Axion string dynamics

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1806.05566, PTEP 2018 (2018) 091E01 “with preliminary update”

(Hiramatsu, Kawasaki, Sekiguchi, MY, Yokoyama, 1012.5502, PRD 83, 123501(2011).
MY, Kawasaki, Yokoyama, hep-ph/9811311, PRL 82, 4578(1999).)

$$c = \hbar = M_G^2 = 1/(8\pi G) = 1$$

Contents

- **Introduction**

  What is a DM ? What is an axion ?

- **Axion abundances**

  (Axion) string dynamics

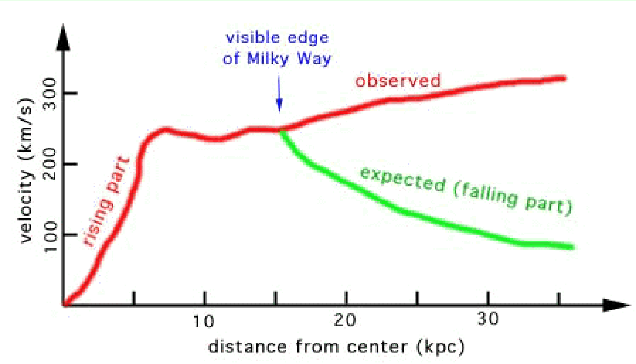
  Spectrum of axions radiated from strings.

- **Discussion and conclusions**

Introduction

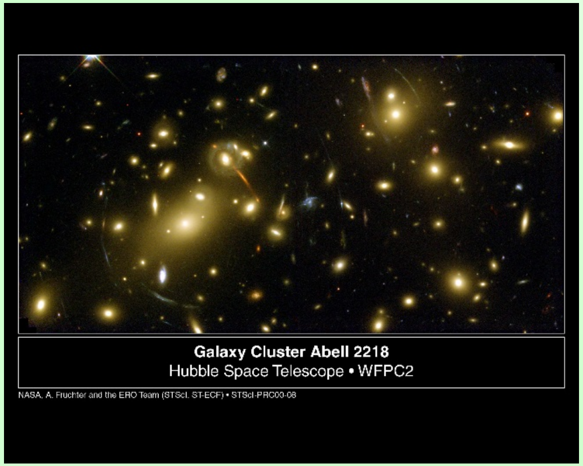
Evidences of the presence of (cold) DM

Rotation Curves of Milky Way



http://personal.psu.edu/mxe17/A020S/pages/darkmatter.html

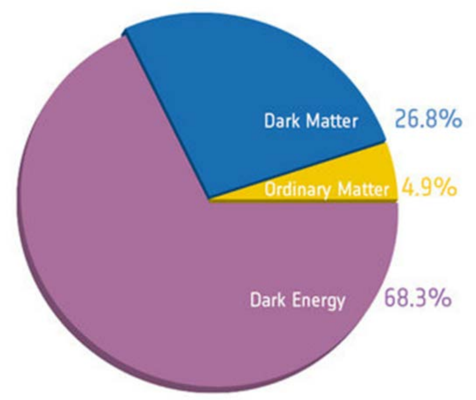
Abell 2218, z = 0.175



HST

Bullet Cluster

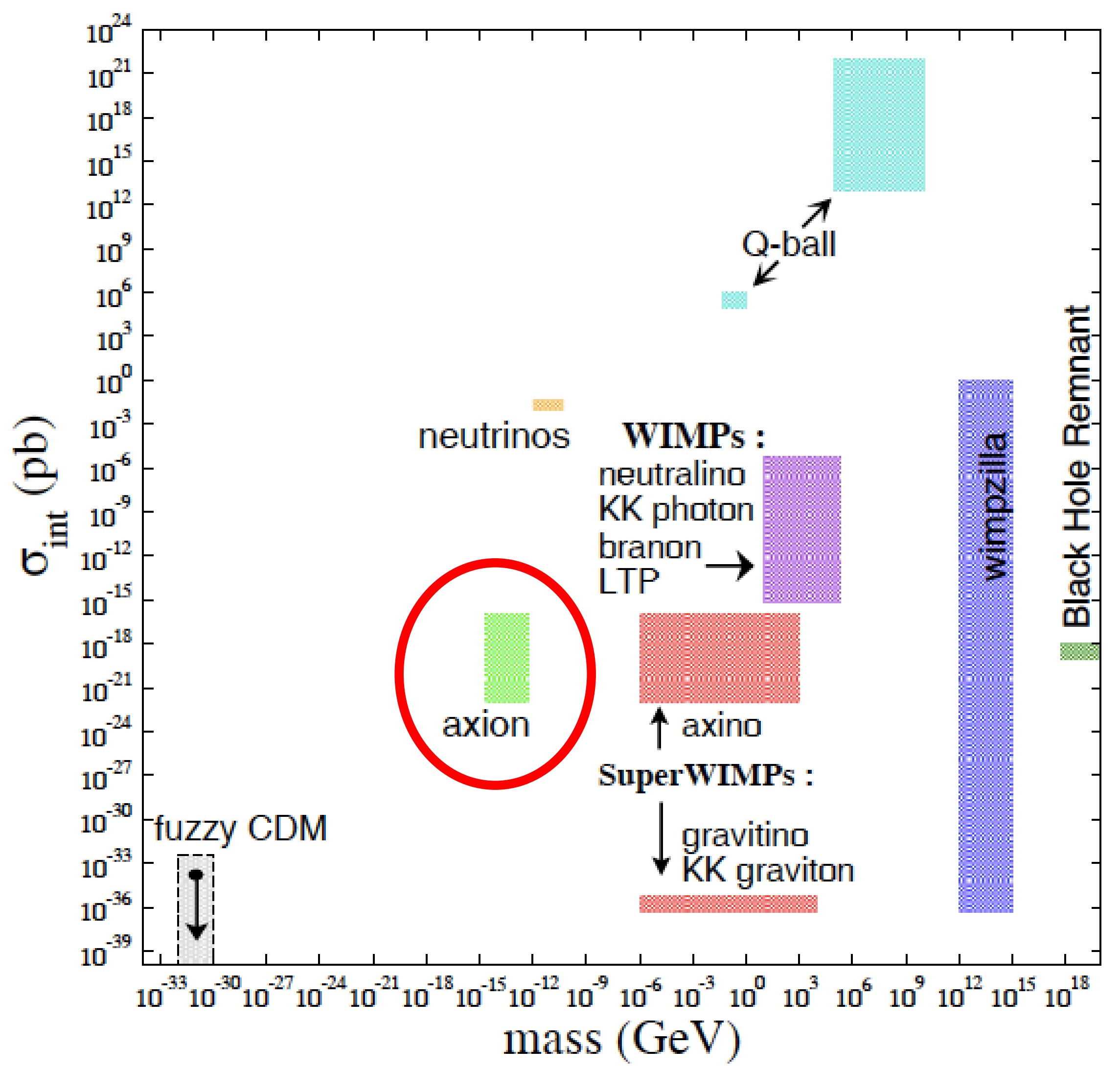
X-ray: NASA/CXC/CfA/ M.Markevitch et al.;
Lensing Map: NASA/STScI; ESO WFI; Magellan/U.Arizona/ D.Clowe et al
Optical: NASA/STScI; Magellan/U.Arizona/D.Clowe et al.



PLANCK

All of these data suggest
the presence of CDM.

Candidates of DM



Gardner & Fuller,
1303.4758
Prog. Part. Nucl. Phys.
71 (2013) 167

What’s an axion (in our community) ?

- **Strong CP problem in QCD**

$$\mathcal{L}_{\mathrm{QCD}} = \mathcal{L}_{\mathrm{pert}} + \bar{\theta}\frac{g_s^2}{32\pi^2}\epsilon^{\mu\nu\alpha\beta}F^a_{\mu\nu}F^a_{\alpha\beta}, \qquad \bar{\theta} = \theta_{\mathrm{QCD}} - \theta_F$$

$$\theta_F \equiv \arg\det\left(Y_d Y_u\right)$$

↑ CP conserving ↑ CP non-conserving

↑ Yukawa matrices

Theoretically, all of $\bar{\theta}\,(0 \le \bar{\theta} < 2\pi)$ **are equivalent.**

But, a neutron electric dipole moment:

$$d_N = (5.2 \times 10^{-16} e\,\mathrm{cm})\,\bar{\theta} < 2.9 \times 10^{-26} e\,\mathrm{cm}$$

⇒ $\bar{\theta} < 10^{-10}$ **Why so small ?**

Peccei-Quinn (anomalous) U(1)$_{PQ}$ symmetry

Effectively makes the parameter $\overline{\theta}$ dynamical

↓

Such a dynamical field : Axion, a(x)

↓

Observable: $\theta(x) = \overline{\theta} - a(x)/f_a$

↕

Dynamically $\overline{\theta} \longrightarrow 0.$

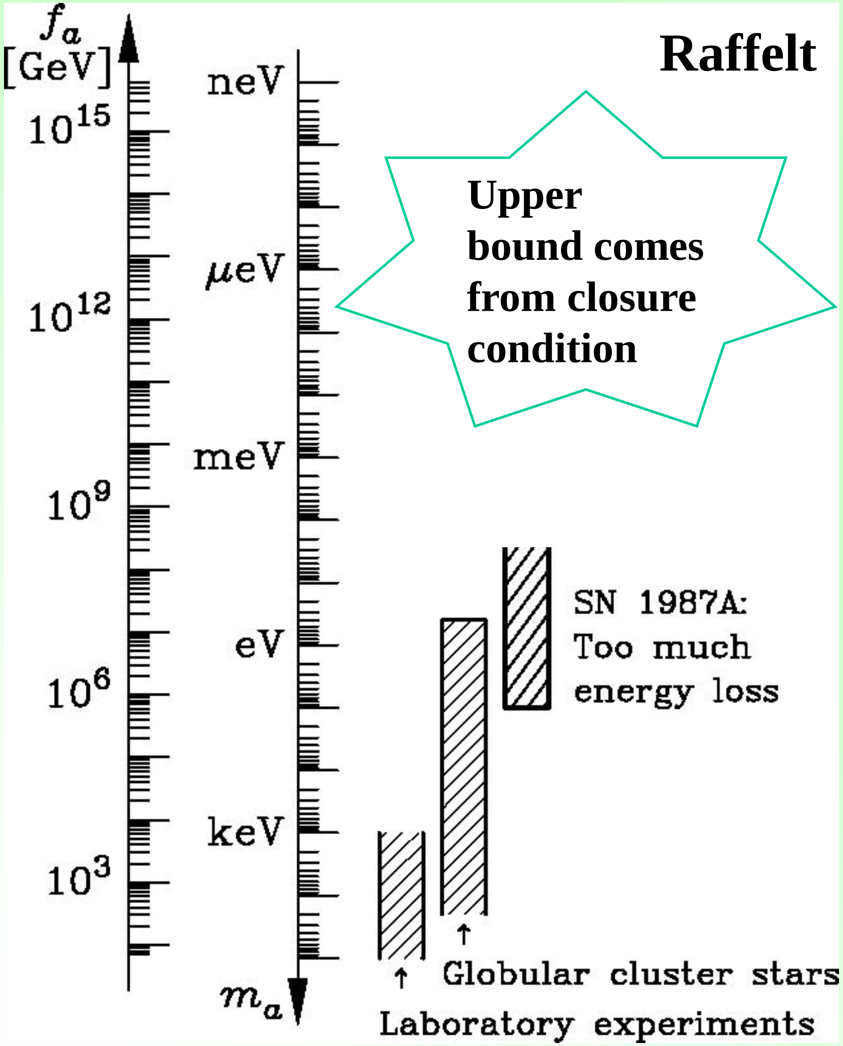
$$V(\overline{\theta}, a) = F_\pi^2 m_\pi^2 \left[1 - \cos\left(\overline{\theta} - \frac{a(x)}{f_a}\right)\right]$$

Peccei & Quinn : introduce a global U(1) symmetry

↓ **"spontaneously broken"**

Axion is identified with NG boson of U(1)$_{PQ}$ symmetry

Constraints on f_a



Upper bound comes from closure condition

Strength of a coupling of an axion

$$\propto \frac{1}{f_a} \propto m_a$$

Lower bounds on fa come from astrophysical objects:

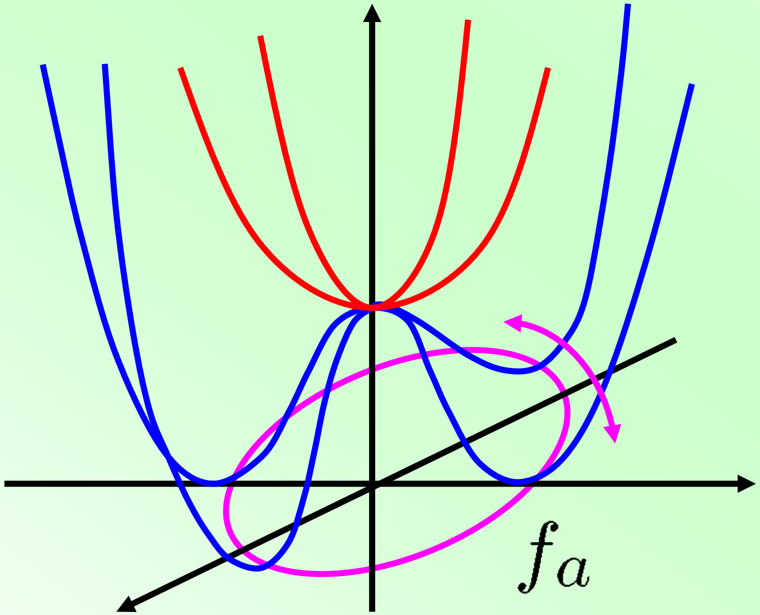
$f_a \gtrsim 4 \times 10^8$ **GeV (SN 1987A)**

(N.B. Bar et al. 1907.05020 casts doubt on this bound.)

On the other hand, upper bound comes from the (over)closure condition.

Production mechanism of axions

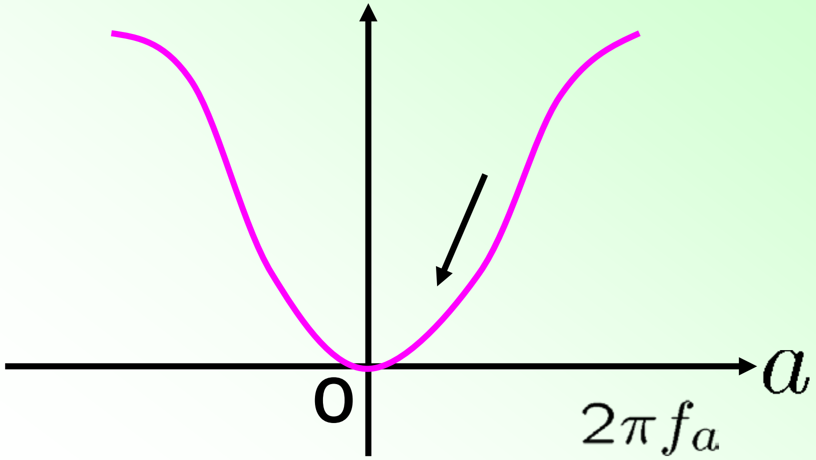
Potential of Peccei-Quinn scalar



(i) U(1) symmetry breaking => strings

Axions are emitted from strings.

In this talk, we concentrate on this abundance.



(ii) After QCD phase transition
=> string-wall system

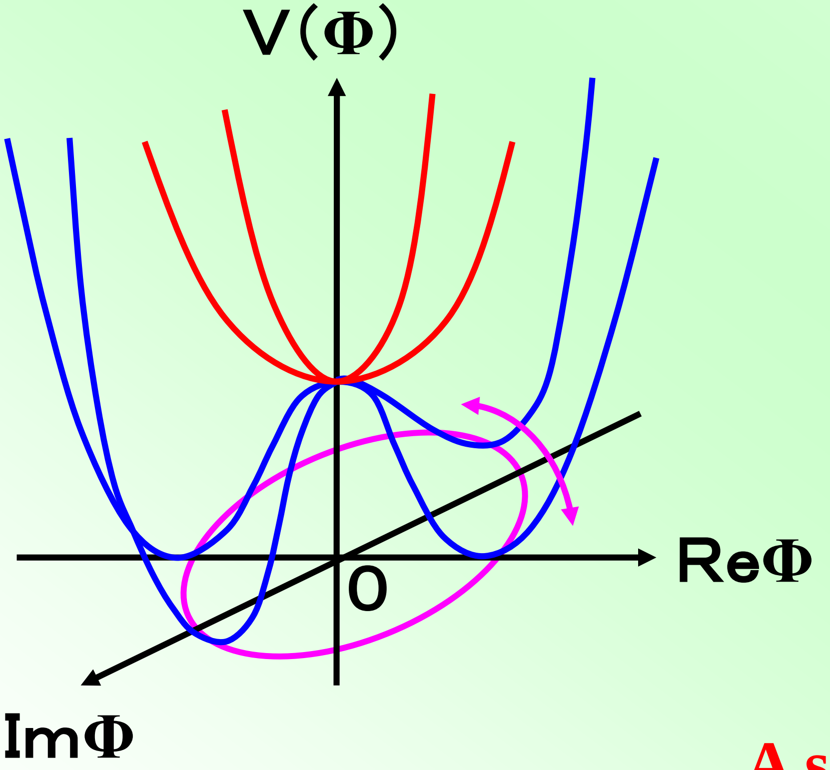
Axions are emitted from such wall system.

In addition, coherent oscillation of an axion contributes the energy density of the Universe.

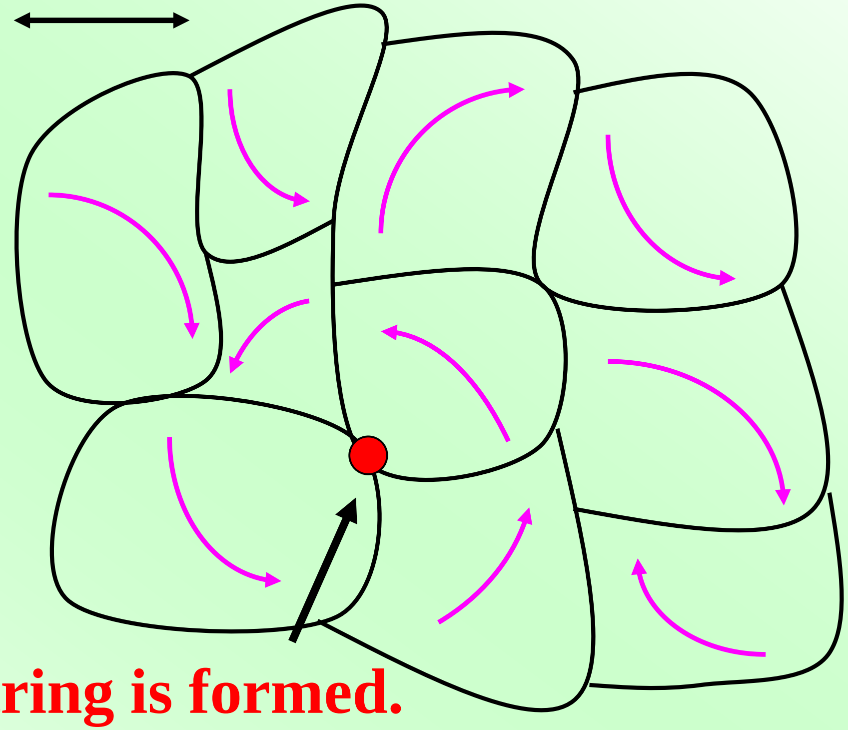
(In case the PQ symmetry is broken only before inflation, only the last contributes.)

Axion strings

Cosmic strings are formed after the U(1) symmetry breaking.



Horizon (correlation) scale at $T \simeq f_a$



A string is formed.

Axions are emitted from such axion strings.

Abundance of axions emitted from axion strings

In order to estimate the abundance of these axions, we need the following information:

i) How many axion strings exist at each time ?
⇔ How much energy is stored in axion strings ?

ii) How many axions are emitted from a string ?
(spectrum ⇔ average energy of emitted axion)

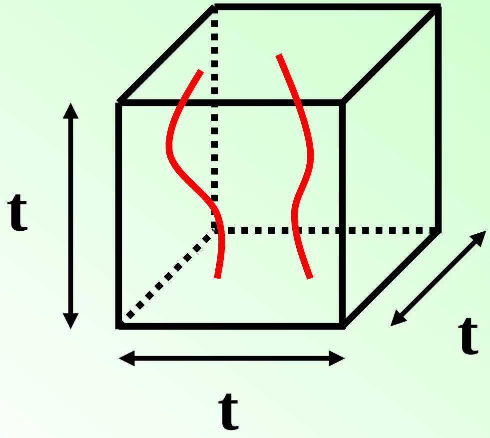
These issues have had long history and debates, and still have.

(My 1999 paper gave the result of the first realistic simulation, but …)

i) How many axion strings exist at each time ?

Scaling property (evolution of cosmic strings in an expanding Universe)

The number of long string per horizon is constant irrespective of cosmic time.



This number is constant, ξ.

（this constant number depends on the background dynamics.）

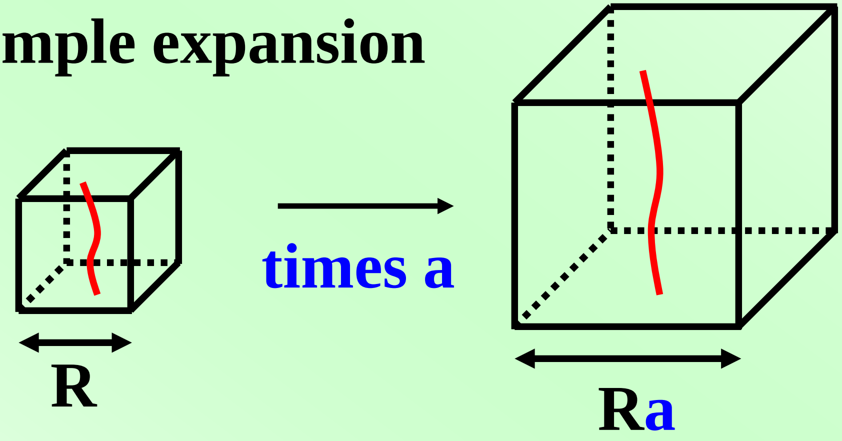
$$\rho_s = \frac{\xi \cdot \mu \cdot t}{t^3} = \xi \frac{\mu}{t^2} \propto t^{-2} \propto a^{-4}(\mathrm{RD}),\ \ a^{-3}(\mathrm{MD})$$

μ : tension (energy per unit length of a string)

How can we understand this property intuitively ?

Intuitive understanding of scaling property

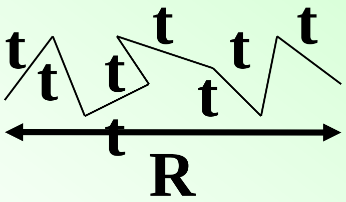
- simple expansion



$$\rho_s = \frac{\mu \cdot Ra}{(Ra)^3} \propto a^{-2}$$

- Tension works only inside horizon, then a string configuration is random beyond horizon.

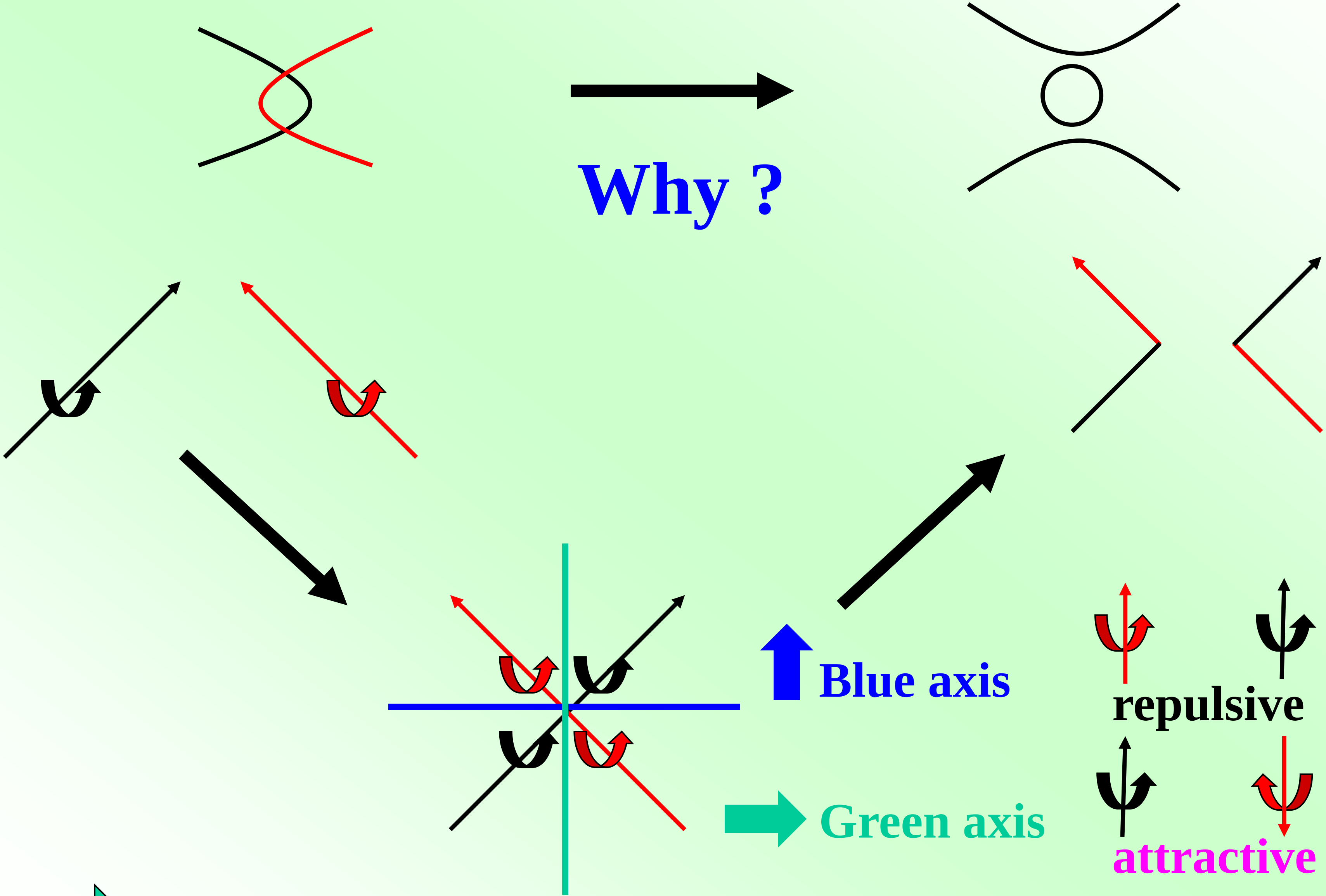
Total length L within interval R : $L \sim \frac{R^2}{\zeta}$



random walk with persistence length $\zeta \sim t$

$$\rho_s = \frac{\mu L}{R^3} = \mu \frac{1}{R\zeta} \propto \frac{1}{at} \propto a^{-3}(\mathrm{RD}),\ a^{-5/2}(\mathrm{MD})$$

Intercommutation



Why ?

⬆ Blue axis

➡ Green axis

repulsive

attractive

➡ Scaling property

One scale model

(Kibble, many others)

$$\dot{\rho}_\infty = -2H\left(1+\langle v^2\rangle\right)\rho_\infty$$ ⟸ **EOM from NG action**

↑ **cosmic expansion** ↑ **random walk**

We assume the presence of typical scale L(t) characterizing the system:

⟹ $$\rho_\infty = \frac{\mu}{L(t)^2}$$

(a string with its length L in the box L^3)

By taking into account of the intercommutation effect, $\dot{\rho}_{\infty\to\mathrm{loop}} = c\frac{\rho_\infty}{L}$

⟹ $$\dot{\rho}_\infty = -2H\left(1+\langle v^2\rangle\right)\rho_\infty - c\frac{\rho_\infty}{L}$$

$L(t) \equiv \gamma(t)\cdot t$

⟹ $$\frac{\dot{\gamma}}{\gamma} = -\frac{1}{2t}\left[\left(1-\langle v^2\rangle\right)-\frac{c}{\gamma}\right]$$ ⟹ $$\gamma_{\mathrm{fix}} = \frac{c}{1-\langle v^2\rangle}$$ **: stable fixed point**

⟹ $$L(t) = \gamma_{\mathrm{fix}}t \propto t$$ ⟹ $$\rho_\infty = \frac{1}{\gamma_{\mathrm{fix}}^2}\frac{\mu}{t^2} \propto \frac{1}{t^2}$$

Local string (based on gauge symmetry breaking)

Gradient energy around a string core is cancelled by gauge field.

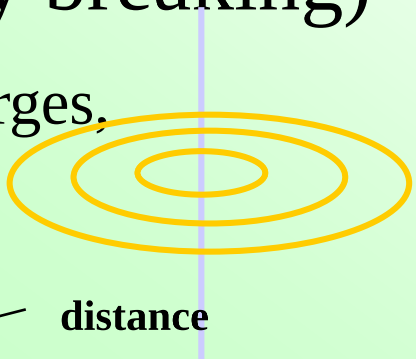
String is thin (energy is stored mostly in the core) and can be well approximated by Nambu-Goto action.

Albrecht & Turok
Bennet & Bouchet
Allen & Shellard
Hindmarsh et al.
Hiramatsu et al.

Scaling is confirmed ⇒ $\xi \sim 10$

◎Global string (based on global symmetry breaking)

Gradient energy around a string core apparently diverges, but has a cutoff given by a neighborhood string.



(~t)

$$\mu \sim \int d^2r|\nabla\Phi|^2 = 2\pi\int dr\,r\left|\frac{1}{r}\frac{\partial\Phi}{\partial\theta}\right|^2 = 2\pi f_a^2 \ln\frac{r_{\rm cut}}{\delta}$$

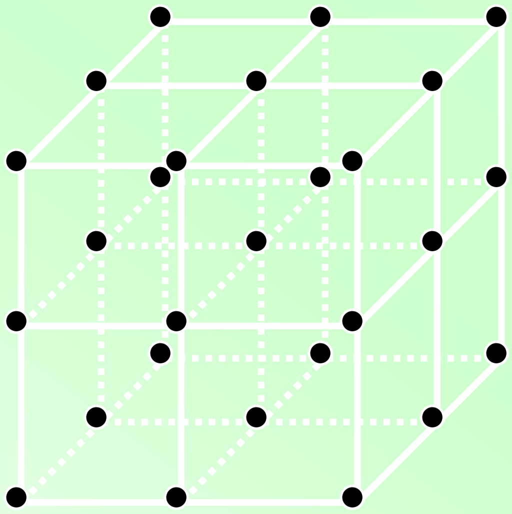
distance

core radius

A force proportional to the inverse separation works between strings.

Try to confirm the scaling property by solving the dynamics of complex scalar fields.

Numerical simulations



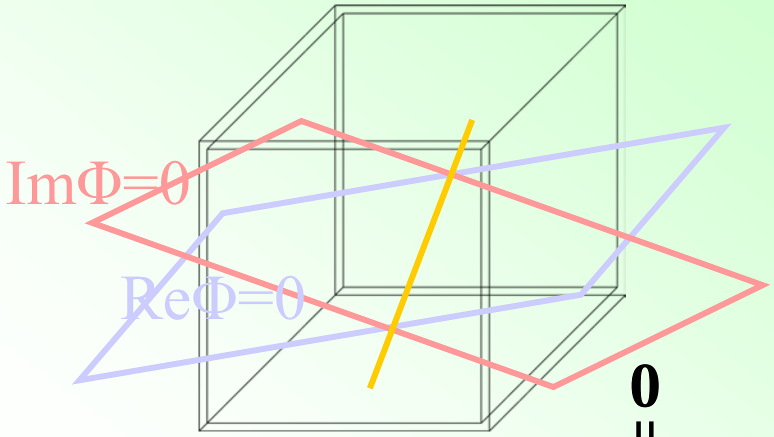
Solve the EOMs of a complex scalar field in the expanding Universe and follow the formation and evolution of strings.

(The string width is fixed, not fat string)

A big obstacle is how to identify a string.

(MY & Yokoyama 2002, Hiramatsu et al. 2011)

4096^3 lattices in this work (256^3 in 1999 paper)



String core is identified with the intersect of two planes with ReΦ=0 and ImΦ= 0, which enables to evaluate the exact position.

We can follow the dynamics of strings, and hence evaluate their velocities.

$$v = \frac{(\nabla\Phi \times \nabla\Phi^*) \times (\Phi'\nabla\Phi^* - \Phi'^*\nabla\Phi)}{(\nabla\Phi \times \nabla\Phi^*)^2}.$$

$$\left(\overset{\mathbf{0}}{\overset{\shortparallel}{\Phi(x,t)}} \simeq \overset{\mathbf{0}}{\overset{\shortparallel}{\Phi(x_0,t_0)}} + \nabla\Phi(x_0,t_0)\cdot(x-x_0) + \dot{\Phi}(x_0,t_0)(t-t_0) \right)$$

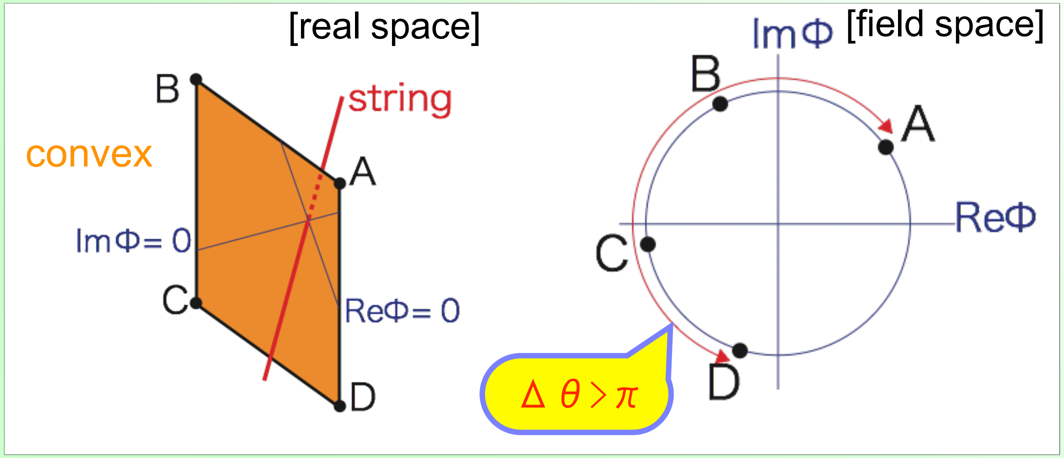
Identification of strings

Identification of strings — non-trivial due to discrete nature of lattice

Our method (Hiramatsu et al. 2010):

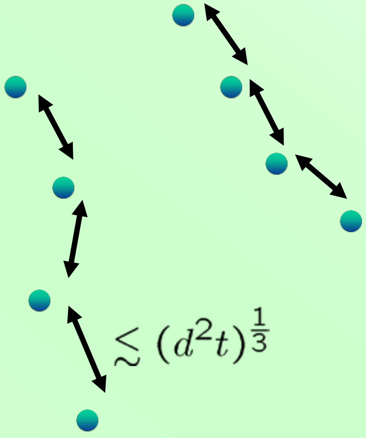
Phase of Φ on all the vertices of a convex penetrated by a string ranges > π.

OK even if string is moving.

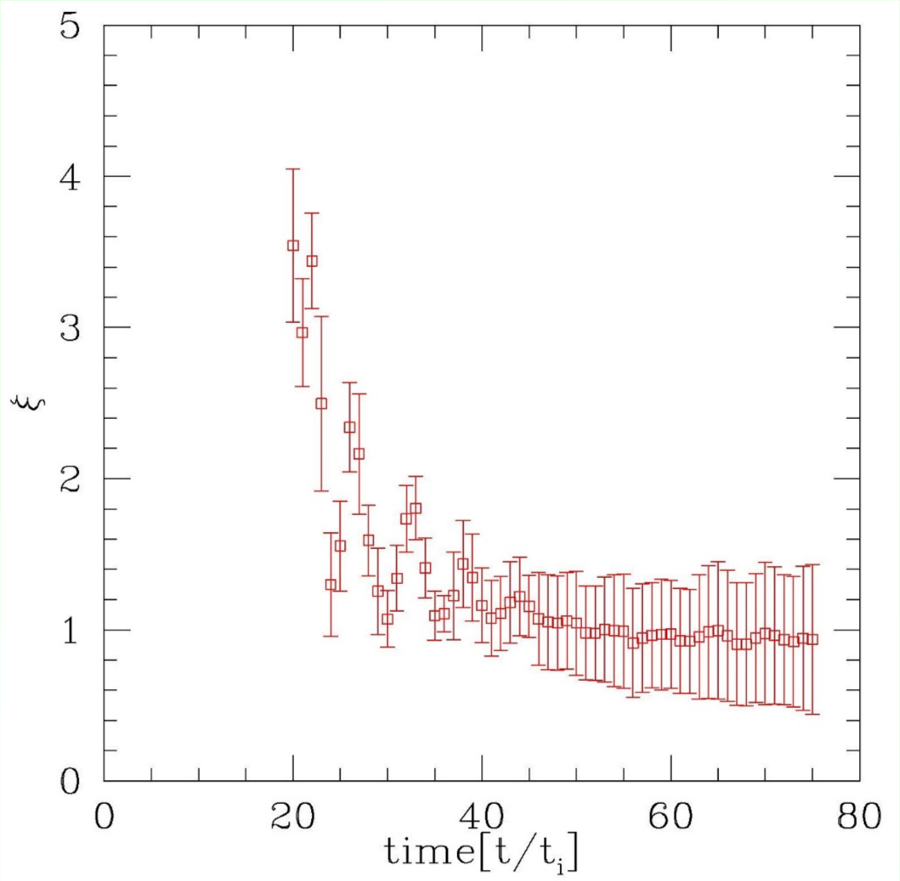


Separation of loop and infinite strings

- Grouping string points via Friends-of-Friends algorithm.
- A group of string points with L < 2 π t is regarded as a loop; otherwise as an infinite string.
- Loops/infinite strings are separated automatically in field theoretic simulation of axion strings.



Is the scaling property confirmed ?



My old paper's results

$$\rho_S = \xi \frac{\mu}{t^2}$$

Intercommutation happens more because of the long-range force.

The initial state is a thermal equilibrium and phase transition happened.

Scaling is almost confirmed ⇒ $\xi \sim 1 << 10$

(Yamaguchi et al. 1999, Yamaguchi & Yokoyama 2002, Hiramatsu et al. 2011)

Simulation setup

Starting from thermal equilibrium state : $T > T_c = \sqrt{6}v$

$$V_{\text{eff}}[\Phi; T] = \lambda(|\Phi|^2 - v^2)^2 + \frac{\lambda}{3}T^2|\Phi|^2,$$

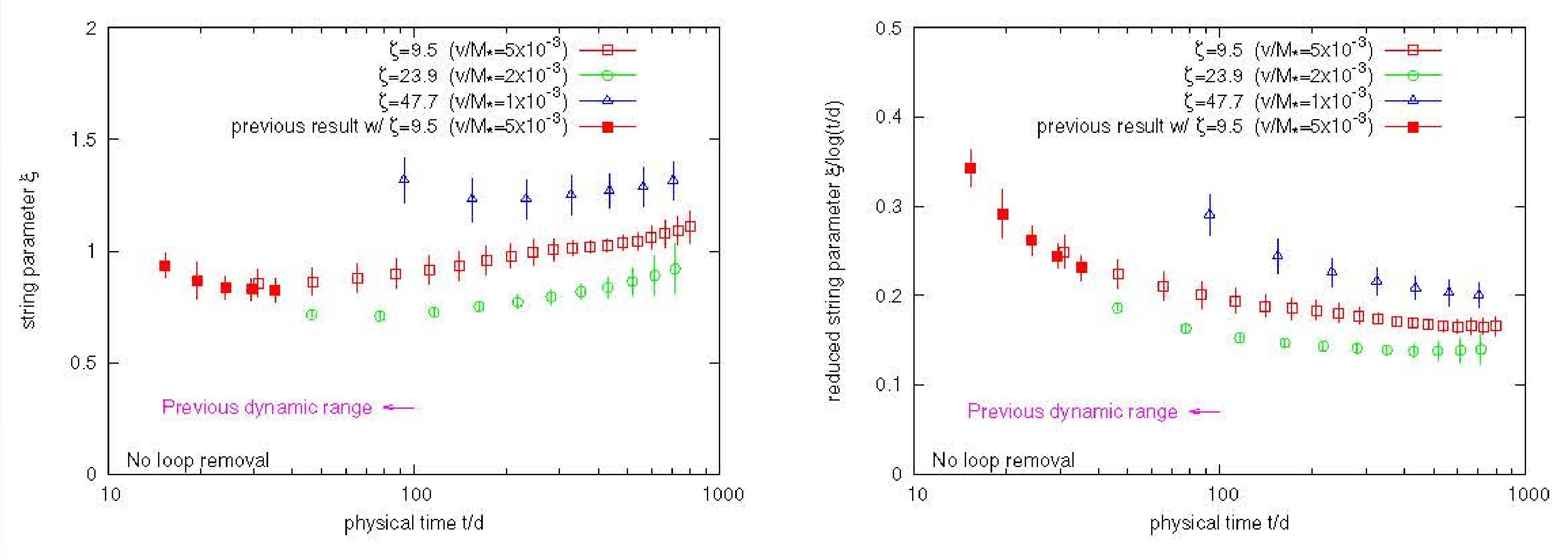
$$\zeta = \left(\frac{45\lambda M_*^2}{2\pi^2 g_* v^2}\right)^{1/2} . \quad g_* = 1000, \quad \lambda = 1. \quad M_* = 1/\sqrt{8\pi G}$$

$$v/M_* = 0.005, 0.002, 0.001 \leftrightarrow \zeta = 9.5, 23.9, 47.8$$

At final time, the string width $d \simeq (\sqrt{\lambda}v)^{-1}$ coincides with the lattice spacing and the box side is comparable to 2 H_f^{-1} = 4 t_f.

$$\rho_s = \xi \frac{\mu}{t^2}$$

Update



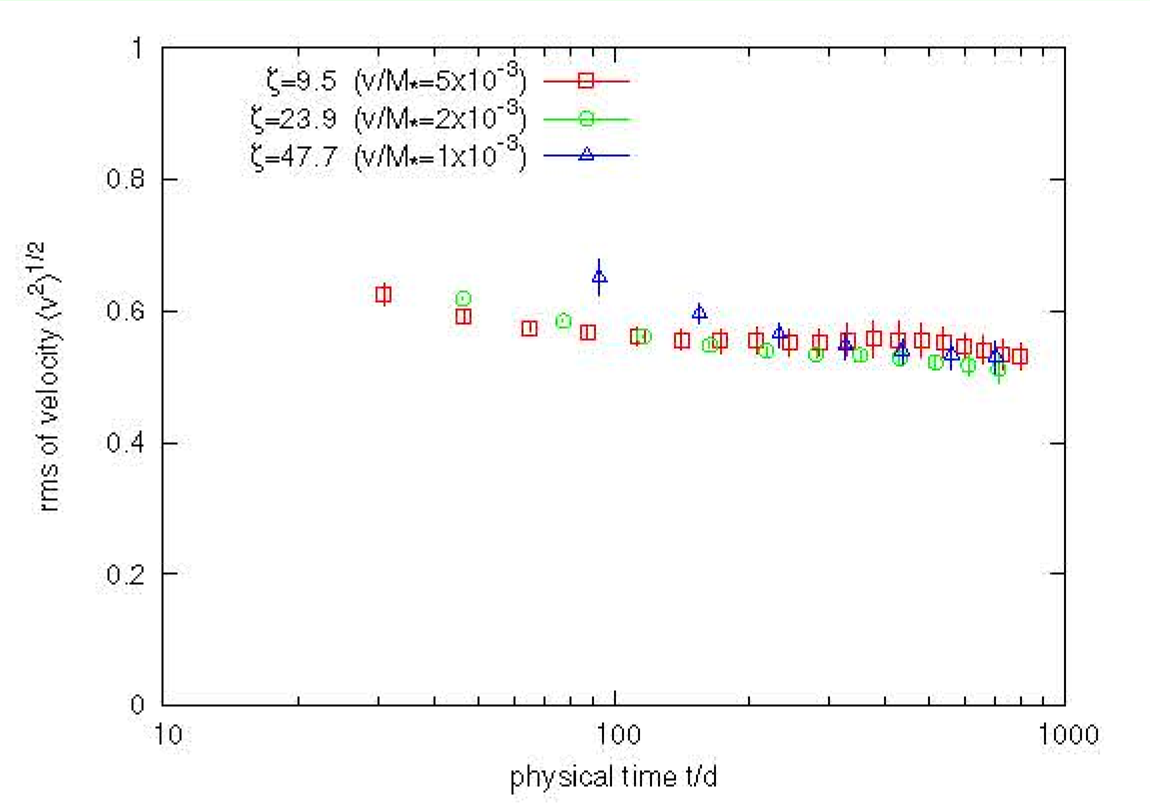
We observe the log-dependence of ξ !!!

Scaling property might be slightly broken due to the logarithmic dependence.

c.f. Fleury et al. 1509.00026
Klaer & Moore 1707.05566
Klaer & Moore 1708.07521
Gorghetto et al.1806.04677
Vaquero et al. 1809.09241
Buschmann et al. 1906.00967

See also Hindmarsh 1908.03522

String velocities



Initially, rms of velocity ~ 0.7 consistent with previous works,
However, as time advances, it decreases gradually and reads around 0.5 at the later time of the simulation.

This is substantially smaller compared to the velocity estimated from simulation of local-string based on the Nambu-Goto action,
but consistent with field-theoretic simulation of local strings.

At this moment, we are unable to tell if the velocity has settled down already within the simulation time or continues to decrease subsequently.

Present abundance of axions (radiated from axion string)

The dynamics of (long) axion strings

(Loops finally disappear by emitting axions)

Energy density of radiated axions
（axions are relativistic at emission）

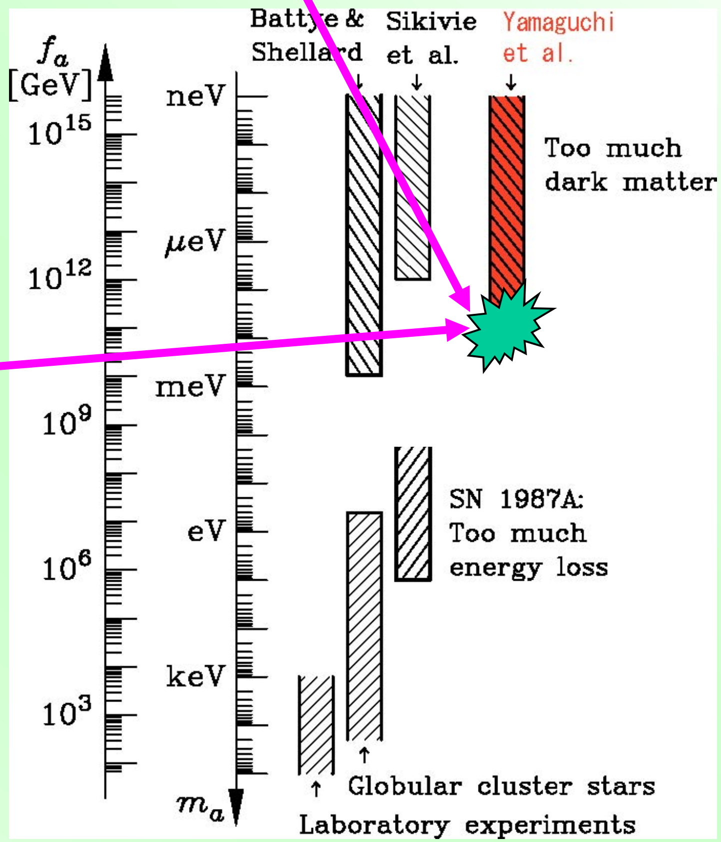
Spectrum, average energy

Current number density

(Axions are now non-relativistic.)

Multiplying its mass leads to the current energy density.

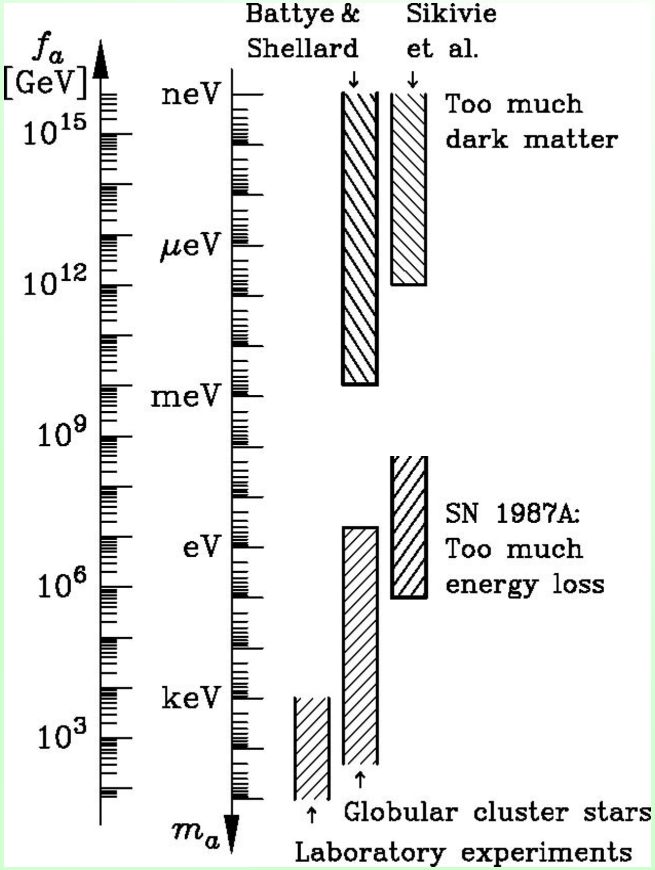
Raffelt



ii) Energy spectrum of axions radiated from axion strings

(We would like to know the number of axions to evaluate the final abundance.)

Raffelt



- Davis, Battye, and Shellard

Spectrum has a sharp peak around the horizon.

Strings oscillate many times before emission.

- Sikivie et al

Spectrum decreases in proportional to wavenumber.

Average energy is enhanced by the factor log(t_0/d) ~ 70, which leads to less number of axions.

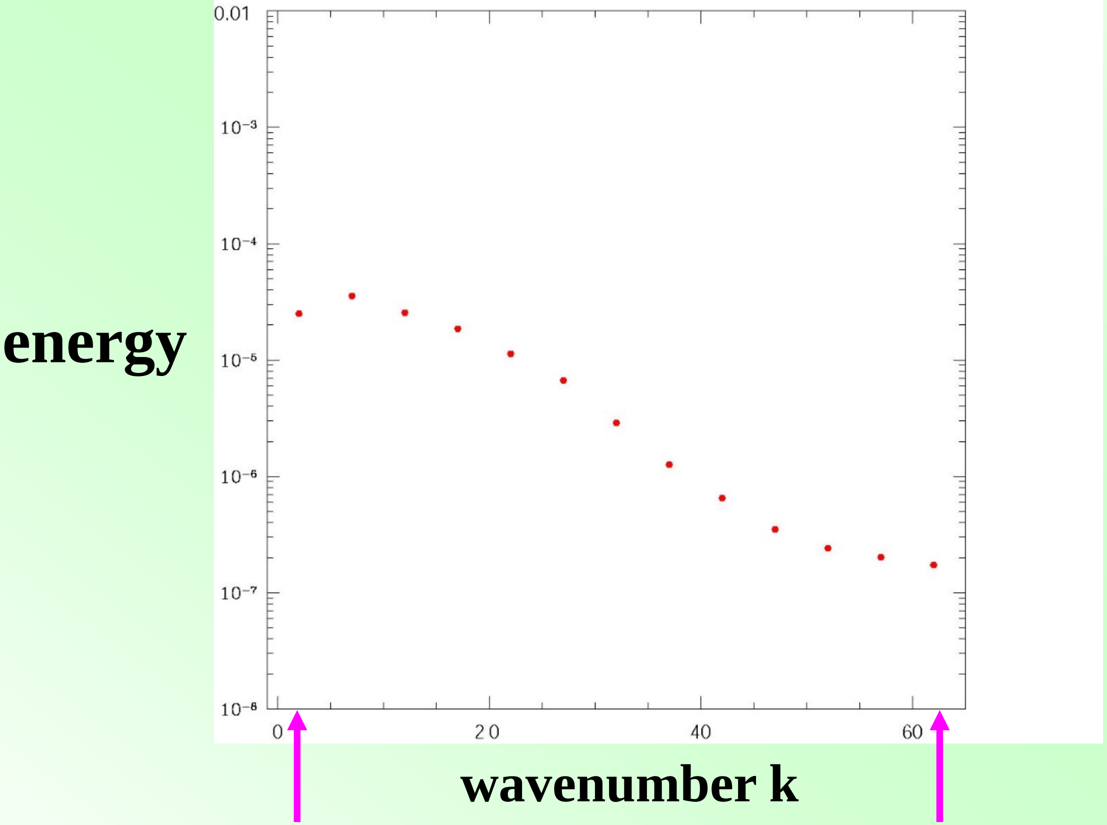
Strings oscillate almost once before emission.

$$\mu \sim \int d^2r |\nabla\Phi|^2 = 2\pi \int drr \left|\frac{1}{r}\frac{\partial\Phi}{\partial\theta}\right|^2 = 2\pi f_a^2 \ln\frac{r_{\rm cut}}{\delta}$$

$$\Longrightarrow \mu \propto \ln(1/d) \propto \ln k \Longrightarrow \frac{d\mu}{dk} \propto \frac{1}{k}$$

Which is correct ?

Spectrum axions emitted from strings



Yamaguchi et al. 1999

(inverse) Horizon scale

width of string

A sharp peak around the horizon scale

(But, not enough dynamic range)

Estimation for the spectrum of emitted axions

(Hiramatsu et al. 2011)

- Remove contamination from strings

$$\dot{a}(\vec{x},t) = \dot{a}_{\mathrm{free}}(\vec{x},t) + (\text{contamination from strings}).$$

$$\frac{1}{2}\langle \dot{a}_{\mathrm{free}}(\vec{k},t)^{*}\dot{a}_{\mathrm{free}}(\vec{k}',t)\rangle = \frac{(2\pi)^3}{k^2}\delta^{(3)}(\vec{k}-\vec{k}')P_{\mathrm{free}}(k,t)$$

- Pseudo power spectrum estimator (PPSE)

(Hiramatsu et al. 2011)

  - masking

window function : $W(\vec{x}) = \begin{cases} 0 & \text{(near strings)} \\ 1 & \text{(elsewhere)} \end{cases}$

➡ $\tilde{\dot{a}}(\vec{x}) \equiv W(\vec{x})\dot{a}(\vec{x}) = W(\vec{x})\dot{a}_{\mathrm{free}}(\vec{x}).$

convolved power spectrum :

$$\langle \tilde{P}(k)\rangle \equiv \int \frac{d\hat{k}}{4\pi}\frac{k^2}{V}\frac{1}{2}\langle|\tilde{\dot{a}}(\vec{k})|^2\rangle = \int \frac{dk'}{2\pi^2}\frac{k^2}{V}M(k,k')P_{\mathrm{free}}(k') \neq P_{\mathrm{free}}(k)$$

$$M(k,k') \equiv \frac{1}{V^2}\int \frac{d\Omega_k}{4\pi}\frac{d\Omega_{k'}}{4\pi}\left|W(\vec{k}-\vec{k}')\right|^2$$

  - deconvolution

with

$$\langle \hat{P}(k)\rangle \equiv \frac{k^2}{V}\int \frac{dk'}{2\pi^2}M^{-1}(k,k')\langle \tilde{P}(k')\rangle = P_{\mathrm{free}}(k)$$

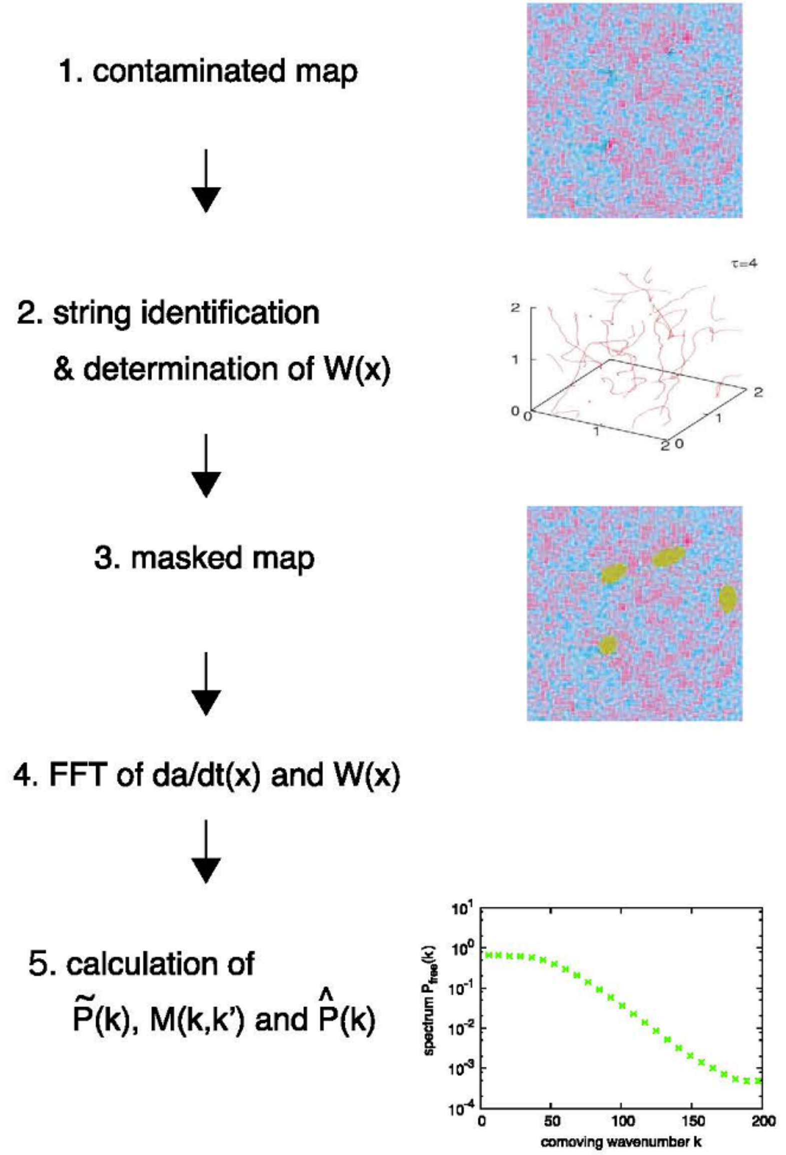


FIG. 2 (color online). Schematic overview of our pipeline.

Contamination removal

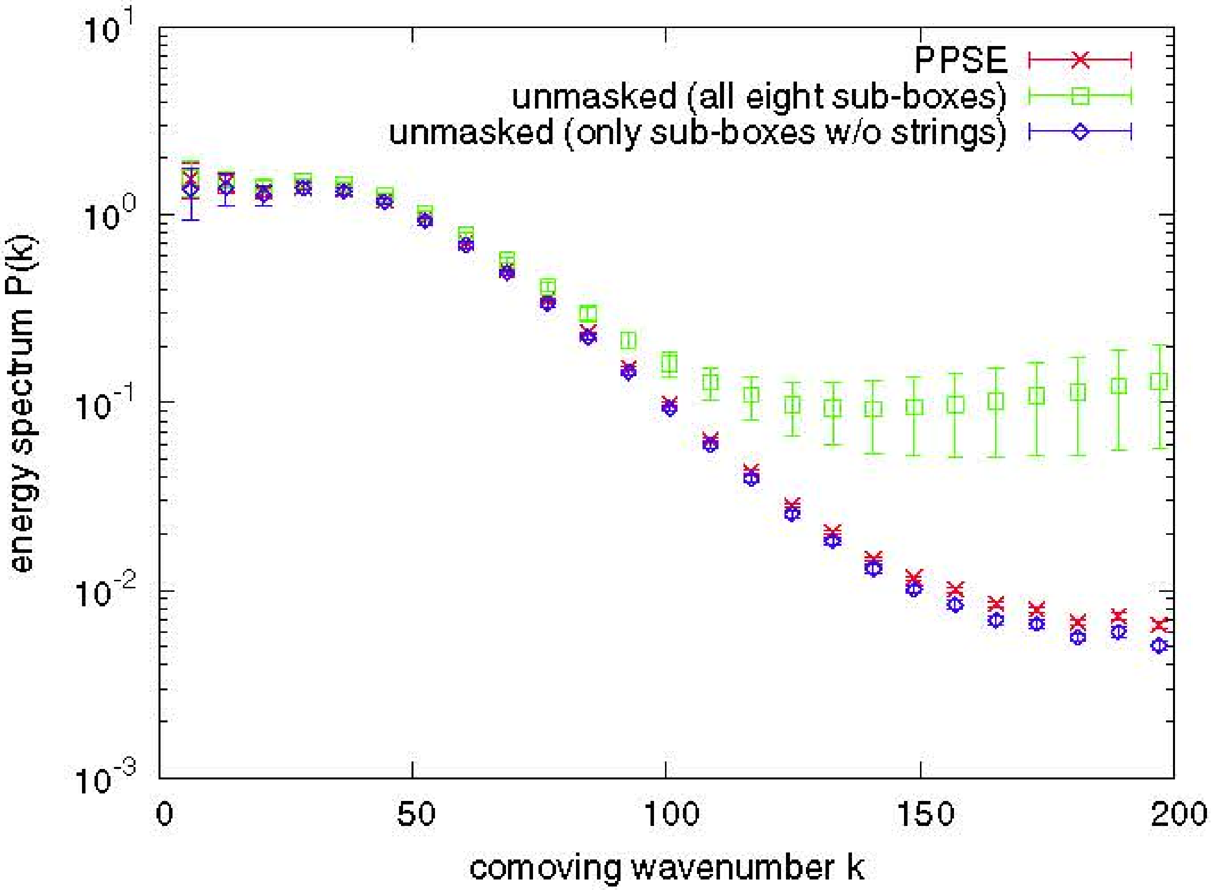
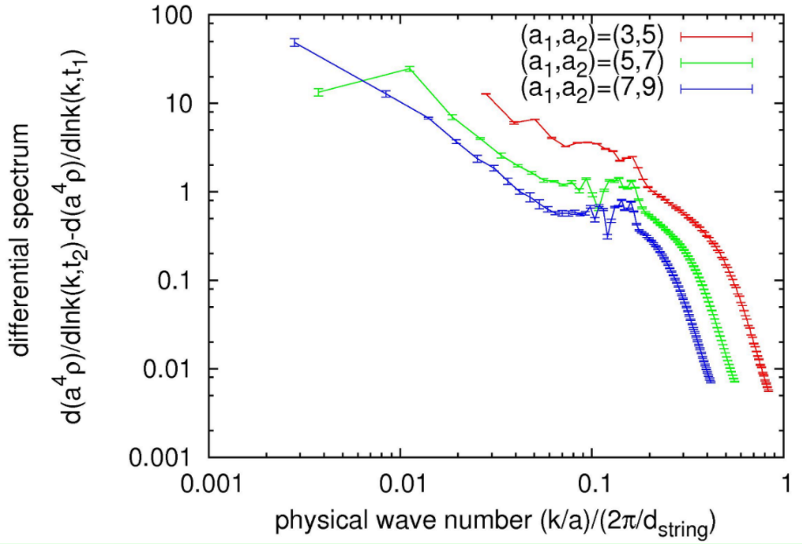


FIG. 3 (color online). Validity check of our estimation method using PPSE. Three different spectra are plotted (see text for details). Only statistical errors are shown; bars corresponds to the square root of the diagonal components of covariance matrices.

update

Spectrum

$$\frac{d\rho}{d\ln k} = k\frac{d\rho}{dk}$$



c.f.
Other groups claim
more flat spectrum

Gorghetto et al.1806.04677
Vaquero et al. 1809.09241

Average momentum / H

$$\overline{k^{-1}} \equiv \frac{\frac{d}{dt}\int dk k^{-1} P_{\rm rad}(k,t)}{\frac{d}{dt}\int dk P_{\rm rad}(k,t)}$$

$$\epsilon(t) = \frac{t}{2\pi a\overline{k^{-1}}(t)}$$

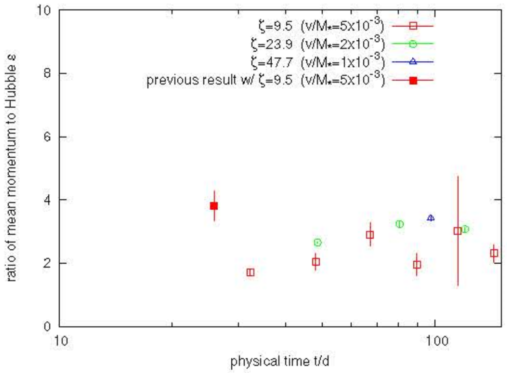


Figure 5: Evolution of the mean momentum of radiated axions in units of the Hubble expansion rate (see Eq. (11)). Color and symbol are the same as in Figure 1.

Average momentum is still around the horizon scale.

Present abundance of axions

~~Strings evolve according to scaling law~~
~~（$\xi\sim1$）~~

↓

Energy density of radiated axions
（axions are relativistic at emission）

Spectrum, average energy

Peak around the horizon scale

↓

Current number density

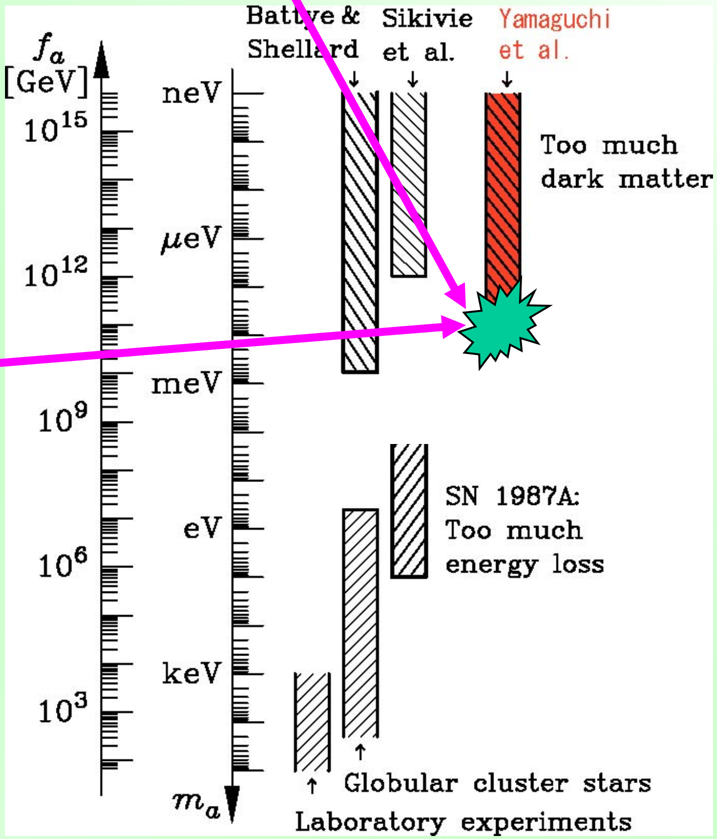
(Axions are now non-relativistic.)

↓

Multiplying its mass leads to the current energy density.

If the log dependence keeps until the QCD phase transition, ξ is enhanced by a factor 2 or larger.

Raffelt



Conclusions

We have tried to estimate axion abundance emitted from axion string.

- Scaling property is slightly broken through the logarithmic dependence.
- Average energy of axion radiated from axion strings is still around the horizon scale in our simulation though there are other claims.

We need to explain this logarithmic dependence analytically.

Unfortunately, we have yet known neither whether this logarithmic dependence continues or stops at some later time nor the final value of ξ.
And, the spectrum of axions needs to be clarified.