Sub-GeV Dark Matter in a Fraternal Twin Higgs Model

arXiv:1805.12139

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Outline

- What's Sub-GeV DM and Why?
- The FTH Model
- Thermal History, Dynamics and Relic Density
- Numerical Benchmarks
- (In)direc Detection and Other Constraints

Long Story in Short

Twin Higgs Model (As a benchmark)

$\Rightarrow$ Twin leptons charged under twin gauges (Sub-GeV range)

$\Rightarrow$ Different twin leptons have hierarchical couplings

$\Rightarrow$ Coscattering/Coannihilation mechanism

$\Rightarrow$ Correct relic density and small annihilation rate

WIMP DM

The DM once in thermal equilibrium with the SM. The averaged self annihilation $\langle \sigma v \rangle \sim 10^{-26}\text{cm}^3/\text{s}$ for a wide mass range.

"Classic" WIMP mass range

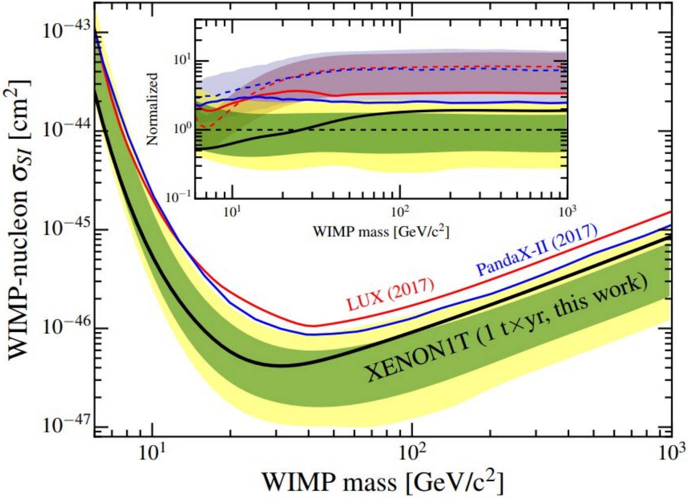
- $m_{DM} \lesssim 100$ TeV (Unitarity Bound)
- $m_{DM} \gtrsim 2$ GeV (Lee-Weinberg Bound)

Z boson is a heavy mediator $\Rightarrow$ For lighter thermal DM, a new light mediator is needed!

Sub-GeV DM

- Not a well defined term, but usually DM with $(O)(\text{MeV}) \lesssim$ mass $\lesssim (O)(\text{GeV})$ fits well.
- Various models: Thermal, Freeze-in, Asymmetric, etc.
- For thermal relics, a light mediator is needed, such as the dark photon.

Why Sub-GeV DM



Xenon1T Collaboration [1805.12562]

Why Sub-GeV DM

Most current DM direct detection experiments are based on heavy nuclei recoiling when the nuclei scatter with the DM particles.

Recoiling Energy $\sim \frac{\mu^2 v^2}{m_{nuclei}} \propto m_{DM}^2$ when $m_{DM} \ll m_{nuclei}$. TASI Lecture Notes, Tongyan Lin

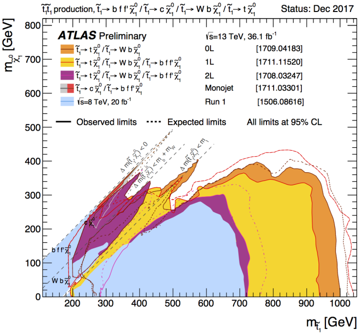
e.g. Xenon threshold $\sim \mathcal{O}(10)$ keV $\Rightarrow$ Not sensitive to DM lighter than $\mathcal{O}(10)$ GeV.

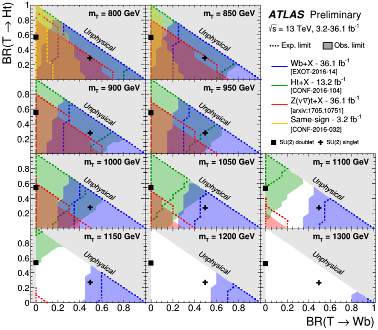
Naturalness of the Weak Scale

Requires top partners not far beyond the EW scale to cancel the quadratic contribution to m_h^2 from the top loop.

$SU(3)$ charged top partners

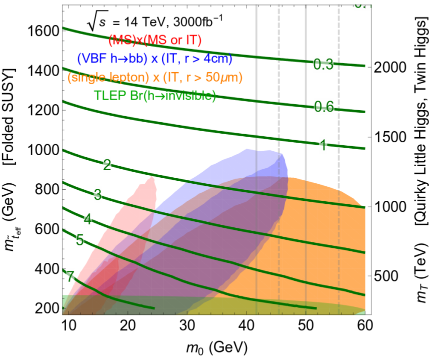
Negative reuslts for both SUSY/global symmetry models.





Neutral Naturalness

There has been a surge of interests in models where the top contribution to the Higgs mass is cut off by states that do not carry color, thus their production rates are low at hadron colliders.



Chacko, Curtin, Verhaaren [1512.05782]

TH Model

- Extended $SO(8)(SU(4)) \to SU(2)_{SM} \times SU(2)_{twin} \times Z_2$
- Gauge structure: $SU(3)^2 \times SU(2)^2 \times U(1)^{(2)}$
- Higgs sector: (H_A, H_B)
- Twin fermions are charged under twin gauge groups

The VEV:

$$\begin{pmatrix} H_A \\ H_B \end{pmatrix} = \frac{f}{\sqrt{2}} \begin{pmatrix} 0 \\ \sin\frac{h}{f} \\ 0 \\ \cos\frac{h}{f} \end{pmatrix} \simeq \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \\ 0 \\ f \end{pmatrix}$$

$f/v \gtrsim 3$ in order to satisfy experimental bounds.

FTH Model

- Exact mirror content of the SM sector and the couplings respect the Z_2 symmetry.

  $\Rightarrow$ Many light degrees of freedom (photon, electron, neutrinos) in the twin sector.

  $\Rightarrow$ They can cause cosmological problems such as a large $\Delta N_{\rm eff}$.

- Fraternal TH contains only 3rd generation fermions, since light generations have small Yukawa and hence play no important roles in the hierarchy problem.

- The lepton sector contains good DM candidates. Craig, Katz [1505.07113], Garcia, Lasenby, March-Russell [1505.07410], Farina [1506.03520], etc.

The Lepton Sector

Twin tau and twin neutrino have Dirac masses:

$$
\begin{aligned}
-\mathcal{L} \;&\supset\; y_{\tau_B}\, L_B\, \widetilde{H}_B\, \tau^c_{B,R} + y_{\nu_B}\, L_B\, H_B\, \nu^c_{B,R} + \text{h.c.} \\
&\supset\; \frac{y_{\tau_B} f}{\sqrt{2}}\, \tau_{B,L}\tau^c_{B,R} + \frac{y_{\nu_B} f}{\sqrt{2}}\, \nu_{B,L}\nu^c_{B,R} + \text{h.c.}.
\end{aligned}
$$

Respecting the twin lepton number, no Majorana mass terms.
Assume the twin neutrino has a smaller mass and becomes the DM.

Lepton Mixing and Hierarchial Coupling

We gauge the broken twin $U(1)$ hypercharge.

- The twin hypercharge breaking can be parameterized by a spurion field S which is a singlet under $SU(3)_B \times SU(2)_B$ but carries $+1$ twin $U(1)$ charge
- Spuronic terms mix the twin tau and twin neutrino, but still respect the twin lepton number:

$$-\mathcal{L} \supset \frac{d_1}{\Lambda} S L_B \widetilde{H}_B \nu^c_{B,R} + \frac{d_2}{\Lambda} S^\dagger L_B H_B \tau^c_{B,R} \,.$$

Lepton Mixing and Hierarchial Coupling(2)

The mass matrix:

$$\left(\begin{array}{cc} \tau_{B,R}^c & \nu_{B,R}^c \end{array} \right) \left(\begin{array}{cc} m_{\tau_B} & \mu_2 \\ \mu_1 & m_{\nu_B} \end{array} \right) \left(\begin{array}{c} \tau_{B,L} \\ \nu_{B,L} \end{array} \right).$$

Assuming $\mu_1 \gg \mu_2$ for simplicity, the mass matrix gives two mass eigenstates $\hat{\tau}$ and $\hat{\nu}$ with mixing angles $\theta_{1,2}$ which are close to each other:

$$\sin\theta_1 = \frac{\mu_1 m_{\tau_B}}{m_{\tau_B}^2 - m_{\nu_B}^2 + \mu_1^2}, \quad \sin\theta_2 = \frac{m_{\nu_B}}{m_{\tau_B}} \sin\theta_1\,,$$

in the small mixing limit. We then use θ_1 as the parameter for mixing.

The Twin(Dark) Photon

Kinematic mix with the SM photon, the effective Lagrangian with conventional parameterization:

$$\mathcal{L} \supset -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}F'^{\mu\nu}F'_{\mu\nu} + \frac{1}{2}m_{\hat{\gamma}}^2 A'^{\mu}A'_{\mu} - \frac{\epsilon}{2}F^{\mu\nu}F'_{\mu\nu}$$

- SM photon is still massless.
- SM photon doesn't couple to twin $U(1)$ charge.
- Mixing with Z is negligible in this mass range.
- ϵ can be conducted by exotic fermion loops.

(Co)annihilation

As the twin sector in thermal contact with the SM, the relic density is dominated by annihilating processes:

- $$\hat{\nu}\hat{\nu} \to \hat{\gamma}\hat{\gamma} \ (A)$$

  "Normal" Annihilation, θ_1^4 suppressed. Plays no significant role when $\theta_1 \ll 1$)

- $$\hat{\tau}\hat{\nu} \to \hat{\gamma}\hat{\gamma} \ (C_A)$$

  "Asymmetric" Coannihilation, suppressed by θ_1^2

- $$\hat{\tau}\hat{\tau} \to \hat{\gamma}\hat{\gamma} \ (C_S)$$

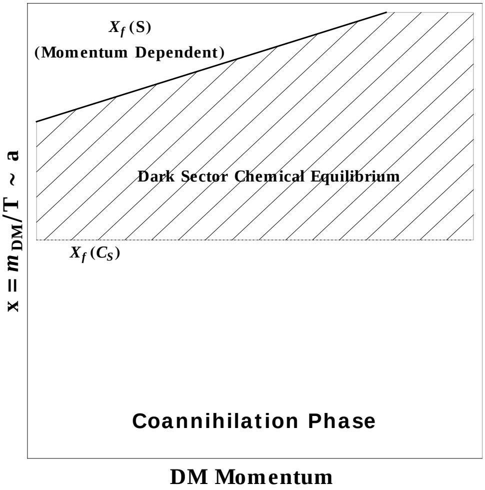
  "Sterile/Symmetric" Coannihilation, Boltzmann suppression only

(Co)annihilation(2)

- When $\theta_1 \ll 1$, the rate of C_S still wins even with the smaller $\hat{\tau}$ number density
- DM freezes out when the rate C_S goes below the time scale of universe $\sim H^{-1}$.

Need $\langle \sigma v \rangle_{C_A/C_S} \gtrsim 10^{-26}\mathrm{cm}^3/\mathrm{s}$ in order to produce the correct Ωh^2.

Cartoons



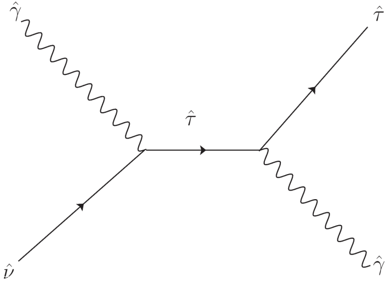
$X_f\,(S)$
(Momentum Dependent)
Dark Sector Chemical Equilibrium
$X_f\,(C_S)$
Coannihilation Phase
$\mathbf{x = m_{DM}/T \sim a}$
DM Momentum

Coscattering

First discussed in D'Agnolo, Pappadopulo, Ruderman [1705.08450] and Garny, Heisig, Llf, Vogl [1705.09292].

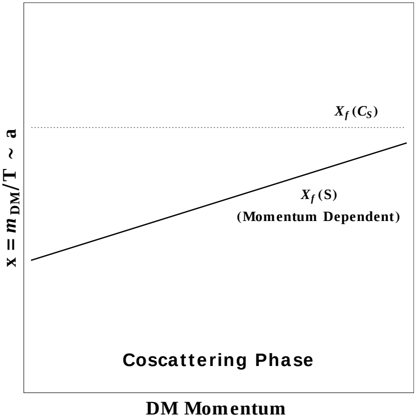
Basic Idea

Since $\hat{\nu}$ coupling is always suppressed, it is possible to decouple it from the rest of the twin sector.



$\hat{\nu}$ decouples from the twin sector as the "flavour changing" scattering process (S) decouples.

Cartoons(2)



Assuming C_A decouples earlier than S due to the fact that $\hat{\gamma}$ has much higher number density. Also assuming $\hat{\tau} \to \hat{\nu}\hat{\gamma}$ decay is not on-shell.

Momentum-Dependent Freeze-out

Kinematic Threshold

For $\hat{\nu}$, the inelastic scattering process $\hat{\nu}\hat{\gamma} \to \hat{\tau}\hat{\gamma}$ has a mass gap.

$\Rightarrow$ Only a $\hat{\gamma}$ with enough energy can turn $\hat{\nu}$ into $\hat{\tau}$.

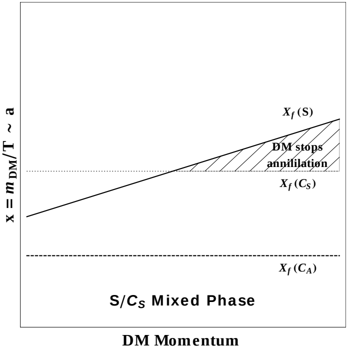
$\Rightarrow$ High momentum modes will have a significantly larger interaction rate and freeze-out later.

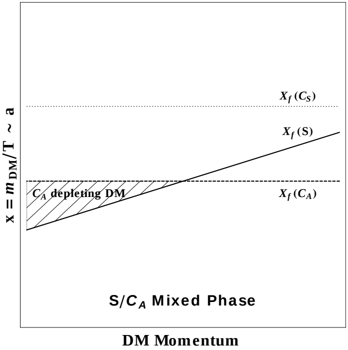
- True as long as the elastic scattering process $\hat{\tau}\hat{\nu} \to \hat{\tau}\hat{\nu}$ is suppressed.
- Further correction for the coscattering phase, larger Ωh^2.

Mixed Phase

- Symmetric coannihilation C_S and coscattering S can happen simultaneously.
- Not negligible in parameter space.
- Ωh^2 can be orders of magnitude off from both ends.
- Not fully discussed in literatures.

Cartoons For Mixed Phases





When θ_1 is of moderate size.

When $m_{\hat{\gamma}}$ is close to $m_{\hat{\nu}}$.

Parameterization

Following the references, we define:

$$\Delta \equiv \frac{m_{\hat{\tau}} - m_{\hat{\nu}}}{m_{\hat{\nu}}},$$

$$r \equiv \frac{m_{\hat{\gamma}}}{m_{\hat{\nu}}},$$

together with lepton mixing θ_1, dark photon mixing ϵ, masses and couplings.

Enumerating the Coannihilation Phase

Since the twin sector is in equilibrium, we can keep track of the net twin lepton number density only:

$$\dot{n}_{\rm tot} + 3Hn_{\rm tot} = -\langle\sigma v\rangle_{C_S}\left(n_{\hat{\tau}}^2 - (n_{\hat{\tau}}^{\rm eq})^2\right) - \langle\sigma v\rangle_{C_A}\left(n_{\hat{\tau}} n_{\hat{\nu}} - n_{\hat{\tau}}^{\rm eq} n_{\hat{\nu}}^{\rm eq}\right)$$

$$-\langle\sigma v\rangle_{A}\left(n_{\hat{\nu}}^2 - (n_{\hat{\nu}}^{\rm eq})^2\right).$$

- $n_{\hat{\tau}}/n_{\hat{\nu}} = n_{\hat{\tau}}^{\rm eq}/n_{\hat{\nu}}^{\rm eq}$
- The C_S term contributes the most.
- Easy to solve for the $n_{\hat{\nu}}(T)$.

The Coscattering Phase

In this case, the number density of $\hat{\tau}$ will be negligible. If the threshold for coscattering is also negligible, then:

$$\dot{n}_{\hat{\nu}} + 3Hn_{\hat{\nu}} = - \langle \sigma v \rangle_S (n_{\hat{\nu}} - n_{\hat{\nu}}^{\rm eq}) n_{\hat{\gamma}}^{\rm eq}.$$

However, need to include the effect of coscattering threshold when $\hat{\nu}$ is not in kinematic equilibrium.

Kinematic Inequilibrium

- Due to the large coupling/mixing, $\hat{\tau}$ and $\hat{\gamma}$ are in kinematic equilibrium with the SM, sharing the same temperature:

$$\frac{f_{\hat{\tau}}(T,p)}{f_{\hat{\tau}}(T,p)^{\rm eq}} = \frac{n_{\hat{\tau}}(T)}{n_{\hat{\tau}}(T)^{\rm eq}},$$

$$\frac{f_{\hat{\gamma}}(T,p)}{f_{\hat{\gamma}}(T,p)^{\rm eq}} = \frac{n_{\hat{\gamma}}(T)}{n_{\hat{\gamma}}(T)^{\rm eq}}.$$

- As $\hat{\nu}$ decouples from $\hat{\tau}$ and $\hat{\gamma}$, it can deviate from thermal distribution:

$$\frac{f_{\hat{\nu}}(T,p)}{f_{\hat{\nu}}(T,p)^{\rm eq}} \neq \frac{n_{\hat{\nu}}(T)}{n_{\hat{\nu}}(T)^{\rm eq}}.$$

The Coscattering Phase(2)

The Boltzmann Eq. for phase space density $f(p, T)$.

$$(\partial_t - Hp\partial_p)\, f_{\hat{\nu}}(p, t) = \frac{1}{E_p} C[f_{\hat{\nu}}](p, t),$$

where the collision operator is:

$$C[f_{\hat{\nu}}](p, t) = \frac{1}{2} \int d\Omega_k d\Omega_{p'} d\Omega_{k'} |\overline{\mathcal{M}}|^2$$

$$[f_{\hat{\tau}}(p', t) f_{\hat{\gamma}}(k', t) - f_{\hat{\nu}}(p, t) f_{\hat{\gamma}}(k, t)](2\pi)^4 \delta^4(\sum p^\mu).$$

The Coscattering Phase(3)

In a more convenient variables a and comoving momentum $q \equiv ap$, we further simplify the Boltzmann Eq. as:

$$Ha\partial_a f_{\hat{\nu}}(q,a) = [f_{\hat{\nu}}^{\text{eq}}(q,a) - f_{\hat{\nu}}(q,a)]\widetilde{C}(q,a),$$

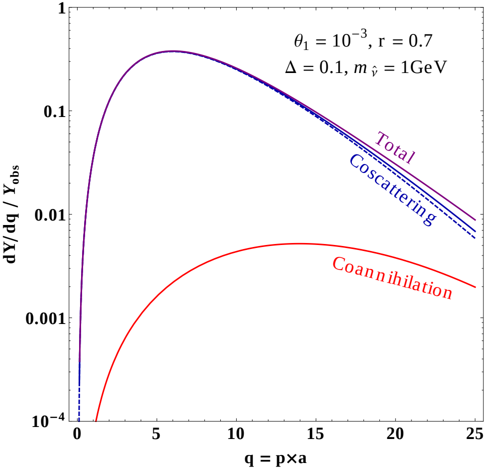
which gives the solution

$$f_{\hat{\nu}}(q,a) = f_{\hat{\nu}}^{\text{eq}}(q,a) - \int_{a_0}^{a} da' \frac{df_{\hat{\nu}}^{\text{eq}}(q,a')}{da'} e^{-\int_{a'}^{a} \frac{\widetilde{C}(q,a'')}{Ha''} da''}.$$

Momentum-Dependant freeze-out

Each mode freezes out when $\widetilde{C}(q,a) \simeq H(a)$, lower q freezes out earlier.

The Coscattering Phase(4)



Phase space volume $\propto q^3$

The Mixed Phase

In the mixed phase, the calculation is more complicated as all processes have to be take in to account.

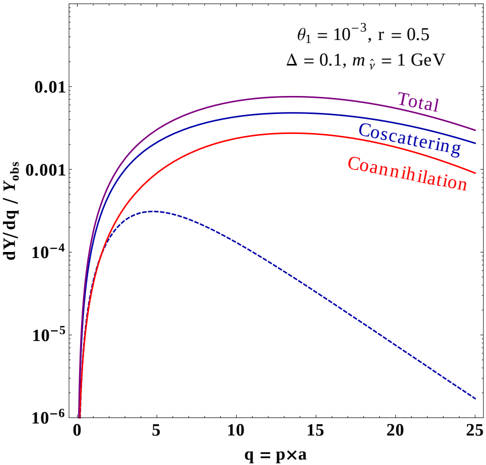
Can be done by solving the two equations together:

- $$\dot{n}_{\hat{\tau}} + 3Hn_{\hat{\tau}} = -\langle \sigma v \rangle_{C_S} (n_{\hat{\tau}}^2 - (n_{\hat{\tau}}^{\text{eq}})^2) - \langle \sigma v \rangle_{C_A} (n_{\hat{\tau}} n_{\hat{\nu}} - n_{\hat{\tau}}^{\text{eq}} n_{\hat{\nu}}^{\text{eq}}) - \langle \sigma v \rangle_{IS} \, n_{\hat{\gamma}}^{\text{eq}} \left(n_{\hat{\tau}} - n_{\hat{\tau}}^{\text{eq}} \frac{n_{\hat{\nu}}}{n_{\hat{\nu}}^{\text{eq}}} \right),$$

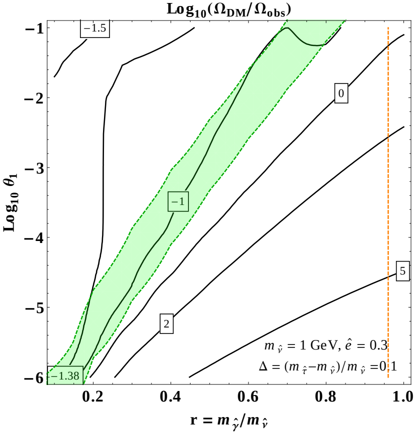
- $$Ha\partial_a f_{\hat{\nu}}(q,a) = [\frac{Y_{\hat{\tau}}(a)}{Y_{\hat{\tau}}^{\text{eq}}(a)} f_{\hat{\nu}}^{\text{eq}}(q,a) - f_{\hat{\nu}}(q,a)]\widetilde{C}(q,a).$$

The Mixed Phase(2)

Using the $n_{\hat{\tau},\hat{\nu}}$ from coannihilation result as our input:

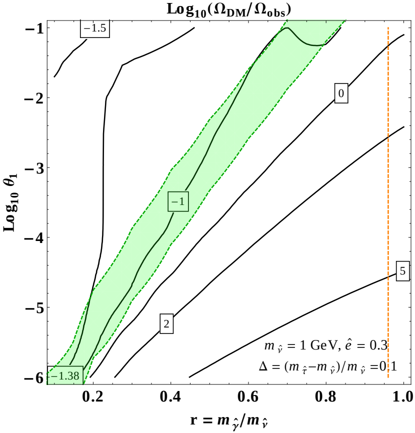


The Benchmark



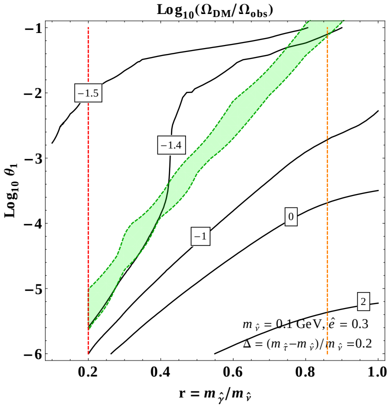
- Fixing $\hat{e} = 0.3$, $m_{\hat{\nu}} = 1$ GeV, $m_{\hat{\tau}} = 1.1$ GeV (Δ=0.1)
- Changing mixing angle θ_1 and $m_{\hat{\gamma}}$ (r)
- Semi-analytical result matches numerical approach
- 4-5 different phases

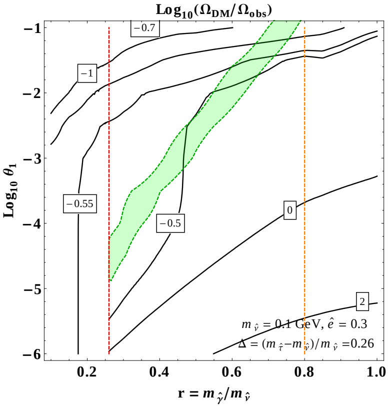
A Closer Look



- Lower Right: Coscattering
- Upper Left: "Traditional" Coannihilation (C_S and C_A)
- Green Shades: Coannihilation-Coscattering Mixed Phase (C_S and S)
- Vertical Contours: "Sterile Coannihilation" (C_S) D'Agnolo, Mondino, Ruderman, Wang, [1803.02901]
- The Right End: C_A/S Mixed Phase (Insignificant Effect)

Other Benchmarks (100 MeV)

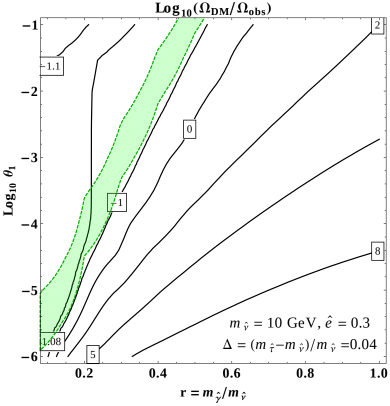




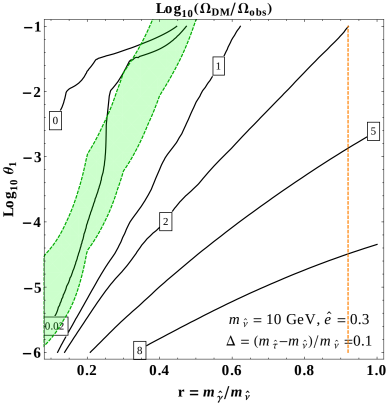
$\Delta = 0.20$

$\Delta = 0.26$

Other Benchmarks (10 GeV)



$\Delta = 0.04$



$\Delta = 0.10$

Different Constraints

- Direct Detection
- Indirect Detection
- Dark Photon Searches
- Collider Constraints
- Cosmological

Direct Detection

- Neuclei recoiling-based experiments (PandaX, Xenon, LUX) are not very constraining now.
- Dark Photon Portal
  Suppressed by $\theta_1^4 \epsilon^2$, not effective.
- Higgs Portal
  Can be relevant since the higgs-baryon coupling is not too small, but suppressed by $(v/f)^4$.
  - A 10 GeV $\hat{\nu}$ would require $f/v \gtrsim 30$ to escape the current detection.
  - For $f/v \gtrsim 3$, the bound for $m_{\hat{\nu}}$ is $\lesssim 6$ GeV.
  - Experiments based on phonon recoiling can detect low mass DM, however, punished by small Yukawa for higgs portal.

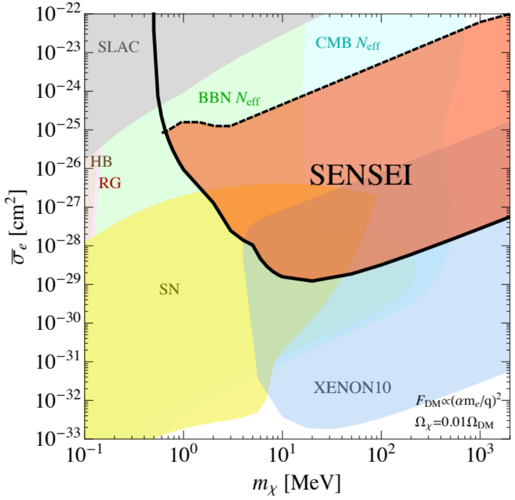
Electron Recoiling

Sensitive to our mass range, but suppressed coupling. For dark photon portal:

$$\begin{aligned}\sigma_{\hat{\nu}e} &\simeq \frac{g^2\hat{e}^2\epsilon^2\theta_1^4}{\pi m_{\hat{\gamma}}^4}\left(\frac{m_e m_{\hat{\gamma}}}{m_e + m_{\hat{\gamma}}}\right)^2 \\ &\simeq 4.3\times 10^{-38}\left(\frac{\hat{e}}{0.3}\right)^2\left(\frac{\epsilon}{10^{-3}}\right)^2\left(\frac{\theta_1}{10^{-1}}\right)^4\left(\frac{10\ \text{MeV}}{m_{\hat{\nu}}}\right)^4\left(\frac{0.5}{r}\right)^4\ \text{cm}^2,\end{aligned}$$

which is much below the current constraints.

Electron Recoiling(2)

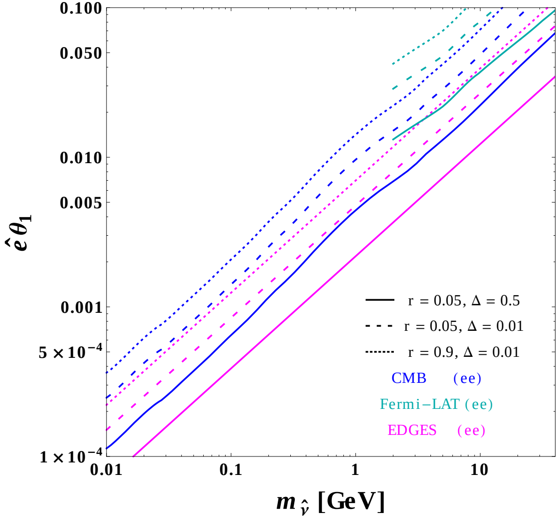


The Sensei Collaboration [1804.00088]

Indirect Detection

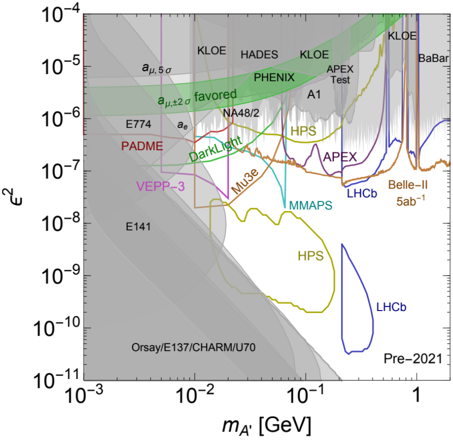
- Can be stringent for sub-GeV DM because of the large number density.
- (Naive) thermal relics that annihilate visibly with mass $\lesssim$ 10 GeV are already ruled out.
- However, in this model, the DM annihilation is dominated by $\hat{\nu}\hat{\nu} \to \hat{\gamma}\hat{\gamma} \to 4f$, which is suppressed by $\hat{e}^4\theta_1^4$.

Indirect Detection(2)



Dark Photon Searches

Based on dark photon visible mode (No on-shell decay to the twin sector)



Battaglieri *et al*. [1707.04951]

Collider Constraints

Like other twin fermions, $\hat{\tau}$ and $\hat{\nu}$ are most likely to be produce at the LHC by $h \to \hat{\nu}\hat{\nu}/\hat{\tau}\hat{\tau}$ decays.

- the current upper bound of $\mathrm{Br}(h \to \text{invisible}) < 24\%$ constrains $m_{\hat{\nu}} \lesssim 22\,(60)$ GeV for $f/v = 3\,(5)$.
- For a future e^+e^- collider, a 0.3% measurement can constrain $m_{\hat{\nu}}$ down to $\lesssim 2\,(6)$ GeV.

Displaced mode(s) of $\hat{\tau}$

$$c\tau(\hat{\tau}) \approx \frac{8.8\times 10^6}{N_f}\left(\frac{0.2}{\Delta}\right)^5\left(\frac{10^{-3}}{\theta_1}\right)^2\left(\frac{0.3}{\hat{e}}\right)^2\left(\frac{10^{-3}}{\epsilon}\right)^2\left(\frac{r}{0.5}\right)^4\left(\frac{1\ \mathrm{GeV}}{m_{\hat{\nu}}}\right)\mathrm{cm}$$

Could be tested by future detectors.

But also causes trouble in cosmology.

Cosmological Constraints

$\tau(\hat{\tau})$ must be short enough in order to satisfy Big Bang Nucleosynthesis (BBN) bounds.

- Small $\hat{\tau}$ number density after freeze-out
- Partial energy injection

Since $\hat{\gamma}$ is in equilibrium with the SM, will contribute to the effective number of neutrinos (N_{eff}) if it is too light.

- Currently: $m_{\hat{\gamma}} \gtrsim 11$ MeV
- After CMB S4: $m_{\hat{\gamma}} \gtrsim 19$ MeV

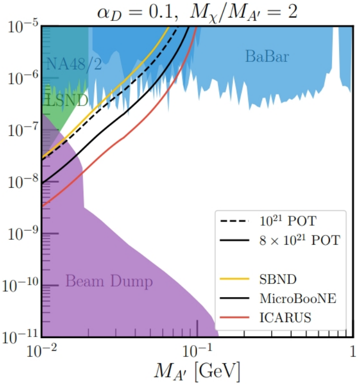
Recent "ν" Ideas (ν experiments, etc.)

We have discussed $\hat{\gamma}$ constraints found in various intensity frontier experiments, under the good approximation that $\hat{\tau}$ is heavy and decouples.

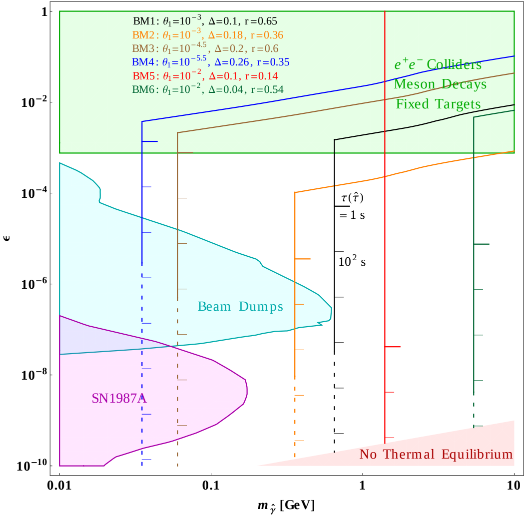
If r ratio is not extreme, $\hat{\tau}$ can be generated by beam lines and leave 'dark trident' signal ($\hat{\tau} + N \to \hat{\tau} + 2\ell + N$)

- Need large beam intensity
- Huge detector and good resolution
- $\Rightarrow$ Neutrino experiments (DUNE as the next generation benchmark?)

Gouvea, Fox, Harnik, Kelly and Zhang [1809.06388]



Constraints Summary



- Colored bulks for various $\hat{\gamma}$ constraints (approximately)
- 6 DM benchmarks (curves), the l.h.s have smaller Ω_{DM}
- Ticks for $\hat{\tau}$ lifetime and dashed part excluded by BBN.
- Turning due to 3-body decays

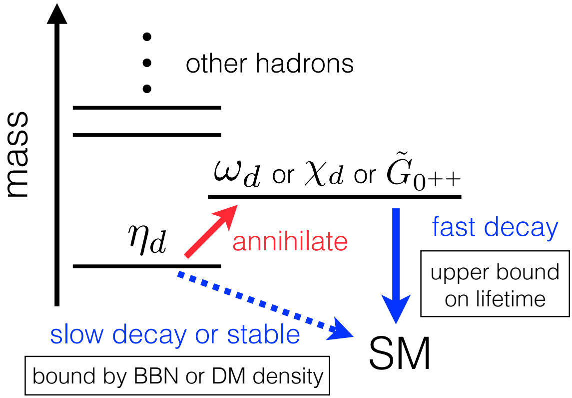
Conclustion

- DM naturally fits in to the FTH scenario, also addresses the hierarchy problem.
- Sub-GeV DM, compatible with current constraints.
- Different regimes with diverse thermal histories.
- Uniform, computationally cheap and precise evaluation for all phases.

Related Works: Cosmological Constraints on Strongly Coupled Relics/ DM

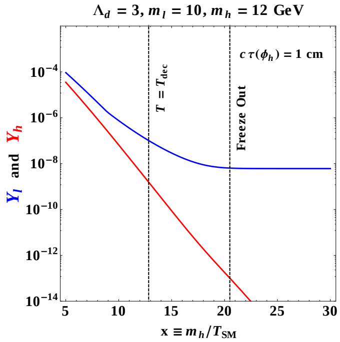
It is also natural to ask what if the lightest dark sector is strongly coupled.

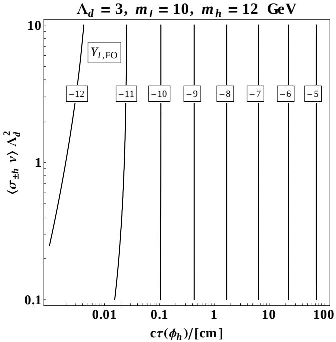
In FTH like models with a dark $U(1)$, the decay of the lightest twin meson ($\hat{\eta}$) is suppressed by the small couplings and the loop factor. However, the vector (scalar) decays much faster than 1 sec.



Related Works: Cosmological Constraints on Strongly Coupled Relics/ DM (2)

The relic density/ cosmological constraints strongly depends on the dark hadron spectrum, not (too) much on couplings.



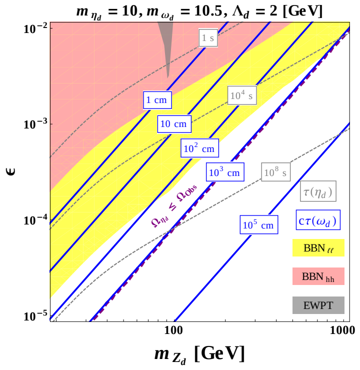


Large population of slow decaying $\hat{\eta}$ after dark sector chemically decouples.

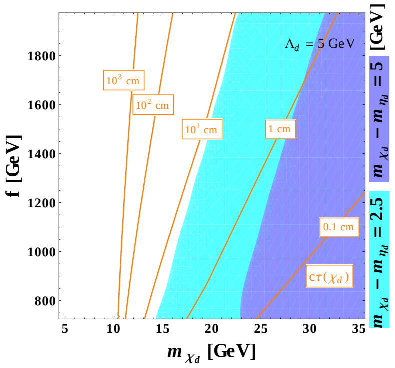
The effect is not sensitive to coupling change within 1-2 orders of magnitude.

Related Works: Cosmological Constraints on Strongly Coupled Relics/ DM (3)

A work with Yuhsin Tsai [arXiv:1901.09936]
The decay length of the heavier component usually falls within collider reach, especially LHCb.



Dark photon as mediator case.



Higgs as mediator case.

Mixing Angles in Detail

$$\begin{pmatrix} \hat{\tau}_R^c \\ \hat{\nu}_R^c \end{pmatrix} = \begin{pmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} \tau_{B,R}^c \\ \nu_{B,R}^c \end{pmatrix},$$

$$\begin{pmatrix} \hat{\tau}_L \\ \hat{\nu}_L \end{pmatrix} = \begin{pmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{pmatrix} \begin{pmatrix} \tau_{B,L} \\ \nu_{B,L} \end{pmatrix}$$

$\hat{\gamma}$ couple to $\hat{\tau}\hat{\nu}$ is basically vector-like, with coupling $\propto \hat{e}\left[(\theta_1+\theta_2)+(\theta_1-\theta_2)\gamma^5\right]$

Effective Two Body Decays

If the decay is two body (on-shell), it becomes much more effective than the coscattering because:

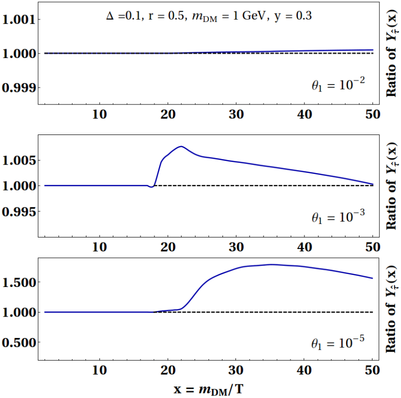
- The threshold is lower, for instance, when $p = 0$, we have:

$$\frac{k_T(S)}{k_T(ID)} = \frac{\sqrt{\Delta(\Delta+2)(\Delta+2r)(\Delta+2r+2)}}{|r^2 - \Delta(\Delta+2)|} > 1.$$

- Less phase space suppression.
- Less $\hat{e}^2$ suppression

Therefore, when $r < \Delta$, the (inverse) decay will keep $\hat{\nu}$ and $\hat{\tau}$ in chemical equilibrium and makes the relic density follow the coannihilation result.

Estimating the Error



$Y_{\hat{\tau}}$ is not as sensitive to the early coannihilation decoupling as $Y_{\hat{\nu}}$ is.

$\Rightarrow$ the coannihilation calculation gives a very good approximation.
No further iteration needed.