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Dark Matter Sommerfeld-enhanced annihilation and bound-state decay at finite temperature

Tobias Binder

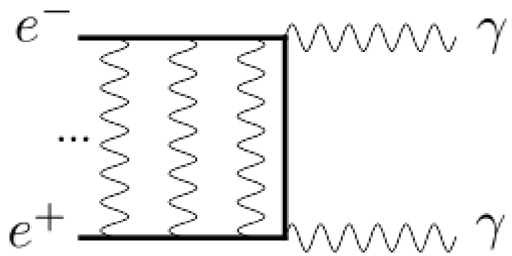
based on

[arXiv:1808.06472] Phys.Rev. **D98** (2018) no.11, 115023

in collaboration with

Laura Covi (Göttingen) and **Kyohei Mukaida** (DESY).

Non-relativistic Positronium annihilation



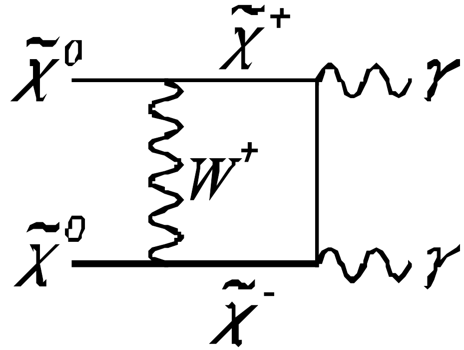
- Bound-state decay **[J. Wheeler 1946]**:

$$\Gamma_n = 4(\sigma v)_0 \times |\psi_n(r=0)|^2$$

- Sommerfeld-enhanced annihilation **[A. Sakharov 1948]**:

$$(\sigma v) = (\sigma v)_0 \times |\psi(r=0)|^2$$
$$\propto (\sigma v)_0\, (\alpha / v_{\rm rel})\,, \;\text{for } v_{\rm rel} \lesssim \alpha$$

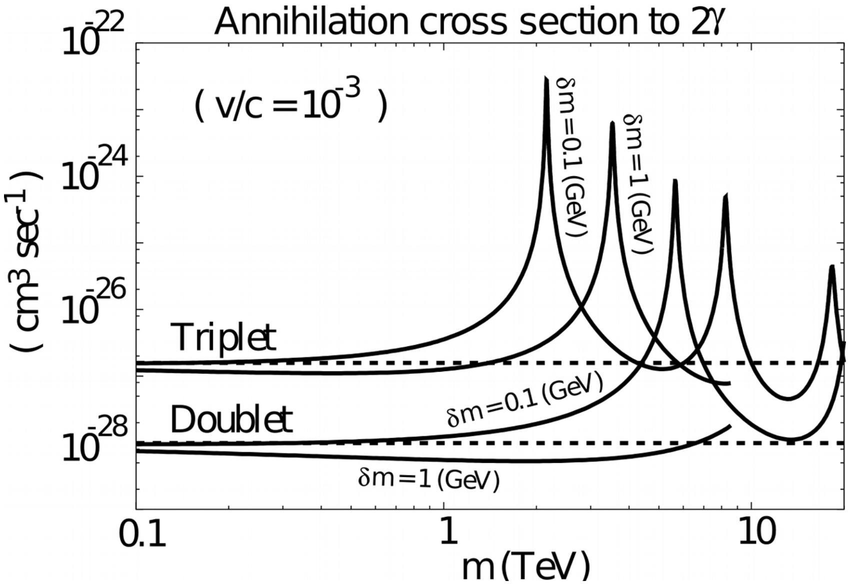
Sommerfeld enhancement for Wino-neutralino



Galactic center

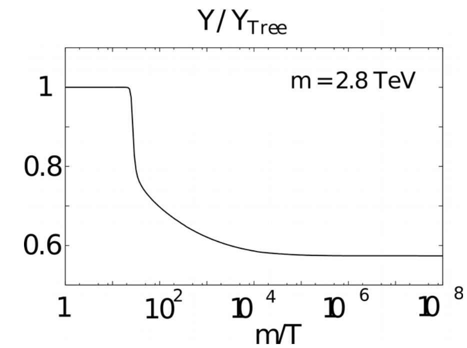
[J. Hisano *et al.* '03, '05]

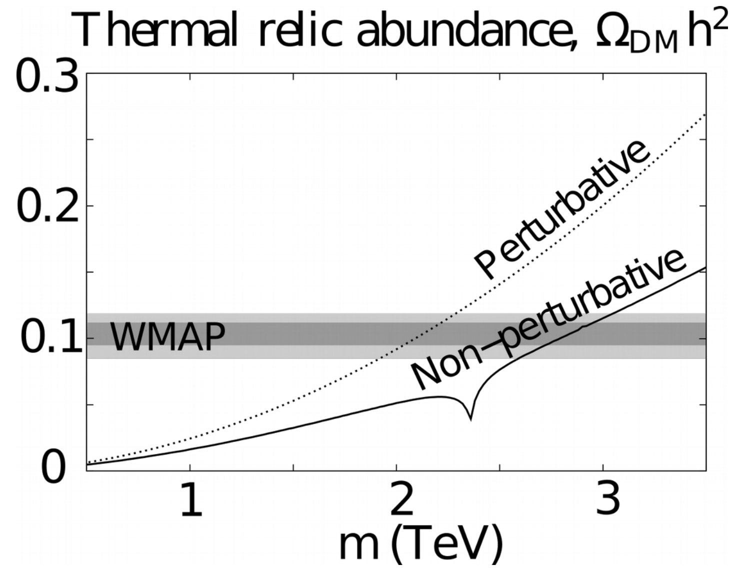
"Explosive DM"



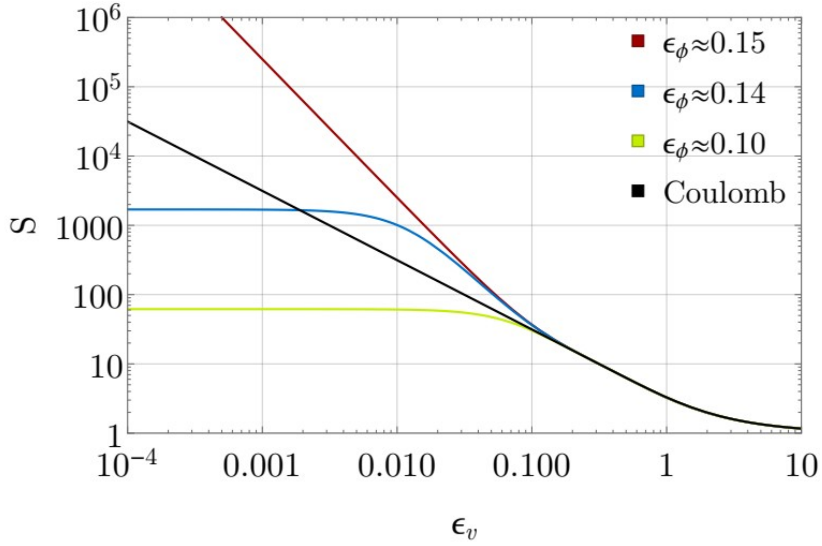
Relic abundance

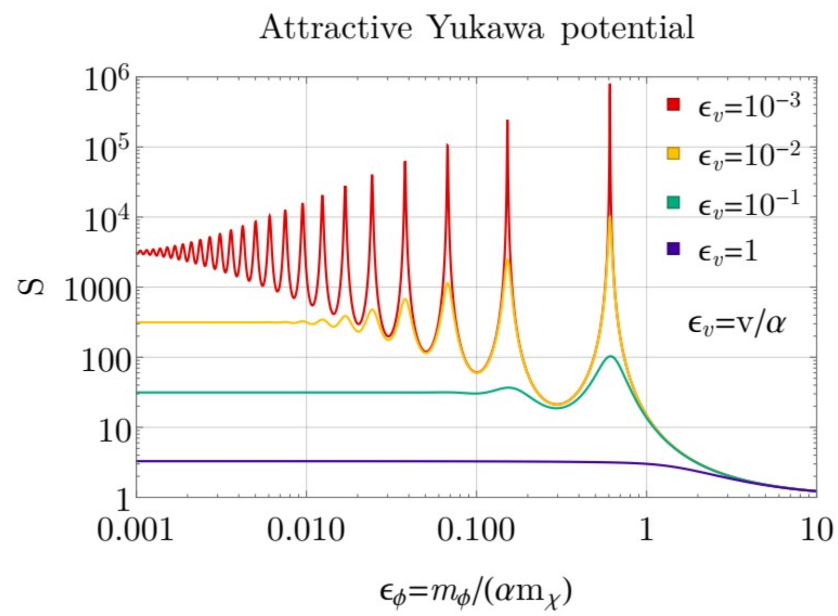
[J. Hisano *et al.* '06]

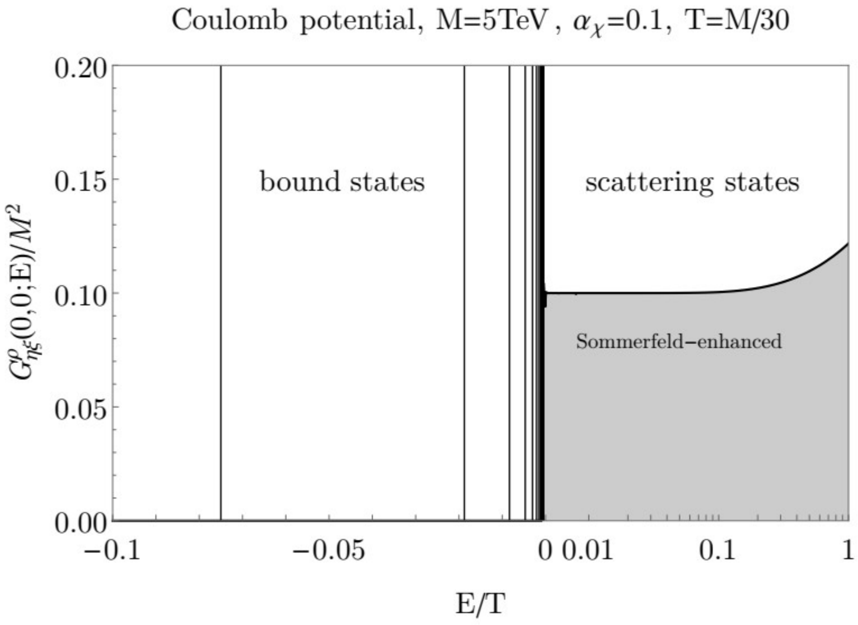


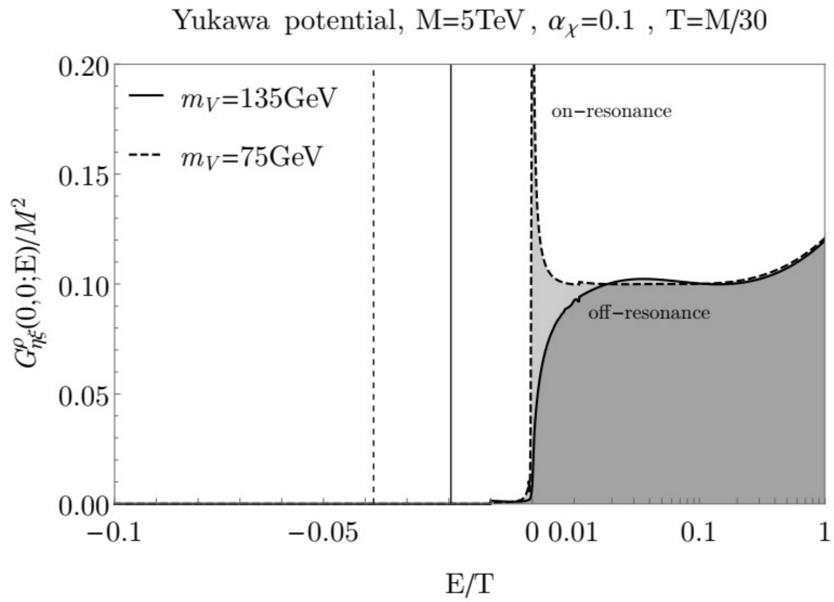


Attractive Coulomb vs. Yukawa potential

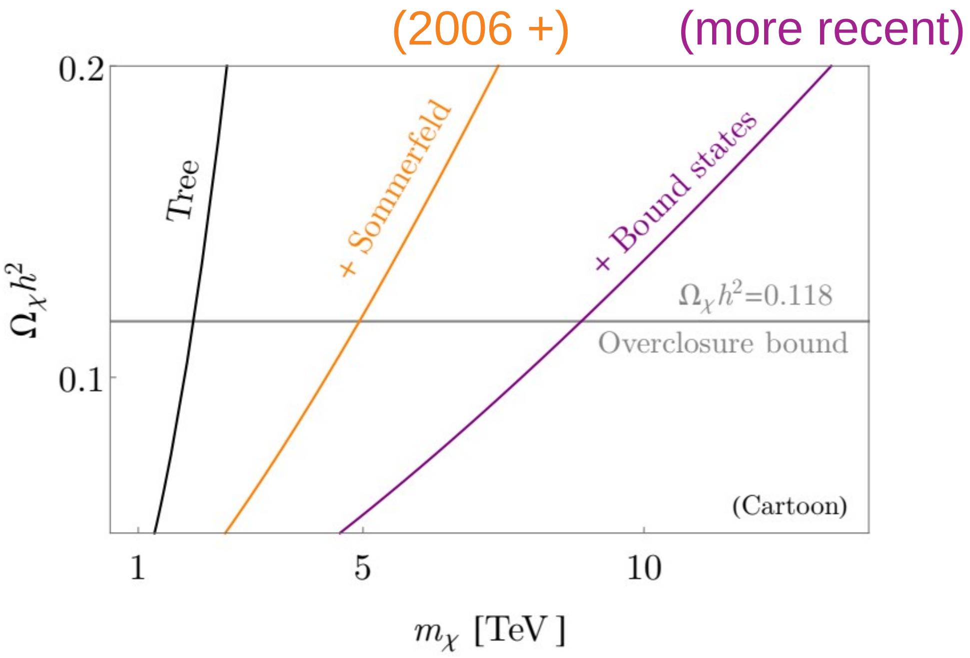








Sommerfeld enhancement + Bound states



Ex.: Classical Wino, Minimal dark matter, WIMPoniums, Neutralino DM co-annihilating with colored charged particles, Higgs mediated bound states, U(1) hidden charged dark sectors, SIDM with light mediators, ... (~ $\mathcal{O}(100)$ publications)

[J. Hisano *et al.* '06, ..., J. Feng *et al.* '09, Harling&Petraki '14, ... , Harz&Petraki '19, probably more to come]

Bound-state formation:

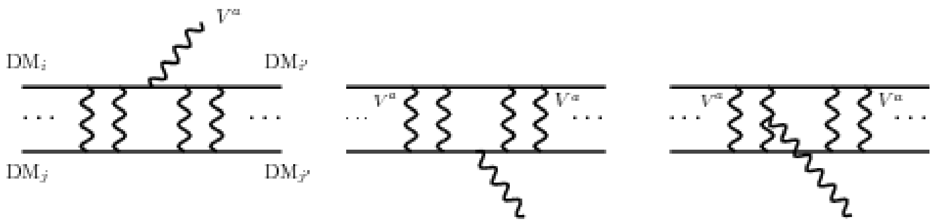


Fig. taken from [Mitridate et al. '17]

Outline

Part 1:

1) Long-range effects in vacuum

SE, Bound-state decay, BS formation, dissociation, level-transitions

2) Boltzmann equations including long-range interactions (vacuum)

Ionization equilibrium, ...

3) Perturbative Non-equilibrium QFT

Keldysh-Schwinger formalism, **EOM of correlation functions**,
NLO Collision term, ...

Part 2:

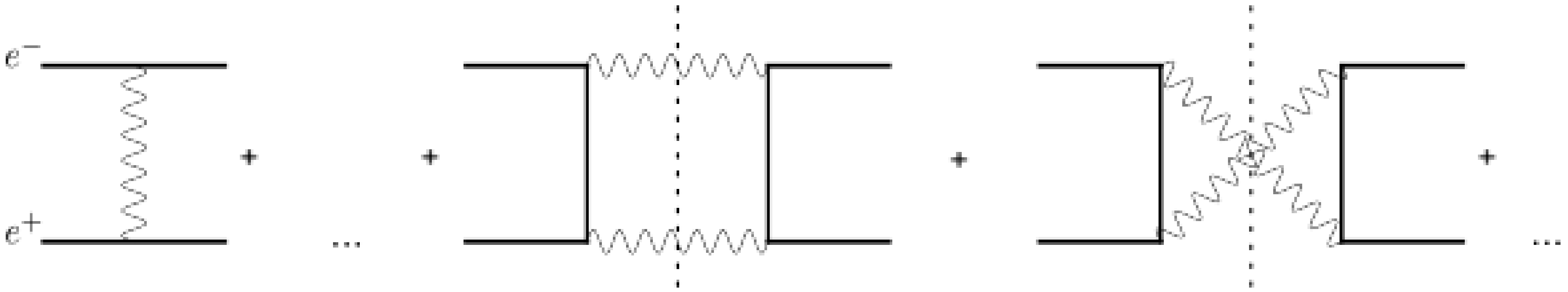
Sommerfeld enhanced annihilation and bound-state decay at finite temperature

Non-relativistic QED (NRQED)

$$S_{\rm NR} \supset \int \mathrm{d}^4x\ \eta^\dagger(x)\left[i\partial_t + \frac{\nabla^2}{2m}\right]\eta(x) + \xi^\dagger(x)\left[i\partial_t - \frac{\nabla^2}{2m}\right]\xi(x)$$

$$+\int \mathrm{d}^4x\mathrm{d}^4y\ J(x)\left[-\frac{1}{2}\delta(x^0-y^0)\frac{\alpha}{|\mathbf{x}-\mathbf{y}|}\right]J(y) + O^\dagger(x)\left[\frac{i\pi\alpha^2}{m^2}\delta^4(x-y)\right]O(y),$$

where $J(x) \equiv \eta^\dagger(x)\eta(x) + \xi^\dagger(x)\xi(x)$ and $O(x) \equiv \xi^\dagger(x)\eta(x)$.



W. E. Caswell and G. P. Lepage, "Effective lagrangians for bound state problems in QED, QCD, and other field theories", Phys. Lett. B 167, 437 (1986).

From NRQED to wave-function formalism

$$S_{\rm NR} = \int \mathrm{d}^4x\, \eta^\dagger(x) \left[i\partial_t + \frac{\nabla^2}{2m_e} \right] \eta(x) + \xi^\dagger(x) \left[i\partial_t - \frac{\nabla^2}{2m_e} \right] \xi(x)$$
$$+ \int \mathrm{d}^4x \mathrm{d}^4y\, J(x) \left[-\frac{1}{2}\delta(x^0 - y^0) \frac{\alpha}{|\mathbf{x}-\mathbf{y}|} \right] J(y) + O^\dagger(x) \left[\frac{i\pi\alpha^2}{m_e^2} \delta^4(x-y) \right] O(y).$$

Acting H on two-body state $|\psi(t)\rangle = \frac{1}{\sqrt{N}} \int \mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{y}\, \psi(\mathbf{x},\mathbf{y},t) \eta^\dagger(\mathbf{x}) \xi(\mathbf{y}) |0\rangle\,,$

leads to Schrödinger eq.:

$$\left[-\frac{\Delta}{2\mu} - \frac{\alpha}{r} - 2i\frac{\pi\alpha^2}{m^2}\delta^3(\mathbf{r}) \right] \psi(x) = E\psi(x).$$

Imaginary part leads to violation of current $\vec{j}(\mathbf{x}) = \frac{2}{m} \Im \left[\psi^\star(\mathbf{x}) \vec{\nabla} \psi(\mathbf{x}) \right]$, i.e.:

$$-\vec{\nabla}\cdot\vec{j}(\mathbf{x}) = 4\frac{\pi\alpha^2}{m_e^2} |\psi(\mathbf{0})|^2 \delta^3(\mathbf{r})$$

$$-\int \mathrm{d}^3x\, \vec{\nabla}\cdot\vec{j}(x) = 4\frac{\pi\alpha^2}{m^2} |\psi(x=0)|^2 \begin{cases} = 4(\sigma v_{\rm rel}) & \text{if } E > 0 \\ = \Gamma_n & \text{if } E < 0 \end{cases}$$

Only l=0 survives. Typically imaginary part is treated as perturbation, however, for Yukawa potential some care must be taken.

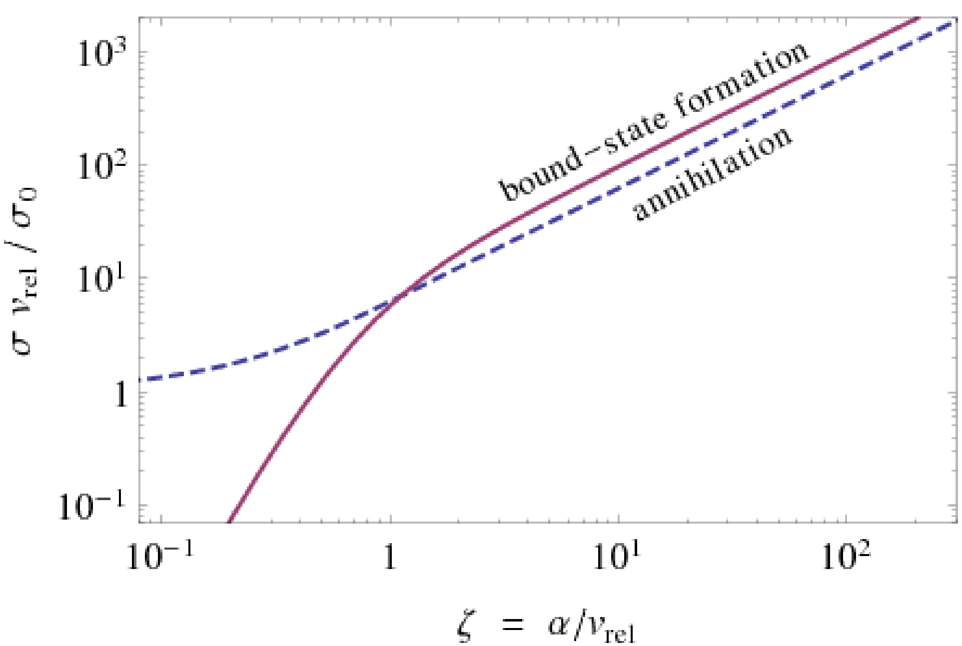
Radiative processes

E.g., consider direct capture into the ground state via single massless mediator emission:

$$(\chi\bar{\chi})_{\rm sc} \;\to\; \phi + (\chi\bar{\chi})_{100}$$

$$(\sigma v_{\rm rel}) = \frac{\alpha_\chi^2 \pi}{m^2} S(\xi) \frac{2^9 \xi^4}{3(1+\xi^2)^2} e^{-4\xi \operatorname{acot}(\xi)}$$

[Petraki et al. '15] Vector mediator



... but wait, there is also dissociation (reverse process)!
So shouldn't the bound-states get immediately destroyed, back to the scattering states?

(old argument why we should NOT care about bound states)

Generically, one has to consider the following coupled network:

$$(\chi\bar{\chi})_{\rm sc} \rightleftharpoons \;\mathrm{SM\ SM}', \qquad \textbf{annihilation}$$

$$\left.\begin{aligned}(\chi\bar{\chi})_{\rm sc} &\rightleftharpoons \phi + (\chi\bar{\chi})_{B_i}\,,\\ (\chi\bar{\chi})_{B_i} &\rightleftharpoons \phi + (\chi\bar{\chi})_{B_j}\,,\end{aligned}\right\} \qquad \textbf{radiative processes}$$

$$(\chi\bar{\chi})_{B_i} \rightleftharpoons \mathrm{SM\ SM}'. \qquad \textbf{decay}$$

Boltzmann equations

In principle, we just have to solve:

$$\begin{aligned}\dot{n}_s + 3Hn_s =& - \langle(\sigma v)_{\rm an}\rangle \left[n_s^2 - (n_s^{\rm eq})^2\right] \\ & - \sum_i \langle(\sigma v)_i\rangle \left[n_s^2 - n_i (n_s^{\rm eq})^2/n_i^{\rm eq}\right] , \\ \dot{n}_i + 3Hn_i =& - \Gamma_i \left[n_i - n_i^{\rm eq}\right] \\ & + \langle(\sigma v)_i\rangle \left[n_s^2 - n_i (n_s^{\rm eq})^2/n_i^{\rm eq}\right] \\ & - \sum_j \Gamma_{i\to j} \left[n_i - n_j n_i^{\rm eq}/n_j^{\rm eq}\right]\end{aligned}$$

(+ co-annihilation)

No public code exists, even model-by-model analysis relies on simplifications of these equations.

(Saha) Ionization equilibrium

$$\dot{n}_s + 3Hn_s = -\langle(\sigma v)_{\rm an}\rangle\left[n_s^2 - (n_s^{\rm eq})^2\right] - \sum_i \langle(\sigma v)_i\rangle\left[n_s^2 - n_i (n_s^{\rm eq})^2/n_i^{\rm eq}\right],$$

$$\dot{n}_i + 3Hn_i = -\Gamma_i\left[n_i - n_i^{\rm eq}\right] + \langle(\sigma v)_i\rangle\left[n_s^2 - n_i (n_s^{\rm eq})^2/n_i^{\rm eq}\right] - \sum_j \Gamma_{i\to j}\left[n_i - n_j n_i^{\rm eq}/n_j^{\rm eq}\right]$$

Assumption: Radiative processes much faster than annihilation or decay

[TB, Covi, Mukaida '18]

$$\left(\frac{n_s}{n_s^{\rm eq}}\right)^2 = \frac{n_i}{n_i^{\rm eq}},\ \forall i \ \Rightarrow 2\mu \equiv 2\mu_s = \mu_i\ \forall i$$

Reduces the system to one degree of freedom, i.e. the TOTAL DM DENSITY.

Remaining task is to express chemical potential as function of total n.

(Saha) Ionization equilibrium

$$n = n_s + \sum_i n_i$$
$$= n_s^{\text{eq}} e^{\beta\mu} + \sum_i n_i^{\text{eq}} e^{2\beta\mu}$$

Quadratic equation has solution:

$$\beta\mu = \ln\left[\frac{\alpha(nK(T))n}{n_s^{\text{eq}}}\right] \;, \alpha(x) = \frac{\sqrt{1+4x}-1}{2x} \;, K(T) = \sum_i \frac{n_i^{\text{eq}}}{n_s^{\text{eq}} n_s^{\text{eq}}} .$$

Inserting chemical potential back into sum of the BEs, leads to:

$$\dot{n} + 3Hn = -\left[\langle(\sigma v)_{\text{an}}\rangle + \sum_i \Gamma_i \frac{n_i^{\text{eq}}}{n_s^{\text{eq}} n_s^{\text{eq}}}\right] \left(\alpha^2 n^2 - n_s^{\text{eq}} n_s^{\text{eq}}\right)$$

[TB, Covi, Mukaida '18]

Note this equation is independent of all radiative cross sections!

(Saha) Ionization equilibrium

$$\dot{n} + 3Hn = -\left[\langle(\sigma v)_{\rm an}\rangle + \sum_i \Gamma_i \frac{n_i^{\rm eq}}{n_s^{\rm eq} n_s^{\rm eq}}\right] \left(\alpha^2 n^2 - n_s^{\rm eq} n_s^{\rm eq}\right)$$

[TB, Covi, Mukaida '18]

$$\Gamma_i \frac{n_i^{\rm eq}}{n_s^{\rm eq} n_s^{\rm eq}} \frac{s}{H} \propto \sqrt{m/T} e^{\beta |E_i|}$$

- Even though radiative processes are balanced, the decay depletes the relic abundance! If radiative processes are efficient for temperature much smaller the binding energy, there is exponential enhancement (ignoring corrections from degree of ionization).
- **At some point** the dissociation rate drops below the decay rate and ionization equilibrium will be broken. Then, the BE reads:

$$\dot{n}_s + 3Hn_s = -\left[\langle(\sigma v)_{\rm an}\rangle + \sum_i \langle(\sigma v)_i\rangle\right] n_s^2$$

➡ Scattering states will decouple from BS **at some point** and freeze-out.

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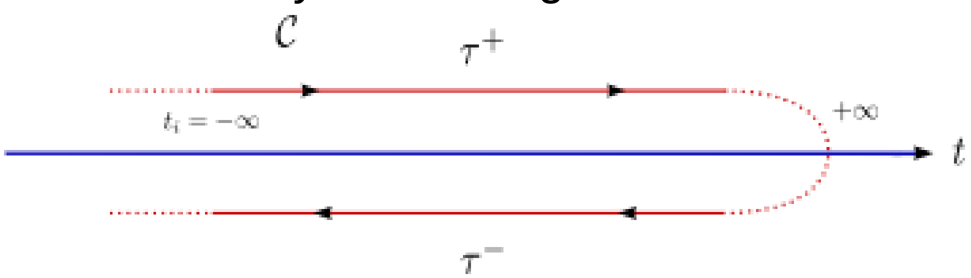
Part 2:

Sommerfeld enhanced annihilation and bound-state decay at finite temperature

Introduction to thermal field theory

- Computation of thermally averaged expectation values (in-in formalism).
- Flattening of the time contour not possible (as in usual vacuum field theory).
- LSZ doesnt work, cross section formally does not exist.

Keldysh-Schwinger Contour



$$G_O(x,y) \equiv \langle T_{\mathcal{C}} O(x) O^\dagger(y) \rangle$$
$$\langle ... \rangle \equiv \mathrm{Tr}[\rho ...]$$

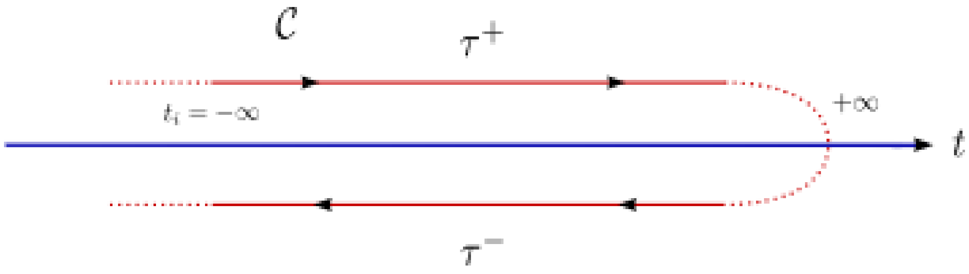
$$G(x,y) \hat{=} \begin{pmatrix} G^{++}(x,y) & G^{+-}(x,y) \\ G^{-+}(x,y) & G^{--}(x,y) \end{pmatrix} .$$

- Information (observables) of the system are contained in G.
- EoM determines dynamics.

Keldysh-Schwinger formalism

$$G(x,y) \equiv \langle T_{\mathcal{C}}\psi(x)\psi^\dagger(y)\rangle = \theta_{\mathcal{C}}(x^0,y^0)\langle\psi(x)\psi^\dagger(y)\rangle \mp \theta_{\mathcal{C}}(y^0,x^0)\left\langle\psi^\dagger(y)\psi(x)\right\rangle \hat{=} \begin{pmatrix} G^{++}(x,y) & G^{+-}(x,y) \\ G^{-+}(x,y) & G^{--}(x,y) \end{pmatrix}.$$

The components are defined as



$$G^{-+}(x,y) \equiv \left\langle\psi(x)\psi^\dagger(y)\right\rangle,$$
$$G^{+-}(x,y) \equiv \mp\left\langle\psi^\dagger(y)\psi(x)\right\rangle,$$
$$G^{++}(x,y) \equiv \theta(x^0-y^0)G^{-+}(x,y) + \theta(y^0-x^0)G^{+-}(x,y),$$
$$G^{--}(x,y) \equiv \theta(x^0-y^0)G^{+-}(x,y) + \theta(y^0-x^0)G^{-+}(x,y),$$

Not all components are independent:

$$G^{++}(x,y) + G^{--}(x,y) = G^{+-}(x,y) + G^{-+}(x,y).$$

Equilibrium and KMS relation

$$G^R(x,y) \equiv \theta(x^0 - y^0)\left[G^{-+}(x,y) - G^{+-}(x,y)\right]$$
$$G^A(x,y) \equiv -\theta(y^0 - x^0)\left[G^{-+}(x,y) - G^{+-}(x,y)\right]$$
$$G^\rho(x,y) \equiv G^R(x,y) - G^A(x,y) = G^{-+}(x,y) - G^{+-}(x,y).$$

$\hat{\rho} \propto e^{-\beta H}$ →

$$G^{-+}(x^0 - y^0) = \mp G^{+-}(x^0 - y^0 + i\beta)$$
$$G^{-+}(\omega, \mathbf{p}) = \mp e^{\beta\omega} G^{+-}(\omega, \mathbf{p})$$

Kubo-Martin-Schwinger (KMS) relations

In equilibrium, all we have to compute is the retarded correlation function:

$$G^{+-}(\omega, \mathbf{p}) = \mp\, n_{\mathrm{F/B}}(\omega) G^\rho(\omega, \mathbf{p}), \quad G^{-+}(\omega, \mathbf{p}) = \left[1 \mp n_{\mathrm{F/B}}(\omega)\right] G^\rho(\omega, \mathbf{p}),$$
$$G^{++}(\omega, p) = \frac{G^R(\omega, \mathbf{p}) + G^A(\omega, \mathbf{p})}{2} + \left[\frac{1}{2} \mp n_{\mathrm{F/B}}(\omega)\right] G^\rho(\omega, \mathbf{p}).$$

Dynamics from Equation of motion

Consider the Dyson eq. in **integral** form:

$$G(x,y) = G_0(x,y) - \int_{w,z\in\mathcal{C}} G_0(x,w)\Sigma(w,z)G(z,y)$$

Kadanoff-Baym Ansatz (motivated from KMS condition):

$$G^{+-}(t,p) = -\rho(t,p)\left[\theta(p^0)f_\chi(t,p^0) + \theta(-p^0)(1 - f_{\bar{\chi}}(t,p^0))\right]$$

Kadanoff-Baym Ansatz + Dyson equation in **differential** form:

$$\partial_t f_\chi(t,\mathbf{p}) = -\frac{1}{4}\int \frac{\mathrm{d}p^0}{(2\pi)}\theta(p^0)\mathrm{Tr}\left[\Sigma^{-+}(t,p)G^{+-}(t,p) - \Sigma^{+-}(t,p)G^{-+}(t,p) + \mathrm{h.c.}\right].$$

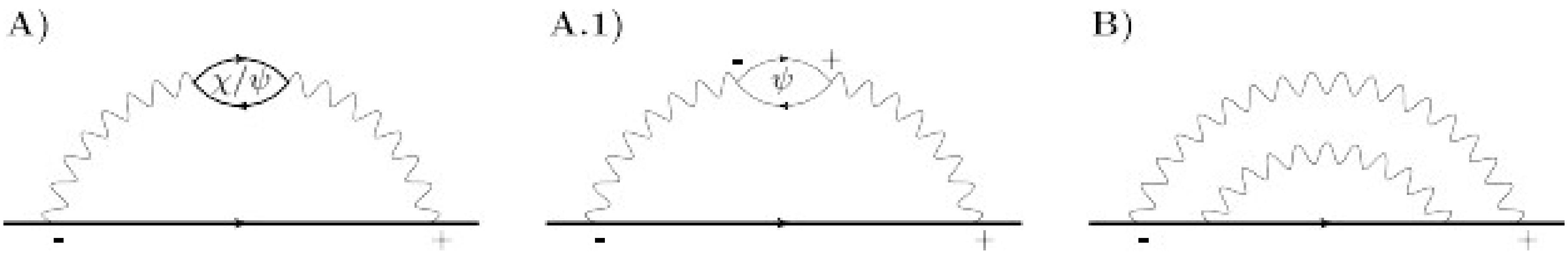
This relates the collision term to two-point correlation function!

To leading order in self-energy expansion this just gives the usual Boltzmann equation!

Kadanoff-Baym equations

$$\mathcal{L} \supset g_\chi \bar{\chi}\gamma^\mu \chi A_\mu + g_\psi \bar{\psi}\gamma^\mu \psi A_\mu$$

At leading order in self-energy expansion:



$$\frac{1}{2}\int \frac{d^4 p_\chi}{(2\pi)^4}\theta(p_\chi^0)\mathrm{Tr}\left[\Sigma^{-+}(t,p_\chi)G_0^{+-}(t,p_\chi) + \mathrm{h.c.}\right]\Bigg|_{\chi\bar{\chi}\to\psi\bar{\psi}}$$

$$= \int \frac{d^4 p_\chi}{(2\pi)^4 2p_\chi^0}\frac{d^4 p_{\bar{\chi}}}{(2\pi)^4 2p_{\bar{\chi}}^0}\frac{d^4 k_\psi}{(2\pi)^4 2k_\psi^0}\frac{d^4 k_{\bar{\psi}}}{(2\pi)^4 2k_{\bar{\psi}}^0}(2\pi)^4\delta(p_\chi^0 - E_\chi)\delta(p_{\bar{\chi}}^0 - E_{\bar{\chi}})\delta(k_\psi^0 - E_\psi)\delta(k_{\bar{\psi}}^0 - E_{\bar{\psi}})$$

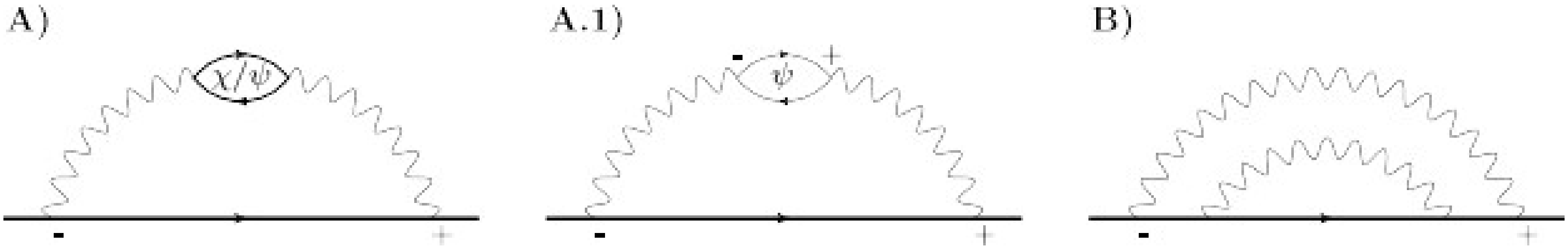
$$\times (2\pi)^4\delta^4(p_\chi + p_{\bar{\chi}} - k_\psi - k_{\bar{\psi}})\left[f_\chi(t,E_\chi)f_{\bar{\chi}}(t,E_{\bar{\chi}})(1-f_\psi(t,E_\psi))(1-f_{\bar{\psi}}(t,E_{\bar{\psi}}))\right]$$

$$\times g_\chi^2 g_\psi^2 \mathrm{Tr}\left[(p_\chi + m_\chi)\gamma^\mu(p_{\bar{\chi}} - m_\chi)\gamma^\nu\right]\frac{g_{\mu\alpha}g_{\nu\beta}}{(p_\chi + p_{\bar{\chi}})^4}\mathrm{Tr}\left[(k_\psi + m_\psi)\gamma^\alpha(k_{\bar{\psi}} - m_\psi)\gamma^\beta\right].$$

Kadanoff-Baym equations

$$\mathcal{L} \supset g_\chi \bar{\chi} \gamma^\mu \chi A_\mu + g_\psi \bar{\psi} \gamma^\mu \psi A_\mu$$

At leading order in self-energy expansion:



In DM dilute limit:

$$\begin{aligned} \dot{n} + 3Hn &= -4 \int \frac{\mathrm{d}^3 \mathbf{p}_\chi}{(2\pi)^3} \frac{\mathrm{d}^3 \mathbf{p}_{\bar{\chi}}}{(2\pi)^3} \frac{(p_\chi p_{\bar{\chi}})}{E_\chi E_{\bar{\chi}}} (\sigma v_{\mathrm{rel}}) \left[f_\chi f_{\bar{\chi}} - f_\chi^{\mathrm{eq}} f_{\bar{\chi}}^{\mathrm{eq}} \right] \\ &= -\langle \sigma v_{\mathrm{rel}} \rangle \left[n^2 - n_{\mathrm{eq}}^2 \right] \end{aligned}$$

Lee-Weinberg equation!

Outlook

- Perturbative expansion of self-energy in Kadanoff-Baym equations can not account for bound-states.
- Idea: Truncate correlation function hierarchy at the 4-point function level.
- Solution of 4-point correlator allows to include resummation of the Coulomb ladders.
- Instead of working with free photon correlator, we take HTL dressed.
  Debye mass, Landau damping, etc.

Summary

- Long-range effects allow for larger DM masses (SE + Bound-state decay)
- Existing literature computes the relic abundance including these effects in “vacuum”.
- Bound-states have a finite size, we expect that in-medium effects can modify bound-state properties.
- A dynamical formulation of SE annihilation and bound-state decay in plasma did not exist in non-equilibrium quantum statistical mechanics (to the best of my knowledge).
- In next talk, we address this gap.

What happens if...