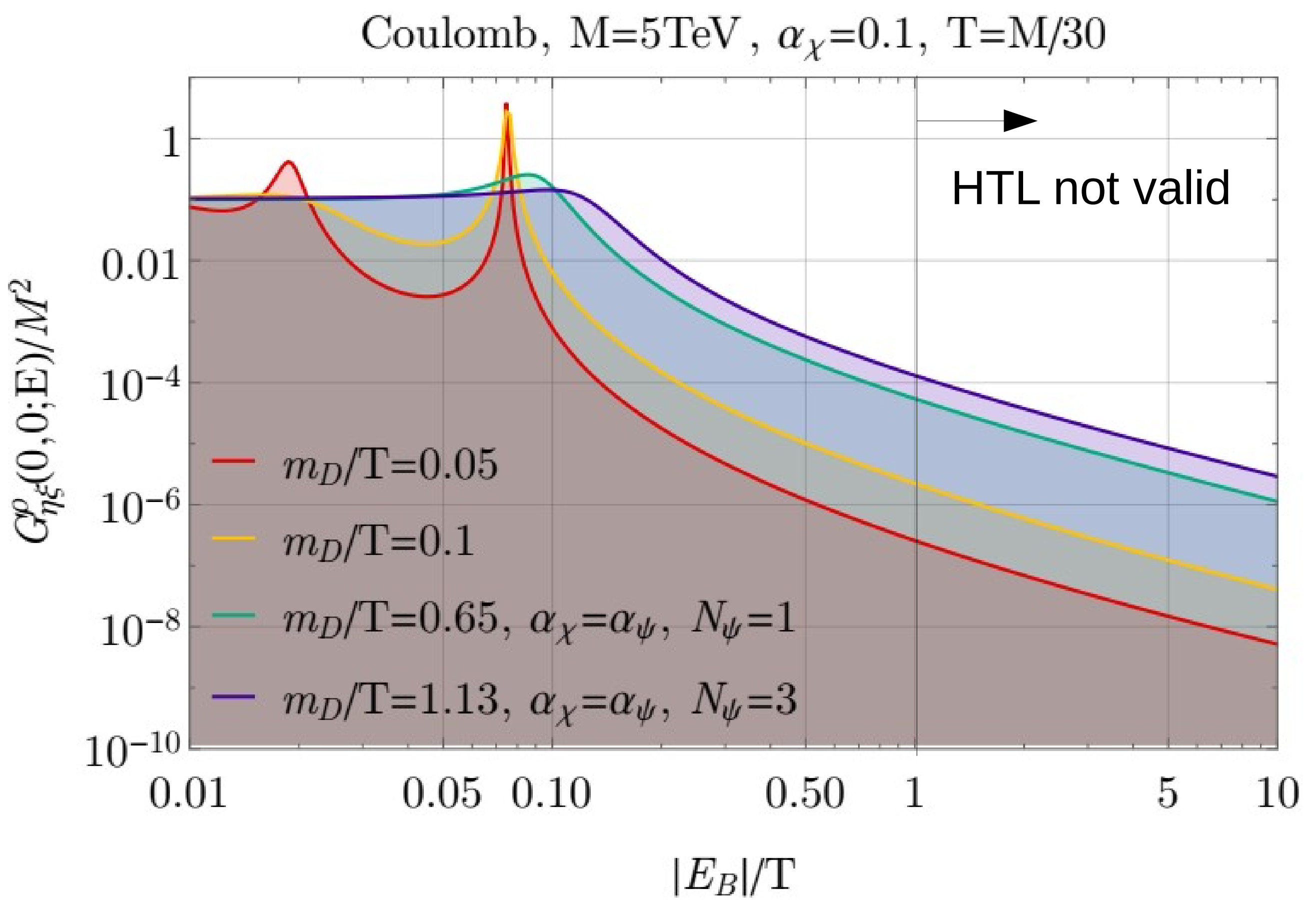
⇒ Rate is exponentially sensitive to changes in spectral function.

Full spectrum





Limitation of HTL approximation



$$G^{++--}_{\eta\xi}\big|_{\mathrm{eq}} = e^{-2\beta M}\int \frac{\mathrm{d}^3\mathbf{P}}{(2\pi)^3} e^{-\beta \mathbf{P}^2/(4M)} \int_{-\infty}^{\infty} \frac{\mathrm{d}E}{(2\pi)} e^{-\beta E} G^{\rho}_{\eta\xi}(\mathbf{0},\mathbf{0};E).$$

Summary consistency checks

$$\dot{n} + 3Hn = -2(\sigma v)_0 G_4^{++--}\Big|_{\text{eq.}} \left[\left(\alpha n / n_{s,0}^{\text{eq}}\right)^2 - 1\right]$$

Consistency check

$V_{\text{eff}}(r,T) = 0$ →

$$\dot{n} + 3Hn = -(\sigma v)_0 \left[n^2 - n_{\text{eq}}^2\right]$$

Lee-Weinberg equation ✓

$\lim_{T \to 0} V_{\text{eff}}(r,T)$ →

$$\dot{n} + 3Hn = -\left(\langle(\sigma v)_0 S\rangle + \sum_i \Gamma_i R_i\right)\left[(\alpha n)^2 - (n_s^{\text{eq}})^2\right]$$

BEs in ionization equilibrium ✓

full $V_{\text{eff}}(r,T)$, $n \sim n_{\text{eq}}$ →

$$\dot{n} + 3Hn = -\Gamma_{\text{chem}}[n - n_{\text{eq}}]$$

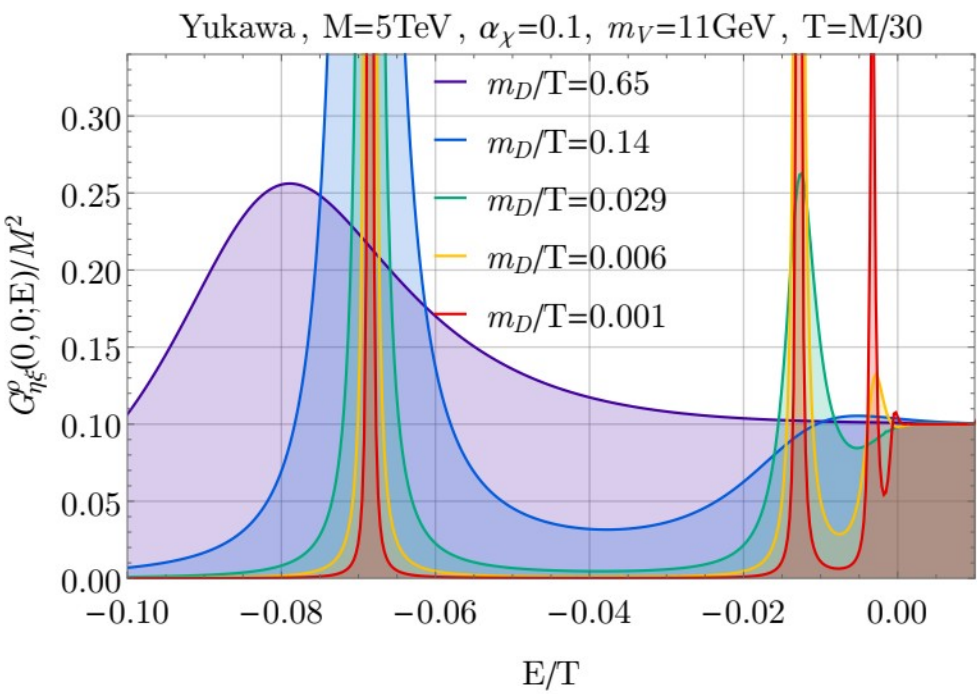
consistent with Langevin approach

in linear regime close to chem. equil. ✓

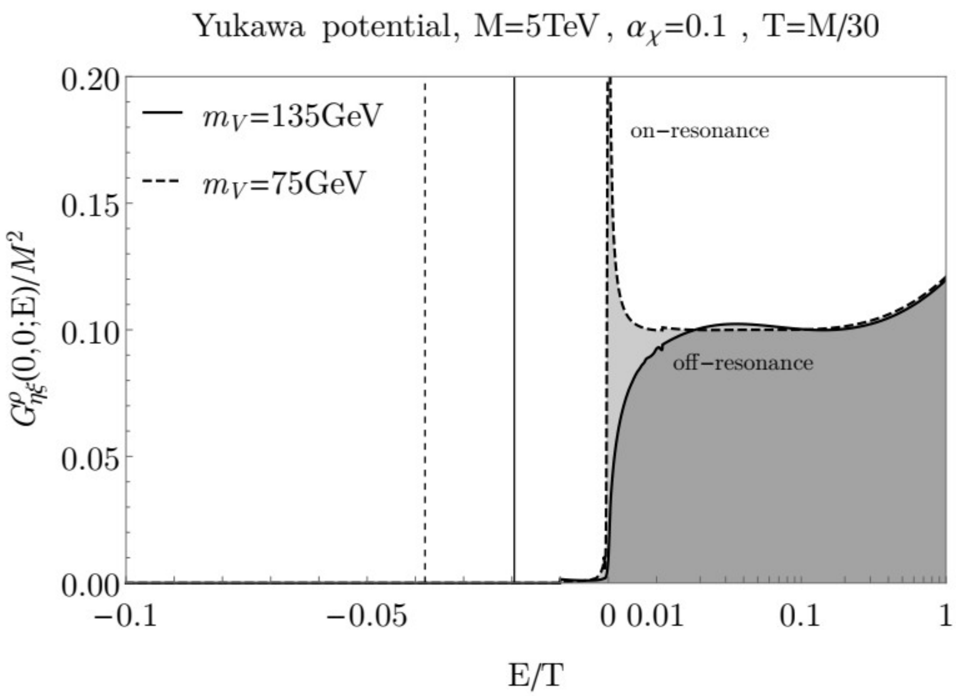
Summary

- Analytically derived new number density equation, accounting for finite temperature corrections to Sommerfeld-enhanced annihilation and bound-state decay.
- Contributed to set a theoretical basis for quantifying the impact of finite temperature corrections in a self-consistent approach.
- Finite temperature corrections arise in spectral function and chemical potential.
- Clear understanding of the limitation (e.g. ionization equilibrium).
- More theoretical developments required to obtain full picture.
- Non-equilibrium QFT promising approach.

Yukawa

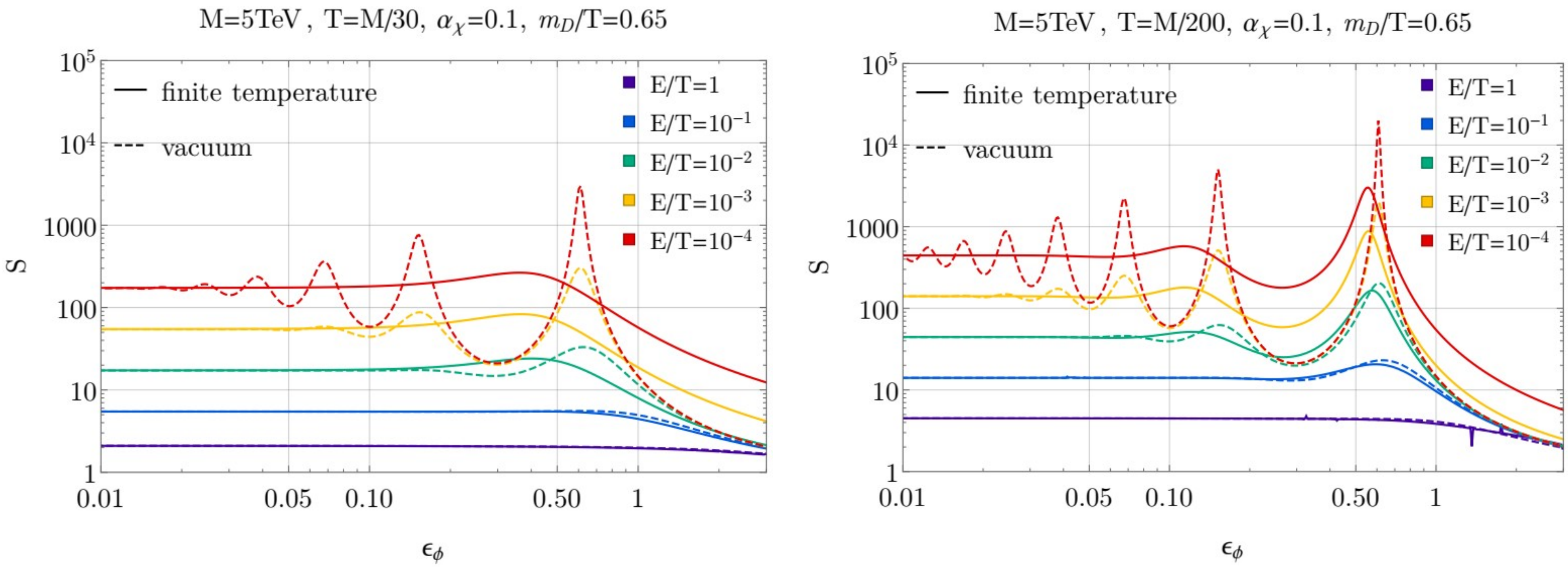


Yukawa , M=5TeV , α_χ=0.1, m_V=11GeV , T=M/30
m_D/T=0.65
m_D/T=0.14
m_D/T=0.029
m_D/T=0.006
m_D/T=0.001
$G^{\rho}_{\eta\xi}(0,0;E)/M^2$
E/T



Yukawa potential, M=5TeV , α_χ=0.1 , T=M/30
m_V=135GeV
m_V=75GeV
on−resonance
off−resonance
$G^{\rho}_{\eta\xi}(0,0;E)/M^2$
E/T

Positive energy solution



Backup: BEs and ionization equilibrium

$$\dot{n}_s + 3Hn_s = -\langle(\sigma v)_{\rm an}\rangle\left[n_s^2 - (n_s^{\rm eq})^2\right] - \sum_i \langle(\sigma v)_i\rangle\left[n_s^2 - n_i K_i^{-1}\right],$$

$$\dot{n}_i + 3Hn_i = -\Gamma_i\left[n_i - n_i^{\rm eq}\right] + \langle(\sigma v)_i\rangle\left[n_s^2 - n_i K_i^{-1}\right] - \sum_j \Gamma_{i\to j}\left[n_i - n_j R_{ij}\right]$$

$$K_i \equiv n_i^{\rm eq}/(n_s^{\rm eq})^2$$

$$R_{ij} \equiv n_i^{\rm eq}/n_j^{\rm eq}$$

Ionization equilibrium:

$$\left(\frac{n_s}{n_s^{\rm eq}}\right)^2 = \frac{n_i}{n_i^{\rm eq}},\ \forall i \ \Rightarrow 2\mu \equiv 2\mu_s = \mu_i\ \forall i$$