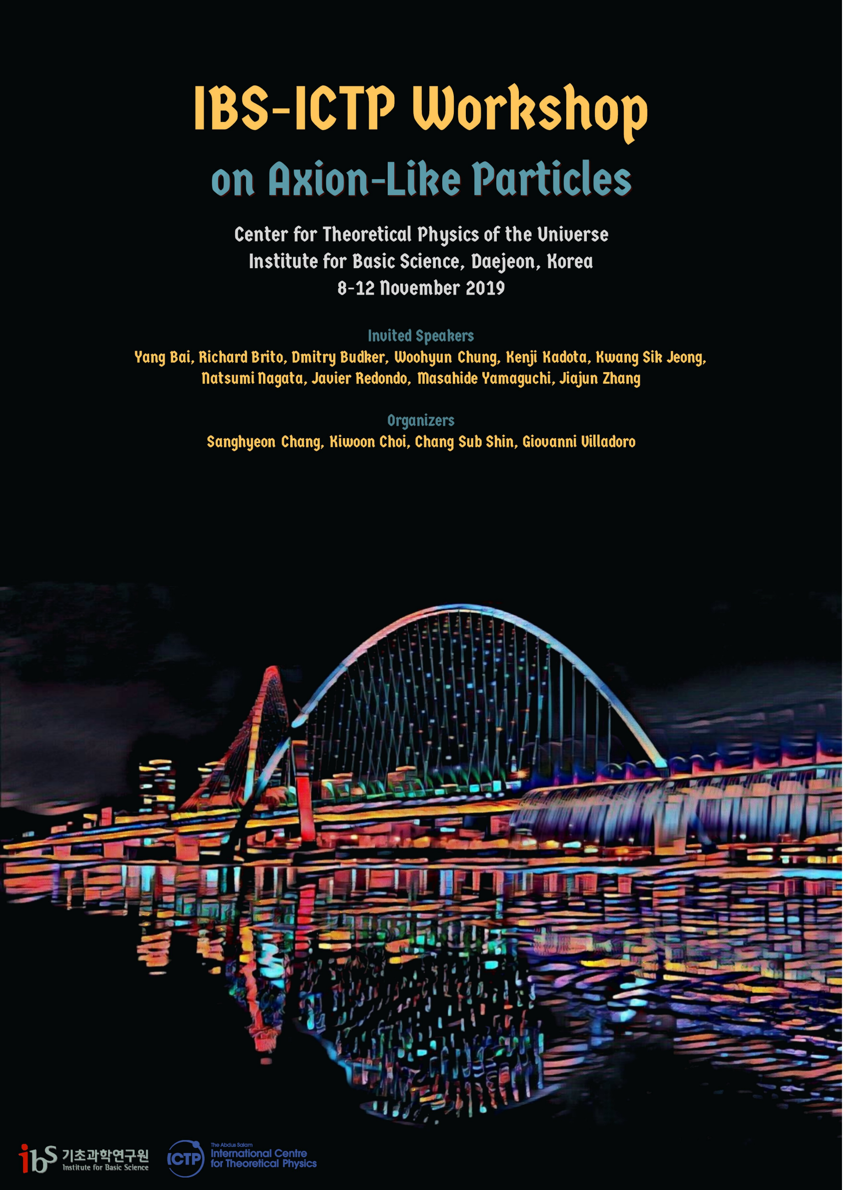

Axions from Strings

Giovanni Villadoro

The Abdus Salam International Centre for Theoretical Physics

Based on 1806.04677 (+ work in progress)
with M. Gorghetto and E. Hardy

QCD Axion

CP violation in SM:

$$\det[y_u y_u^\dagger, y_d y_d^\dagger] \qquad \Rightarrow \qquad \delta$$

$$\det(y_u y_d) e^{i\theta_0} \qquad \Rightarrow \qquad \theta$$

$$\mathcal{L}_{\rm top} = \frac{\theta_0}{32\pi^2} G\widetilde{G}$$

CP violation in SM:

$$\det[y_u y_u^\dagger, y_d y_d^\dagger] \quad \Rightarrow \quad \delta \approx 1.2$$

$$\det(y_u y_d) e^{i\theta_0} \quad \Rightarrow \quad \theta \lesssim 10^{-10}$$

$$\mathcal{L}_{\rm top} = \frac{\theta_0}{32\pi^2} G\widetilde{G}$$

Strong CP Problem!

The QCD Axion Solution

Peccei Quinn '77
Weinberg, Wilczek '78
(KSVZ, DSFZ...)

$$\theta \to \frac{a(x)}{f_a}$$

$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G}$$

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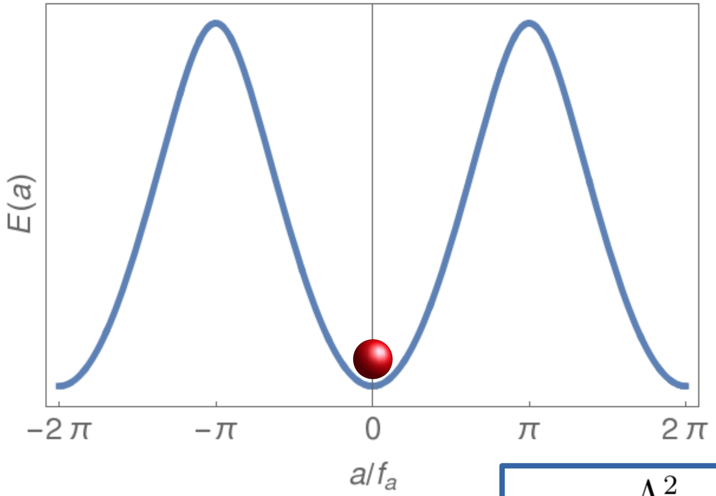


Vafa Witten '84

$$\boxed{m_a \approx \frac{\Lambda_{\rm QCD}^2}{f_a}}$$

QCD Axion Mass

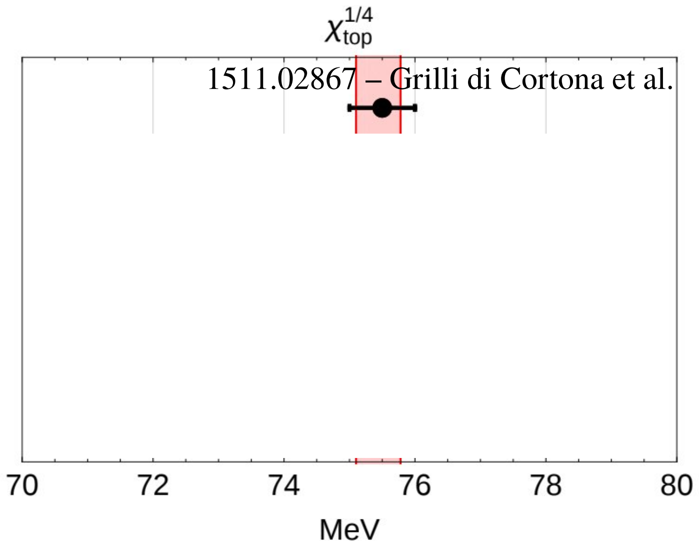
$$m_a = \frac{\chi_{\rm top}^{1/2}}{f_a} = \frac{\sqrt{m_u m_d}}{m_u + m_d}\frac{m_\pi f_\pi}{f_a}$$

Weinberg '78

$$\boxed{V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2}\sin^2\left(\frac{a}{2f_a}\right)}}$$

Di Vecchia Veneziano '80

$$m_a = \frac{\chi_{\rm top}^{1/2}}{f_a}$$



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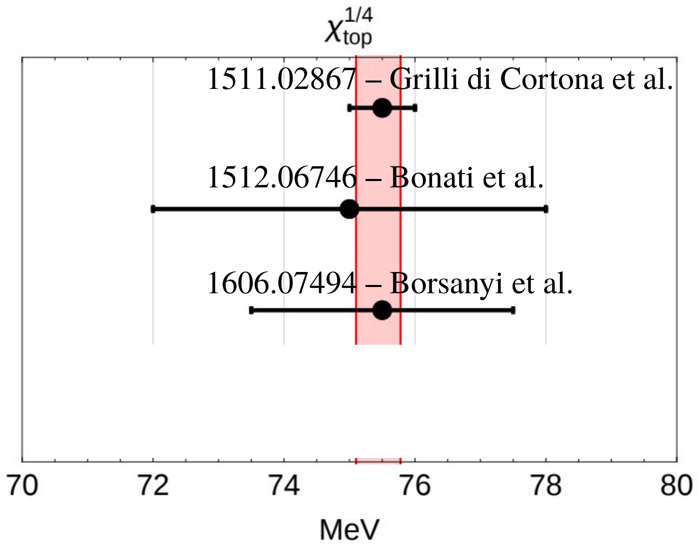

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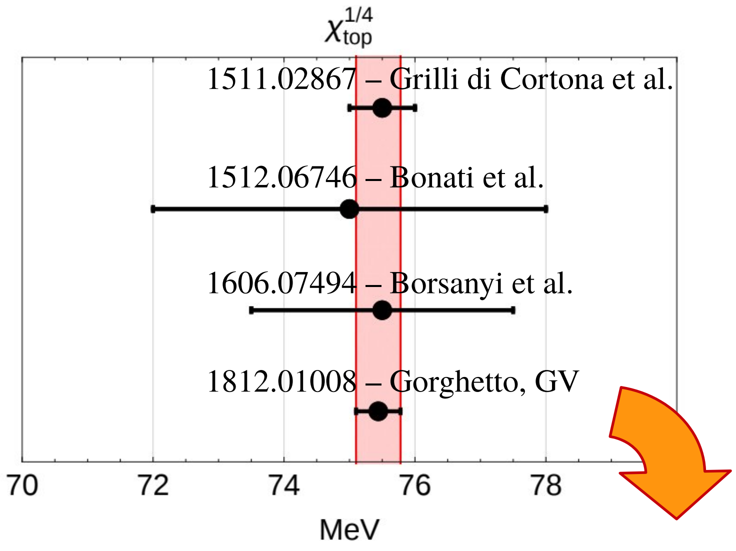
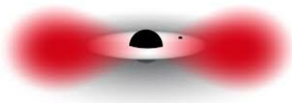
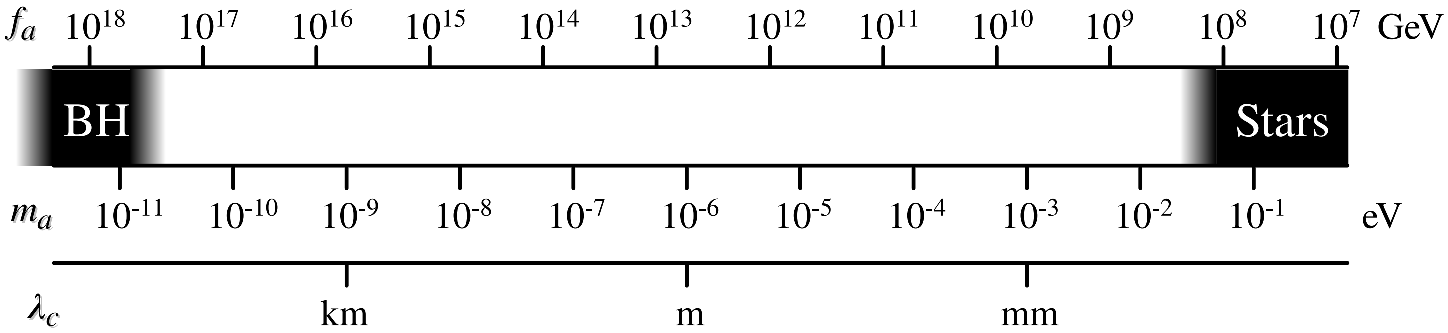


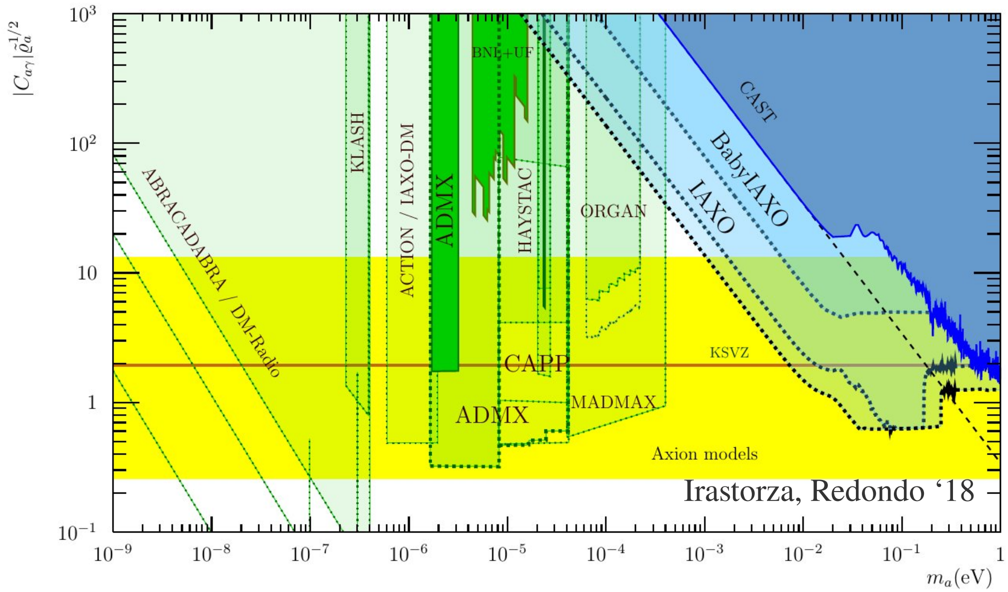
$$m_a = \Big[\underbrace{5.815(22)_z(04)_{f_\pi}}_{\boxed{\rm LO}} \underbrace{-0.121(38)_{\ell_i^r}}_{\boxed{\rm NLO}} \underbrace{-0.022(07)_{\ell_i^r}(05)_{c_i^r}}_{\boxed{\rm NNLO}} \underbrace{+0.019(06)_{k_i^r}}_{\boxed{\rm EM}}\Big]\ \mu\text{eV}\ \frac{10^{12}\ \text{GeV}}{f_a}$$

$$m_a = 5.691(51)\ \mu\text{eV}\ \frac{10^{12}\ \text{GeV}}{f_a} \qquad \chi_{\rm top}^{1/4} = 75.44(34)\ \text{MeV}$$

Axions Searches



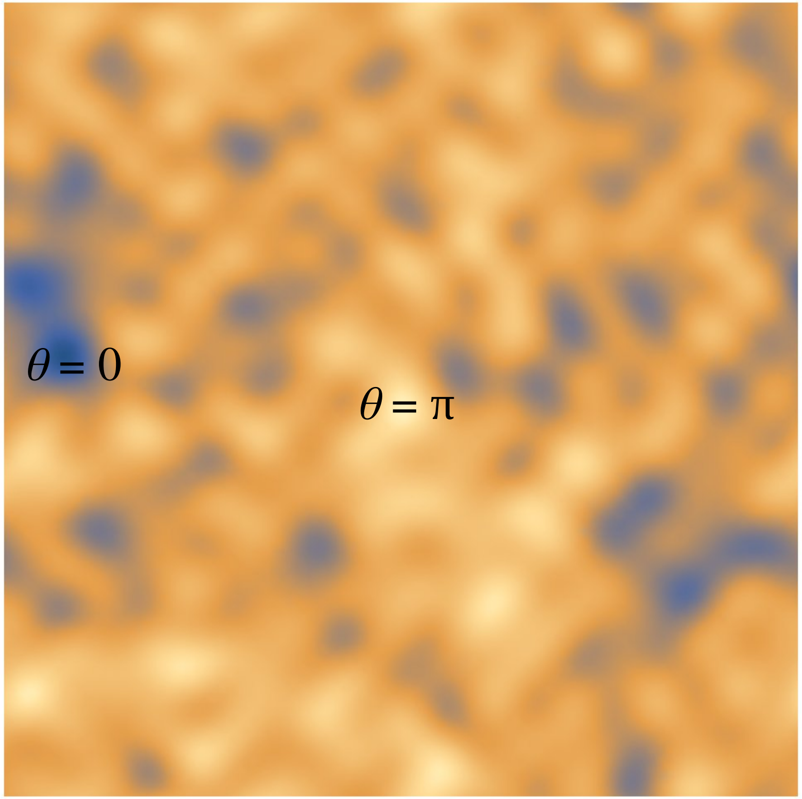




Axion DM

Abbott, Dine, Fischler,
Preskill, Sikivie,Wise,
Wilczek ‘83

Cosmic Evolution of Axions



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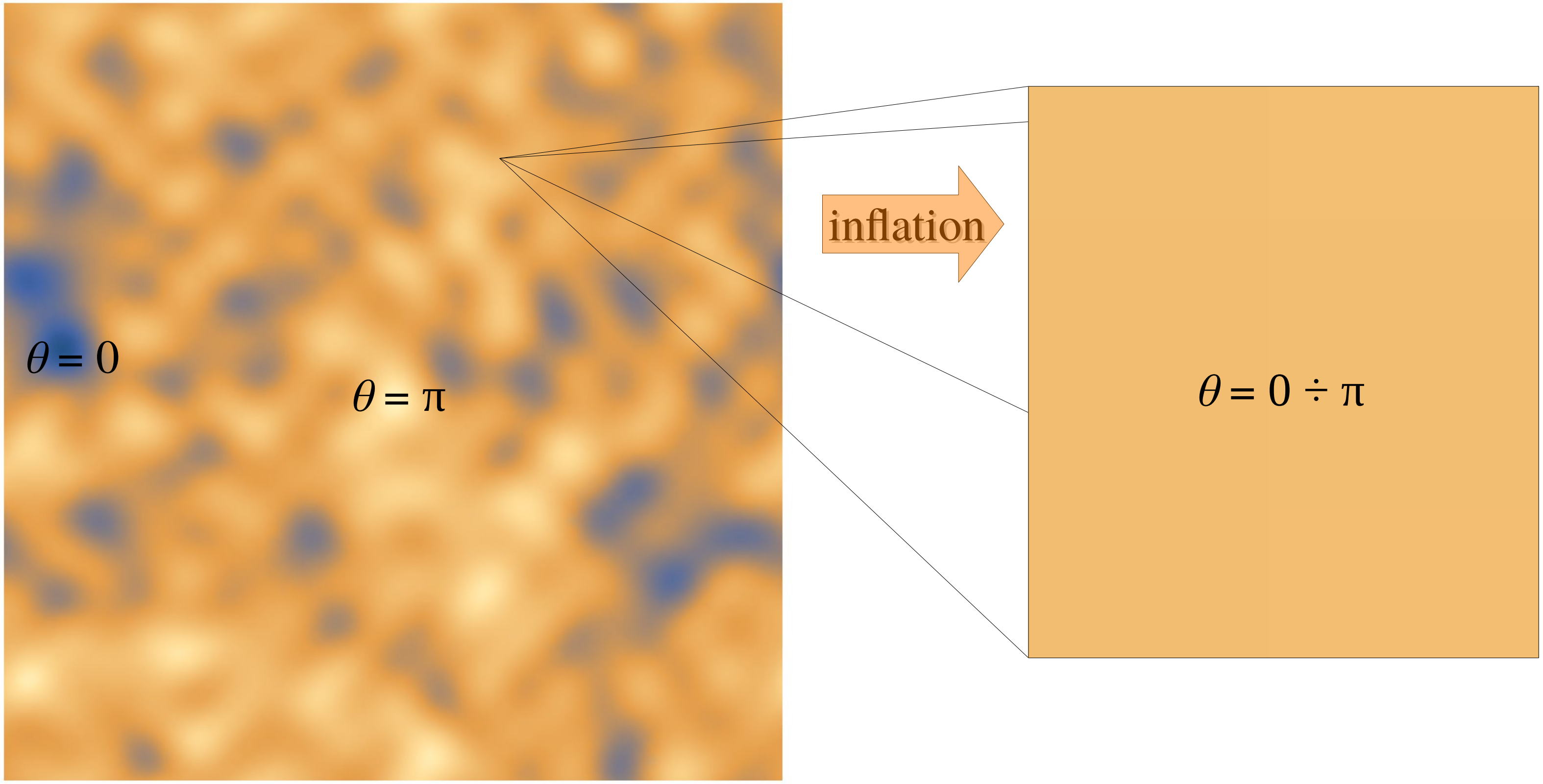

Scenario #1: $(T, H) < f_a$

$\theta = 0 \div \pi$

$$a(t_0) = \text{const} \equiv \theta_0 f_a$$

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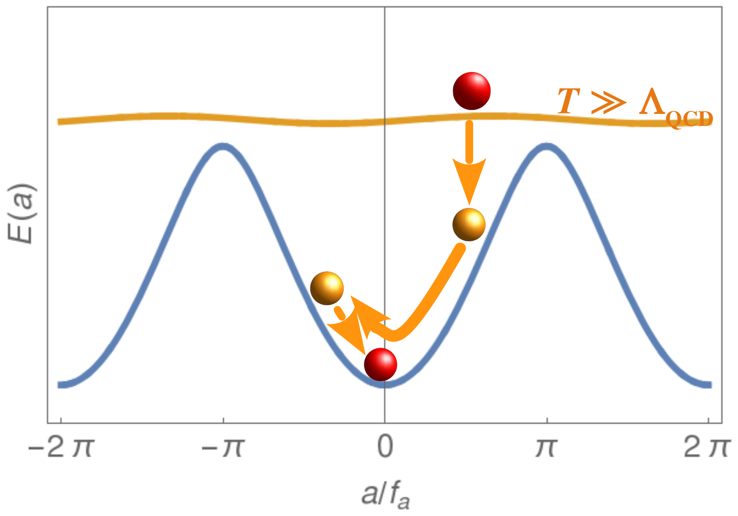
$$\ddot{a} + 3H\dot{a} + m_a^2 a = 0$$



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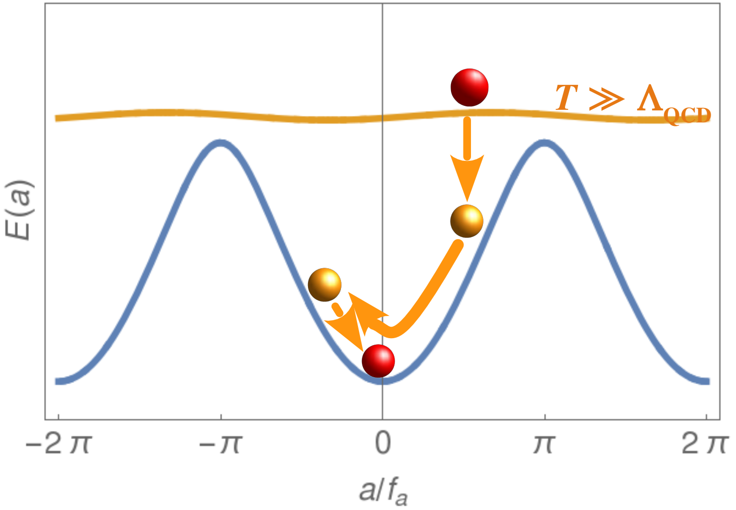

$$\Omega_a \approx 0.1\theta_0^2 \left[\frac{f_a}{10^{12}\mathrm{GeV}}\right]^{1+\epsilon}$$

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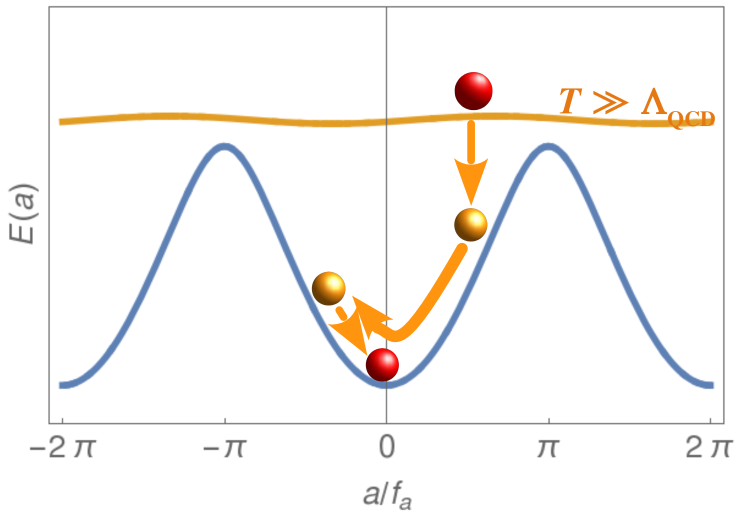
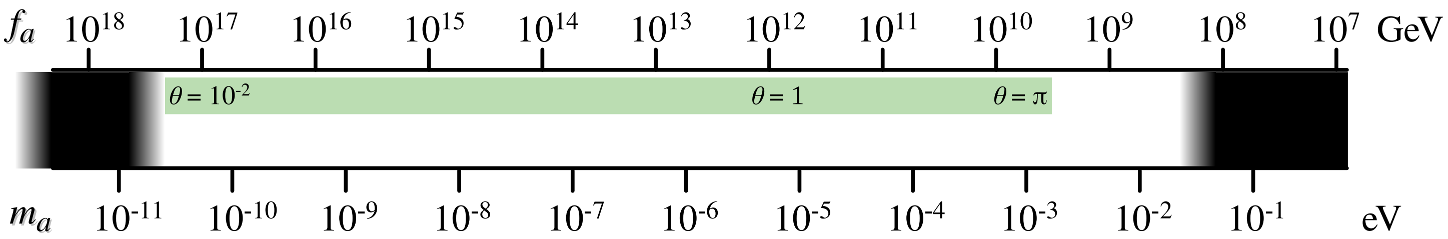


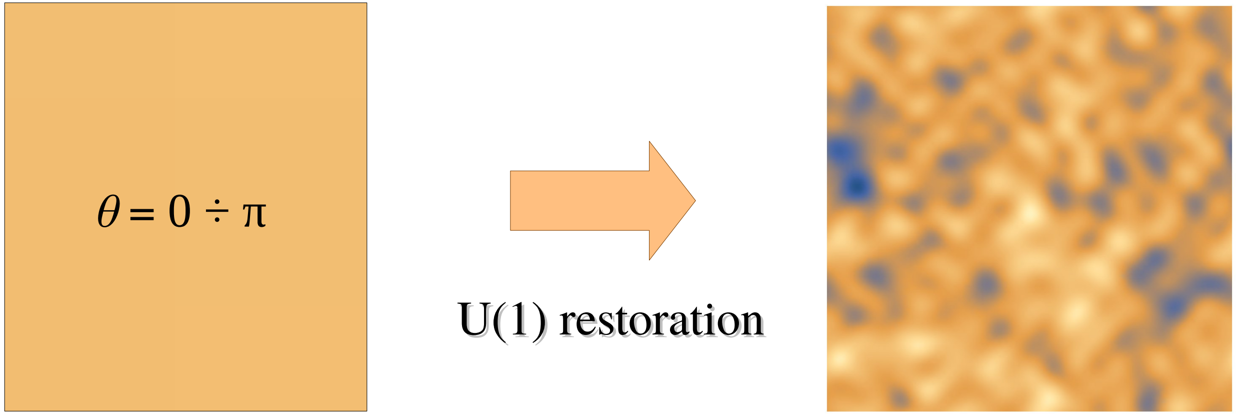
$$\Omega_a \approx 0.1\theta_0^2 \left[\frac{f_a}{10^{12}\text{GeV}}\right]^{1+\epsilon} = \Omega_{DM}$$



Scenario #2: $T\ (or\ H) > f_a$

$\theta = 0 \div \pi$

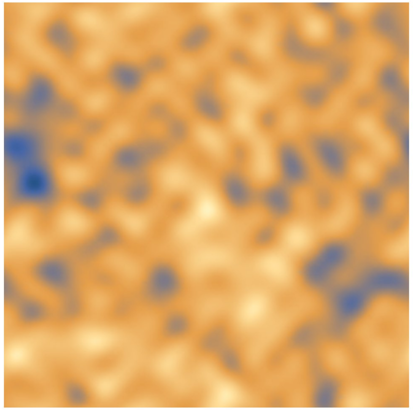
Scenario #2: T (or H) > f_a



Scenario #2: T (or H) > f_a

$\theta = 0 \div \pi$

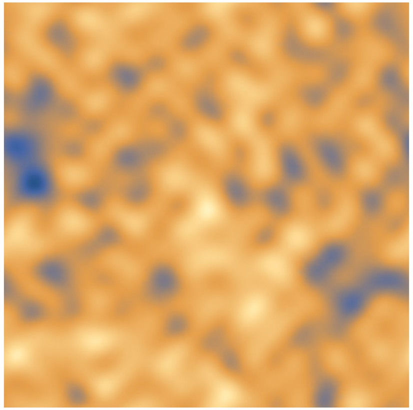
U(1) restoration



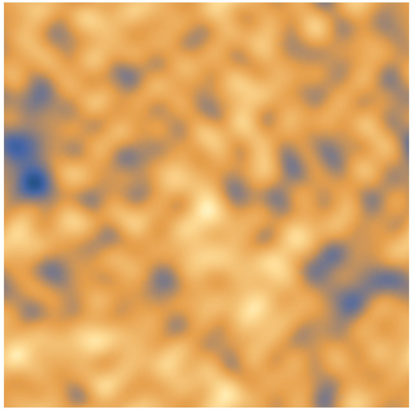
No initial θ dependence

$$\Omega_a = \Omega_{DM}$$

Prediction for m_a !



Complex non-linear evolution



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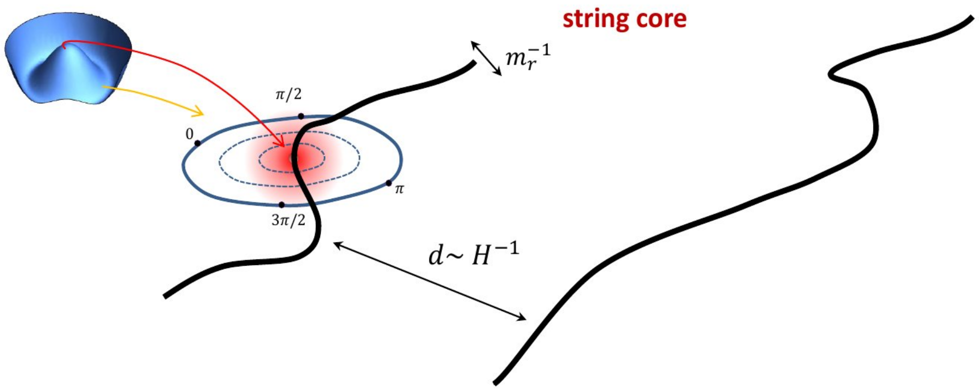

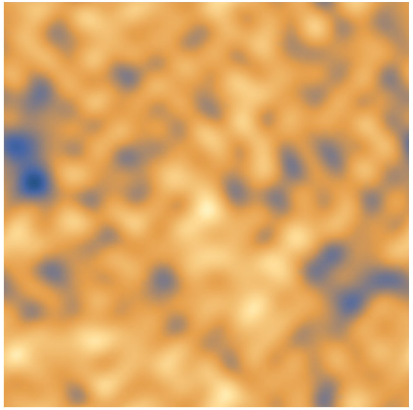

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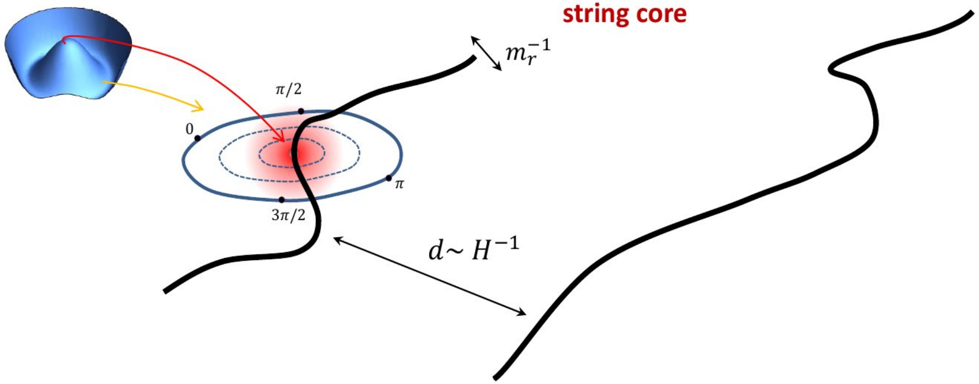
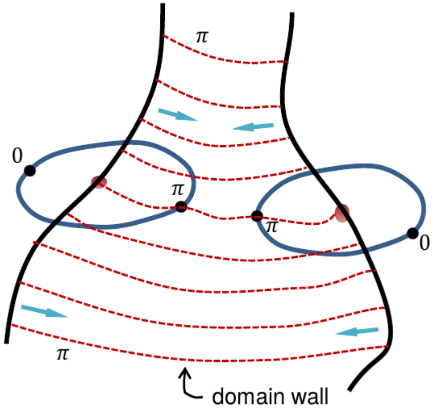
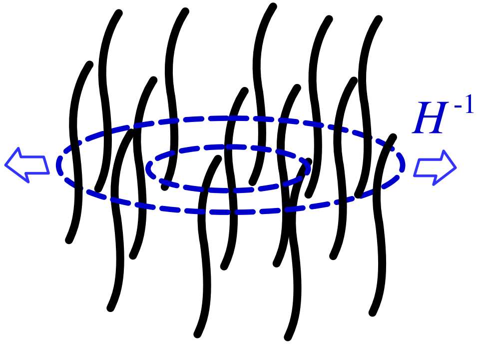


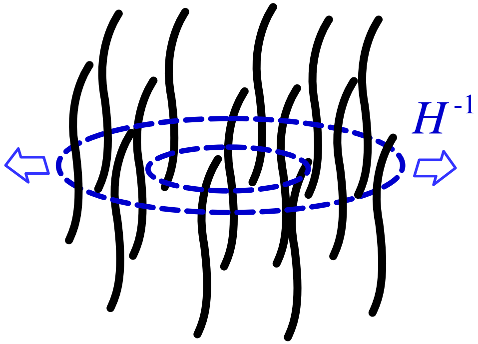


free strings



$$\rho_{\mathrm{free}} \propto \frac{1}{R^2(t)} = \frac{1}{t}$$

free strings



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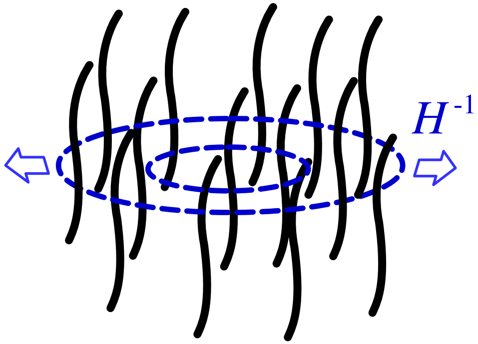

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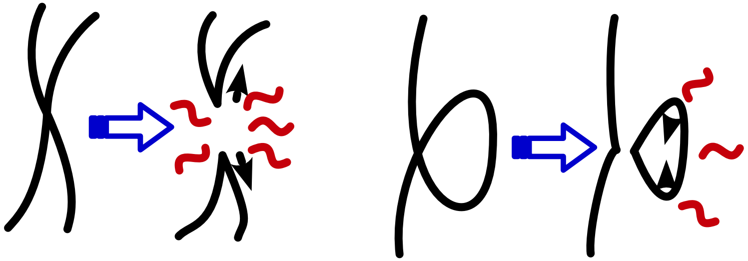
string recombination



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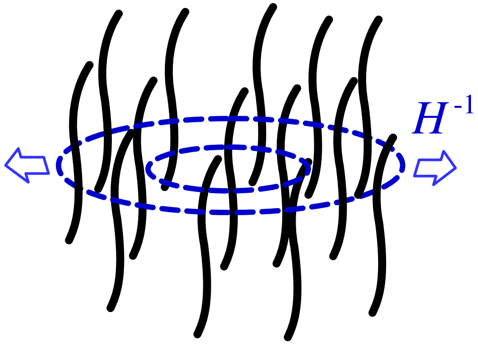

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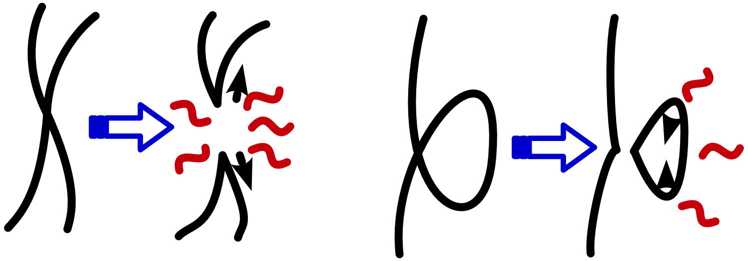
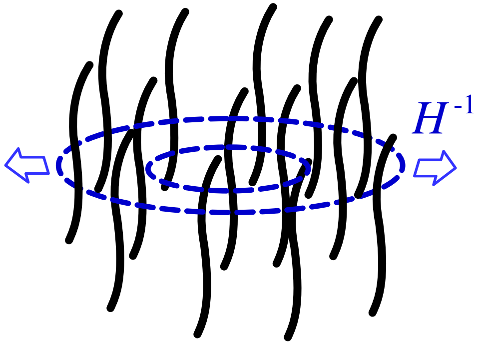


scaling solution

$$\rho_s = \xi \frac{\mu}{t^2}$$

ξ = (# strings) / (Hubble Patch)

free strings



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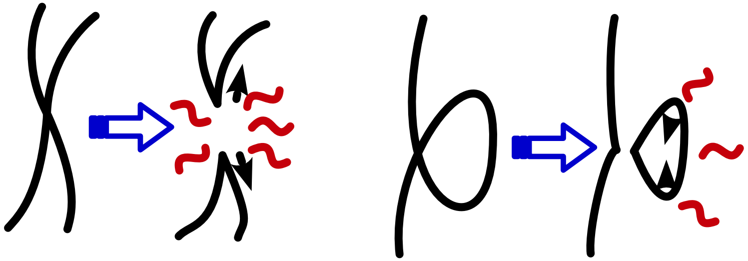

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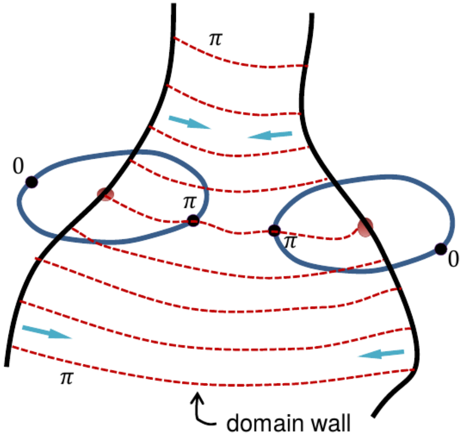
difference goes in radiation

@ $H \sim m_a$ ($T \sim \Lambda_{\rm QCD}$)



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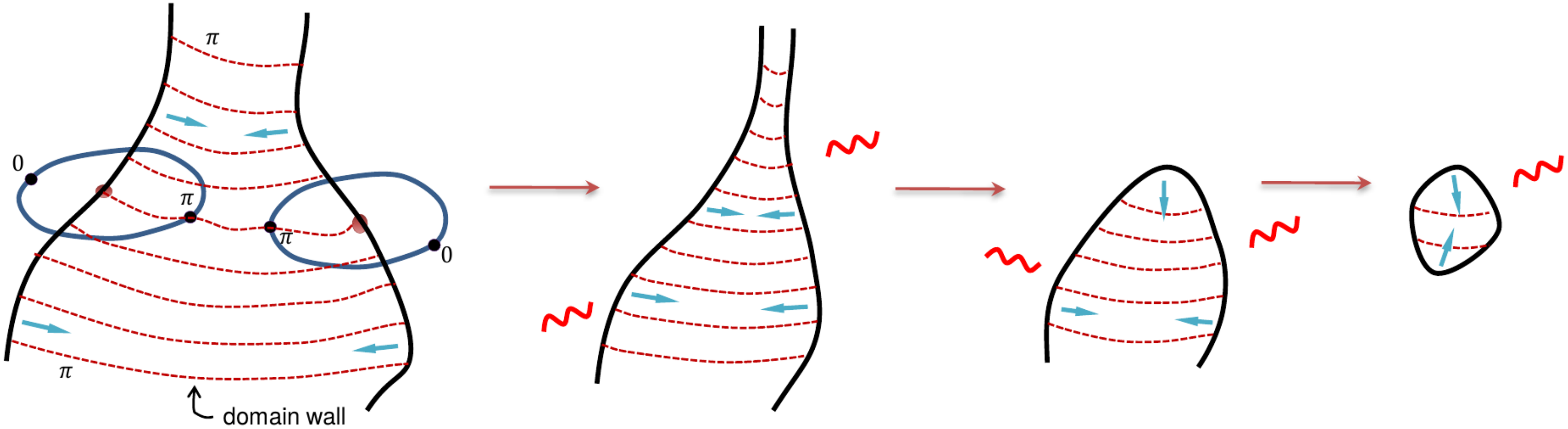

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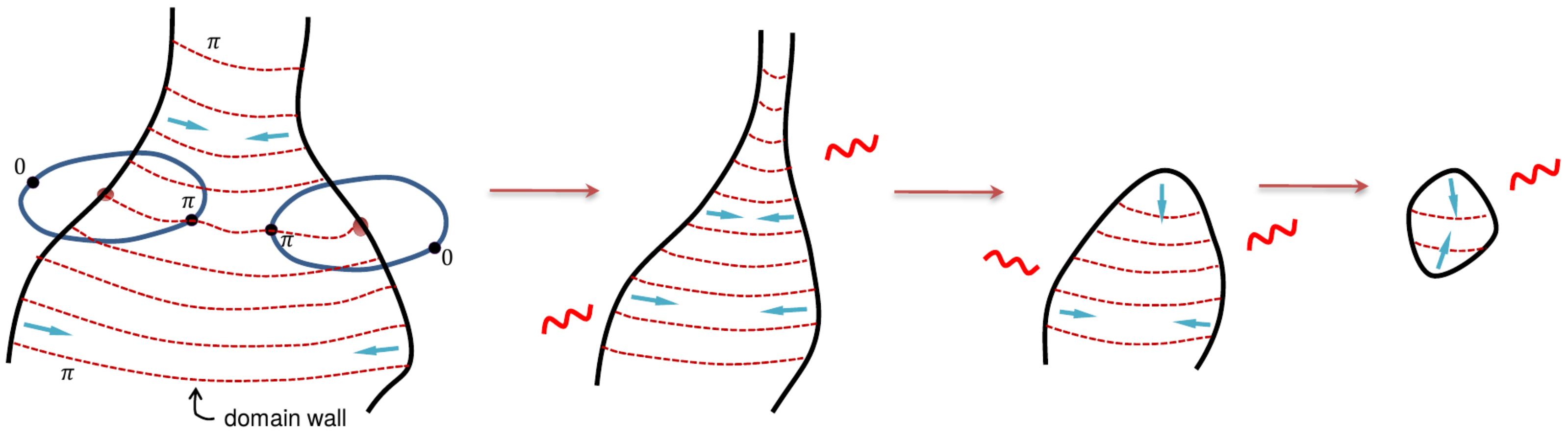
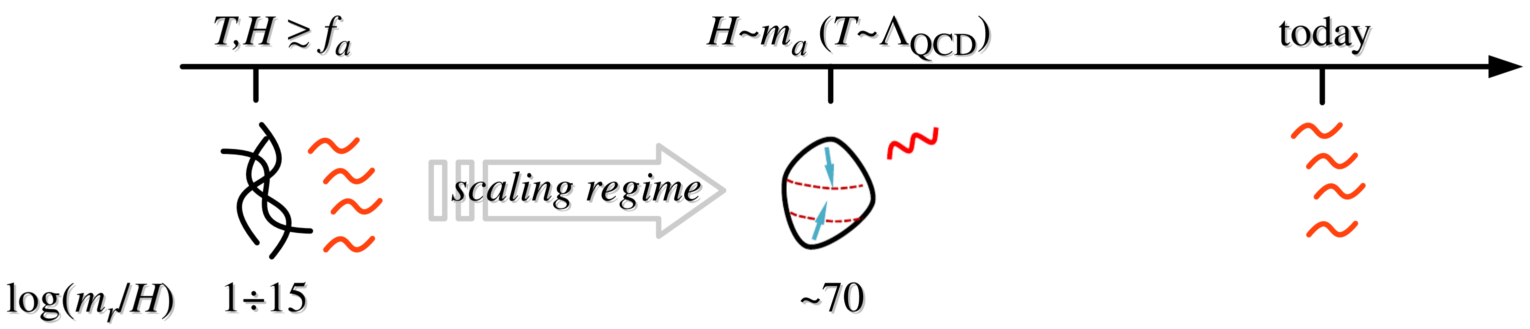
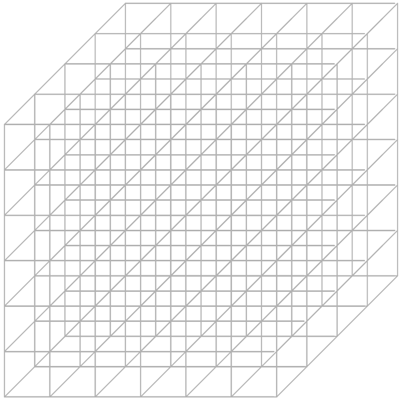




Numerical Simulation

$$V(\Phi) = \frac{\lambda}{4}\left(|\Phi|^2 - \frac{f_a^2}{2}\right)^2$$

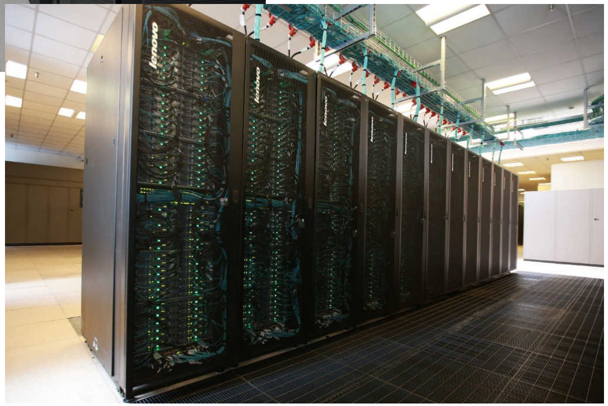
$$\Phi = \frac{f_a + r}{\sqrt{2}}\, e^{ia/f_a}$$



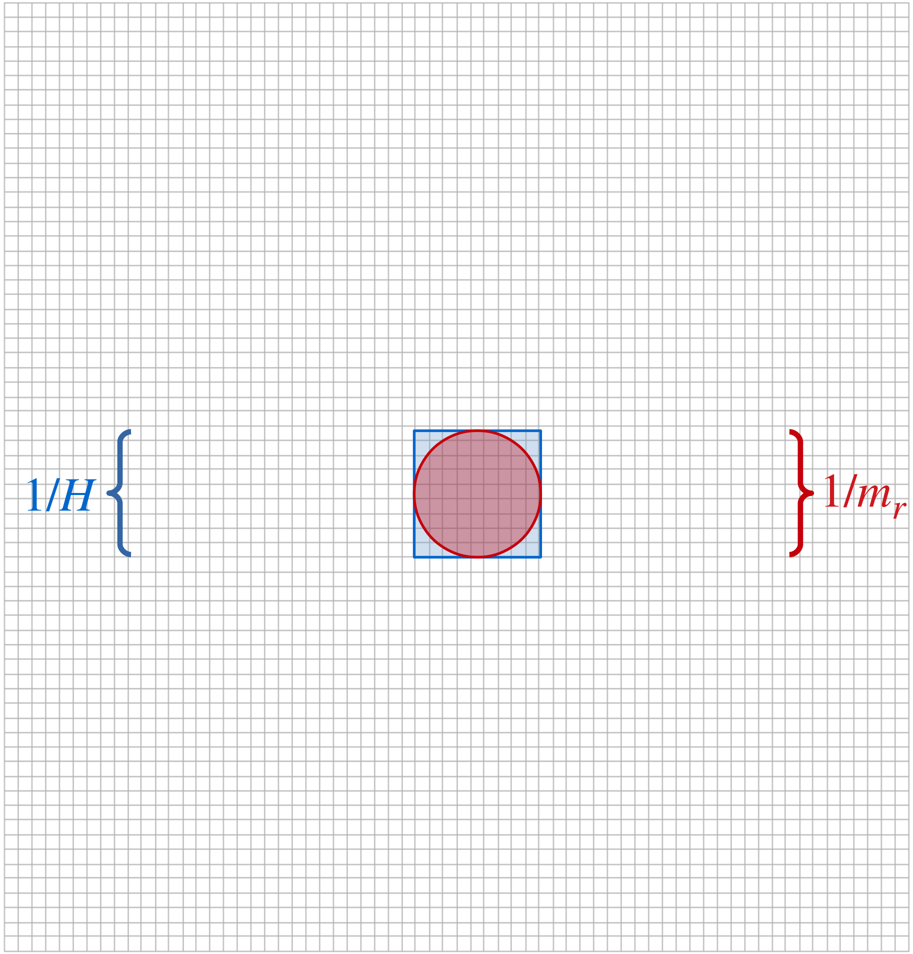
$$L \approx 1250a \;\rightarrow\; 4032a$$

ICTP/SISSA HPC

CINECA



The Bottle Neck



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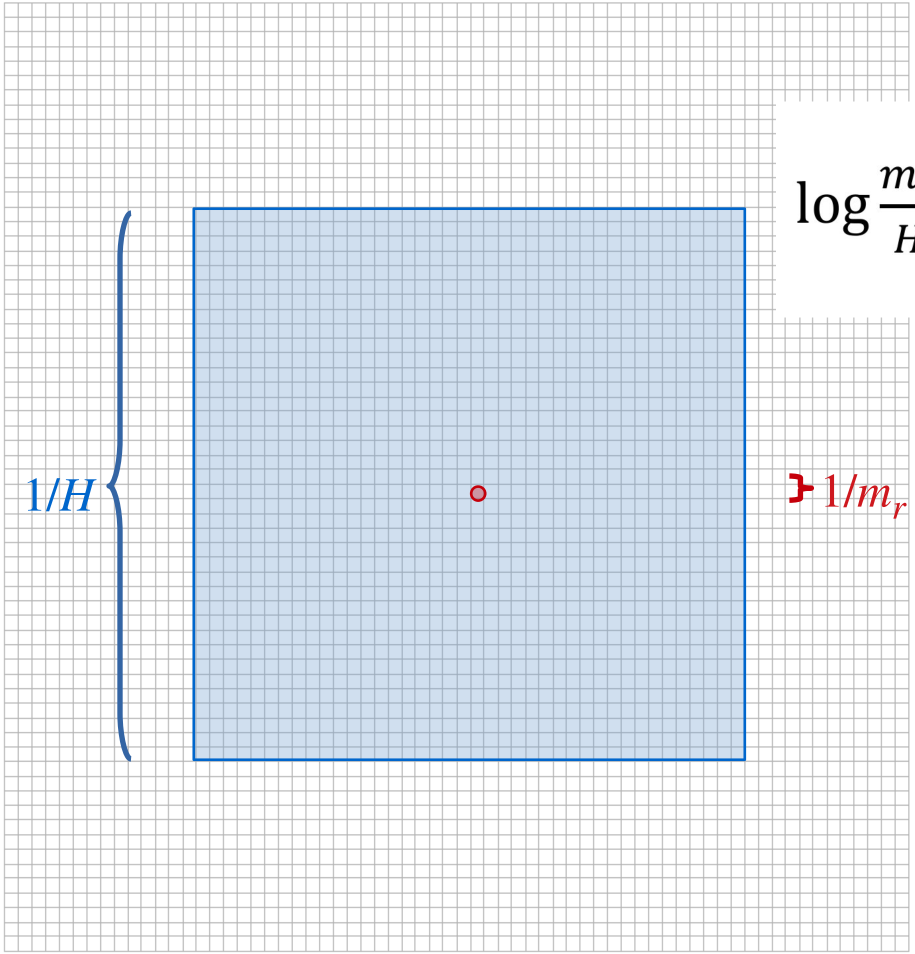
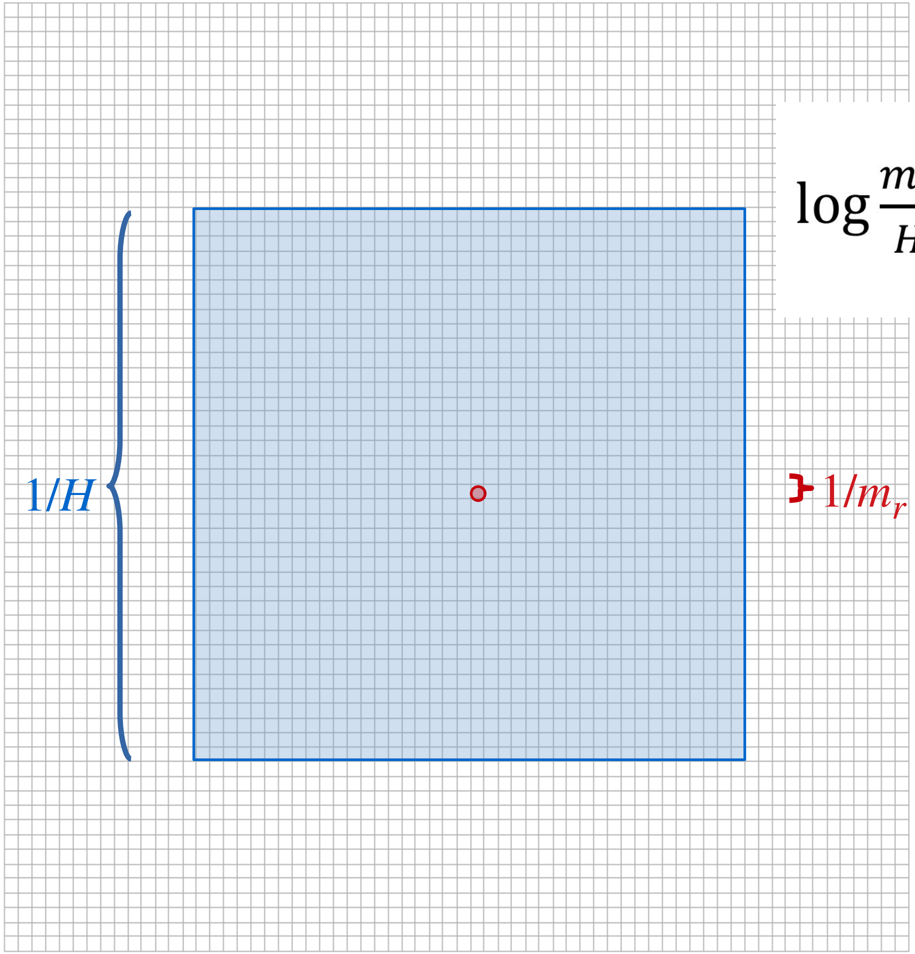


$$\log\frac{m_r}{H} \leq \log\left(\frac{\square}{\bigcirc}\right) \sim 6 \ll 70$$

The Bottle Neck

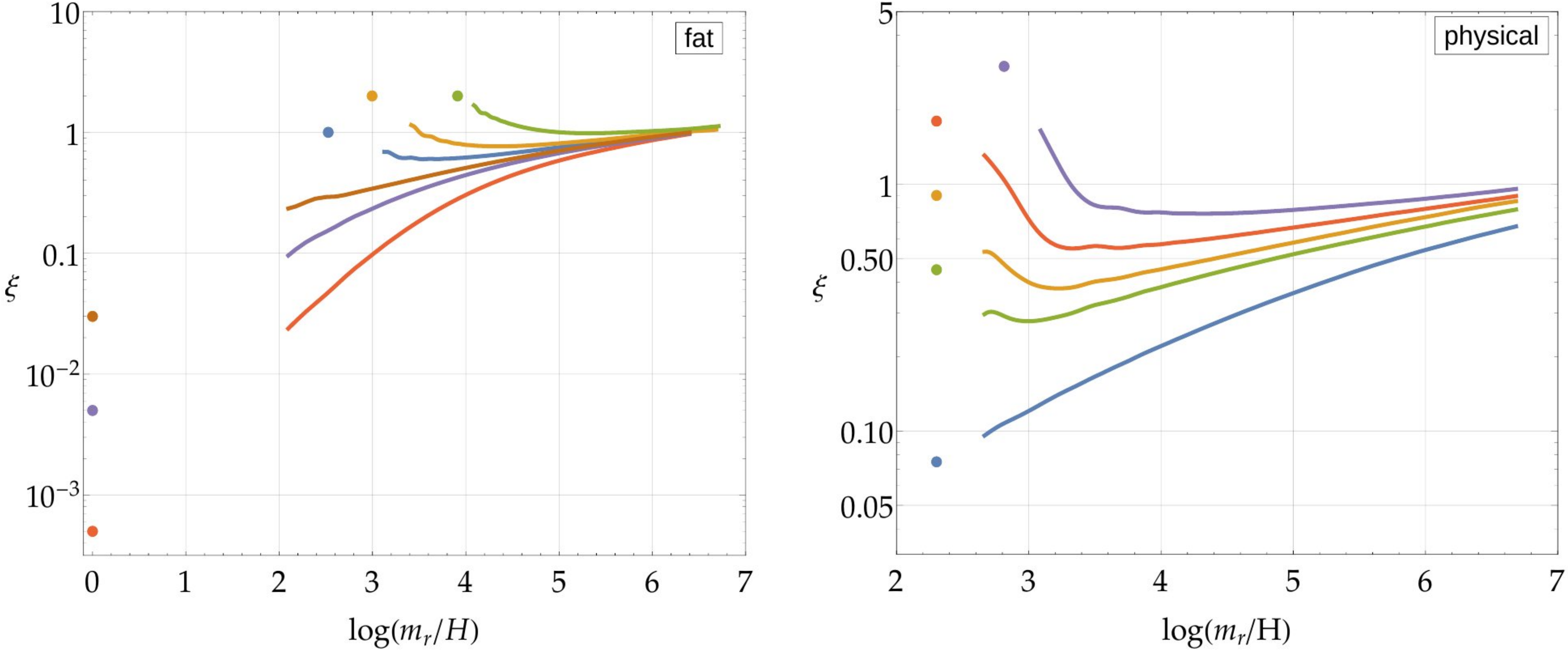


$$\log \frac{m_r}{H} \leq \log\left(\frac{\square}{\bigcirc}\right) \sim 6 \ll 70$$

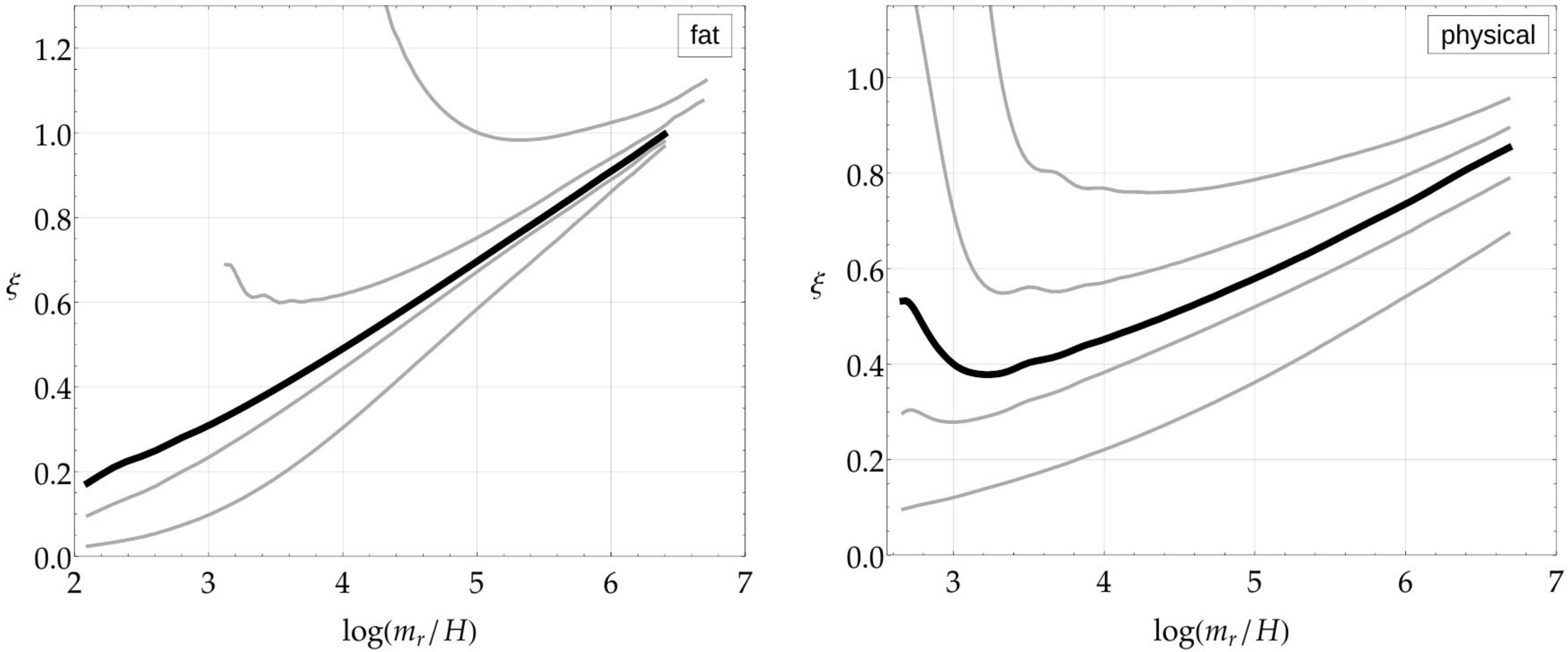
fat string trick

$1/m_r \sim R \sim t^{1/2}$

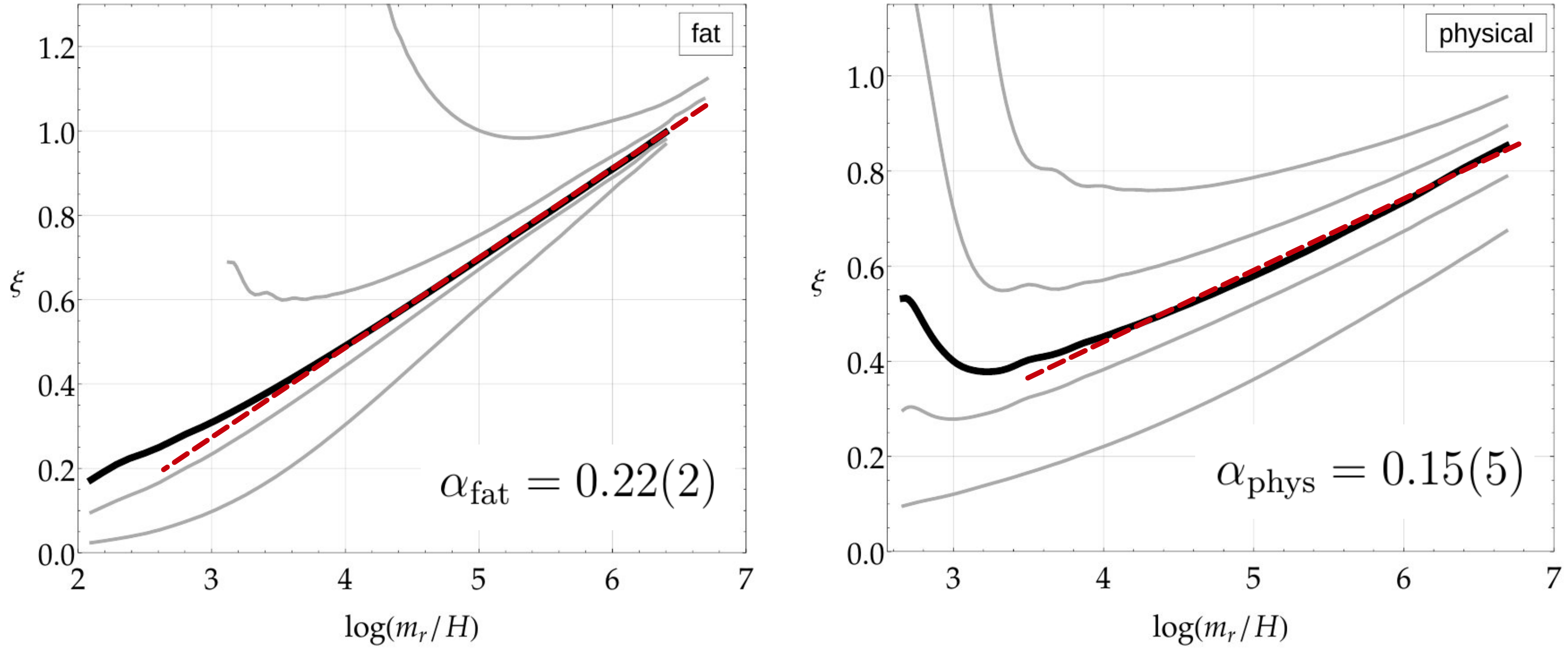
The Scaling Solution is an Attractive Solution



Scaling Violation

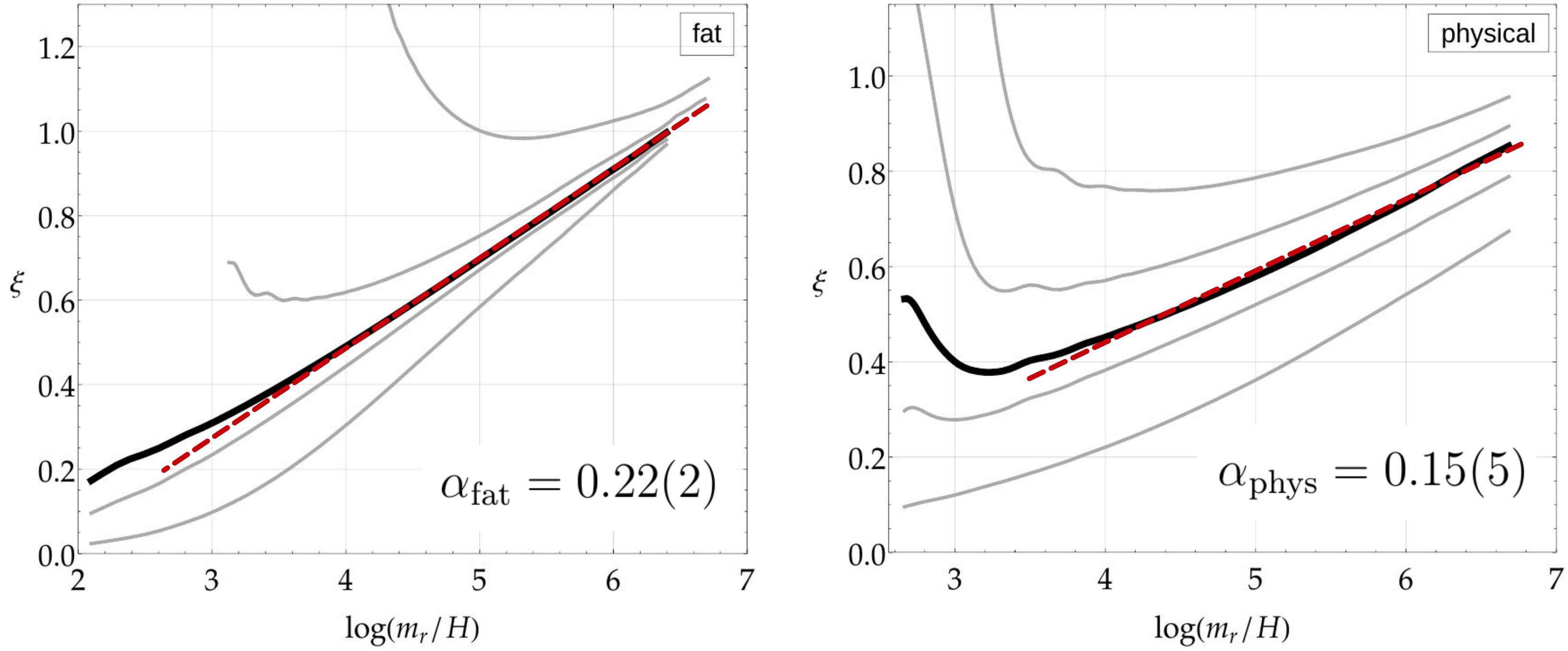


Scaling Violation



$$\xi(t) = \alpha \log\left(\frac{m_r}{H}\right) + \beta$$

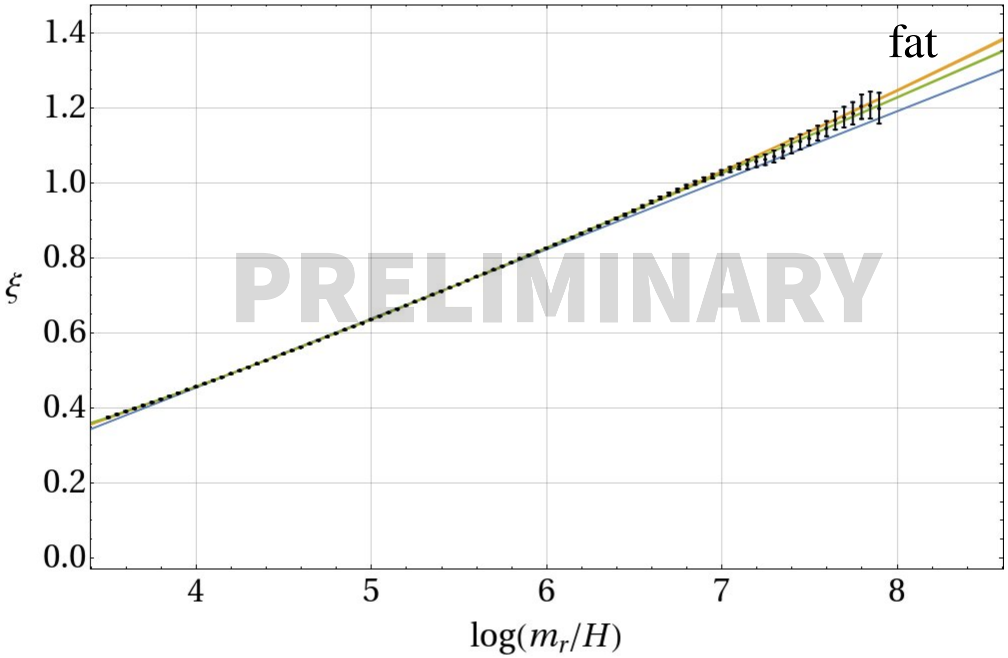
Scaling Violation

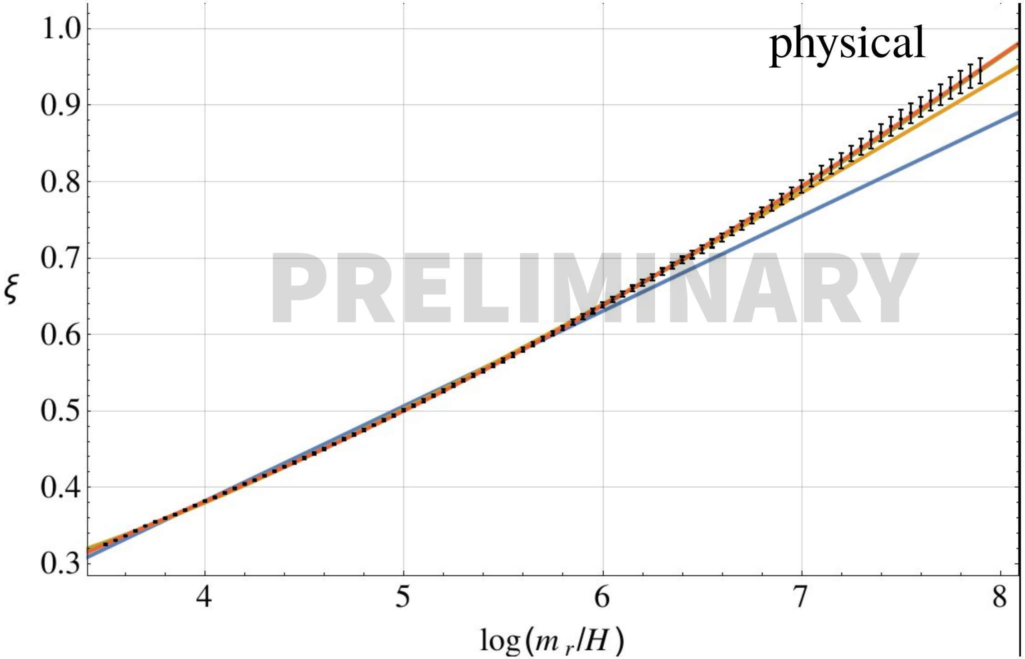


$$\xi(t) = \alpha \log\left(\frac{m_r}{H}\right) + \beta$$

$$\xi \xrightarrow{\log=70} 10$$

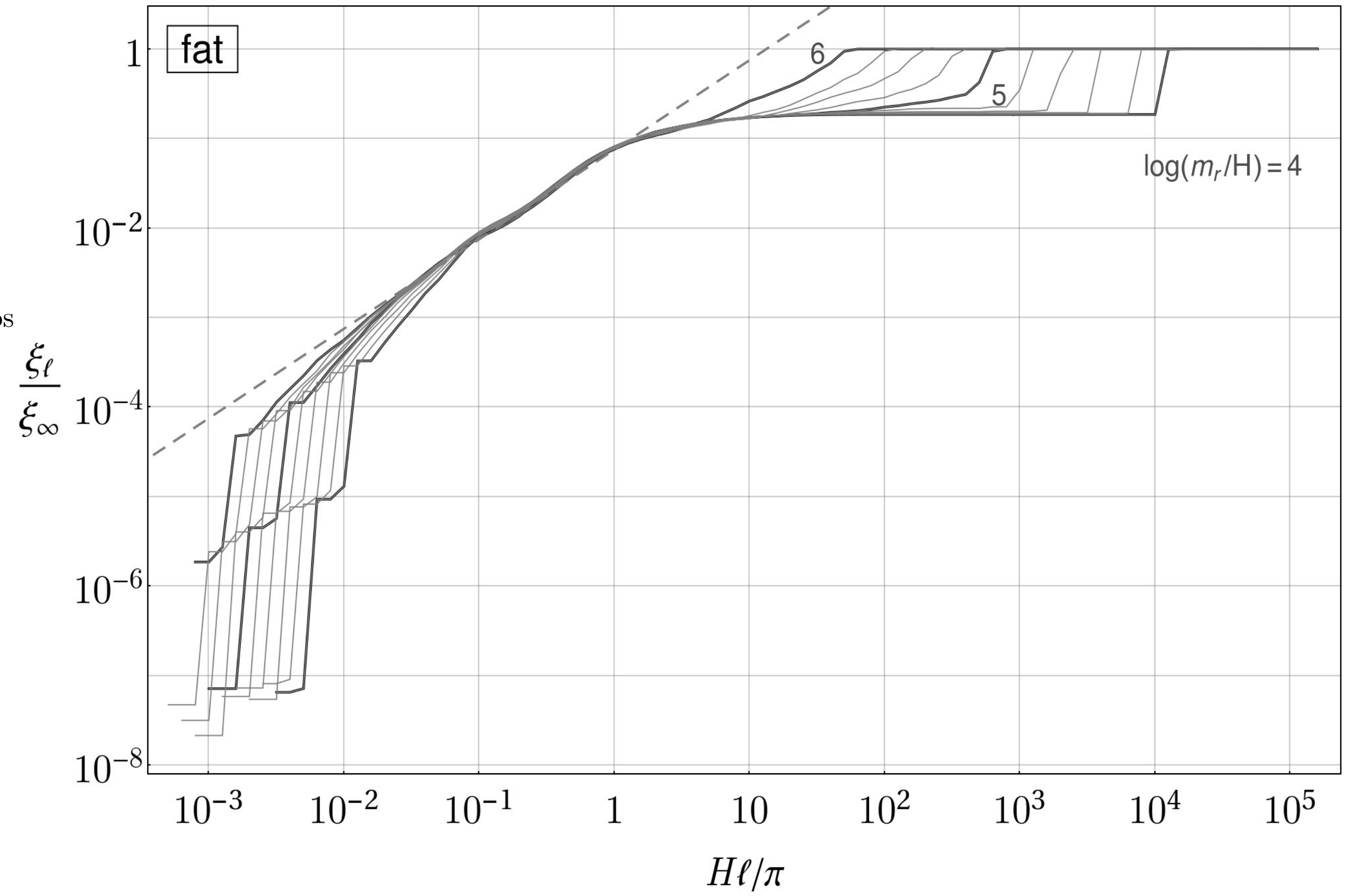
Scaling Violation @4k



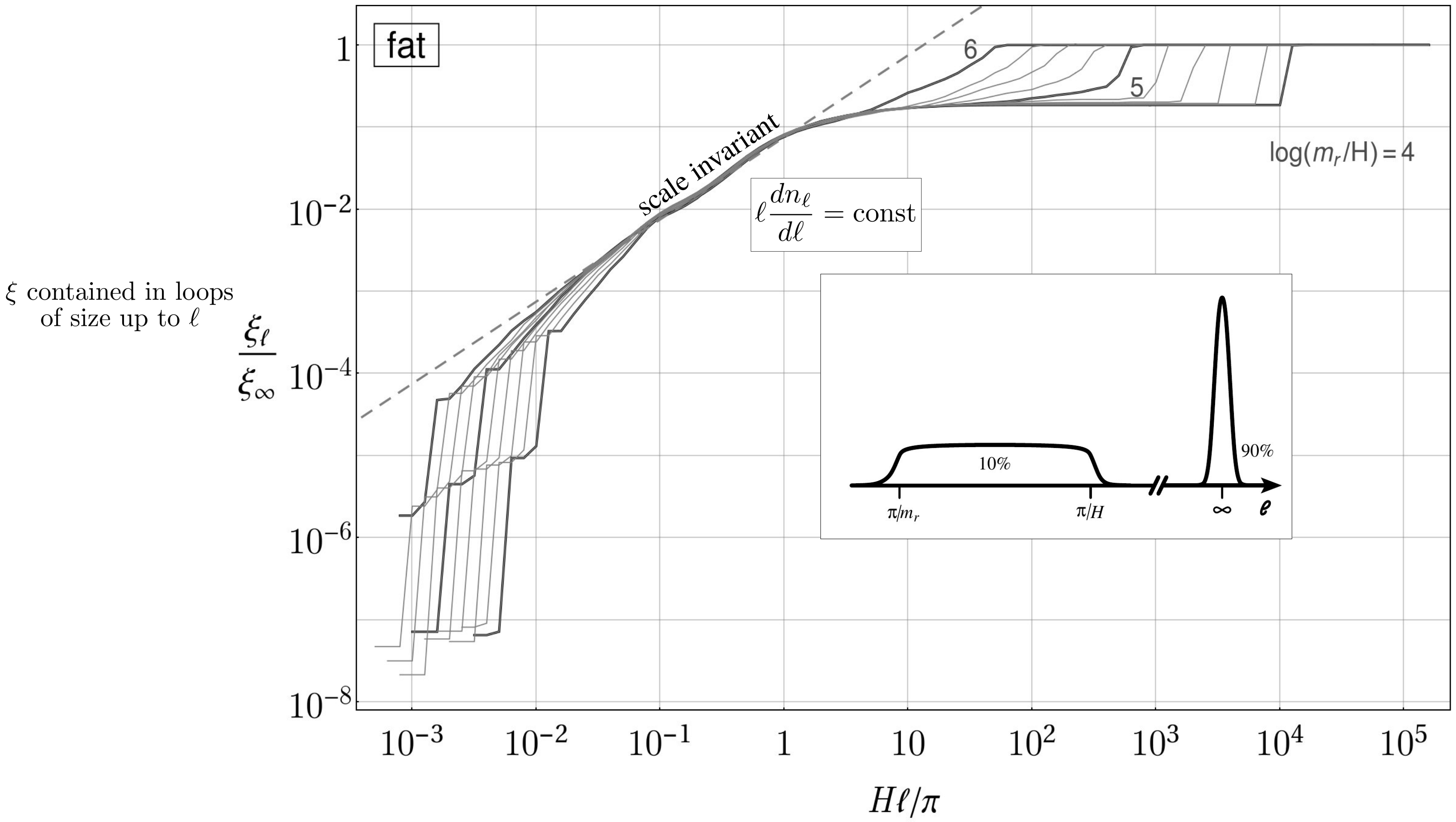


Loop Distribution

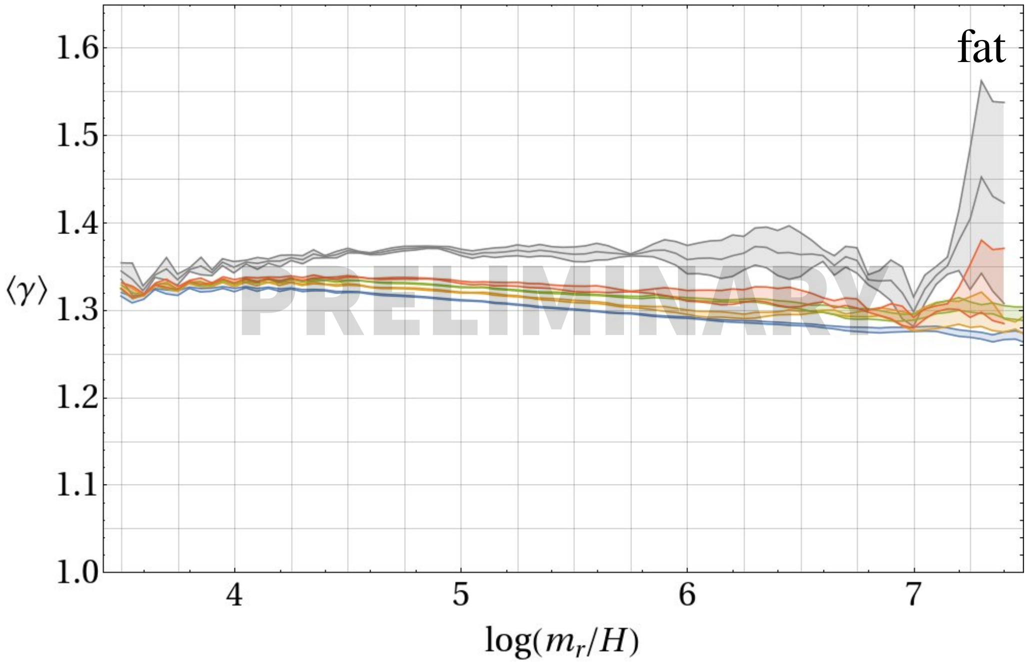
ξ contained in loops
of size up to ℓ

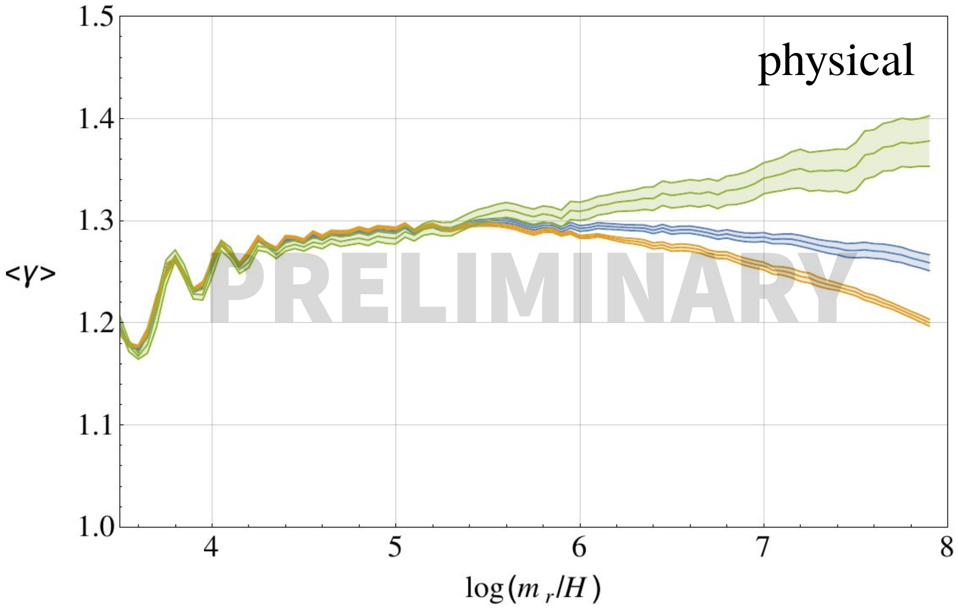


Loop Distribution

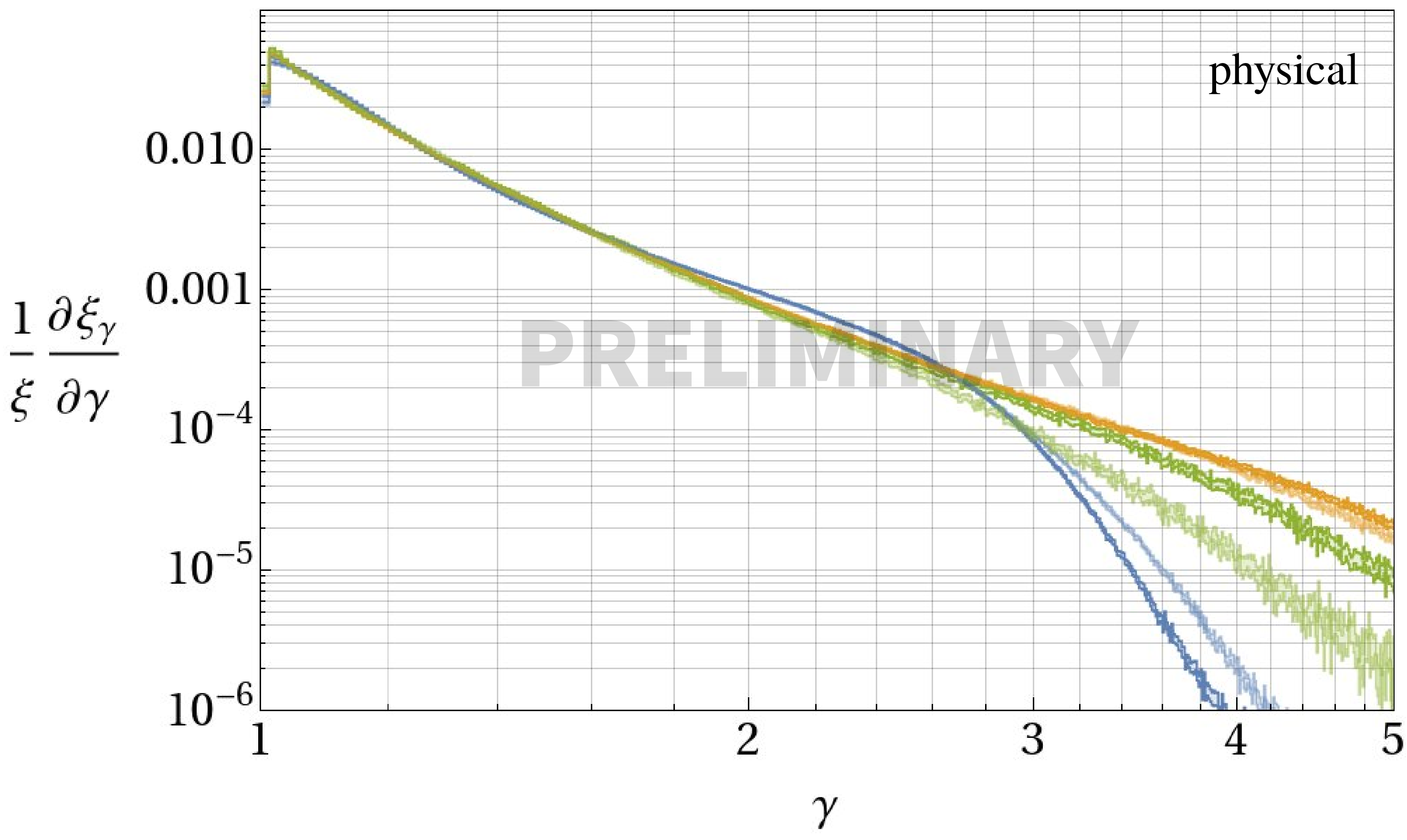


String Velocity Distribution

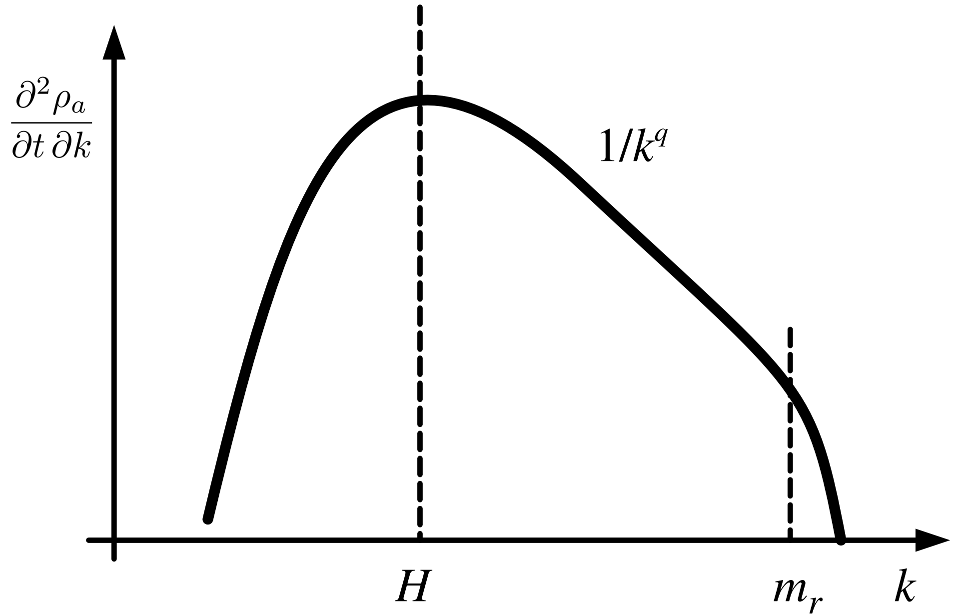




String Velocity Distribution



Axion Spectra VS Axion Number



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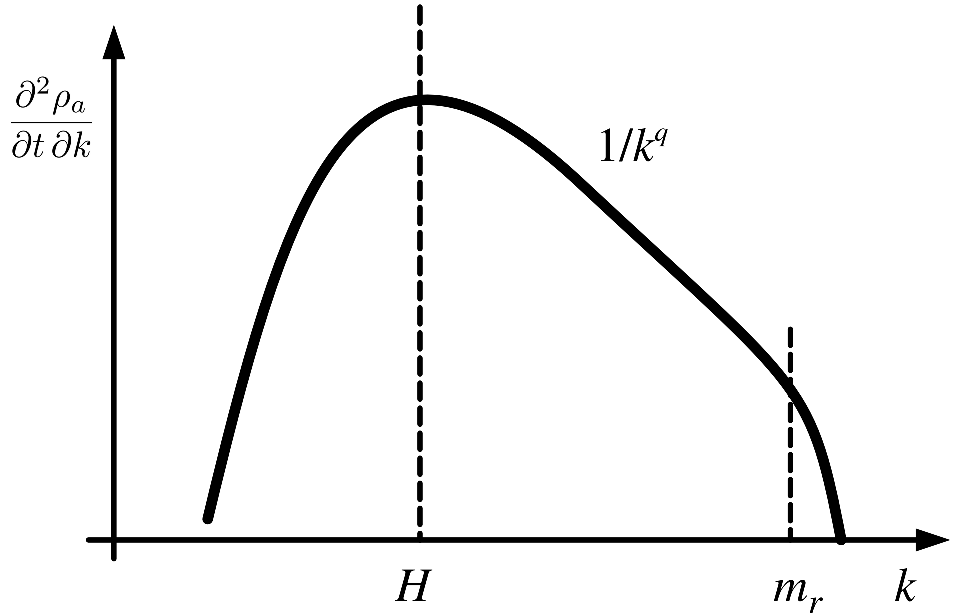
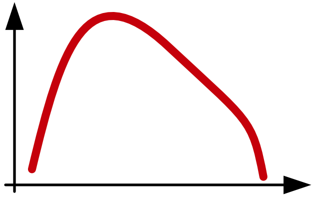
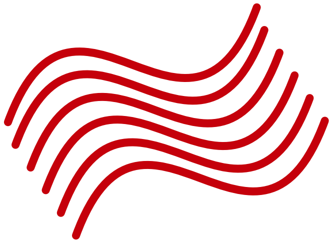


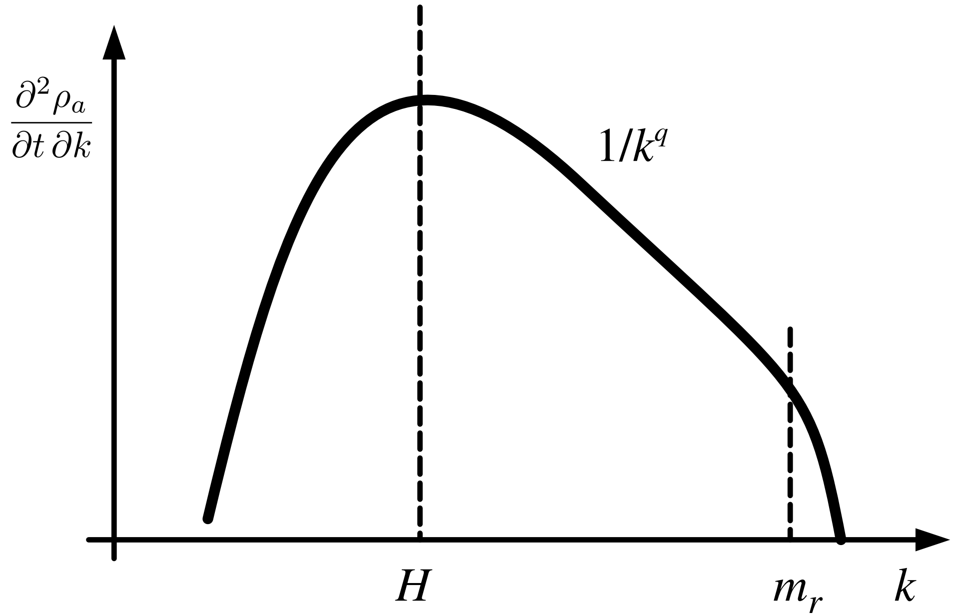
$q > 1$





Davies, Shellard, ...

Axion Spectra VS Axion Number



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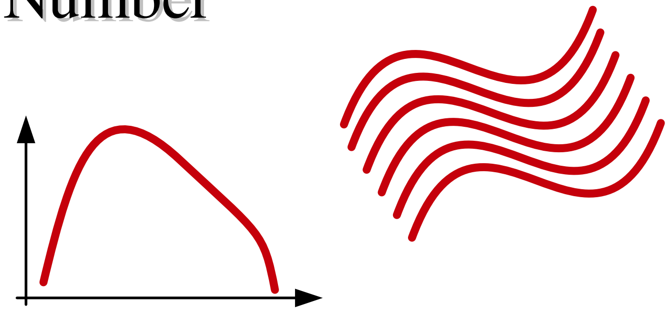
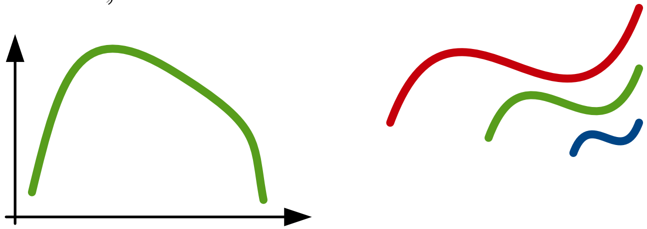


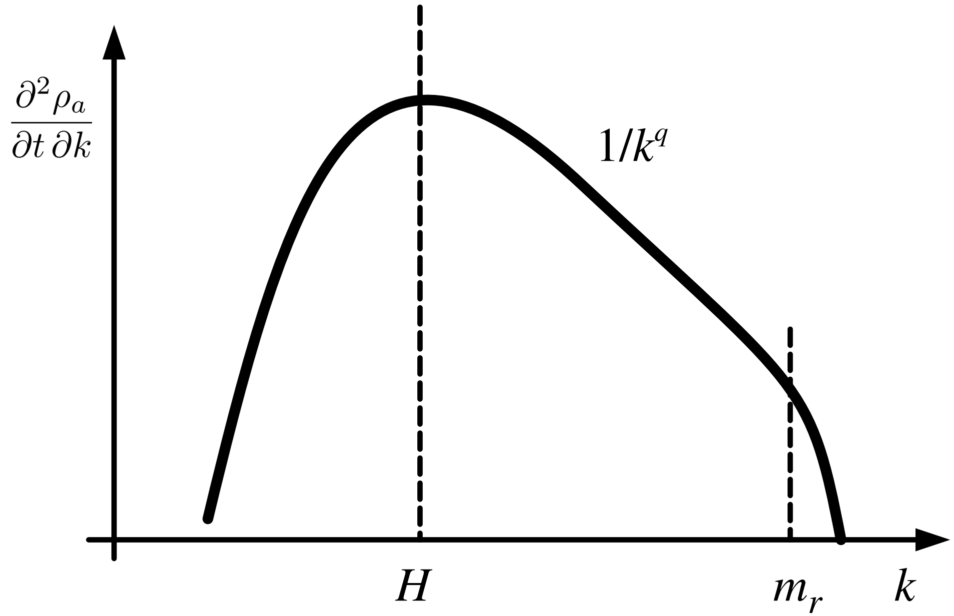
Davies, Shellard, ...

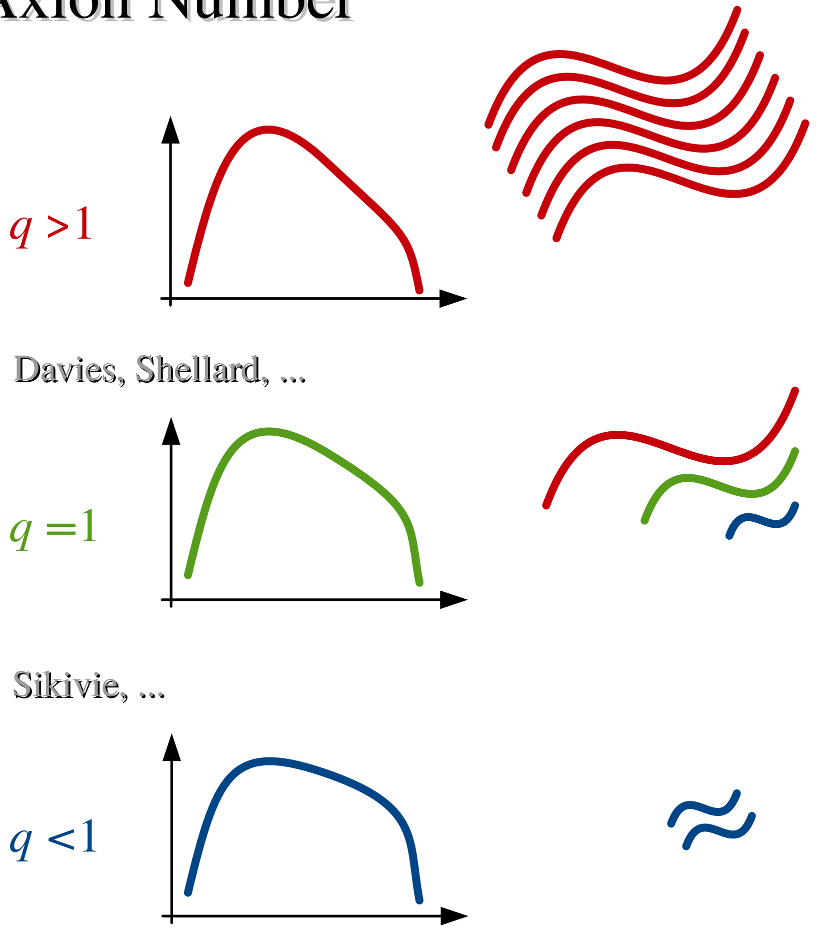
$q = 1$



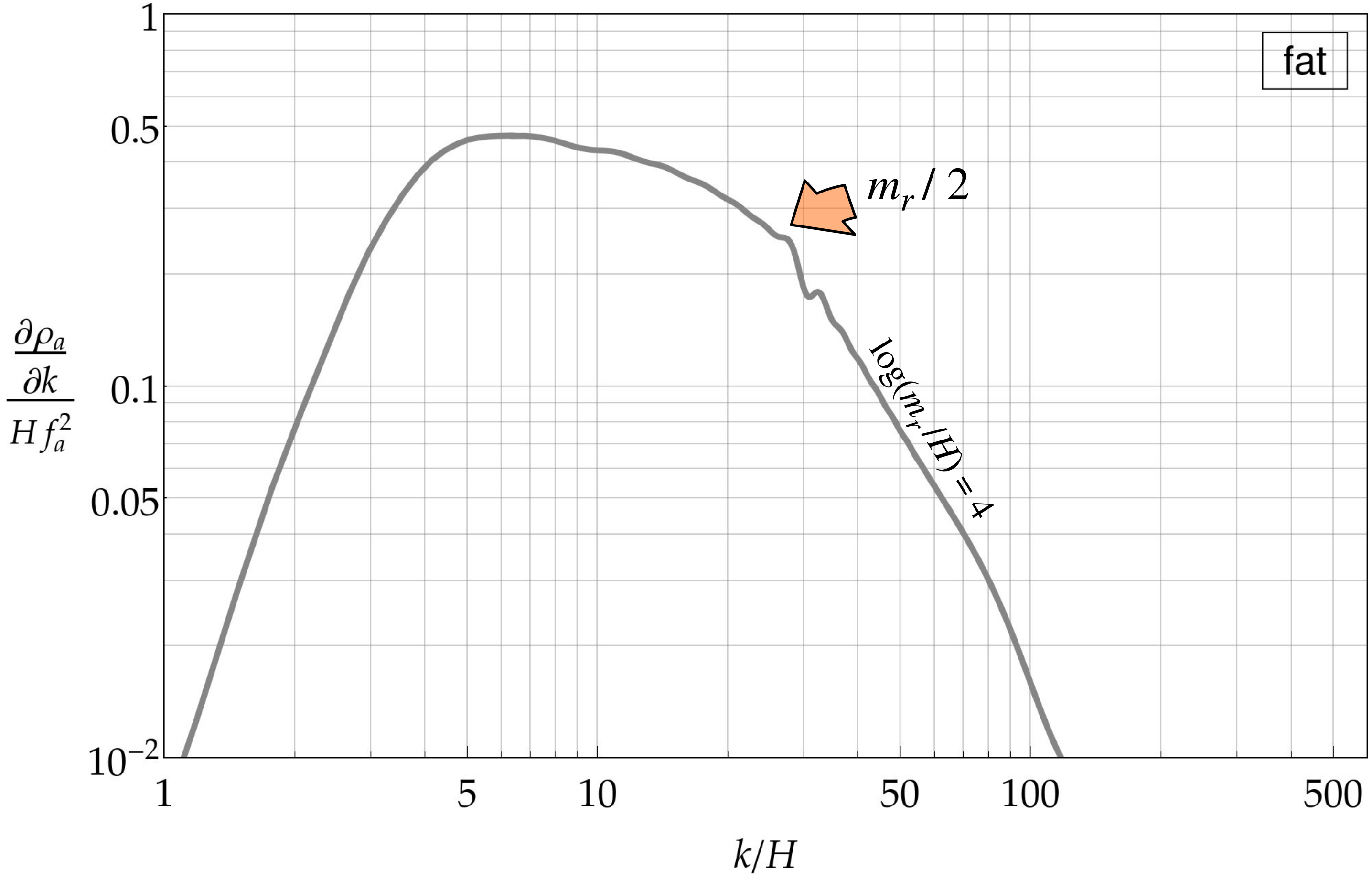
Sikivie, ...

Axion Spectra VS Axion Number

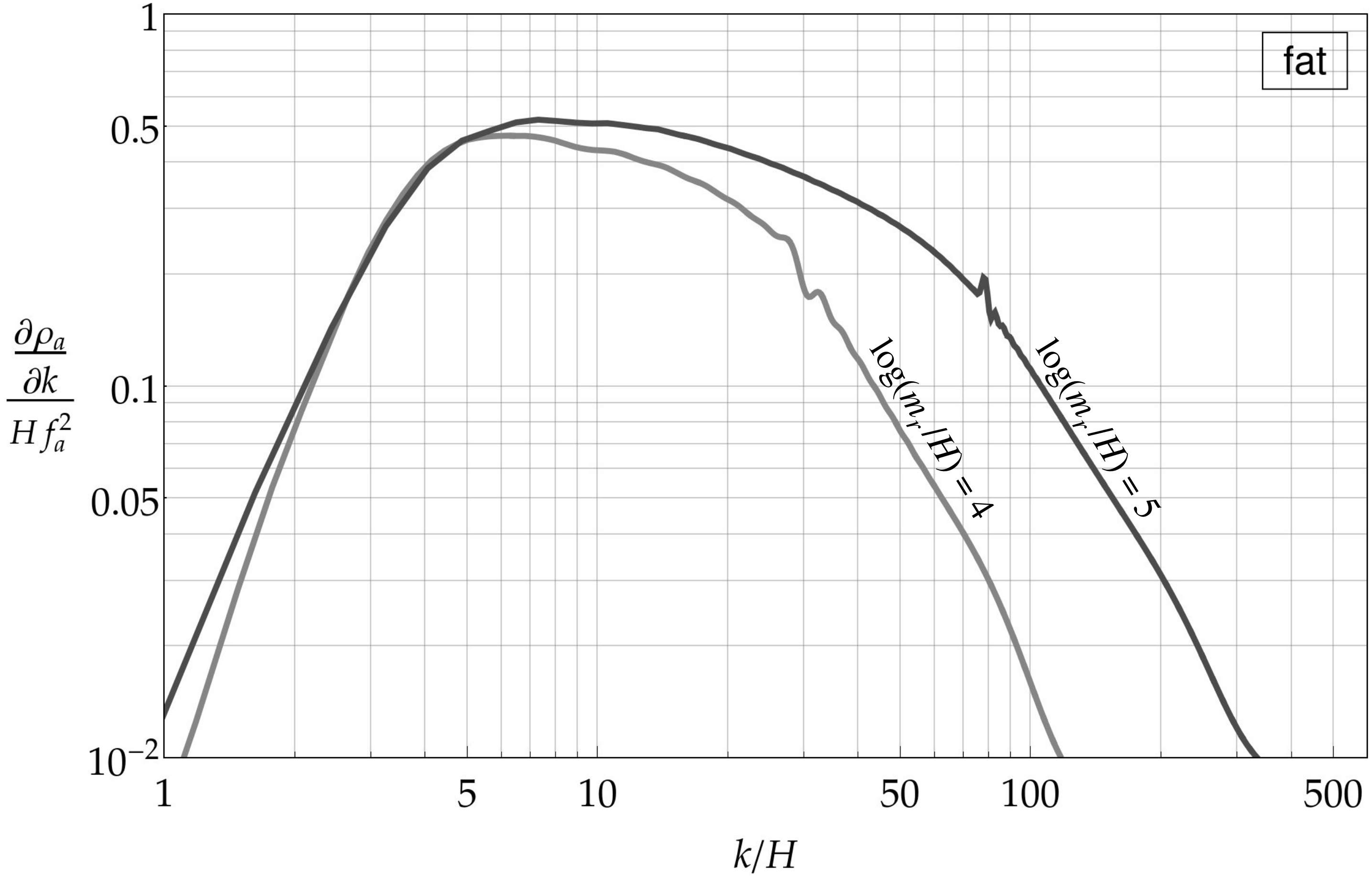




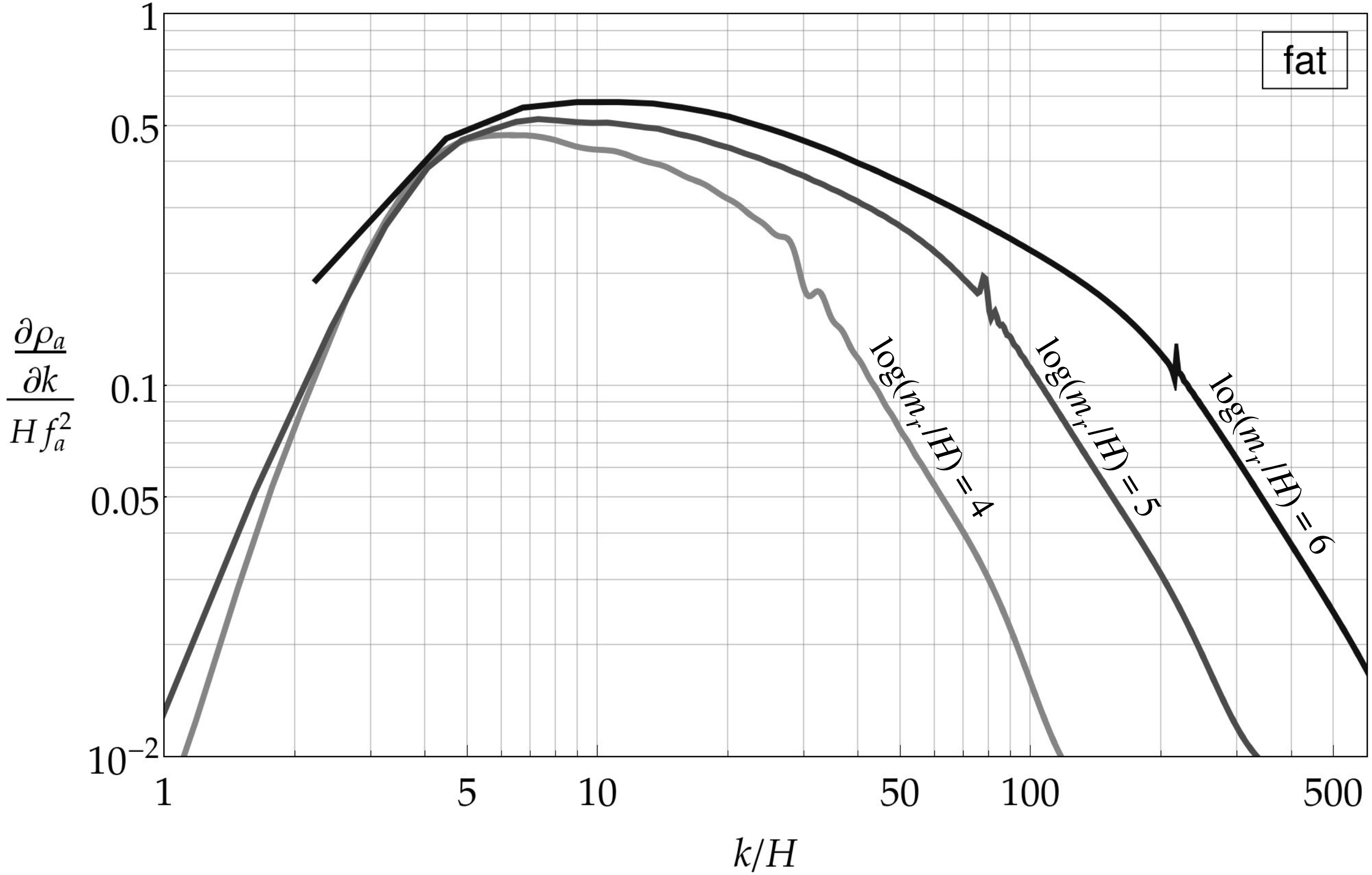
Axion Spectrum



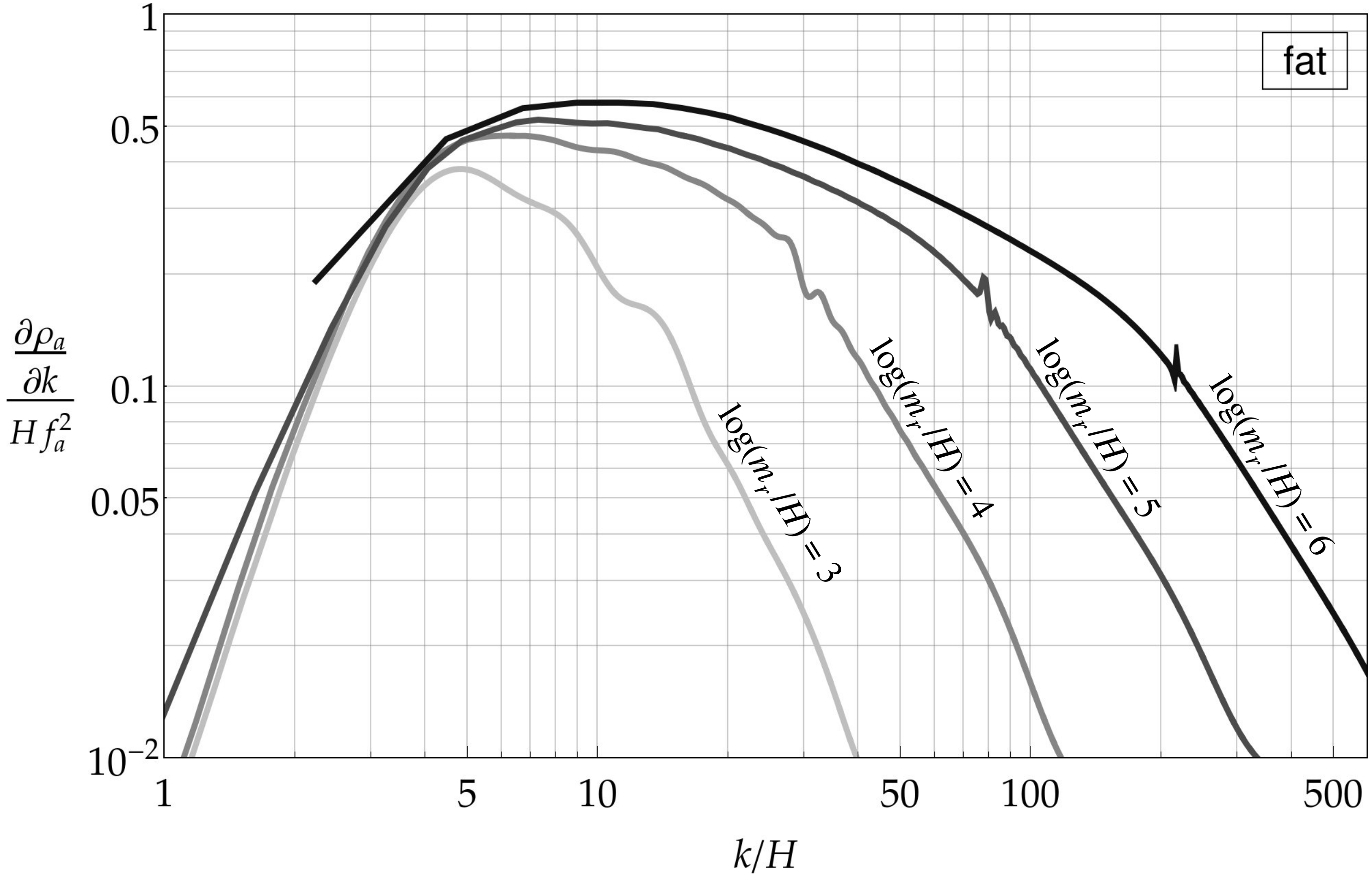
Axion Spectrum



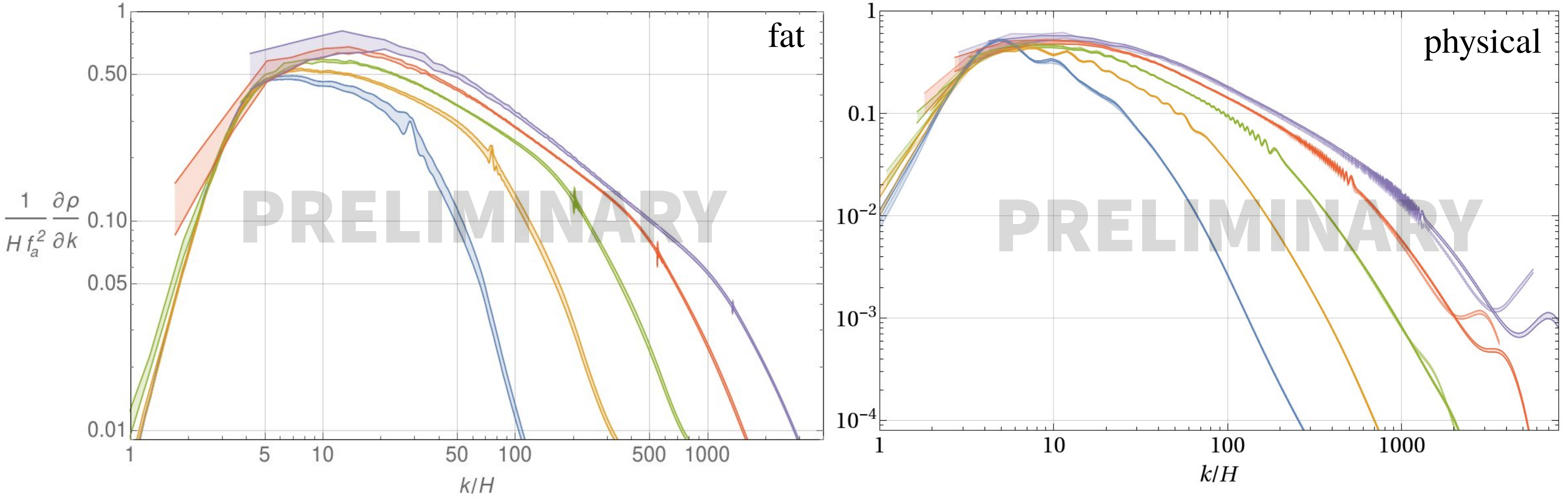
Axion Spectrum



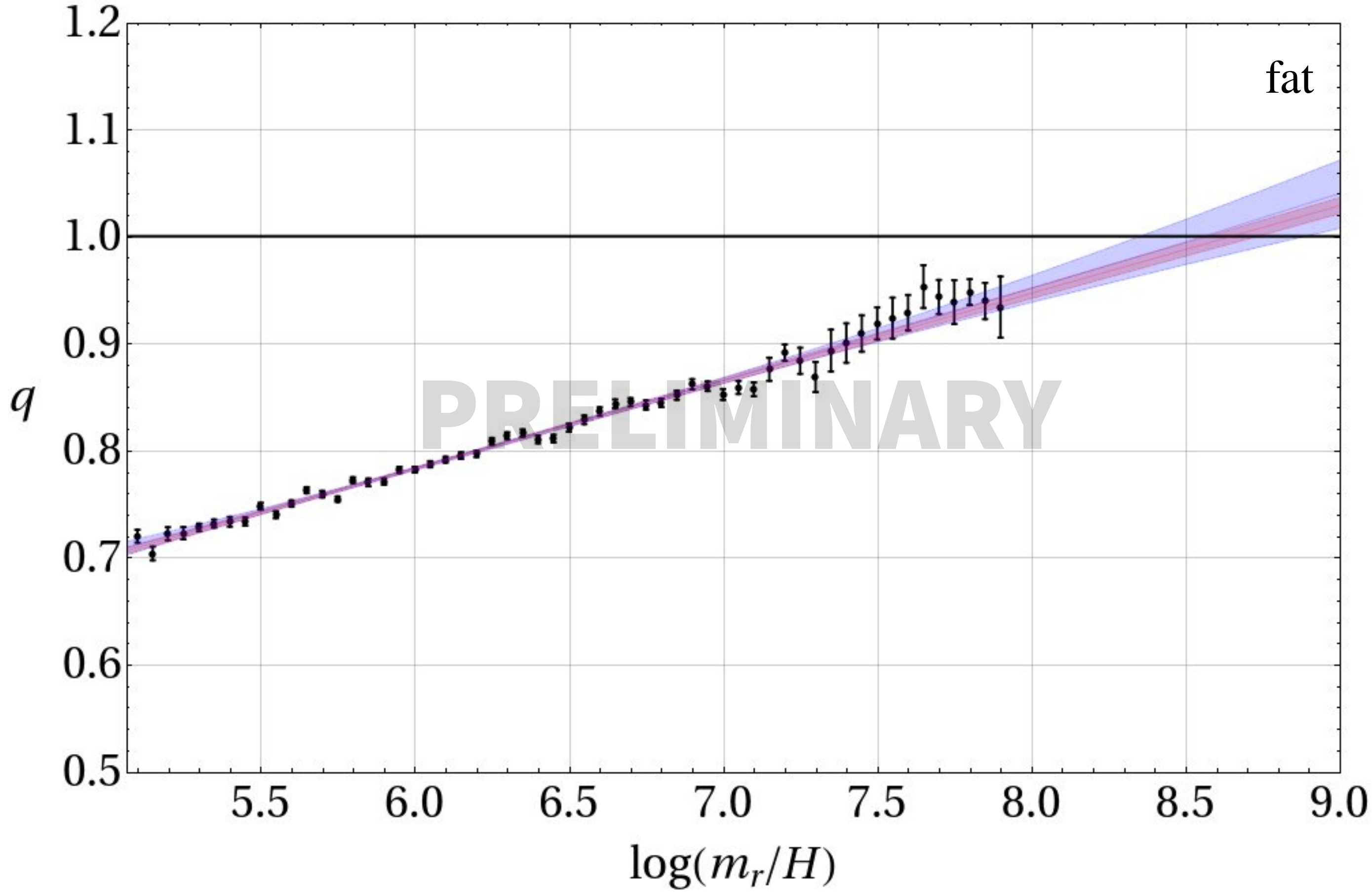
Axion Spectrum



Axion Spectrum @ 4k



Running of the Spectral Index @ 4k



@ $H = m_a$

$$\rho_a^{mis} = \frac{1}{2}\theta_0^2 H^2 f_a^2$$

$$\rho_a^{str} \simeq \xi\mu H^2 \log(m_r/H)$$

@ $H = m_a$

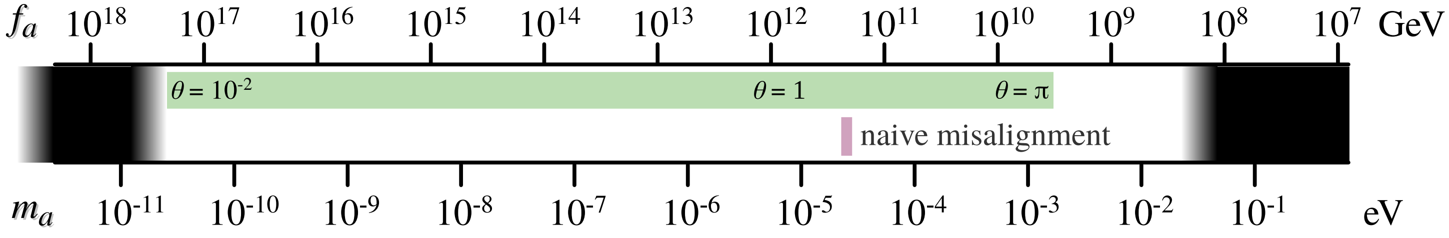
$$\rho_a^{mis} = \frac{1}{2}\theta_0^2 H^2 f_a^2$$

$$\rho_a^{str} \simeq \xi\mu H^2 \log(m_r/H) \simeq 10^5 \rho_a^{mis} \left[\frac{\xi}{15}\right] \left[\frac{\log(m_r/H)}{70}\right]^2$$

$@ \, H = m_a$

$$\rho_a^{mis} = \frac{1}{2}\theta_0^2 H^2 f_a^2$$

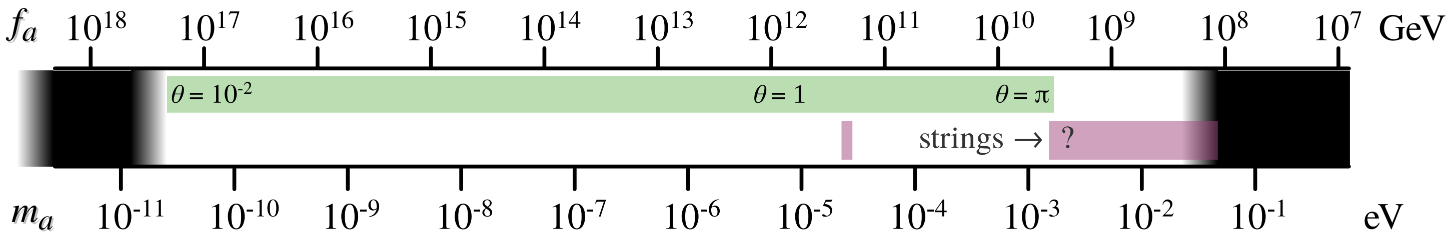
$$\rho_a^{str} \simeq \xi \mu H^2 \log(m_r/H) \simeq 10^5 \rho_a^{mis} \left[\frac{\xi}{15}\right] \left[\frac{\log(m_r/H)}{70}\right]^2$$



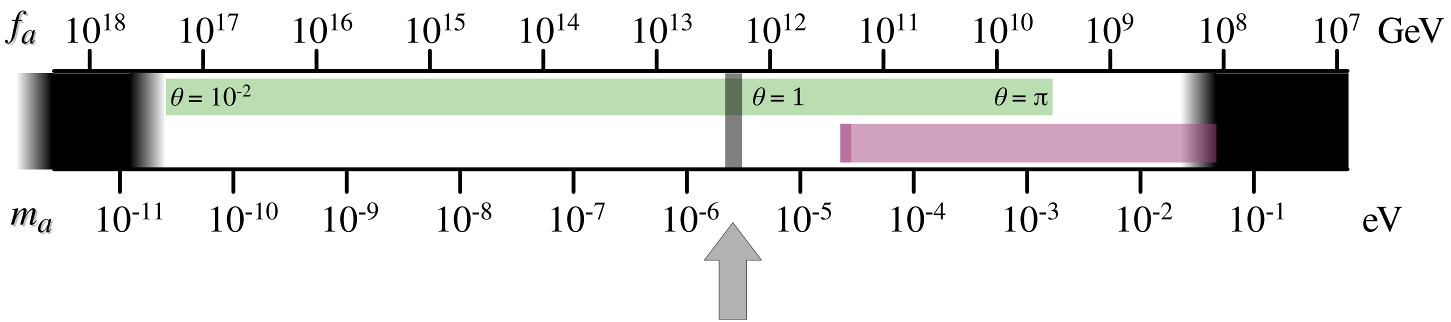
$@ \ H = m_a$

$$\rho_a^{mis} = \frac{1}{2}\theta_0^2 H^2 f_a^2$$

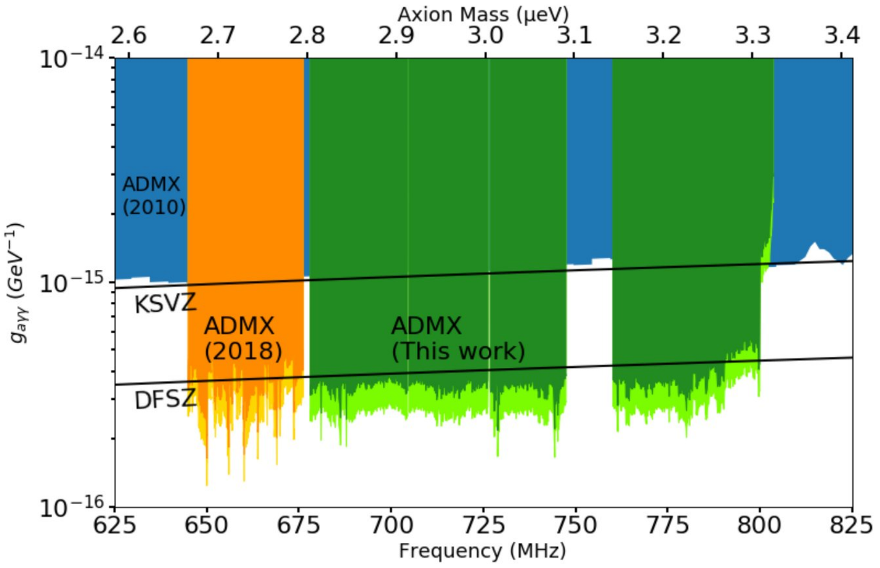
$$\rho_a^{str} \simeq \xi \mu H^2 \log(m_r/H) \simeq 10^5 \rho_a^{mis} \left[\frac{\xi}{15}\right] \left[\frac{\log(m_r/H)}{70}\right]^2$$



Implications for Axion Abundance



new ADMX results:
1910.08638

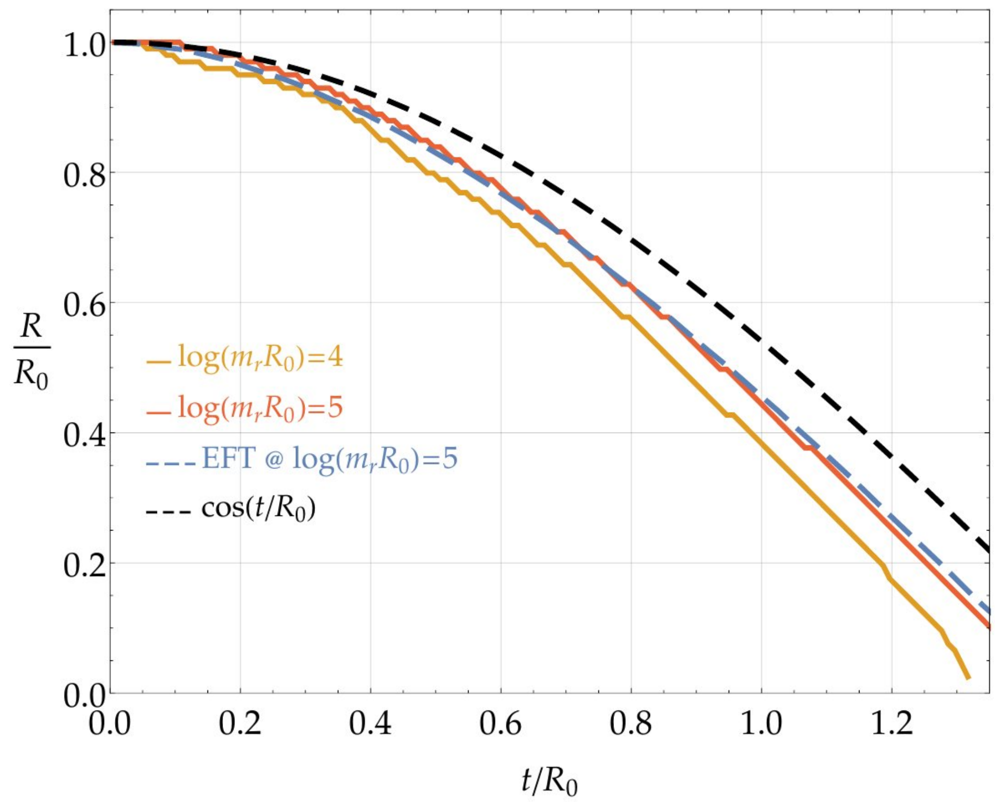


Conclusions:

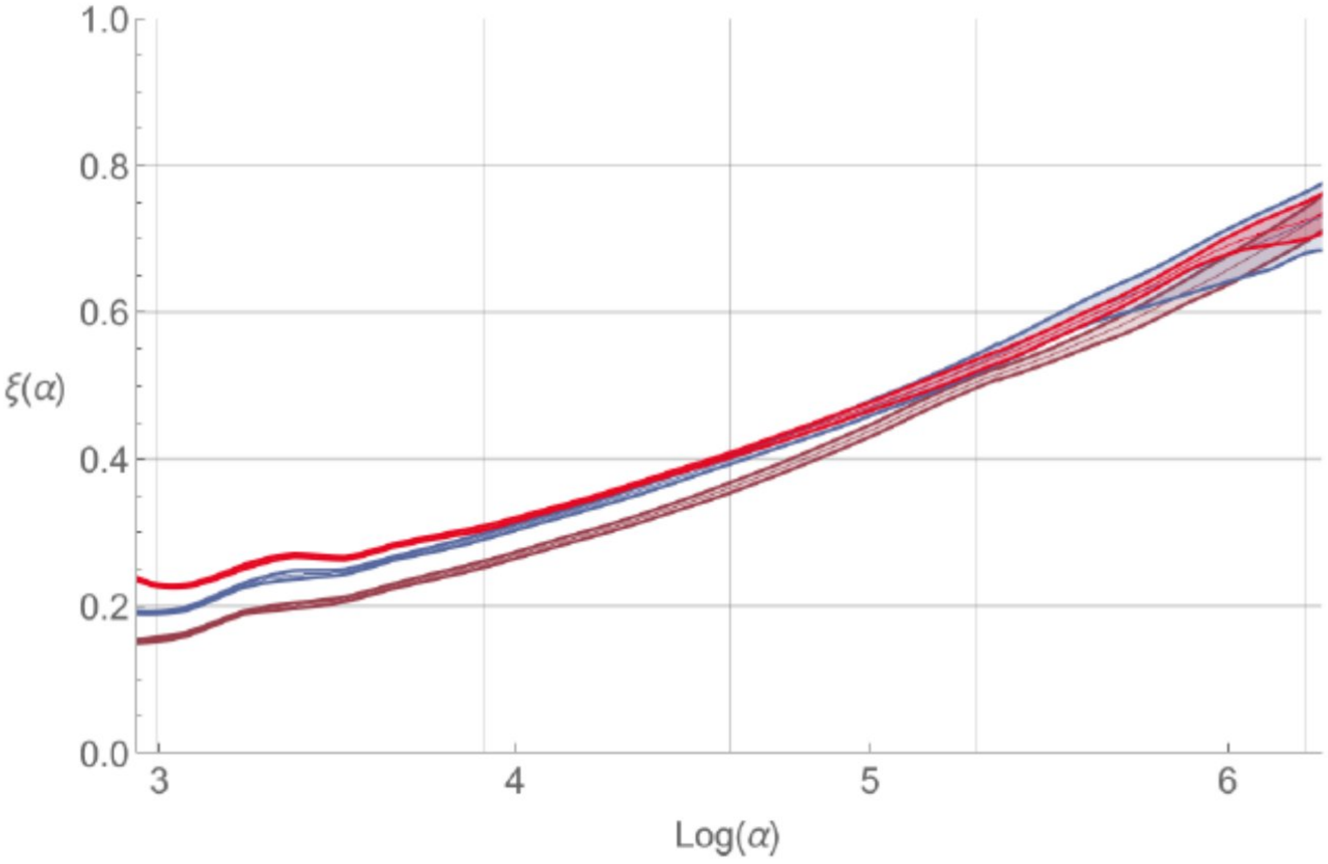
- Attractor solution established in simulations
- Clear Log-violation of scaling properties observed
- Axion Spectrum is UV dominated but evidence from non-trivial evolution
- High precision required for reliable extrapolation
- More To Come!

Thank you!

Back Up

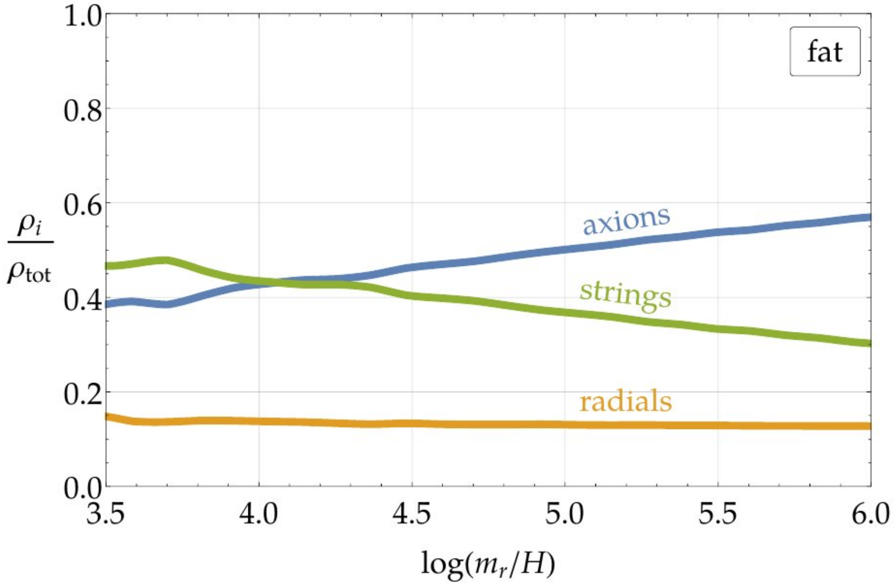


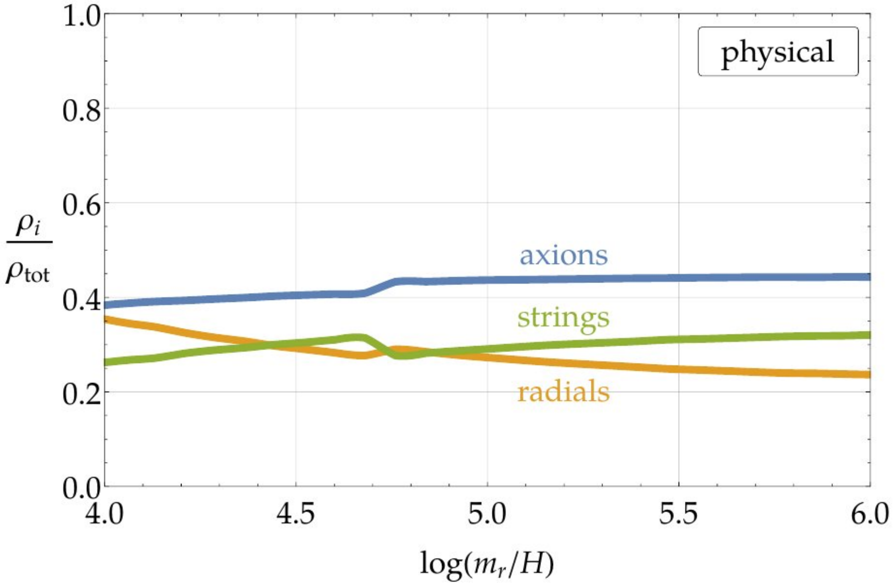
1.0
0.8
0.6
0.4
0.2
0.0
$\frac{R}{R_0}$
$\log(m_r R_0)$=4
$\log(m_r R_0)$=5
EFT @ $\log(m_r R_0)$=5
$\cos(t/R_0)$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
t/R_0



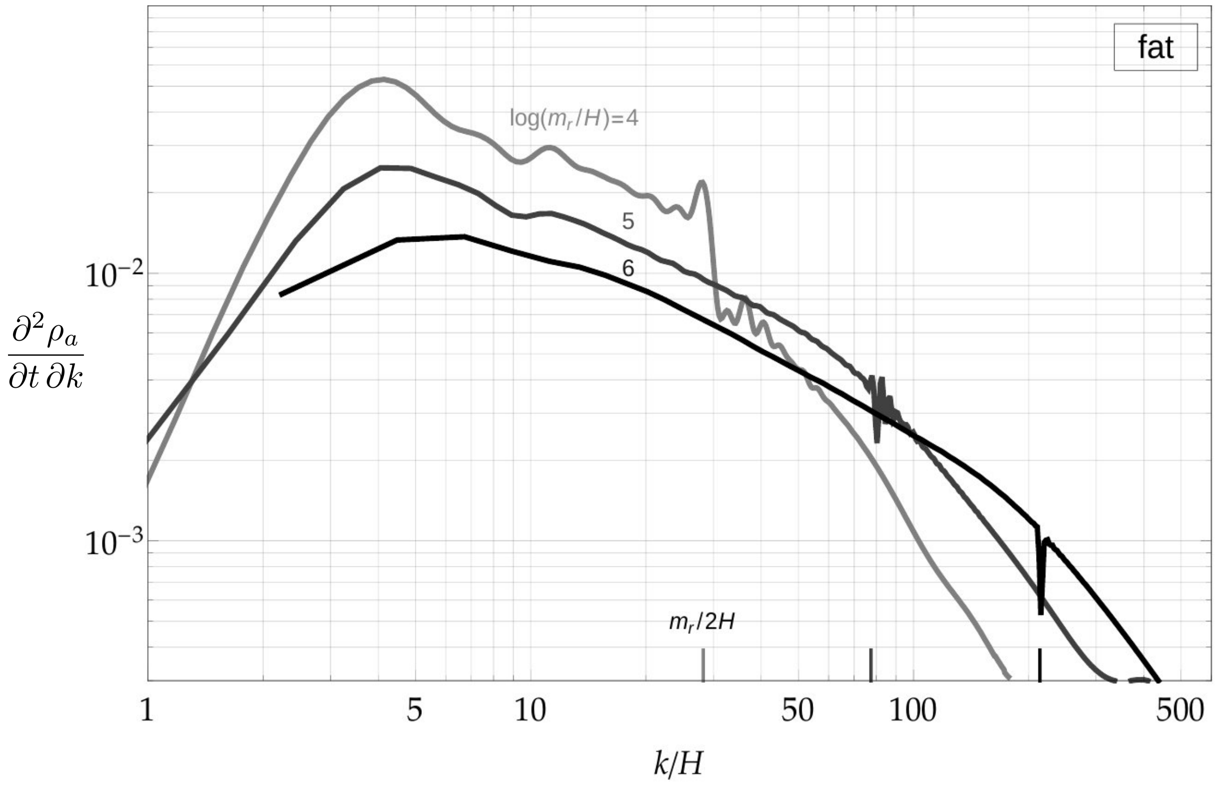
1.0
0.8
0.6
0.4
0.2
0.0
$\xi(\alpha)$
3
4
5
6
Log(α)

Axion, String and Radial Energy Budget

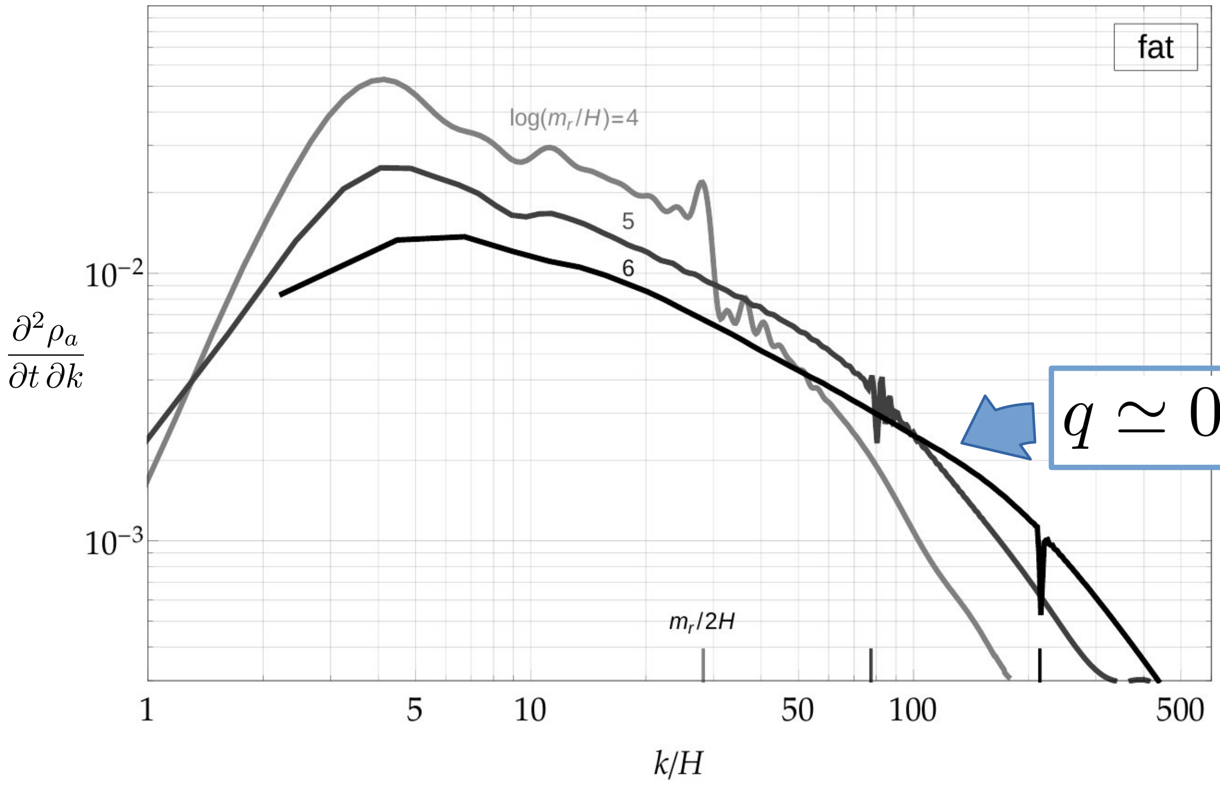




Axion Instantaneous Spectrum

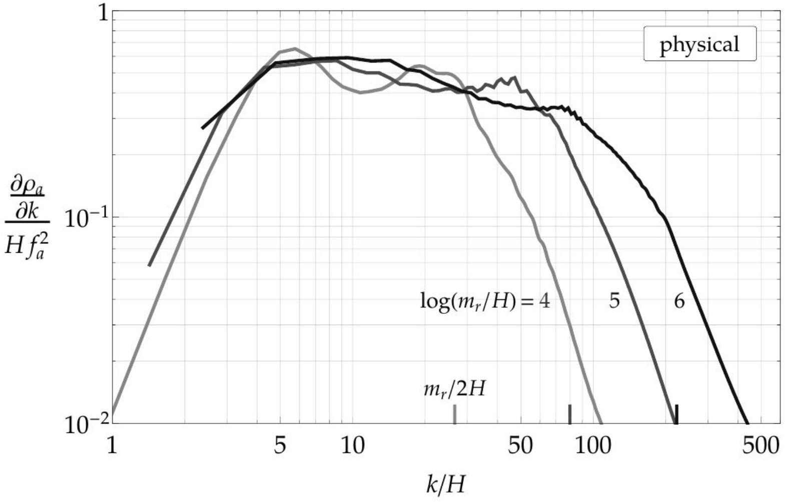


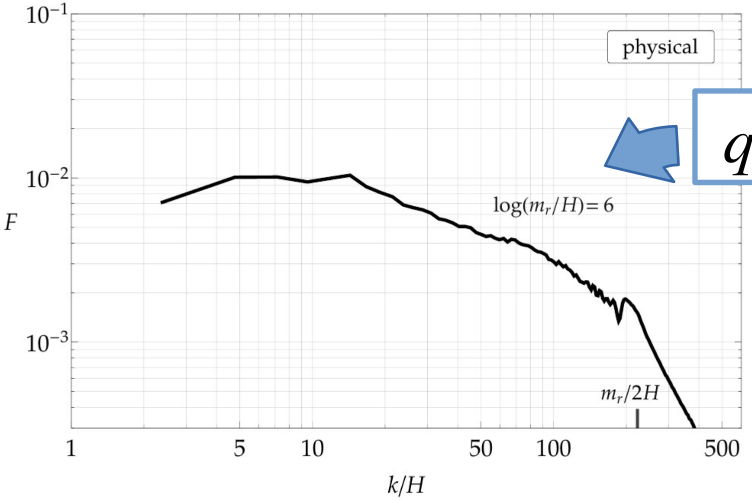
Axion Instantaneous Spectrum



$$q \simeq 0.75(5)$$

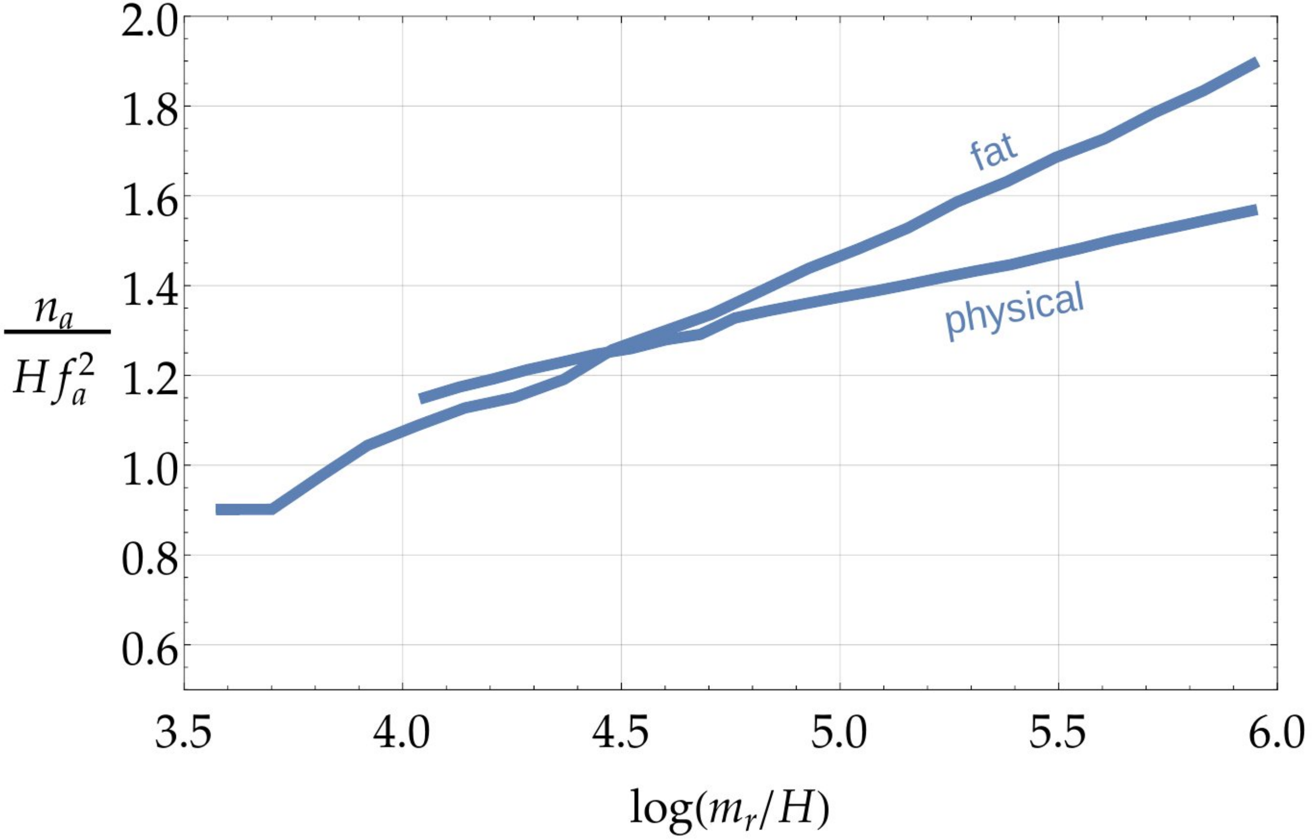
Physical Case



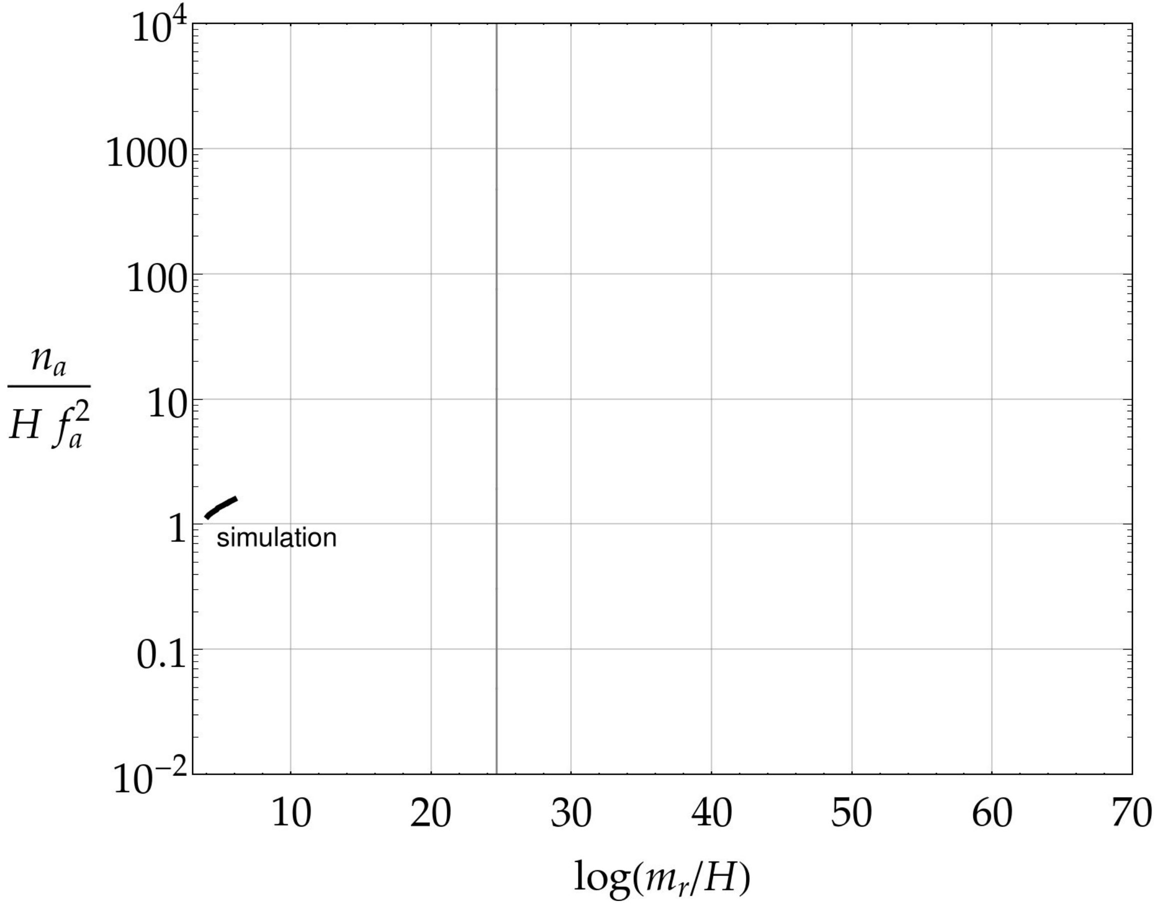


$q \sim 0.7$

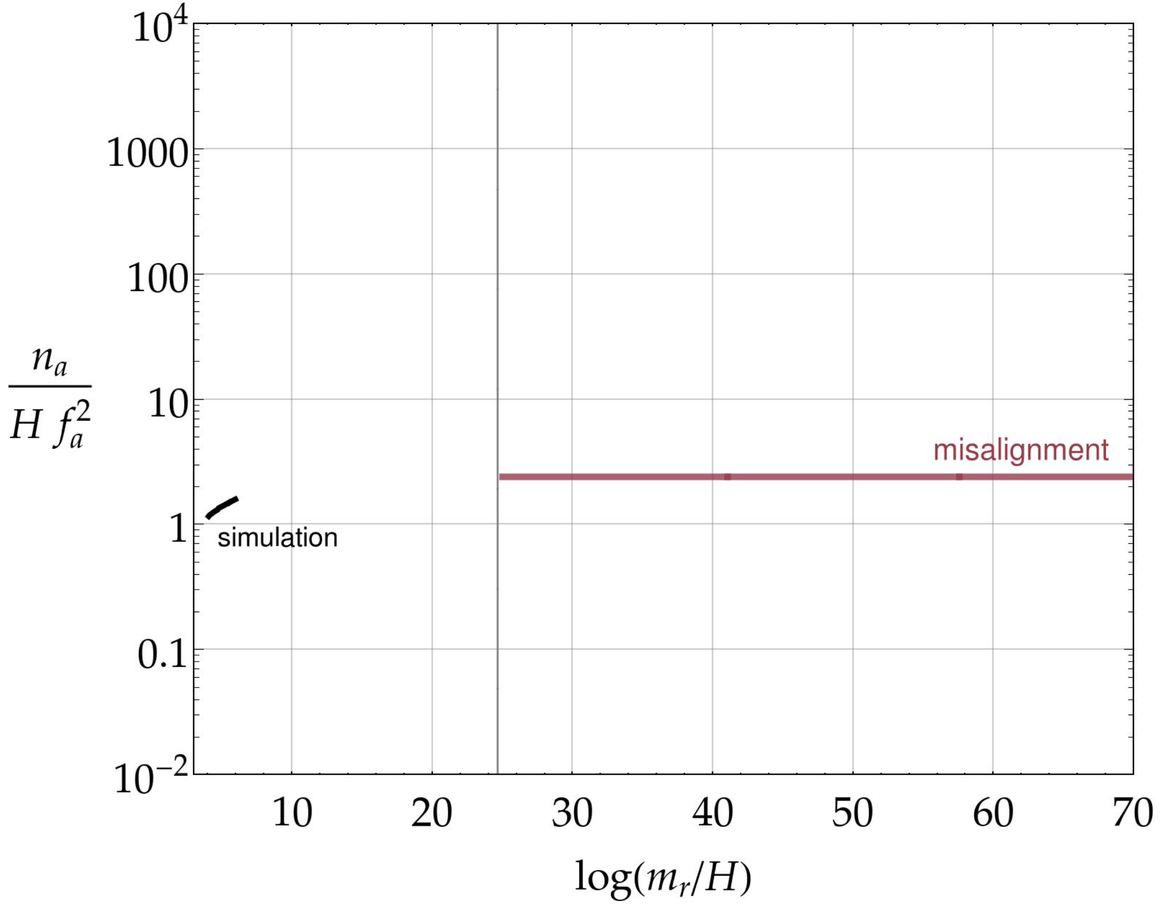
Axion Number Density



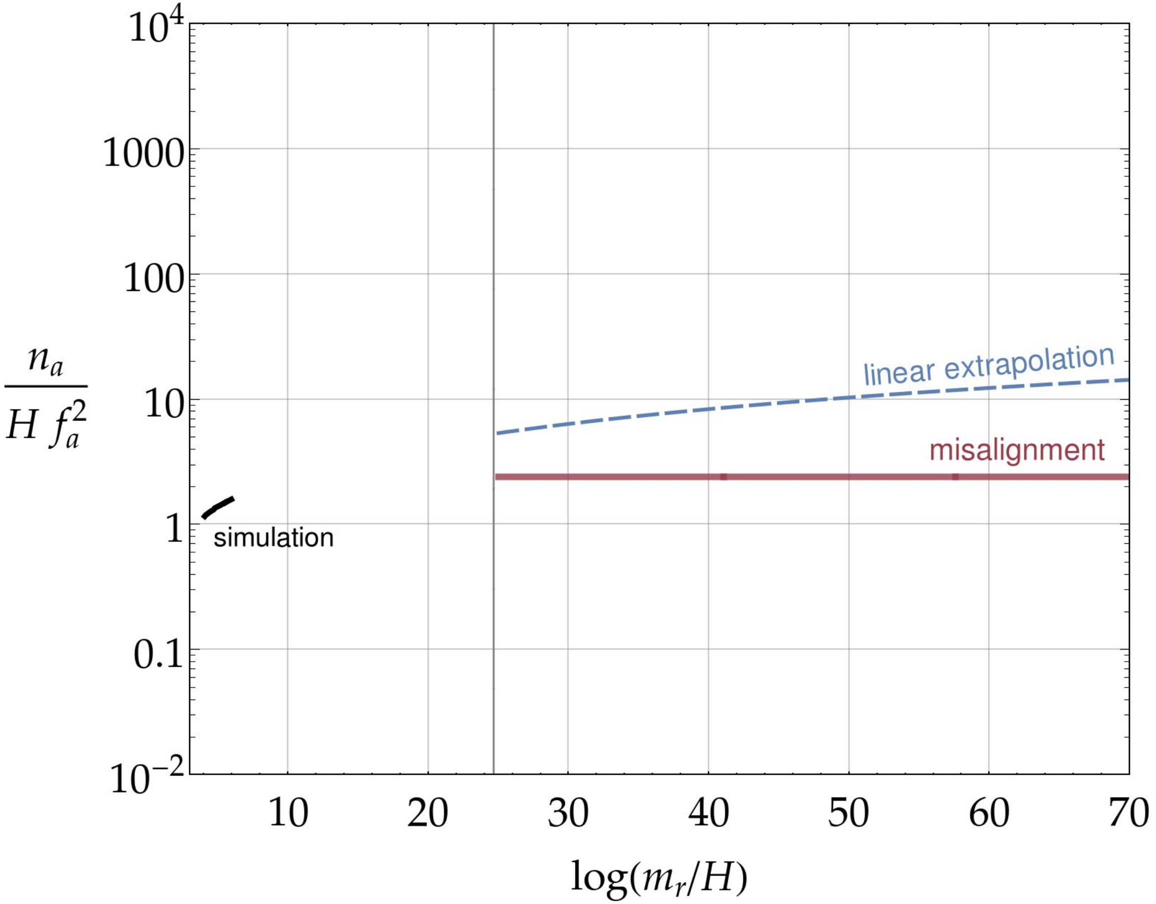
Axion Number Density → Extrapolation



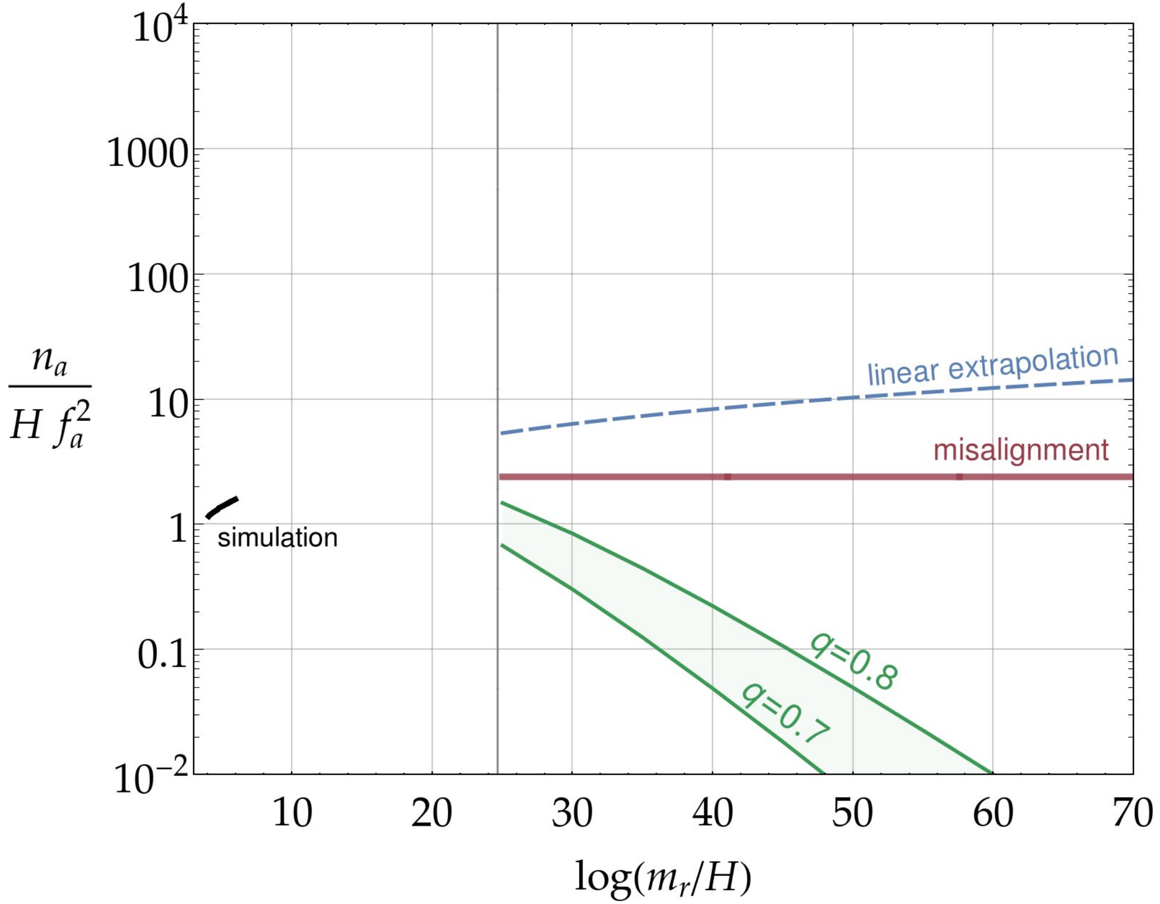
Axion Number Density → Extrapolation



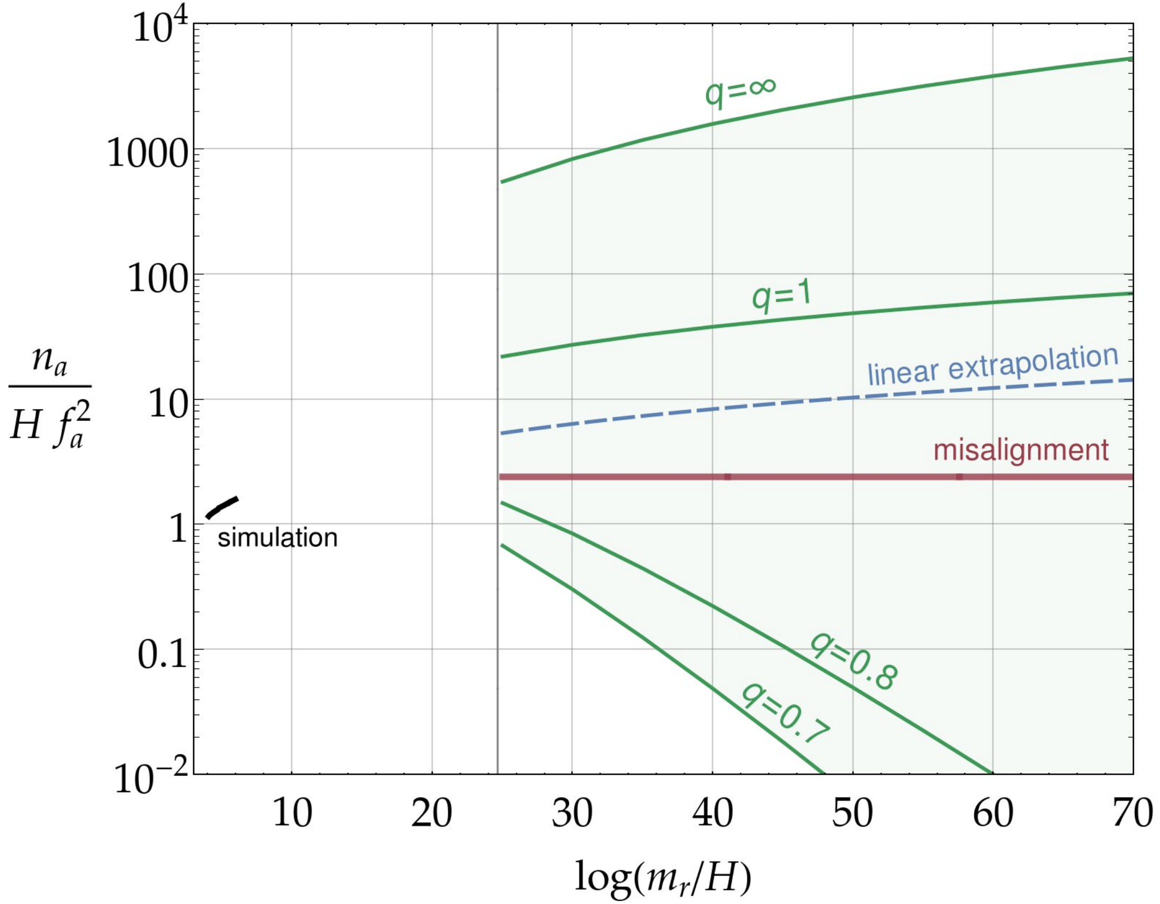
Axion Number Density → Extrapolation



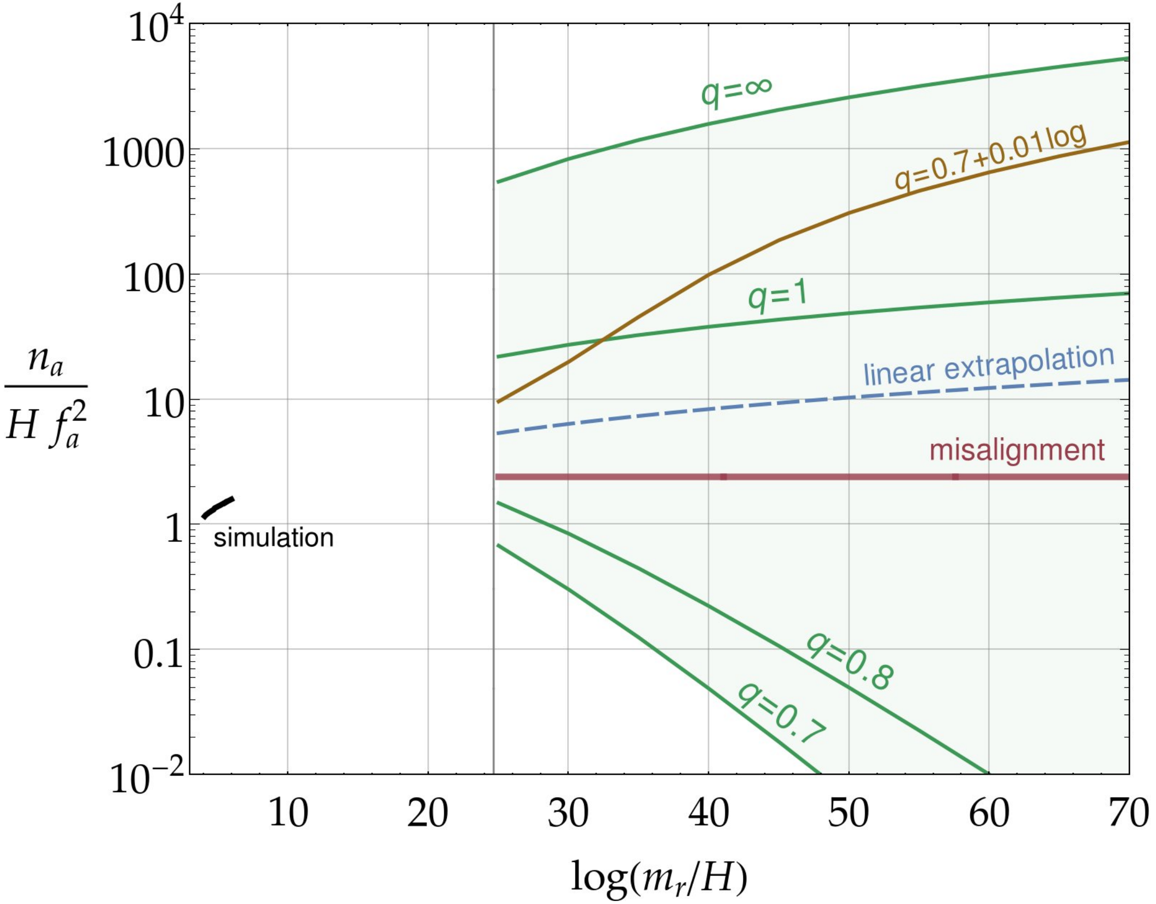
Axion Number Density → Extrapolation



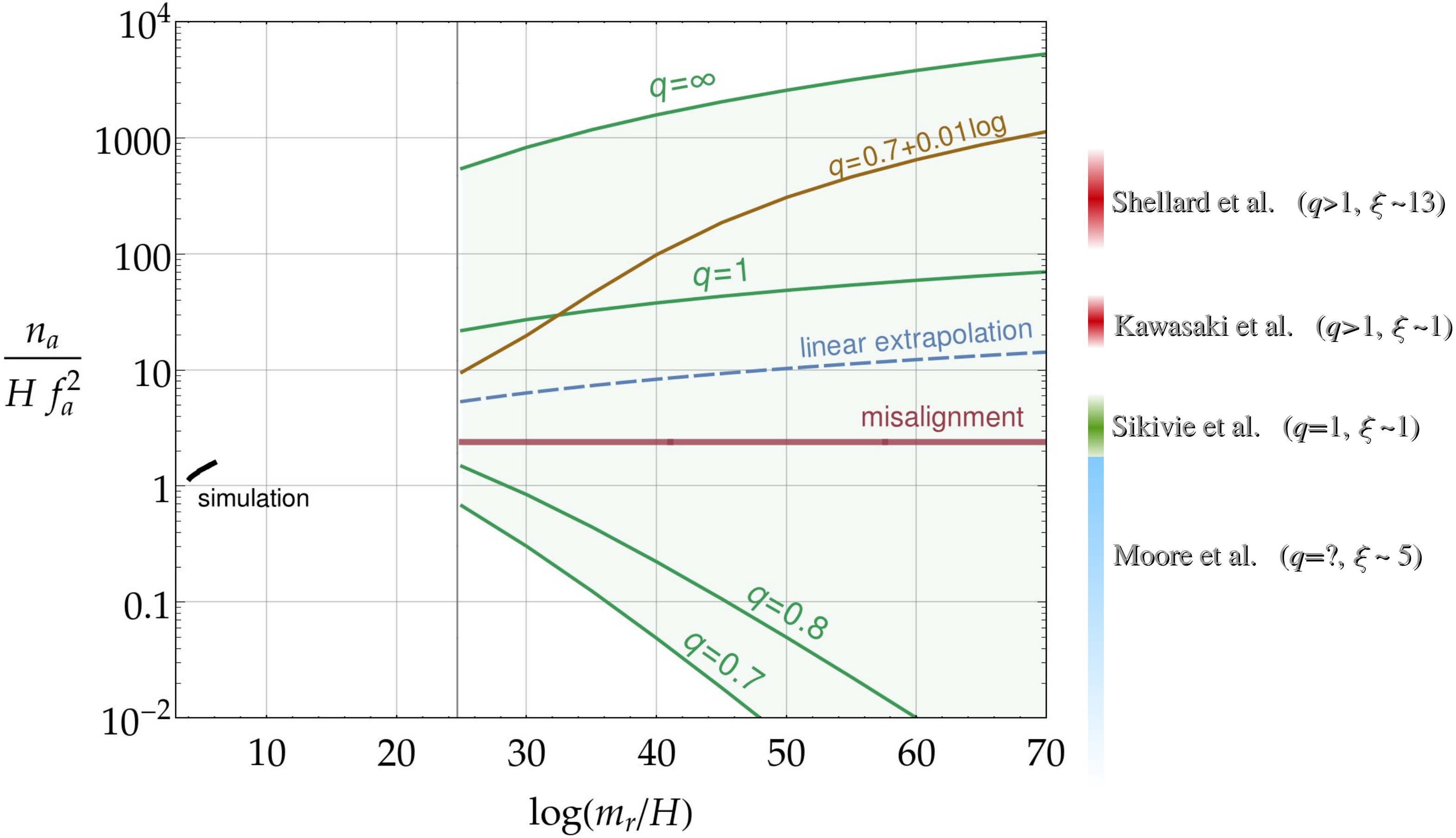
Axion Number Density → Extrapolation



Axion Number Density → Extrapolation



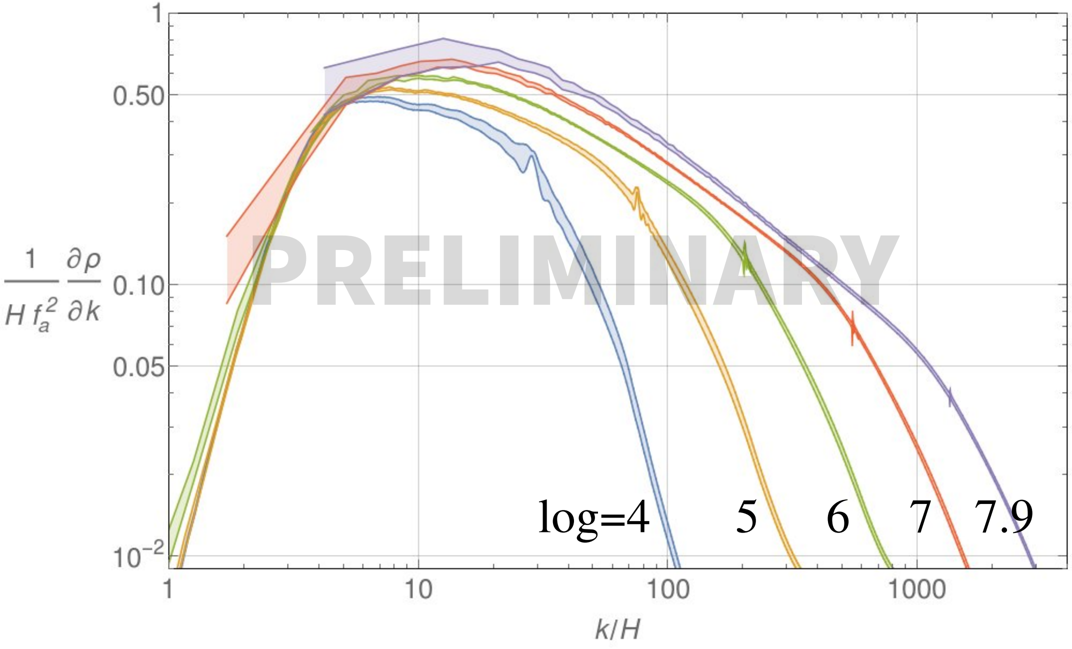
Axion Number Density → Extrapolation

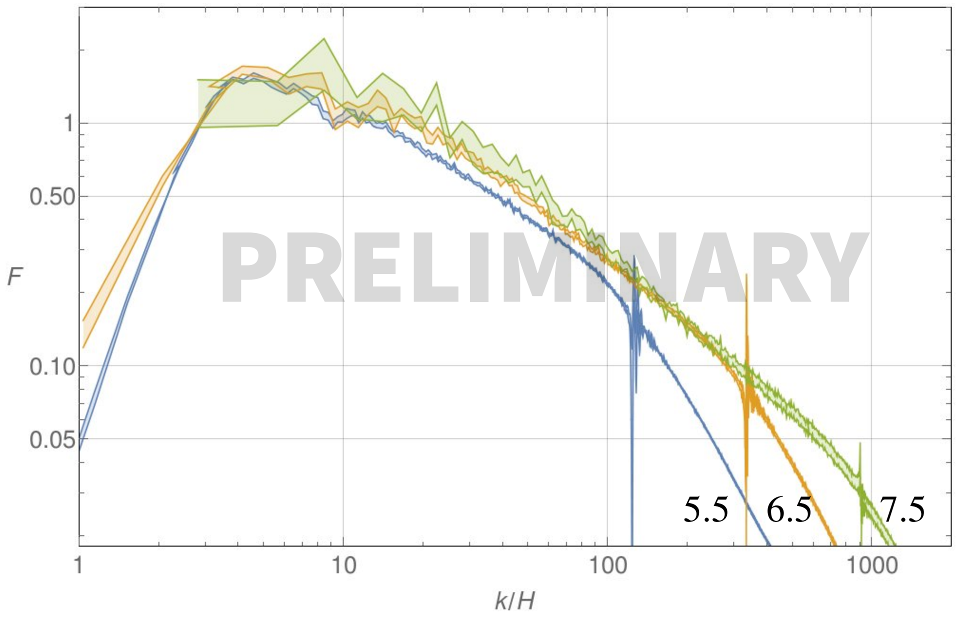


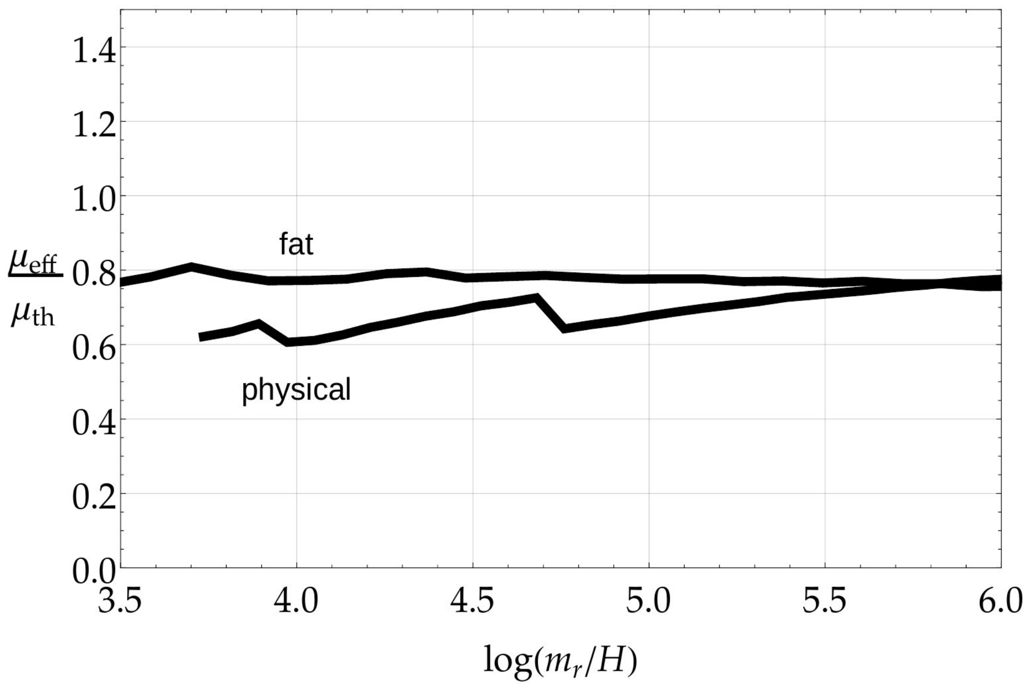
Bigger Grids Preview

1k → 4k

Axion Spectra

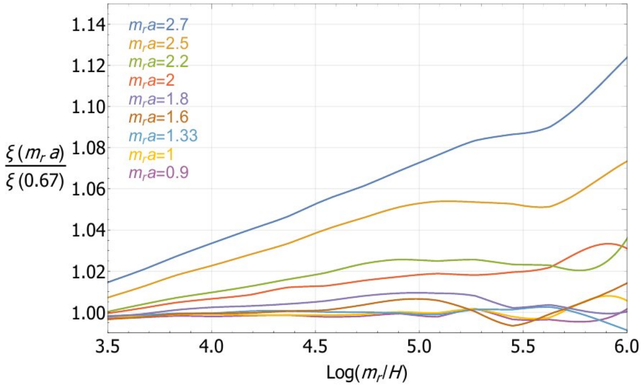


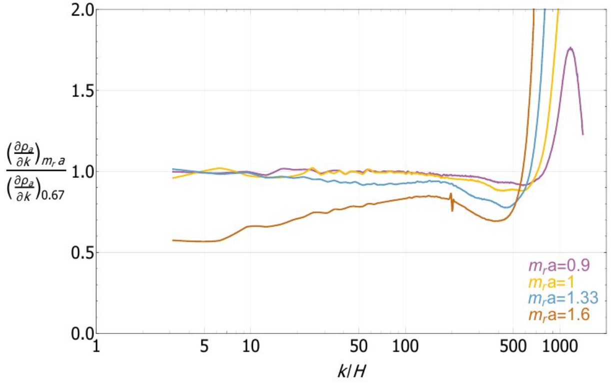


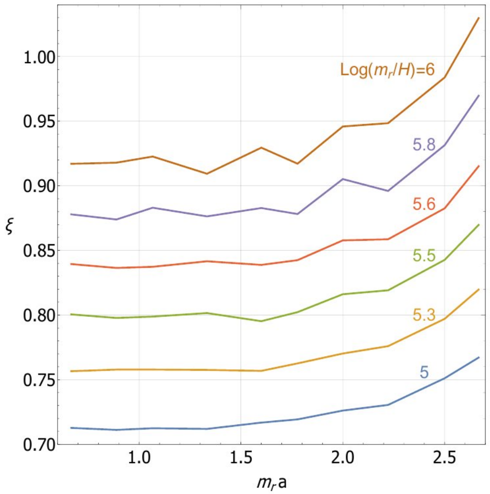


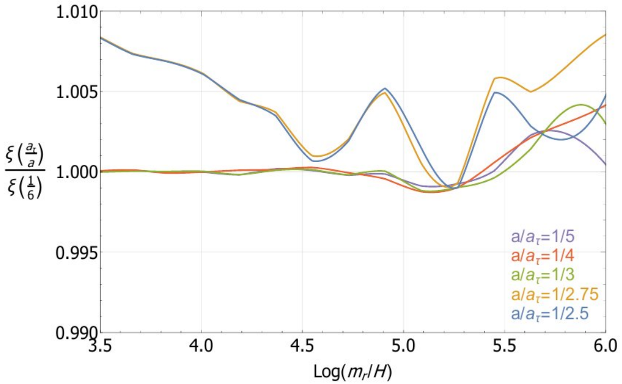
fat
physical
$\frac{\mu_{\rm eff}}{\mu_{\rm th}}$
$\log(m_r/H)$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
3.5
4.0
4.5
5.0
5.5
6.0

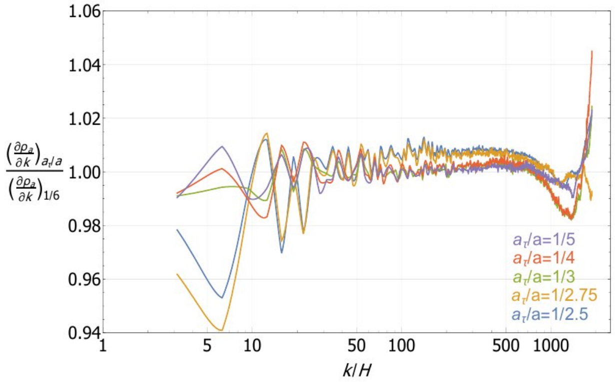
Lattice Spacing



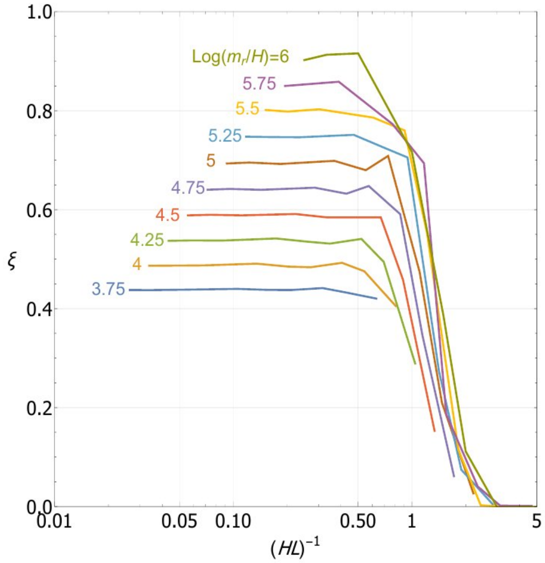


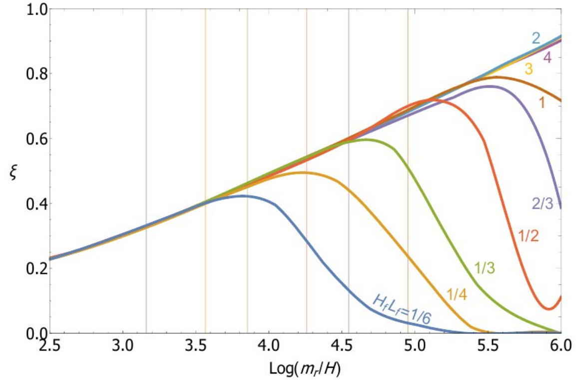


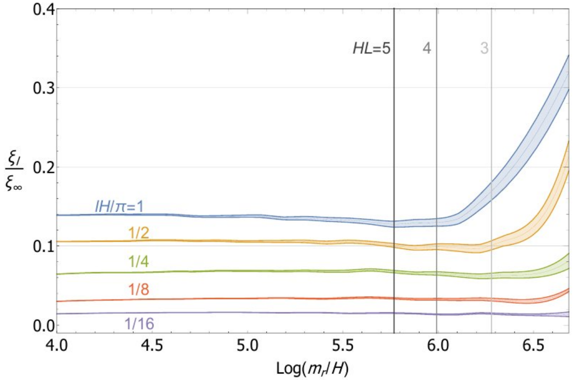


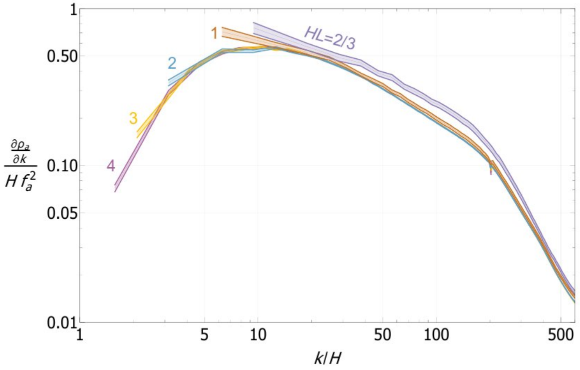


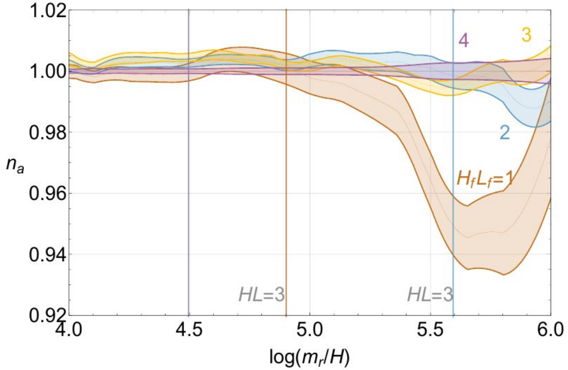
Finite Volume



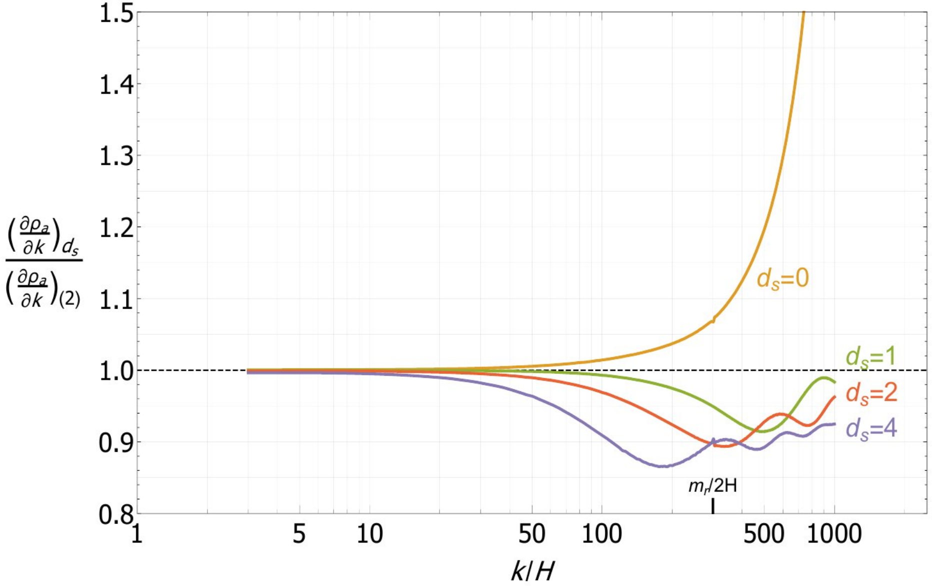




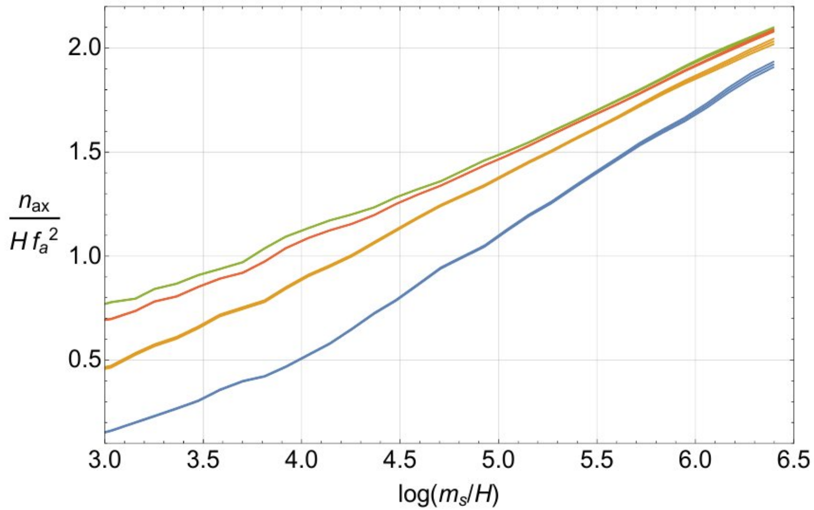


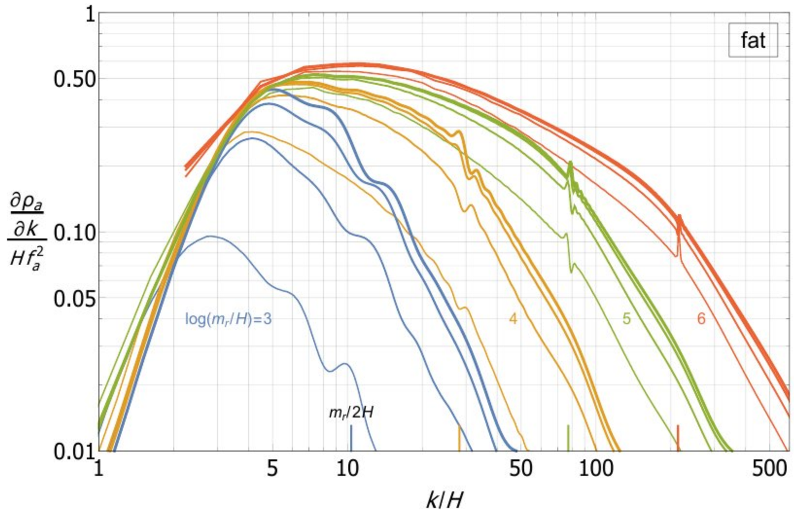


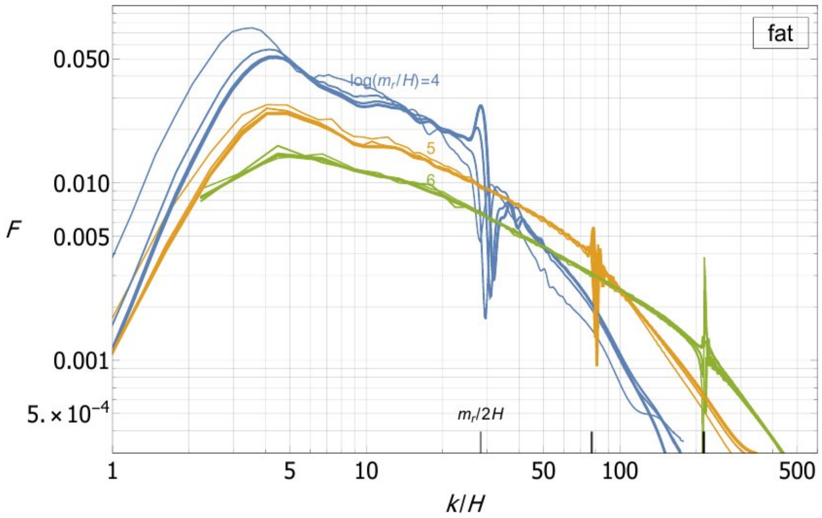
String Screening

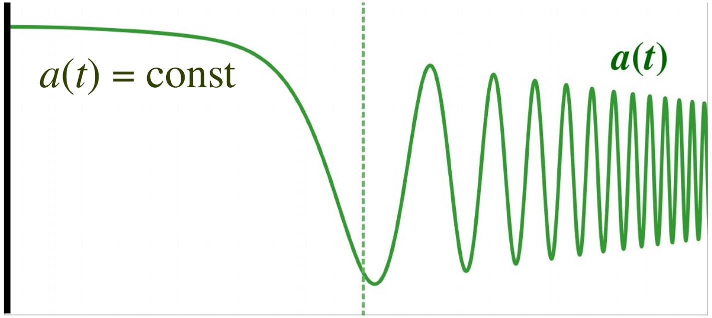


Scaling Solution *vs* Initial Conditions









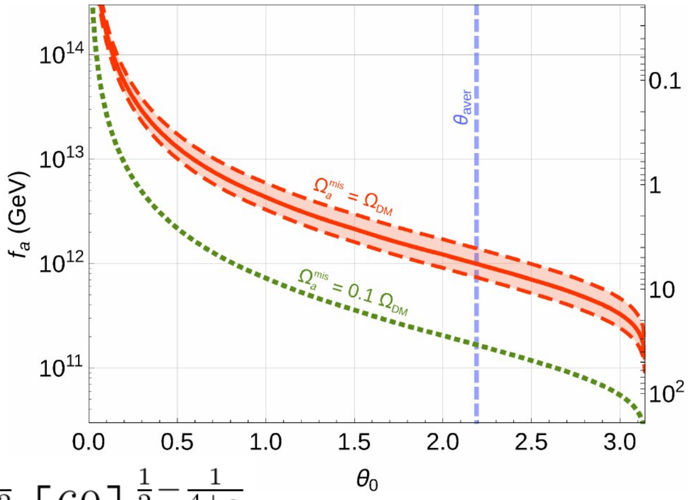
$$\frac{\langle \rho \rangle R^3}{m_a} = \text{const}$$

$$m_a(T_\star) = H(T_\star)$$

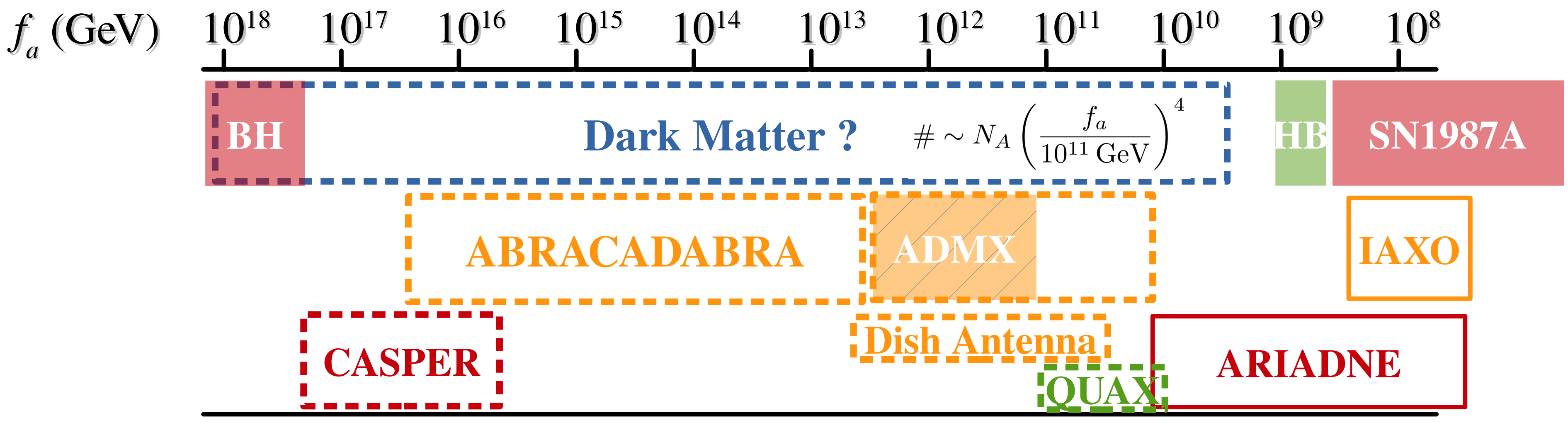
$$m_a(T) = \frac{\Lambda^2}{f_a}\left(\frac{\Lambda}{T}\right)^{\alpha/2} \qquad \frac{T_\star}{\Lambda} = \left[\frac{M_P}{f_a}\frac{3\sqrt{10}}{\pi g_\star^{1/2}}\right]^{\frac{2}{4+\alpha}}$$

$$\Omega_a = \frac{\rho_a^{\text{today}}}{\rho_c} = \Omega_\gamma \frac{m_a}{T_\gamma}\frac{86}{33}\frac{n_{a\star}}{s_\star}$$

$$\frac{n_{a\star}}{s_\star} = \frac{\langle m_a(T_\star) a_0^2(t_\star)\rangle}{\frac{2\pi^2}{45} g_\star T_\star^3} = k\frac{\theta_0^2 f_a^2}{\sqrt{g_\star} M_P T_\star}$$

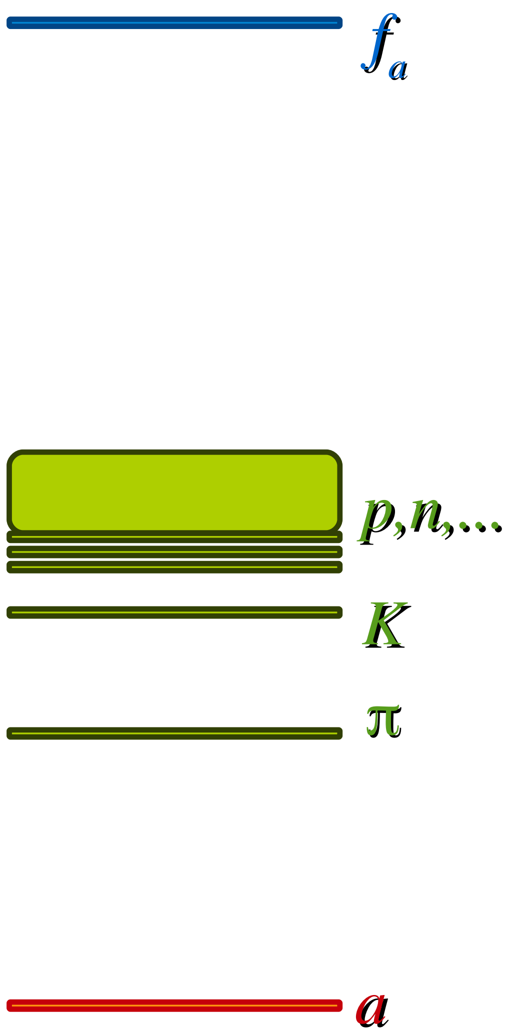


$$\Omega_a = 0.1\ k_\alpha \left[\frac{\theta_0}{2.15}\right]^2 \left[\frac{f_a}{10^{12}\ \text{GeV}}\right]^{1+\frac{1}{2+\alpha/2}} \left[\frac{60}{g_\star}\right]^{\frac{1}{2}-\frac{1}{4+\alpha}}$$

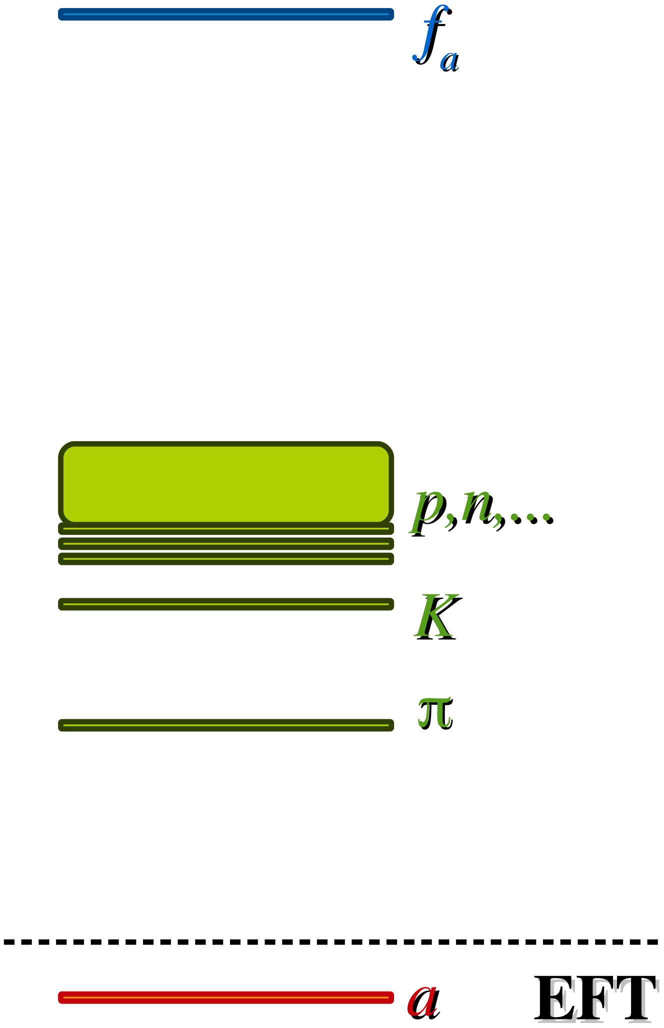


f_a (GeV)
10^{18}
10^{17}
10^{16}
10^{15}
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}
BH
Dark Matter ?
$\# \sim N_A \left(\frac{f_a}{10^{11}\,\mathrm{GeV}}\right)^4$
HB
SN1987A
ABRACADABRA
ADMX
IAXO
CASPER
Dish Antenna
ARIADNE
QUAX

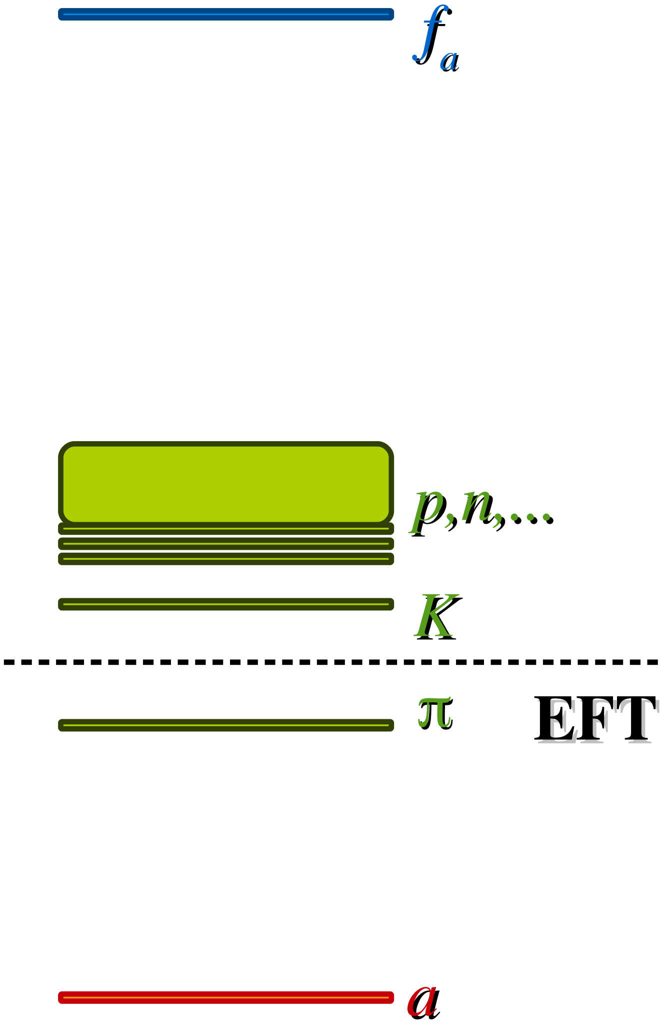
the QCD axion: *and its EFT*



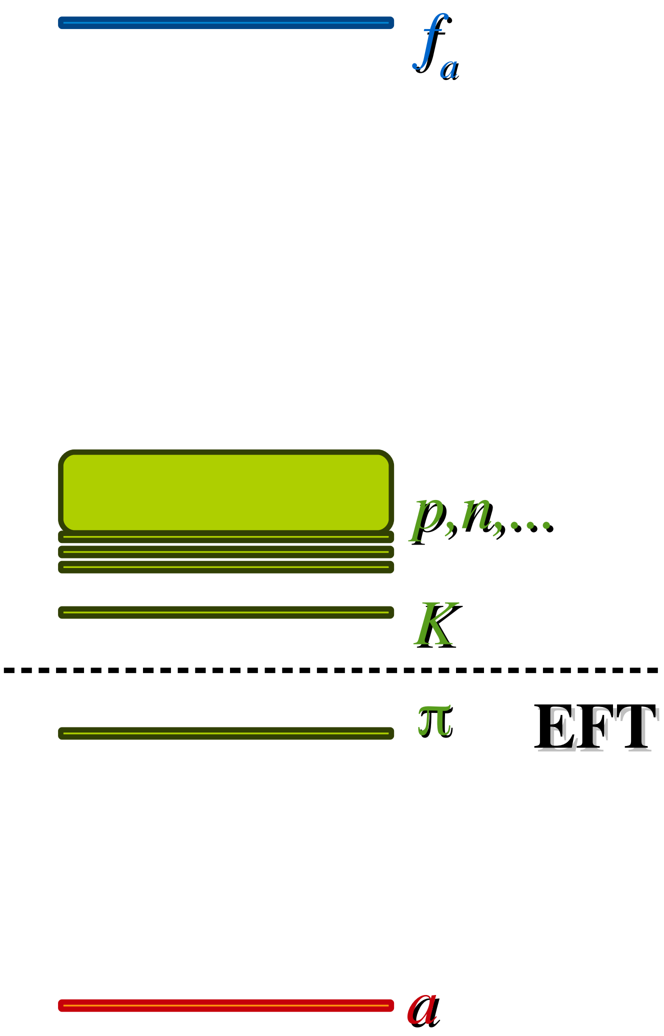
the QCD axion: *and its EFT*



the QCD axion: *and its EFT*

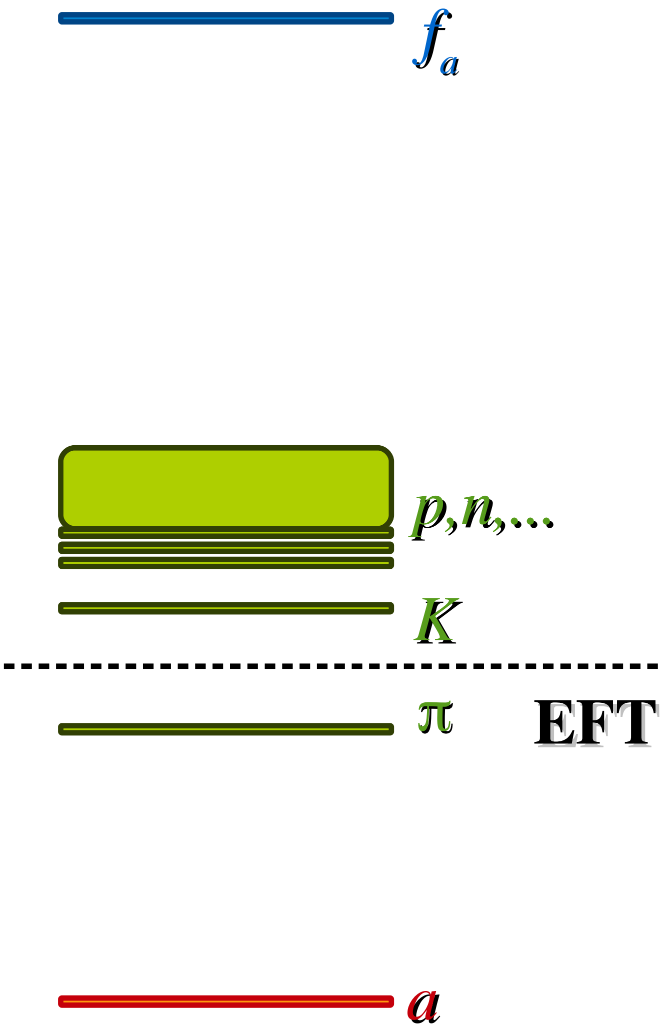


the QCD axion: *and its EFT*



$$\frac{a}{32\pi^2 f_a} G\tilde{G} \;\rightarrow\; \overline{q}_L M_q e^{ia/2f_a} q_R + h.c.$$

the QCD axion: *and its EFT*



$$\frac{a}{32\pi^2 f_a} G\tilde{G} \;\to\; \bar{q}_L M_q e^{ia/2f_a} q_R + h.c.$$

$$\boxed{\mathcal{L}_{\rm QCD}(A_\mu, M_q e^{ia/2f_a}) \to \mathcal{L}_{\rm ChPT}(A_\mu, M_q e^{ia/2f_a})}$$

$$A_\mu = \partial_\mu a/f_a$$

the axion is an external source
in the EFT at LO in $1/f_a$

the QCD axion: *potential*

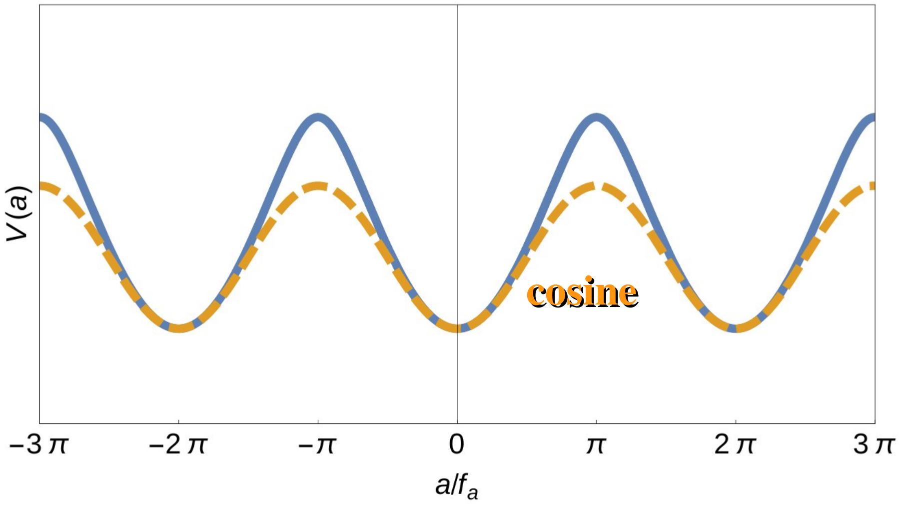
$$V(a,\pi) = -\frac{B_0 f_\pi^2}{2}\langle e^{-i\pi(x)/f_\pi} M_q e^{ia(x)/2f_a} + h.c.\rangle$$

the QCD axion: *potential*

$$V(a,\pi) = -\frac{B_0 f_\pi^2}{2}\langle e^{-i\pi(x)/f_\pi} M_q e^{ia(x)/2f_a} + h.c.\rangle$$

$$\boxed{V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4m_u m_d}{(m_u+m_d)^2}\sin^2\left(\frac{a}{2f_a}\right)}}$$

Di Vecchia Veneziano '80

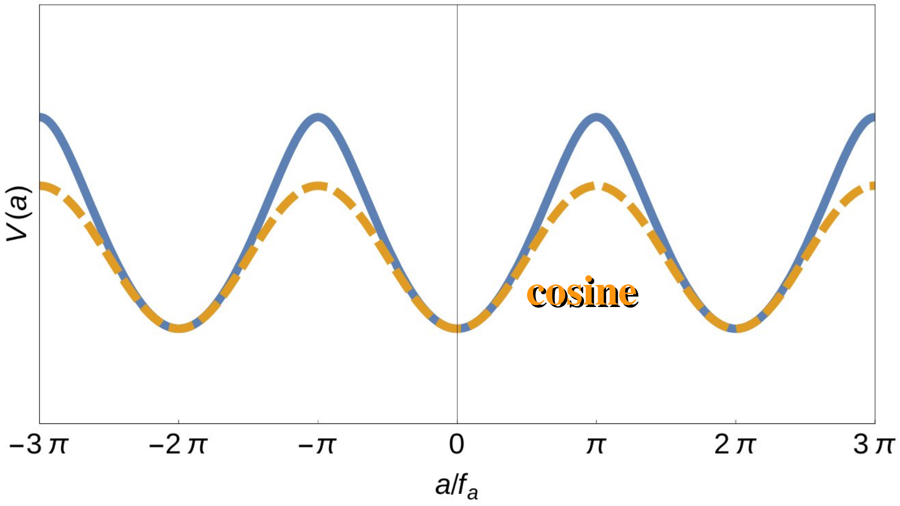


the QCD axion: *potential*

$$V(a,\pi) = -\frac{B_0 f_\pi^2}{2}\langle e^{-i\pi(x)/f_\pi} M_q e^{ia(x)/2f_a} + h.c.\rangle$$

$$\boxed{V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u+m_d)^2}\sin^2\left(\frac{a}{2f_a}\right)}}$$

Di Vecchia Veneziano '80



$$m_a^2 = \frac{m_u m_d}{(m_u+m_d)^2}\frac{m_\pi^2 f_\pi^2}{f_a^2}$$

Weinberg '78

the QCD axion: *the mass @NLO*

$$m_a^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2}$$

the QCD axion: *the mass @NLO*

Grilli, Hardy, Pardo, GV - 1511.02867

$$m_a^2 = \frac{m_u m_d}{(m_u+m_d)^2}\frac{m_\pi^2 f_\pi^2}{f_a^2}\left[1+2\frac{m_\pi^2}{f_\pi^2}\left(h_1^r - h_3^r - l_4^r + \frac{m_u^2 - 6 m_u m_d + m_d^2}{(m_u+m_d)^2} l_7^r\right)\right]$$

the QCD axion: *the mass @NLO*

Grilli, Hardy, Pardo, GV - 1511.02867

$$m_a^2 = \frac{m_u m_d}{(m_u+m_d)^2}\frac{m_\pi^2 f_\pi^2}{f_a^2}\left[1 + 2\frac{m_\pi^2}{f_\pi^2}\left(\underbrace{h_1^r - h_3^r - l_4^r}_{\boxed{4.8 \pm 1.4 \cdot 10^{-3}}} + \frac{m_u^2 - 6m_u m_d + m_d^2}{(m_u+m_d)^2}\underbrace{l_7^r}_{\boxed{7(4)\cdot 10^{-3}}}\right)\right]$$

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lattice average:

$$\boxed{z \equiv \frac{m_u^{\overline{\rm MS}}(2\ \text{GeV})}{m_d^{\overline{\rm MS}}(2\ \text{GeV})} = 0.48(3)}$$

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lattice average:

$$\boxed{z \equiv \frac{m_u^{\overline{\rm MS}}(2\ \mathrm{GeV})}{m_d^{\overline{\rm MS}}(2\ \mathrm{GeV})} = 0.48(3)}$$

$$\boxed{m_a = 5.70(6)(4)\ \mu\mathrm{eV}\left(\frac{10^{12}\mathrm{GeV}}{f_a}\right)}$$

$$(\chi^{top})^{1/4} = \sqrt{m_a f_a} = 75.5(5)\,\mathrm{MeV}$$

the QCD axion: *the mass beyond NLO*

Gorghetto, GV - 1812.01008

$$m_a = \Big[\underbrace{5.815(22)_z(04)_{f_\pi}}_{\text{LO}} \underbrace{-0.121(38)_{\ell_i^r}}_{\text{NLO}} \underbrace{-0.022(07)_{\ell_i^r}(05)_{c_i^r}}_{\text{NNLO}} \underbrace{+0.019(06)_{k_i^r}}_{\text{EM}}\Big]\,\mu\text{eV}\,\frac{10^{12}\ \text{GeV}}{f_a}$$

$$m_a = 5.691(51)\,\mu\text{eV}\,\frac{10^{12}\ \text{GeV}}{f_a}$$

$$\chi_{\text{top}}^{1/4} = 75.44(34)\ \text{MeV}$$

Finite Temperature QCD Axion Mass