Holographic Light Dilaton

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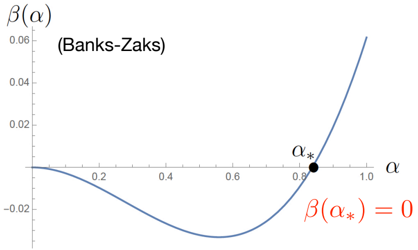
Works under progress with Sang Hui Im and Jong-Wan Lee

See also JHEP 1802 (2018) 102

Near conformal dynamics

- Recently near conformal dynamics has been intensively studied for composite Higgs and for DM as well.
- Consider a Banks-Zaks theory that has a stable IR fixed point.



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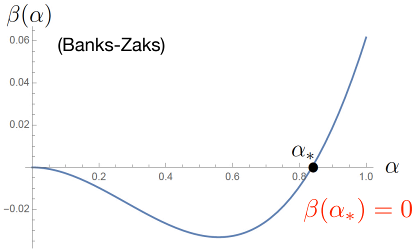
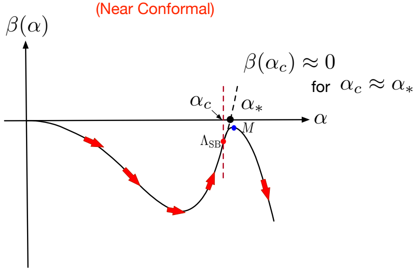


Near conformal dynamics

- The near conformal dynamics may be realized in a deformed BZ theory, having the dynamical generation of fermion mass.



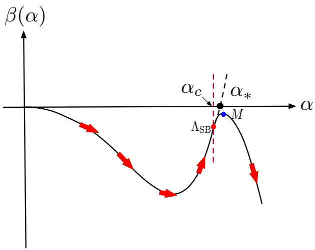
- The theory can be slightly deformed or $\alpha_c \approx \alpha_*$ in the large n_f limit or introducing additional interactions (DKH 2018).

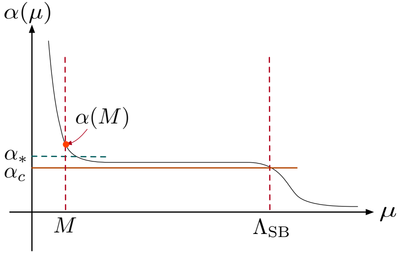
Near conformal dynamics

- How the IR scale M is related to the intrinsic scale of the deformed BZ theory?

Near conformal dynamics

- Because $\beta(\alpha_c) \approx 0$, one expects $M \ll \Lambda_{\rm SB}$, very different from QCD, where $\beta_{\rm QCD} \gg 1$ in IR and $M \sim \Lambda_{\rm SB}$.



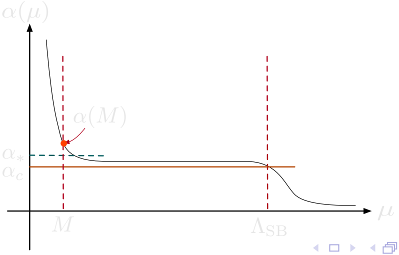


Miransky-BKT scaling

- The dynamical mass M of χSB is argued to be given by the Miransky-BKT Scaling (cf. complex CFT):

$$M(\alpha) = \Lambda_{\mathrm{SB}}(\alpha_c) \exp\left(-\frac{\pi}{\sqrt{\alpha - \alpha_c}}\right) \quad (\alpha > \alpha_c)$$

- The theory is almost scale-invariant for $M < E < \Lambda_{\mathrm{SB}}$, exhibiting walking dynamics, since $\beta(\alpha) \approx 0$.



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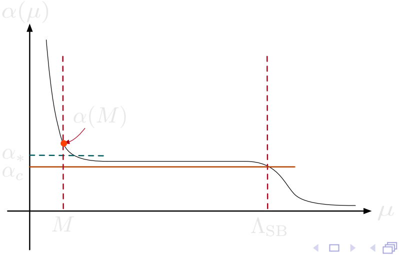

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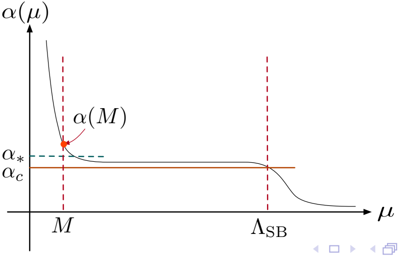

Miransky-BKT scaling

- In the walking region we have approximate scale invariance and ladder approximation is good. The BS equation for the scalar bound-state then becomes

$$\left[P^2 + \partial^2 + \frac{\alpha/\alpha_c}{r^2}\right]\chi_P(x) = 0.$$

- Since the potential is singular, we need to regularize it:

$$V(r) = \begin{cases} -\frac{\alpha/\alpha_c}{r^2} & \text{if } r \geq a, \\ -\frac{\alpha/\alpha_c}{a^2} & \text{if } r \leq a. \end{cases}$$

Very light dilaton

- For bound states to be the cutoff-independent, we require the coupling to depend on the cutoff. (DKH+Rajeev '90)

$$\alpha(a) = \alpha_c + \frac{\pi^2}{[\ln(a\mu)]^2}\,.$$

- The non-perturbative beta function is then

$$\beta^{\rm np}(\alpha) = a\frac{\partial}{\partial a}\alpha(a) = -\frac{2}{\pi}(\alpha-\alpha_c)^{3/2}$$

- The gap equation has a nontrivial solution with this beta function for $\alpha \geq \alpha_c$. (Bardeen et al '86):

$$M \simeq \Lambda(\alpha)\exp\left[\int_{\alpha_0}^{\alpha}\frac{d\alpha}{\beta^{\rm np}(\alpha)}\right] = \Lambda_{UV}\, e^{-\frac{\pi}{\sqrt{\alpha-\alpha_c}}}\,.$$

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- Non-perturbative renormalization requires a new scale.
- In the walking region $\gamma_{\bar{\psi}\psi} \simeq 1$ new marginal operator emerges and therefore generates the new scale, $M \ll \Lambda_{UV}$ (DKH+Rajeev '90):

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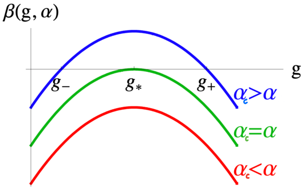
Complex CFT

- Suppose the beta-function of the coupling of the marginal four-Fermi operator is given as (work under progress, DKH+Im+Lee)



$$\beta(\lambda) = -\left(\lambda - \lambda_*\right)^2 - \alpha + \lambda_*^2$$

Marginal deformation of CFT by four-Fermi operator

$$(\lambda = g,\ \alpha_c = \lambda_*^2, \alpha = \alpha_*)$$

- Conformality is lost when the UV fixed point collides with the IR fixed point. (Kaplan-Lee-Son-Stephanov, '09)

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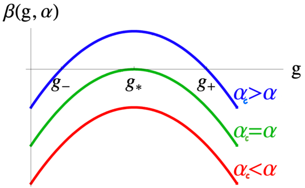

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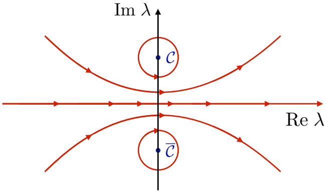
Complex CFT

- The walking dynamics is complex CFT. (V. Gorbenko, S. Rychkov, B. Zan 2018)

$$M = \Lambda \exp\left[-\oint_C \frac{d\lambda}{\beta(\lambda)}\right] = \Lambda \exp^{-\frac{\pi}{\sqrt{\alpha-\alpha_c}}}$$

Near conformal window a new marginal operator rises

whose coupling λ



Very light dilaton

- When χSB occurs at $\alpha = \alpha_c$ or at $\Lambda_{\rm SB}$, generating massless pions, the scale symmetry is also spontaneously broken.

$$0 \neq 3\left\langle\bar{\psi}\psi\right\rangle = \left\langle[D, \bar{\psi}\psi]\right\rangle$$

- In the chirally broken phase therefore we should also have light dilaton, associated the spontaneously broken scale symmetry,

$$\langle 0| \, D_\mu(x) \, |D(p)\rangle = -i f p_\mu e^{-ip\cdot x} \,,$$

where the dilatation current $D_\mu = x^\nu \theta_{\mu\nu}$, if the scale anomaly is small $|\langle\theta^\mu_\mu\rangle| \sim M^4 \ll \Lambda^4_{\rm SB}$, which is the salient feature of near conformal window, unlike QCD.

Very light dilaton

- Consider WT identity:

$$0 = \int_x \partial^\mu \langle 0 | \mathrm{T} D_\mu(x) \theta^\nu_\nu(y) | 0 \rangle = \langle 0 | [D, \theta^\nu_\nu] | 0 \rangle + \int_x \langle 0 | \mathrm{T} \partial^\mu D_\mu(x) \theta^\nu_\nu | 0 \rangle$$

- Partially conserved dilatation current (PCDC) hypothesis:

$$f^2 m_D^2 = -4 \langle \theta^\mu_\mu \rangle \approx -16\, \mathcal{E}_{\rm vac} \sim M^4 \sim m_{\rm dyn}^4 \,.$$

- The scale anomaly is given by the dynamical mass at IR, $m_{\rm dyn}^4$ (Gusynin+Miransky '89; DKH+S.H. Im to appear)

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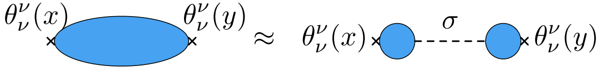

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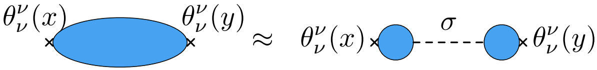

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PCDC and Very light dilaton

- Very light dilaton from quasi-conformal UV sector ($f \sim \Lambda_{SB}$):

$$m_D^2 = -\frac{4\langle\theta^\nu_\nu\rangle}{f^2} \sim \frac{M^4}{f^2} \ll M^2 \,.$$

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$$M = \Lambda_{\rm SB}(\alpha_c)\exp\left(-\frac{\pi}{\sqrt{\alpha-\alpha_c}}\right) .$$

- The dilaton mass $m_D \sim \frac{M^2}{f} \ll M$ if $f \gg M$.
- By the holographic analysis we find (DKH+S.H. Im to appear)

$$m_D = c_1 M\cdot(\alpha-\alpha_c)^{1/4} \quad \text{or} \quad f \sim M\cdot(\alpha-\alpha_c)^{-1/4}$$

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Holographic dilaton - to appear (with S. H. Im)

- Consider a holographic dual of near conformal gauge theory:

$$S = \int_{x,z} \left[\frac{1}{2\kappa^2} (R + 12) + |D_M X|^2 - m_X^2 |X|^2 - \frac{1}{2g_5^2} F_{MN}^2 \right]$$

- Near conformal, the bulk scalar, dual to the fermion bilinear, has $m_X^2 = -4$, saturating BF bound.
- The vacuum solution is then

$$X = \sigma z^2 \quad \text{with} \quad A_L = 0 = A_R$$

- By AdS/CFT σ corresponds to the fermion condensation $\langle \bar{\psi}\psi \rangle$ that breaks chiral symmetry.

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- However, when $\sigma \neq 0$, it will be a deformed AdS_5 with some IR wall :

$$ds^2 = e^{2\varphi(z)}\left[-dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu\right] \; ,$$

- With the vacuum condensation, the geometry is deformed. From the Einstein equation one gets

$$-\partial_z^2\varphi + (\partial_z\varphi)^2 = \frac{4}{3}\kappa^2\sigma^2 z^2 \, .$$

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- Rewriting $e^{\varphi} = e^{\chi}/z$, we find near UV or $z \to 0$

$$\chi(z) = 1 - \frac{1}{15}\kappa^2\sigma^2 z^4 \cdots$$

- By AdS/CFT, χ is dual to the trace of the energy-momentum tensor and then the holographic scale anomaly

$$\langle \theta^{\mu}_{\mu} \rangle = -\frac{1}{15}\kappa^2\sigma^2 \,,$$

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Holographic spectrum

- The chiral condensate deforms the bulk geometry as

$$g_{MN} = e^{2\phi(z)}\eta_{MN}$$

where $\phi(z) \to -\ln z$ for $z \to 0$. (The gravity dual is AAdS_5.)

- To study the dilaton spectrum we consider small fluctuations under the background of the deformed geometry.

$$\tilde{g}_{MN} = e^{2\phi(z)}\left(\eta_{MN} + h_{MN}\right)$$

- We solve the Einstein equation:

$$\tilde{R}_{MN} - \frac{1}{2}\tilde{g}_{MN}\left(\tilde{R} + 12\right) = 8\pi G T_{MN}(\tilde{g})$$

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- Equivalently, keeping the fluctuations of 4d metric traceless the scalar mode satisfies (Kiritsis and Nitti 2007)

$$S'' + \left(3\frac{a'}{a} + 2\frac{\zeta'}{\zeta}\right) + q^2 S = 0$$

where $q^2 = m_s^2$ the on-shell condition, $a = e^{\phi}$ and $\zeta = -2\sigma z^2$.

- Imposing the boundary conditions

$$S(z_{\rm UV}) = 0,\ S'(z_{\rm IR}) = 0$$

we get for the ground-state scalar or dilaton

$$m_D = \frac{\sqrt{2}}{z_{\rm IR}\sqrt{-\ln Z_{\rm UV}}}$$

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Holographic spectrum

- From the Miransky-BKT scaling

$$z_{\mathrm{IR}}^{-1} = z_{\mathrm{UV}}^{-1} e^{-\frac{\pi}{\sqrt{\alpha-\alpha_c}}}$$

we find with $z_{\mathrm{IR}}^{-1} = m_{\mathrm{dyn}}$

$$m_D = \sqrt{\frac{2}{\pi}} (\alpha - \alpha_c)^{1/4} \, m_{\mathrm{dyn}}$$

- The dilaton is therefore parameterically lighter than the typical mass of hadrons.

Holographic spectrum

- From the Miransky-BKT scaling

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Dilaton effective theory (Migdal+Shifman '82: Schechter '80)

- If the scale symmetry is spontaneously broken near the conformal window ($\alpha_* \approx \alpha_c$), the dilaton is very light and the theory is described at low energy, $E < m_{\rm dyn}$, by the dilaton effective Lagrangian:

$$\mathcal{L}_D^{\rm eff} = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - V_A(\chi)\,,$$

where $\chi = fe^{\sigma/f}$ describes the small fluctuations,

$$\theta_\mu^\mu \approx 4\mathcal{E}_{\rm vac}\left(\frac{\chi}{f}\right)^4,$$

with $\langle\chi\rangle = f$ at the vacuum.

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- The dilatation current of the dilaton effective theory becomes

$$\mathcal{D}^\mu = \frac{\partial \mathcal{L}_D^{\text{eff}}}{\partial(\partial_\mu \chi)} (x^\nu \partial_\nu \chi + \chi) - x^\mu \mathcal{L}_D^{\text{eff}} \,.$$

The scale anomaly then takes

$$\partial_\mu \mathcal{D}^\mu = 4 V_A - \chi \frac{\partial V_A}{\partial \chi} \,.$$

- Since χ describes the fluctuations around the vacuum, we take $\partial_\mu \mathcal{D}^\mu = -4\theta^\mu_\mu = -16\mathcal{E}_{\text{vac}}(\chi/f)^4$ to get

$$V_A(\chi) = |\mathcal{E}_{\text{vac}}| \left(\frac{\chi}{f}\right)^4 \left[4 \ln\left(\frac{\chi}{f}\right) - 1\right] \,.$$

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Conclusion

- Near conformal dynamics shows the Miransky-BKT scaling

$$M(\alpha) = \Lambda_{\rm SB}(\alpha_c)\,\exp\left(-\frac{\pi}{\sqrt{\alpha-\alpha_c}}\right) \quad (\alpha > \alpha_c)$$

- The marginal four-Fermi interaction derives the BZ theory into a complex CFT (DKH+Im+Lee to appear).

$$\frac{\lambda}{\Lambda_{\rm SB}^2}\left(\bar{\psi}\psi\right)^2; \quad \beta(\lambda) = -(\lambda-\lambda_*)^2 - \alpha - \alpha_c$$

- By the holographic analysis we find (DKH+S.H. Im to appear)

$$m_D = c_1 M \cdot (\alpha-\alpha_c)^{1/4} \quad \text{or} \quad f \sim M \cdot (\alpha-\alpha_c)^{-1/4}$$

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