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Kobayashi-Maskawa Institute
for the Origin of Particles and the Universe

Fast-rolling relaxion

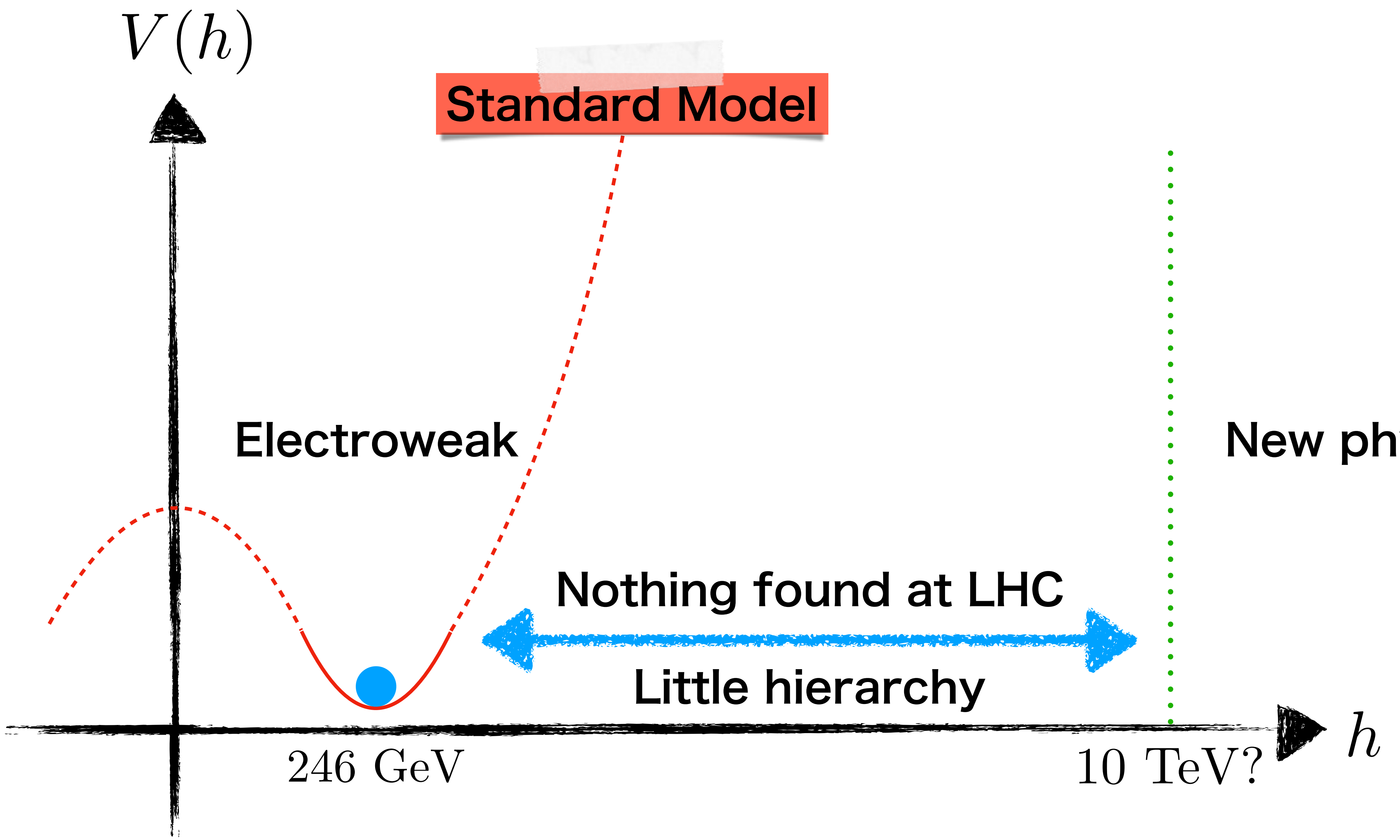
Yutaro Shoji
(KMI, Nagoya University)

1904.02545/hep-ph

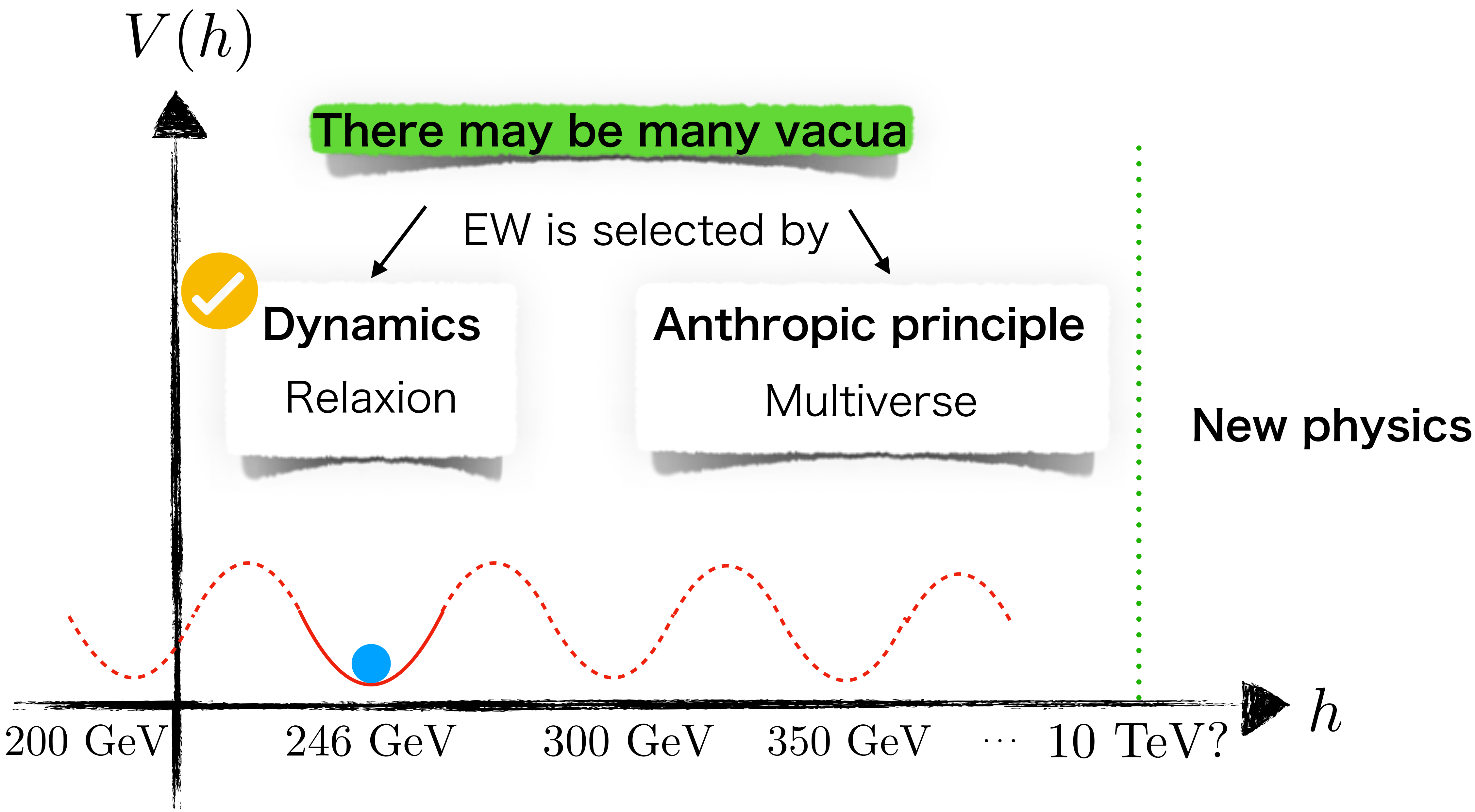
In collaboration with M. Ibe and M. Suzuki in ICRR, IPMU

Introduction

Little hierarchy



Many EW vacua



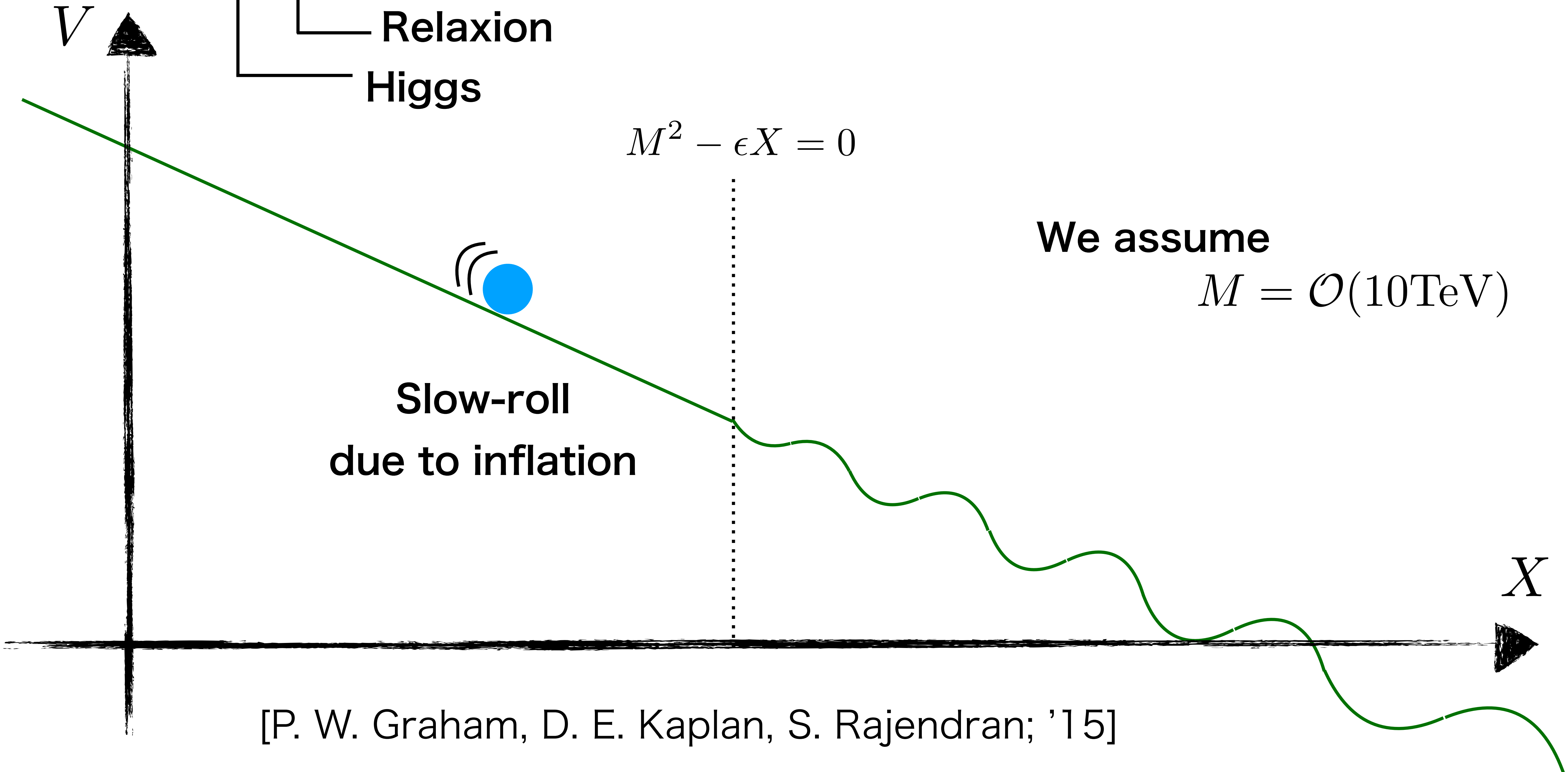
Relaxion mechanism

Higgs mass　　Slope　　Back reaction

$$V(\Phi, X) = (M^2 - \epsilon X)|\Phi|^2 - r\epsilon M^2 X + \Lambda^4(|\Phi|^2)\cos\frac{X}{f} + \frac{\lambda}{4}|\Phi|^4,$$

Relaxion

Higgs

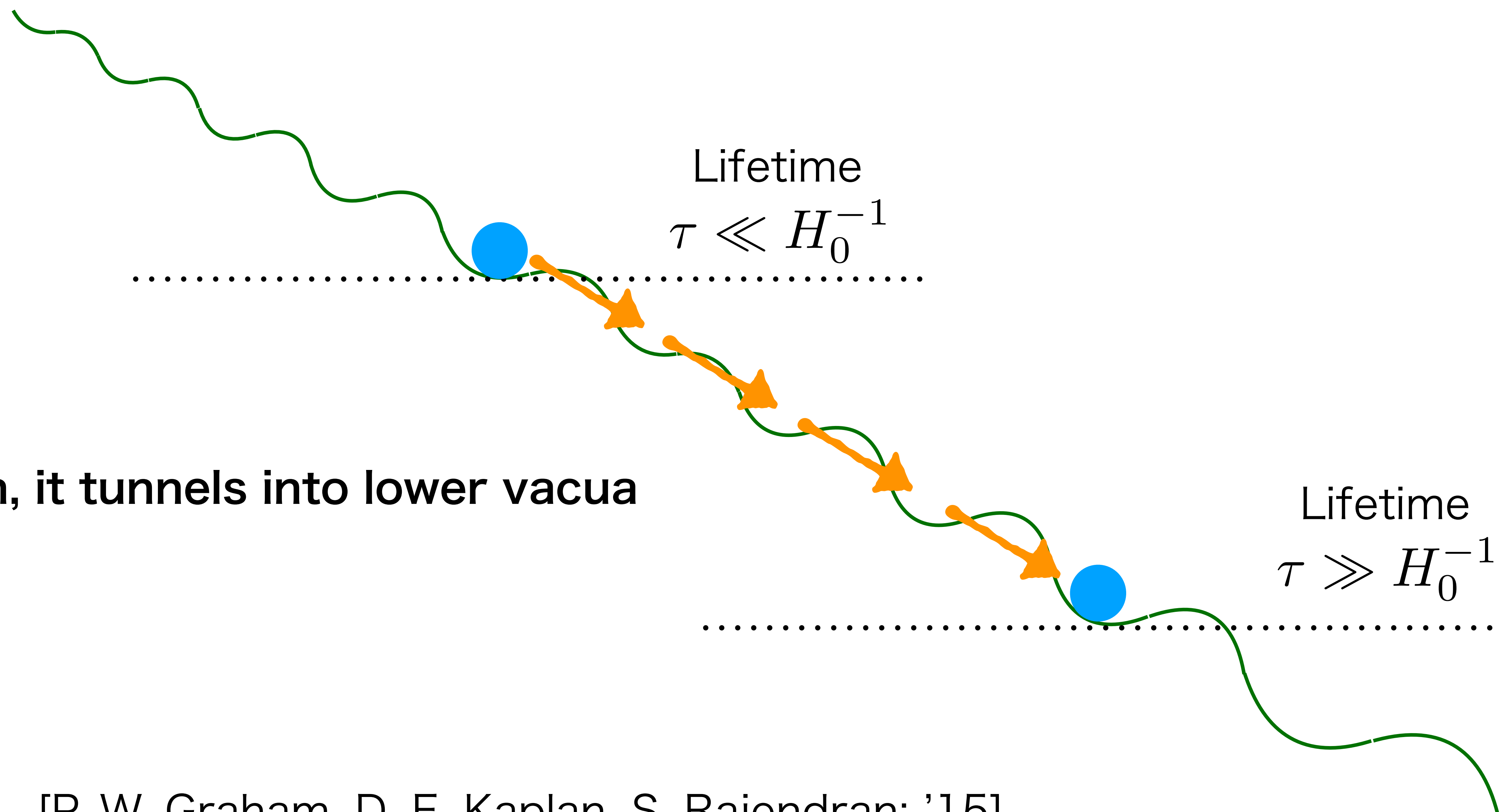


We assume
$M = \mathcal{O}(10\mathrm{TeV})$

[P. W. Graham, D. E. Kaplan, S. Rajendran; '15]

Quantum tunneling

The relaxion stops classically at $\dfrac{dV}{dX} = 0$



Then, it tunnels into lower vacua

[P. W. Graham, D. E. Kaplan, S. Rajendran; ’15]

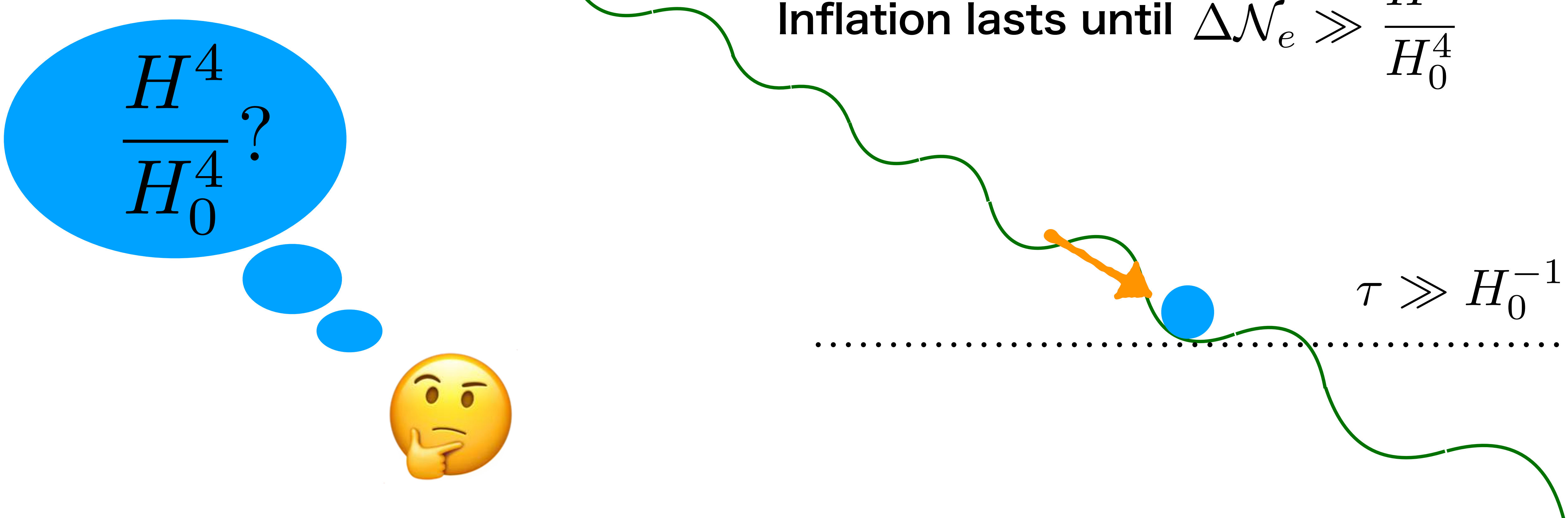
Timescale of tunneling

At the last tunneling, $\gamma \ll H_0^4$

Bubble nucleation rate

The lifetime of each Hubble patch is $\tau \gg \dfrac{H^3}{H_0^4}$

Inflation lasts until $\Delta\mathcal{N}_e \gg \dfrac{H^4}{H_0^4}$



Too large e-folds

$$\mathcal{N}_e \gg \frac{H^4}{H_0^4} \simeq 10^{156} \left(\frac{H}{1 \text{ MeV}} \right)^4$$

It is argued that a too large number of e-folds will cause problems in inflation sector

e.g.) [K. Choi, S. H. Im; '16]

To avoid fine-tuning, we need $\mathcal{N}_e \lesssim 10^{24}$ for slow-roll inflation

(in the context of scanning time)

(possible solution may be N. Kitajima, Y. Tada, F. Takahashi' 19)

[H. Matsui, F. Takahashi; '18, K. Dimopoulos '18,$\cdots$]

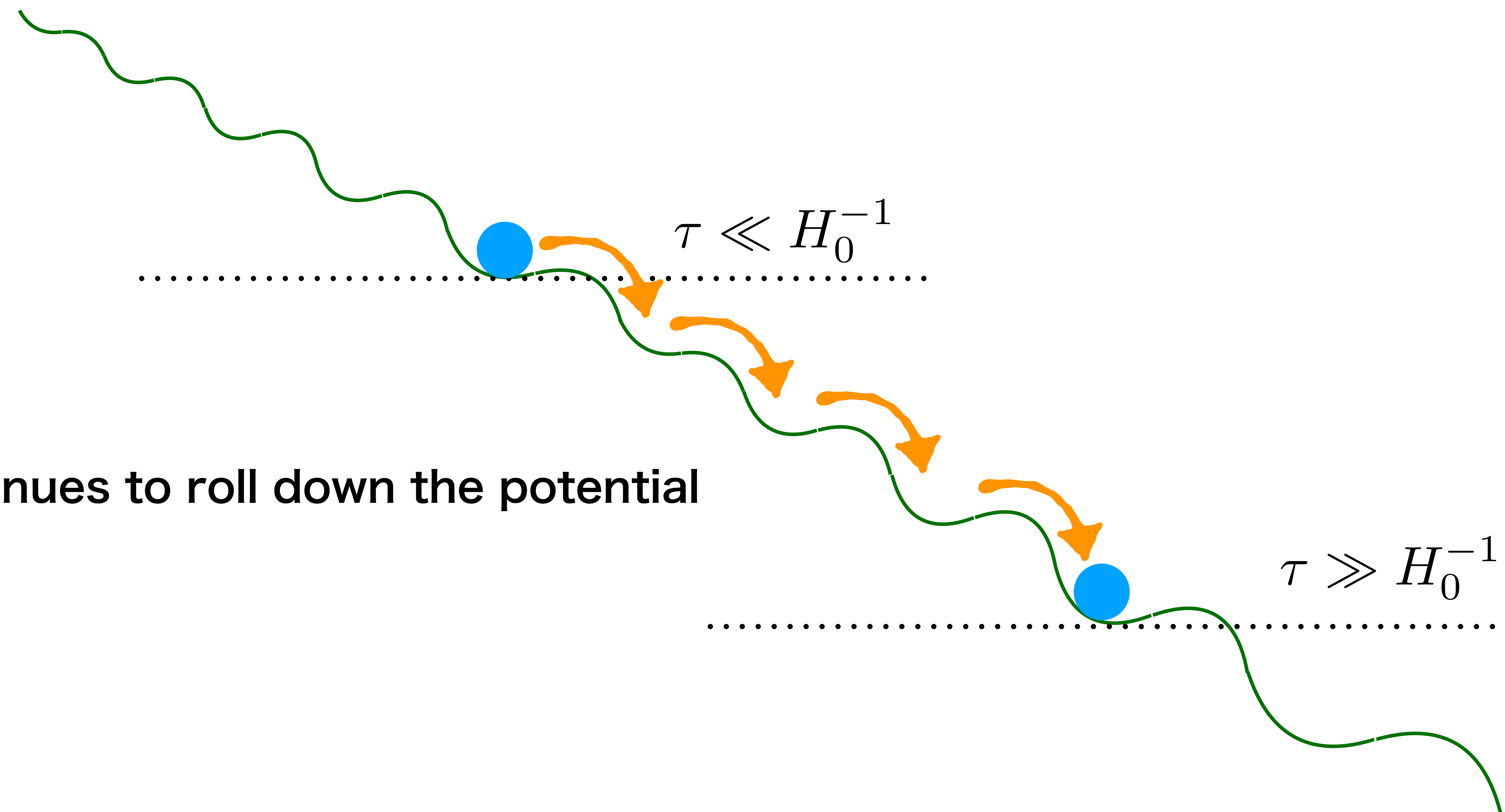
Eternal inflation is generically incompatible with the (refined) de Sitter swampland conjecture

\+ Why not multiverse?

Fast-rolling relaxion

Fast-roll

The relaxion DOES NOT stop classically at $\dfrac{dV}{dX} = 0$



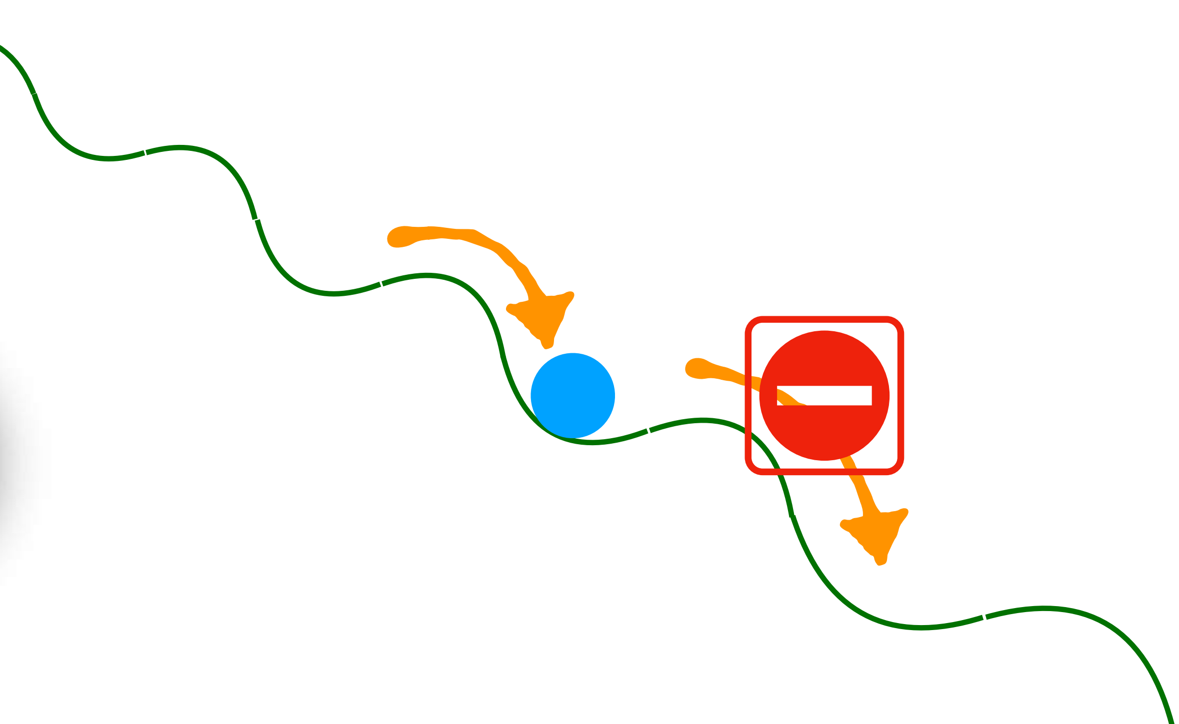
It continues to roll down the potential

Stopping mechanism

We need another stopping mechanism

ex) particle production with $\dfrac{X}{f} F\tilde{F}$

[A. Hook, G. Marques-Tavares; ’16, ⋯]

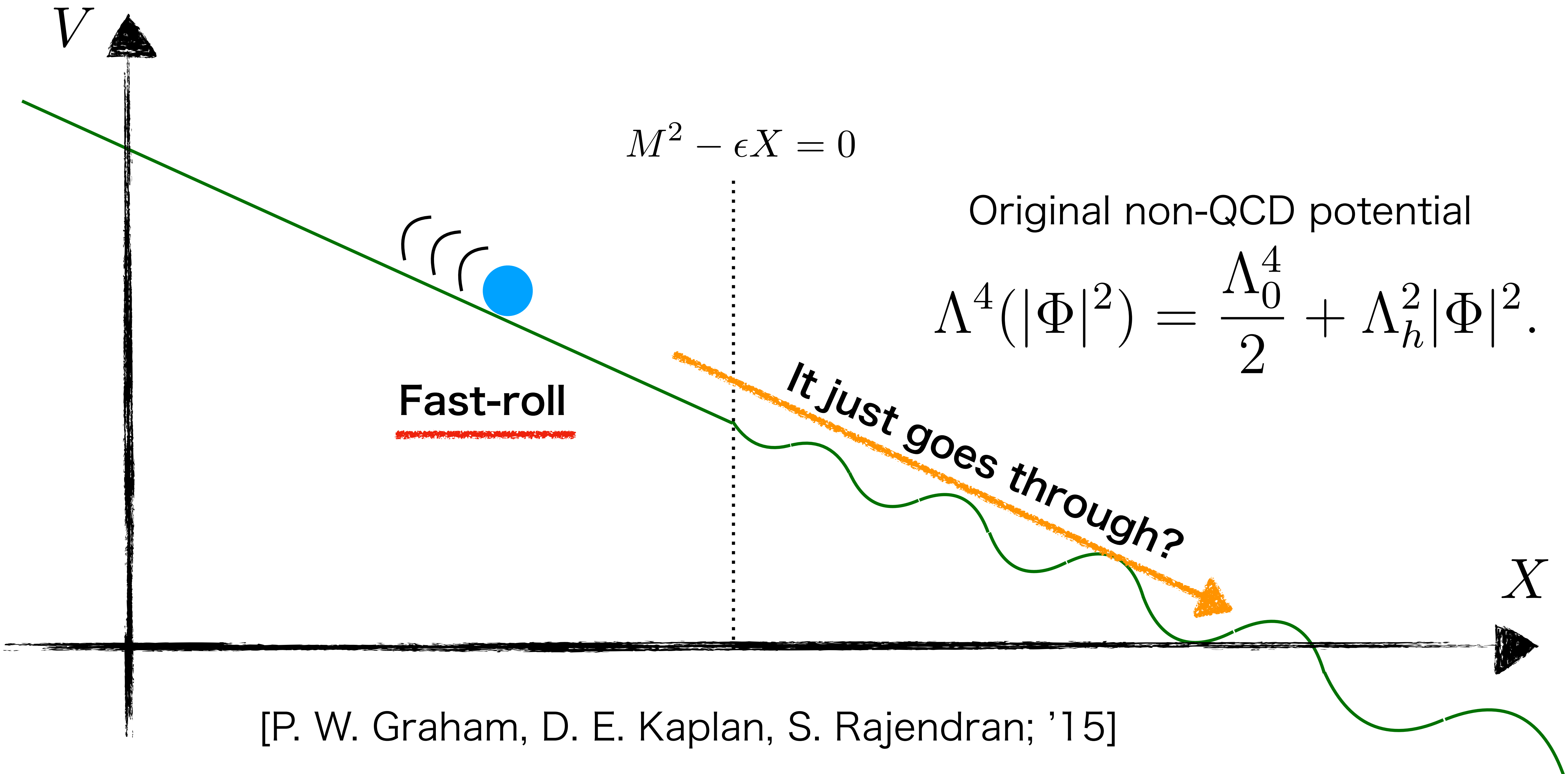


But, we want something

Model

No extension!

$$V(\Phi, X) = (M^2 - \epsilon X)|\Phi|^2 - r\epsilon M^2 X + \Lambda^4(|\Phi|^2)\cos\frac{X}{f} + \frac{\lambda}{4}|\Phi|^4,$$

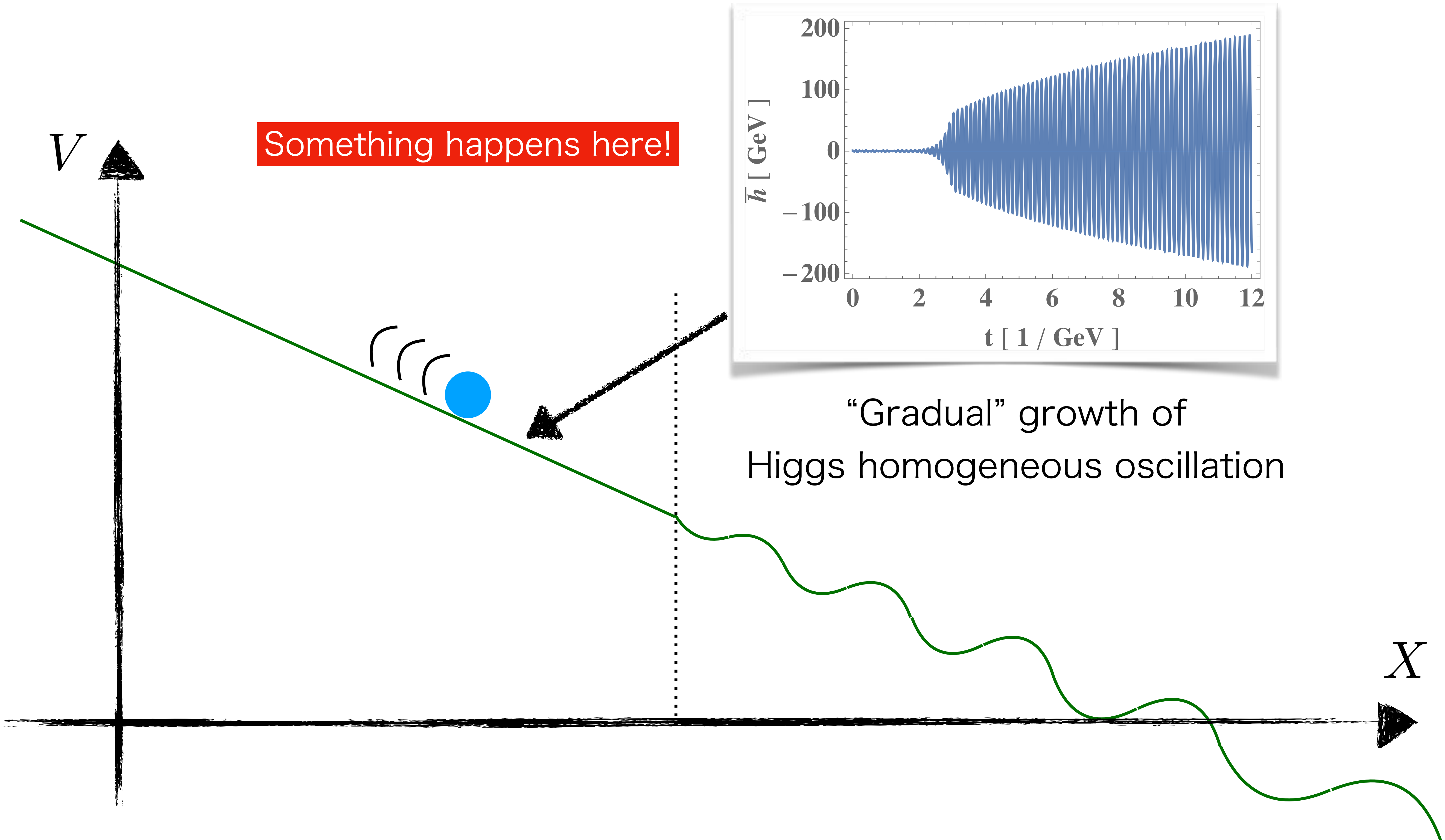


Original non-QCD potential

$$\Lambda^4(|\Phi|^2) = \frac{\Lambda_0^4}{2} + \Lambda_h^2|\Phi|^2.$$

[P. W. Graham, D. E. Kaplan, S. Rajendran; '15]

Higgs homogeneous oscillation



“Gradual” growth of
Higgs homogeneous oscillation

Contents

Edge solution

Equations of motion

$$\ddot{\bar{X}} + 3H\dot{\bar{X}} = \epsilon\left(rM^2 + \frac{\bar{h}^2}{2}\right) + \frac{\cancel{\Lambda_6^4} + \Lambda_h^2\bar{h}^2}{2f}\sin\frac{\bar{X}}{f}\,,$$

$$\ddot{\bar{h}} + 3H\dot{\bar{h}} = -(M^2 - \epsilon\bar{X})\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \Lambda_h^2\bar{h}\cos\frac{\bar{X}}{f}\,,$$

Terminal velocity

$$\boxed{\dot{\bar{X}} \simeq \frac{\epsilon r M^2}{3H}}$$

$$\ddot{\bar{h}} + 3H\dot{\bar{h}} \simeq -(m^2 - \delta m^2 t)\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \Lambda_h^2 h\cos\omega t$$

$$\delta m^2 = f\omega = \frac{\epsilon r M^2}{3H}$$

$$m^2 = M^2 - \epsilon\bar{X}(0)$$

$\bar{X}$: Relaxion homogeneous mode

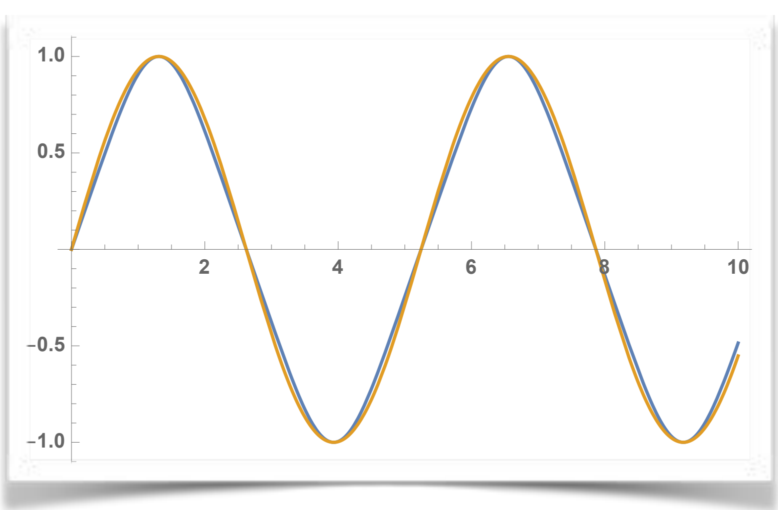
$\bar{h}$: Higgs homogeneous mode

Oscillatory solution

$$\ddot{\bar{h}} + \cancel{3H\dot{\bar{h}}} \simeq -(m^2 - \cancel{\delta m^2 t})\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \cancel{\Lambda_h^2 \bar{h}\cos\omega t}$$

$$\rightarrow \quad \bar{h}(t) = \mathcal{A}\,\mathrm{sn}\left(\sqrt{m^2 + \frac{\lambda}{8}\mathcal{A}^2}\,t, -\frac{\mathcal{A}^2\lambda}{8m^2 + \mathcal{A}^2\lambda}\right)$$

$$\simeq \mathcal{A}\sin(\overline{m}(\mathcal{A})t) \ ,$$



$\mathcal{A}$: Amplitude of oscillation

Effective mass

$$\overline{m}^2(\mathcal{A}) = \frac{\pi^2(8m^2 + \mathcal{A}^2\lambda)}{32\left[K\left(-\frac{\mathcal{A}^2\lambda}{8m^2+\mathcal{A}^2\lambda}\right)\right]^2} \simeq m^2 + \frac{3}{16}\lambda\mathcal{A}^2$$

Parametric resonance

$$\ddot{\bar{h}} + \cancel{3H\dot{\bar{h}}} \simeq -\underbrace{(m^2 - \cancel{\delta m^2 t})\bar{h} - \frac{\lambda}{4}\bar{h}^3}_{\overline{m}^2(\mathcal{A})\bar{h}} - \underline{\Lambda_h^2 h \cos\omega t}$$

Mathieu equation

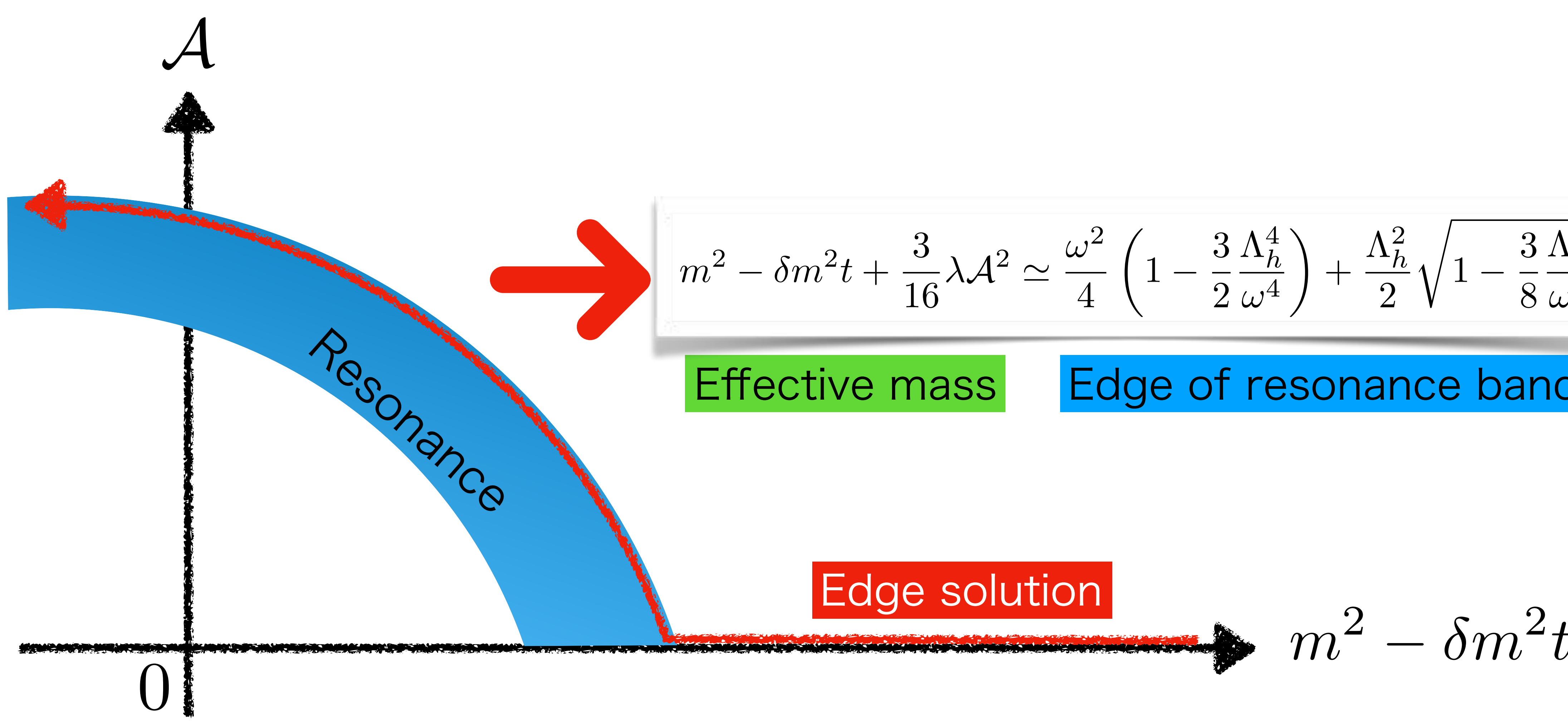
$$\rightarrow \boxed{\ddot{\bar{h}} \simeq -\overline{m}^2(\mathcal{A})\bar{h} - \Lambda_h^2 h \cos\omega t}$$

Exponential growth of the amplitude occurs when

$$\frac{\omega^2}{4}\left(1 - \frac{3}{2}\frac{\Lambda_h^4}{\omega^4}\right) - \frac{\Lambda_h^2}{2}\sqrt{1 - \frac{3}{8}\frac{\Lambda_h^4}{\omega^4}} \lesssim \overline{m}^2(\mathcal{A}) \lesssim \frac{\omega^2}{4}\left(1 - \frac{3}{2}\frac{\Lambda_h^4}{\omega^4}\right) + \frac{\Lambda_h^2}{2}\sqrt{1 - \frac{3}{8}\frac{\Lambda_h^4}{\omega^4}}$$

Edge solution

$$\ddot{\bar{h}} + \cancel{3H\dot{\bar{h}}} \simeq -(m^2 - \underline{\delta m^2 t})\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \Lambda_h^2 h \cos\omega t$$



$$m^2 - \delta m^2 t + \frac{3}{16}\lambda\mathcal{A}^2 \simeq \frac{\omega^2}{4}\left(1 - \frac{3}{2}\frac{\Lambda_h^4}{\omega^4}\right) + \frac{\Lambda_h^2}{2}\sqrt{1 - \frac{3}{8}\frac{\Lambda_h^4}{\omega^4}}$$

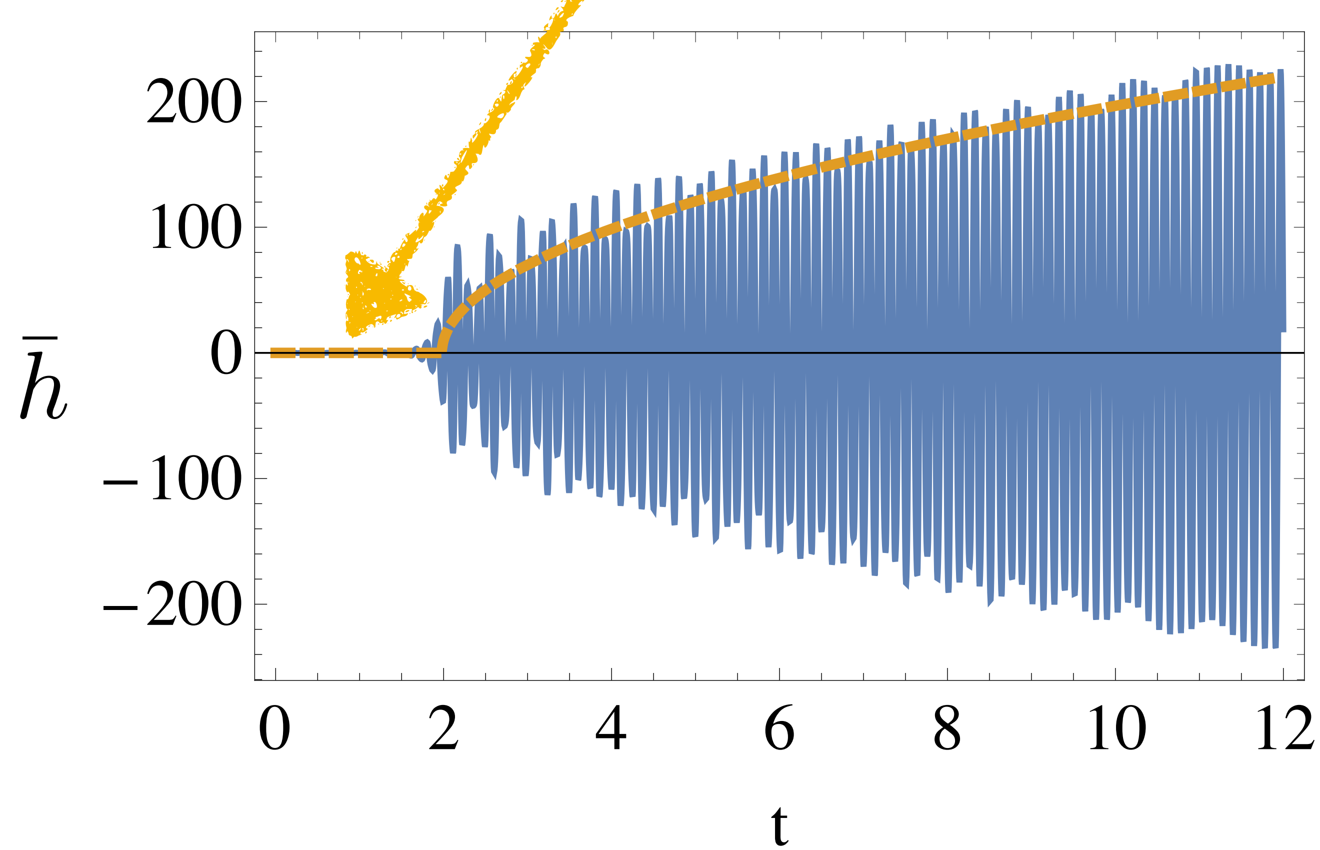
Effective mass

Edge of resonance band

Numerical check

$$\ddot{\bar{h}} + \cancel{3H\dot{\bar{h}}} \simeq -(m^2 - \delta m^2 t)\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \Lambda_h^2 h \cos\omega t$$

$$\boxed{m^2 - \delta m^2 t + \frac{3}{16}\lambda\mathcal{A}^2 \simeq \frac{\omega^2}{4}\left(1 - \frac{3}{2}\frac{\Lambda_h^4}{\omega^4}\right) + \frac{\Lambda_h^2}{2}\sqrt{1 - \frac{3}{8}\frac{\Lambda_h^4}{\omega^4}}}$$

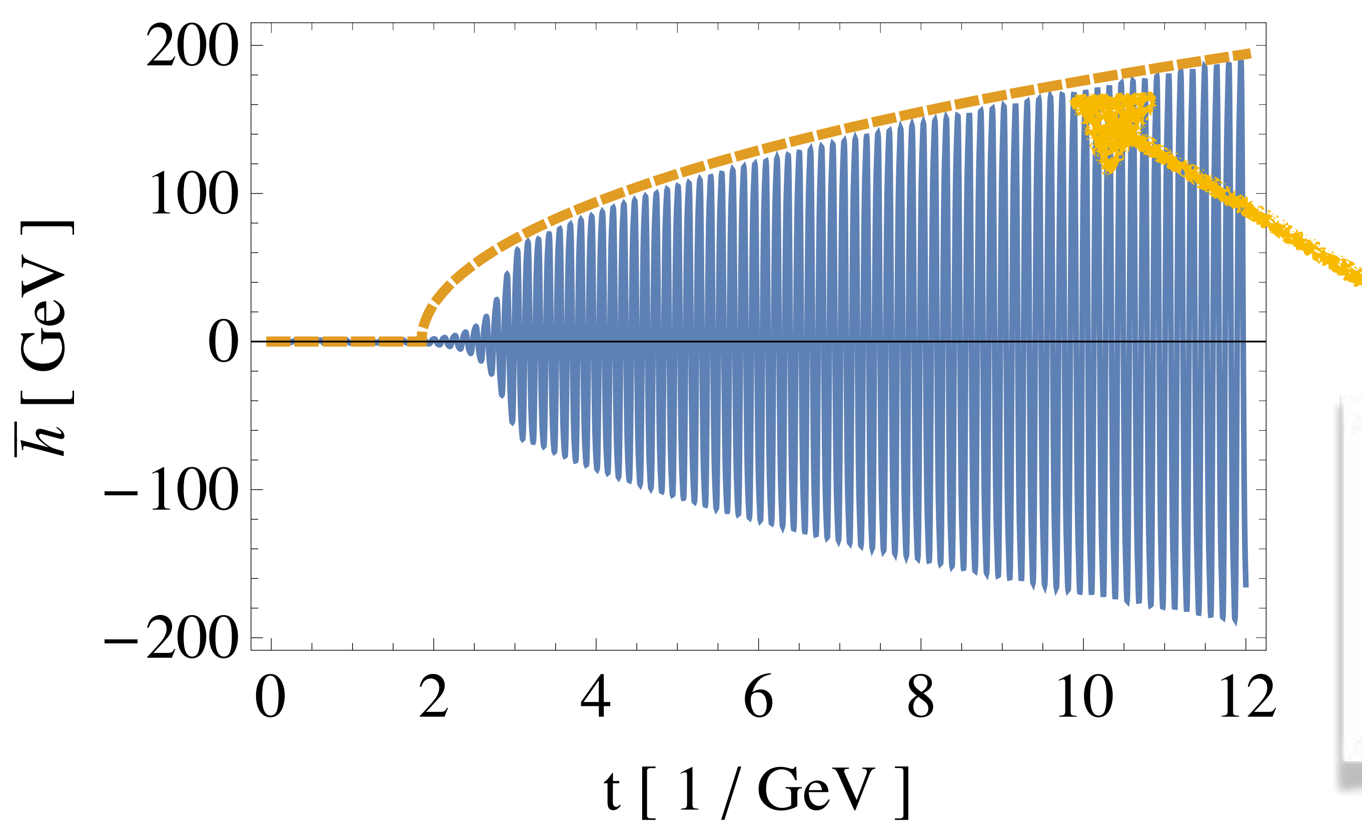


Edge solution (full)

Hubble friction, Back reactions, ⋯

$$\ddot{\bar{X}} + 3H\dot{\bar{X}} = \epsilon\left(rM^2 + \frac{\bar{h}^2}{2}\right) + \frac{\Lambda_0^4 + \Lambda_h^2\bar{h}^2}{2f}\sin\frac{\bar{X}}{f}\ ,$$

$$\ddot{\bar{h}} + 3H\dot{\bar{h}} = -(M^2 - \epsilon\bar{X})\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \Lambda_h^2\bar{h}\cos\frac{\bar{X}}{f}\ ,$$

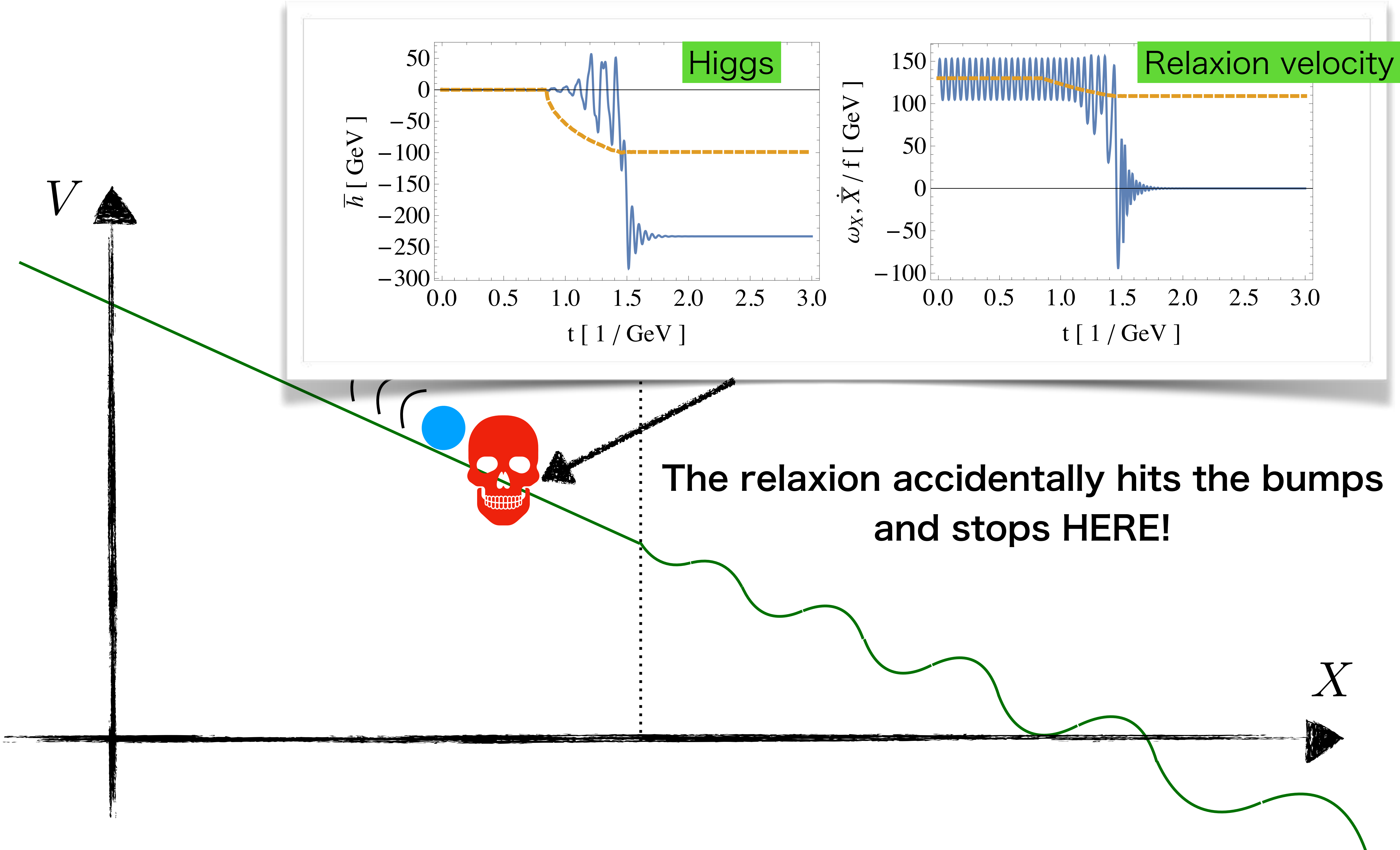


Analytic expression

$$\begin{aligned}
m_\Phi^2 + \frac{3\lambda}{16}\mathcal{A}_h^2 \simeq \frac{\omega_X^2}{4} + \frac{\Lambda_h^2}{2} - \frac{\Lambda_h^2(32\Lambda_0^4 + 17\Lambda_h^2\mathcal{A}_h^2)}{128f^2\omega_X^2}& \\
- \frac{(8\Lambda_h^2 - 3\lambda\mathcal{A}_h^2)(8\Lambda_h^2 - \lambda\mathcal{A}_h^2)}{512\omega_X^2} - \frac{9\omega_X^2}{4\Lambda_h^2}H^2\ ,& \\
\omega_X \simeq \frac{r\epsilon M^2}{3Hf} - \frac{\mathcal{A}_h^2\omega_X}{8f^2} - \frac{\lambda\mathcal{A}_h^4}{128f^2\omega_X} + \frac{3\mathcal{A}_h^2\omega_X^3}{16f^2\Lambda_h^4}H^2\ ,&
\end{aligned}$$

Stopping mechanism

Stopping mechanism

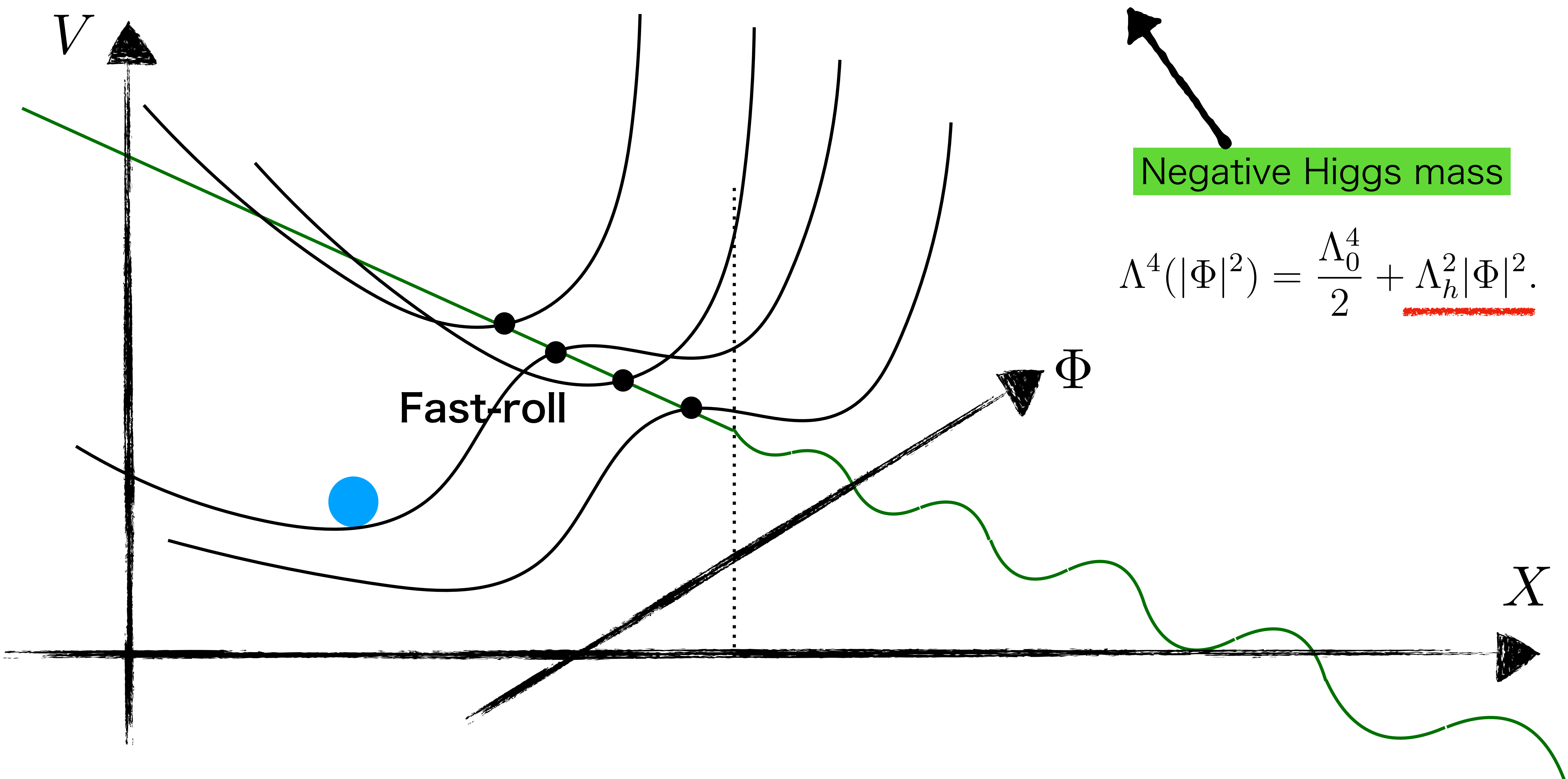


The relaxion accidentally hits the bumps and stops HERE!

Negative Higgs mass

Positive but small

$$V(\Phi, X) = (M^2 - \epsilon X)|\Phi|^2 - r\epsilon M^2 X + \Lambda^4(|\Phi|^2)\cos\frac{X}{f} + \frac{\lambda}{4}|\Phi|^4,$$



Negative Higgs mass

$$\Lambda^4(|\Phi|^2) = \frac{\Lambda_0^4}{2} + \Lambda_h^2|\Phi|^2.$$

Conditions for successful relaxation

Conditions

<u>Relaxion rolling</u>

The relaxion does not slow-roll

The relaxion can go over
the cosine potential
with its terminal velocity

Classical rolling dominates
over quantum fluctuations

<u>Stopping mechanism</u>

The edge solution grows

Oscillation starts before
the relaxion pass through
the EW scale

The Higgs boson obtains
a large enough VEV
after the relaxion stops

<u>EW vacuum</u>

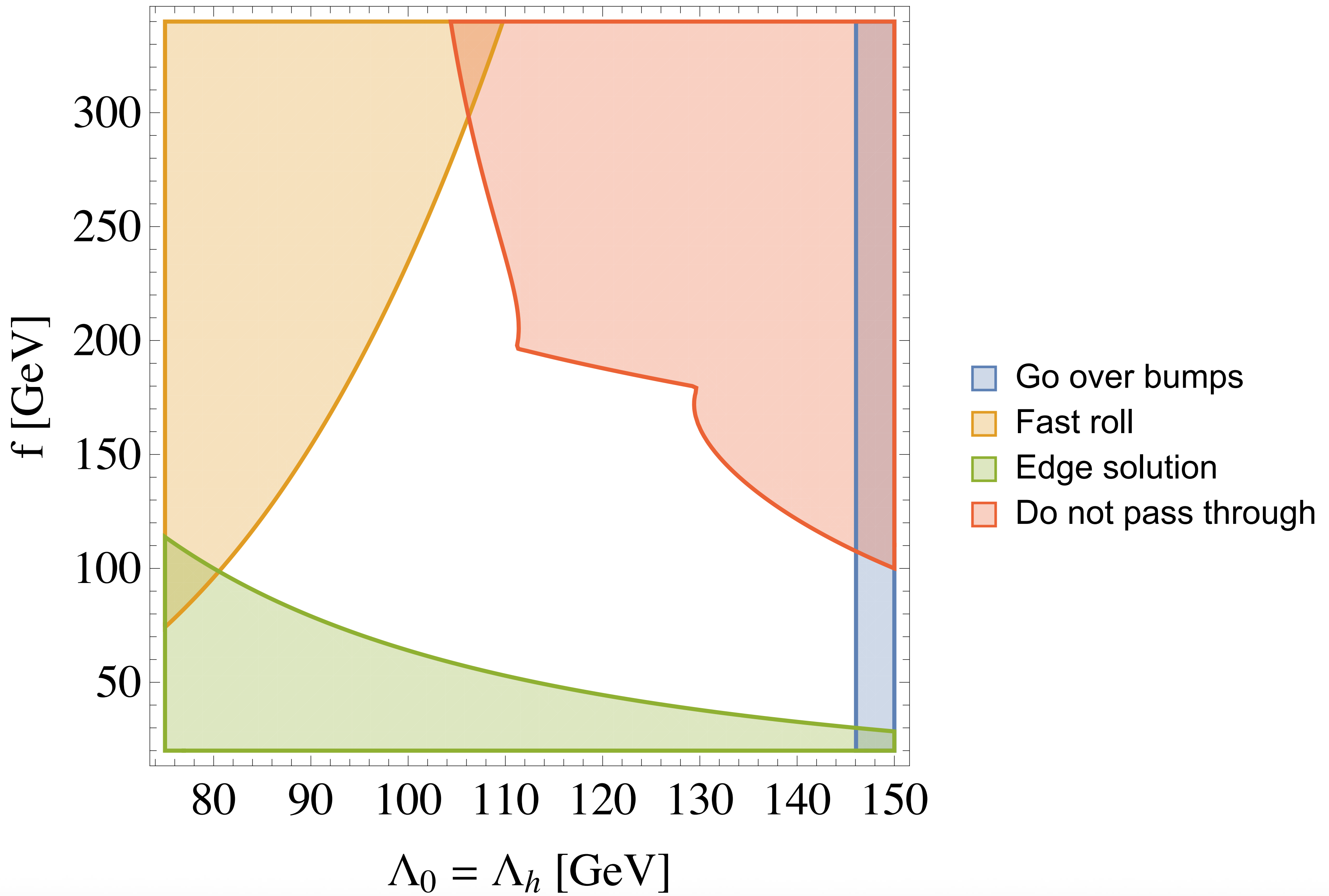
The EW vacuum is a stationary point

The stability of the EW vacuum

There are enough number of vacua around the EW scale

Parameter space

$$H = 10\ \text{GeV},\ \epsilon = 0.8\ \text{GeV},\ r = 0.002,\ M = 20\ \text{TeV}.$$



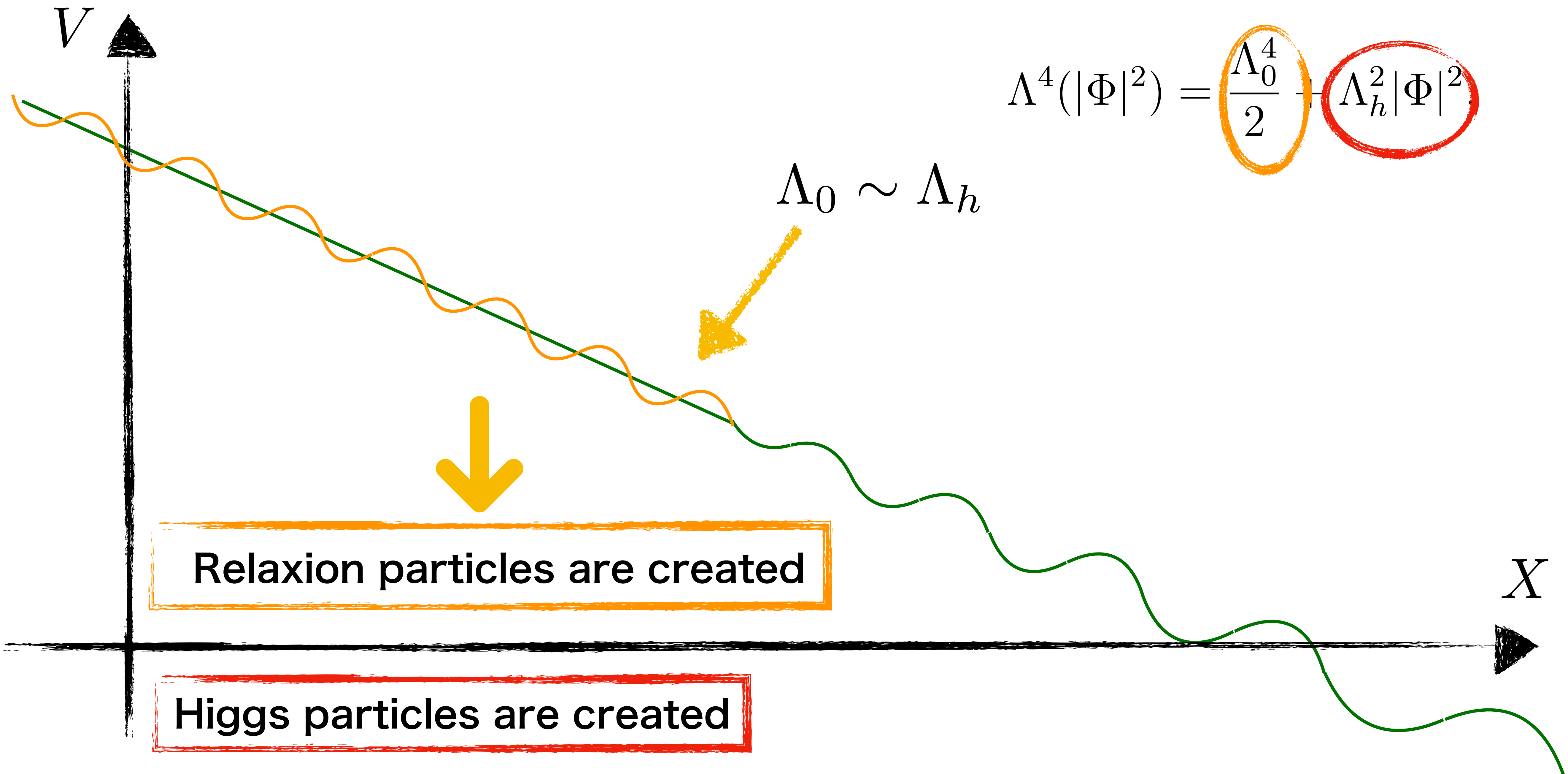
The lifetime of the vacuum is much longer than the age of the universe

Inhomogeneous modes

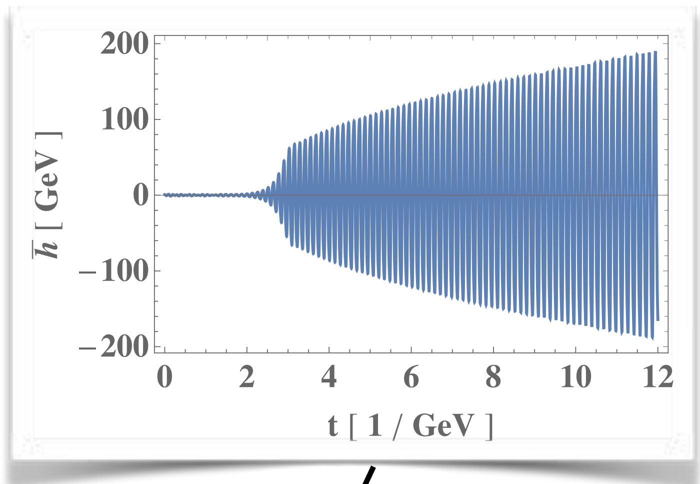
Particle production

$$V(\Phi, X) = (M^2 - \epsilon X)|\Phi|^2 - r\epsilon M^2 X + \Lambda^4(|\Phi|^2)\cos\frac{X}{f} + \frac{\lambda}{4}|\Phi|^4,$$

$$\Lambda^4(|\Phi|^2) = \frac{\Lambda_0^4}{2} + \Lambda_h^2|\Phi|^2$$

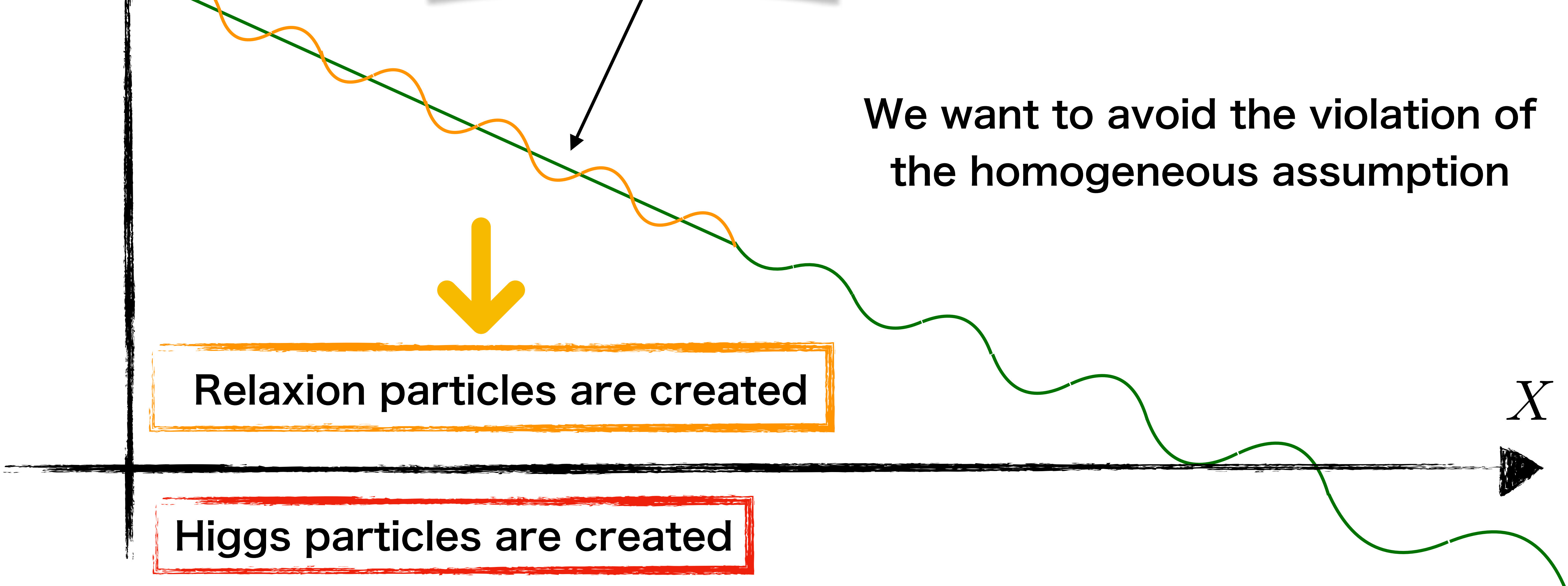


Particle production



Decay into
relaxion, Higgs, gauge, ⋯

We want to avoid the violation of
the homogeneous assumption



Relaxion particles are created

Higgs particles are created

Equations of motion

$$X(x,t) = \bar{X}(t) + \varphi(x) \ , \qquad \varphi(x) = \int \frac{d^3k}{(2\pi)^3} \varphi_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}} \ ,$$

$$h(x,t) = \bar{h}(t) + \chi(x) \ . \qquad \chi(x) = \int \frac{d^3k}{(2\pi)^3} \chi_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}} \ .$$

$$\Downarrow$$

$$\ddot{\varphi}_{\mathbf{k}} + 3H\dot{\varphi}_{\mathbf{k}} + \left(m_\varphi^2 + \frac{|\mathbf{k}|^2}{a^2(t)} \right) \varphi_{\mathbf{k}} + \delta m_\varphi^2 \varphi_{\mathbf{k}} + \delta m_{\text{mix}}^2 \chi_{\mathbf{k}} = 0 \ ,$$

$$\ddot{\chi}_{\mathbf{k}} + 3H\dot{\chi}_{\mathbf{k}} + \left(m_\chi^2 + \frac{|\mathbf{k}|^2}{a^2(t)} \right) \chi_{\mathbf{k}} + \delta m_\chi^2 \chi_{\mathbf{k}} + \delta m_{\text{mix}}^2 \varphi_{\mathbf{k}} = 0 \ ,$$

$$m_\varphi^2 \simeq \frac{\Lambda_h^2 \mathcal{A}_h^2}{8f^2} \ ,$$

$$\delta m_\varphi^2 \simeq -\frac{2\Lambda_0^4 + \Lambda_h^2 \mathcal{A}_h^2}{4f^2} \cos \omega_X t + \frac{\Lambda_h^2 \mathcal{A}_h^2}{8f^2} \cos 2\omega_X t \ ,$$

$$m_\chi^2 \simeq m_\Phi^2 + \frac{3}{8} \lambda \mathcal{A}_h^2 \ ,$$

$$\delta m_\chi^2 \simeq \left(\Lambda_h^2 - \frac{3\lambda}{8} \mathcal{A}_h^2 \right) \cos \omega_X t \ ,$$

$$\delta m_{\text{mix}}^2 \simeq -\frac{\Lambda_h^2 \mathcal{A}_h}{2f} \left(\cos \frac{\omega_X}{2} t - \cos \frac{3\omega_X}{2} t \right) \ .$$

These equations have instabilities like the Mathieu equation

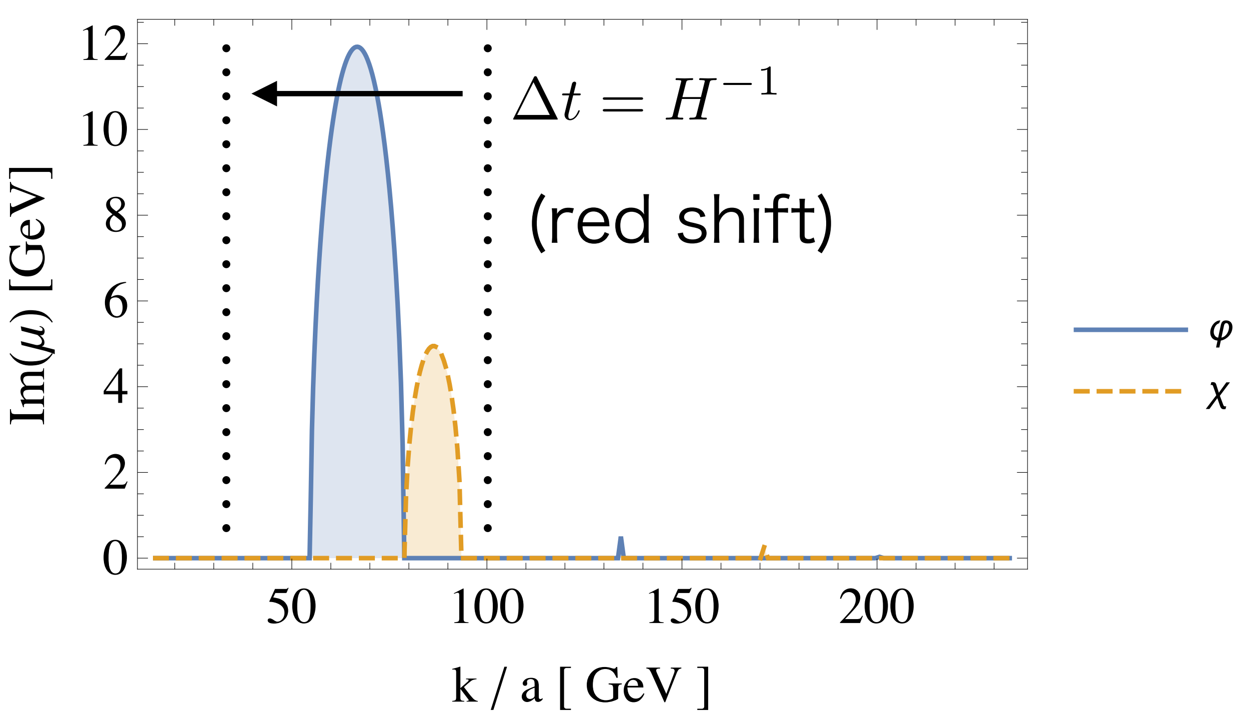
Before the Higgs oscillation

$$\chi_{\mathbf{k}} \simeq \frac{C_\chi}{\sqrt{\mu_\chi(t)}} F_\chi(t) e^{i\int^t \mu_\chi(t')dt' - \frac{3}{2}Ht} \,,$$

$$\varphi_{\mathbf{k}} \simeq \frac{C_\varphi}{\sqrt{\mu_\varphi(t)}} F_\varphi(t) e^{i\int^t \mu_\varphi(t')dt' - \frac{3}{2}Ht} \,,$$

F_χ and F_φ are $\mathcal{O}(1)$ functions

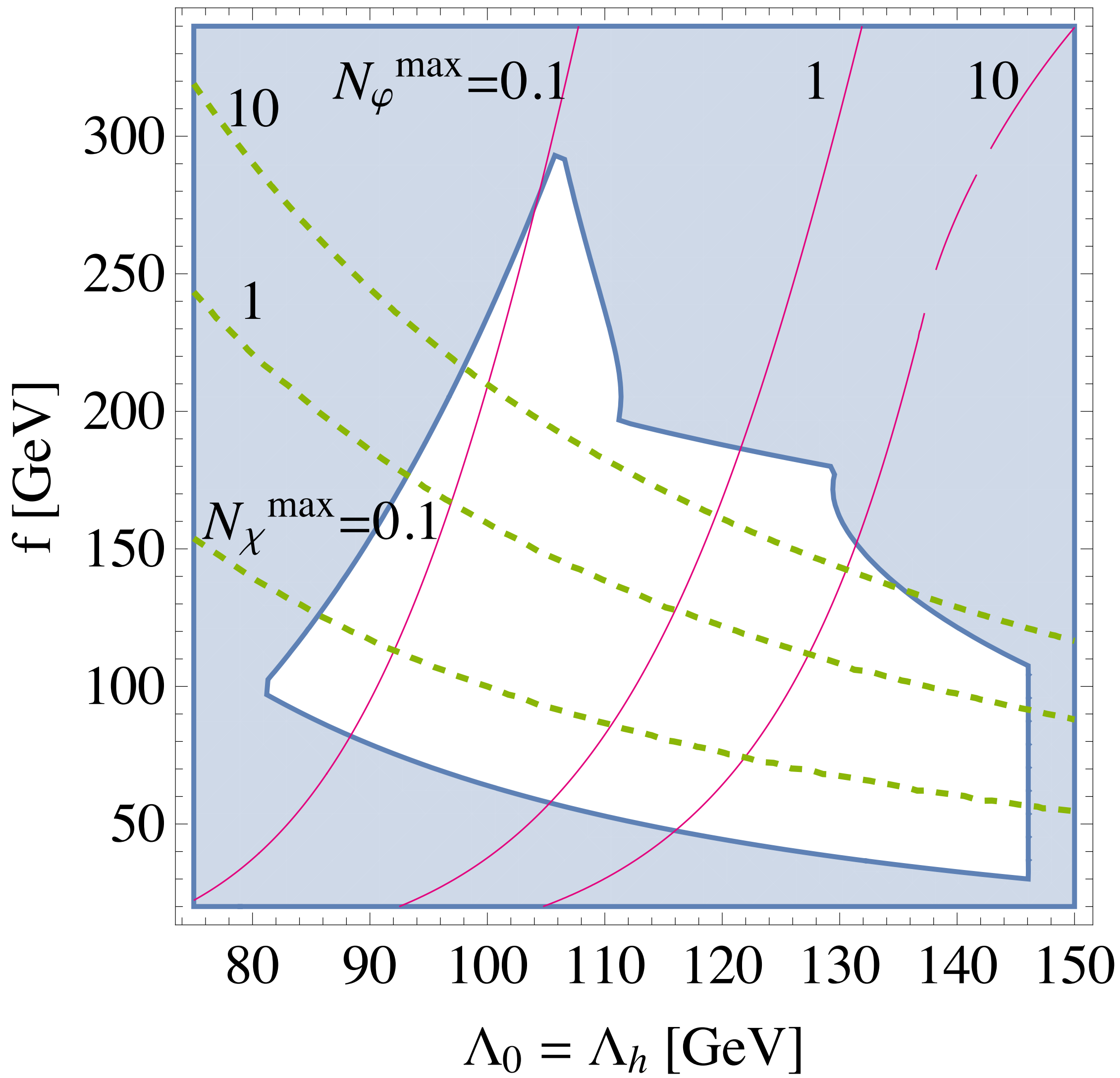
The exponential growth takes place only for a short time



$\lambda = 0.52,\ H = 10\ \text{GeV},\ \Lambda_0 = 80\ \text{GeV},\ \Lambda_h = 100\ \text{GeV},\ \epsilon = 0.8\ \text{GeV},\ r = 0.001,$

$f = 80\ \text{GeV},\ M = 20\ \text{TeV}.$

Occupation number



$N_\varphi^{\rm max}, N_\chi^{\rm max}$

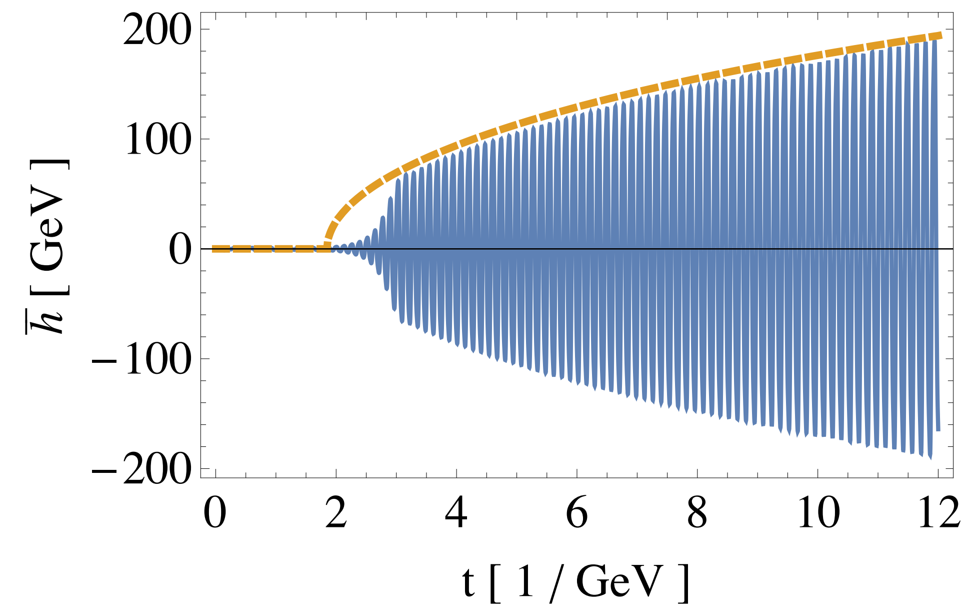
: (maximally amplified) occupation numbers

We require

$$N_\varphi^{\rm max} \lesssim 10\ ,\ \ N_\chi^{\rm max} \lesssim 10\ ,$$

(same order as zero point fluctuation)

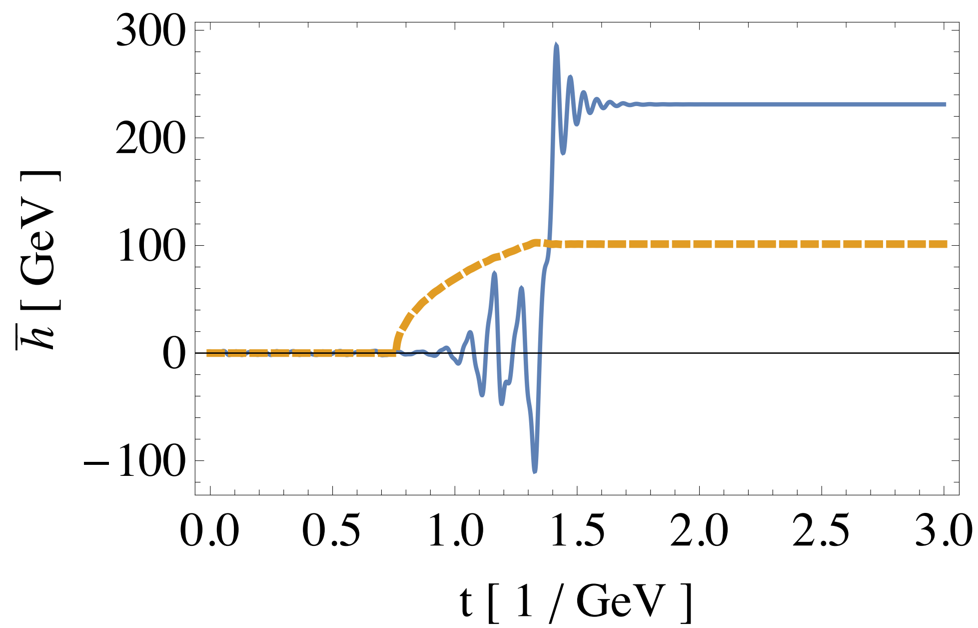
After the Higgs oscillation



Resonances are too strong

Too many particles are produced

Limit the number of oscillations

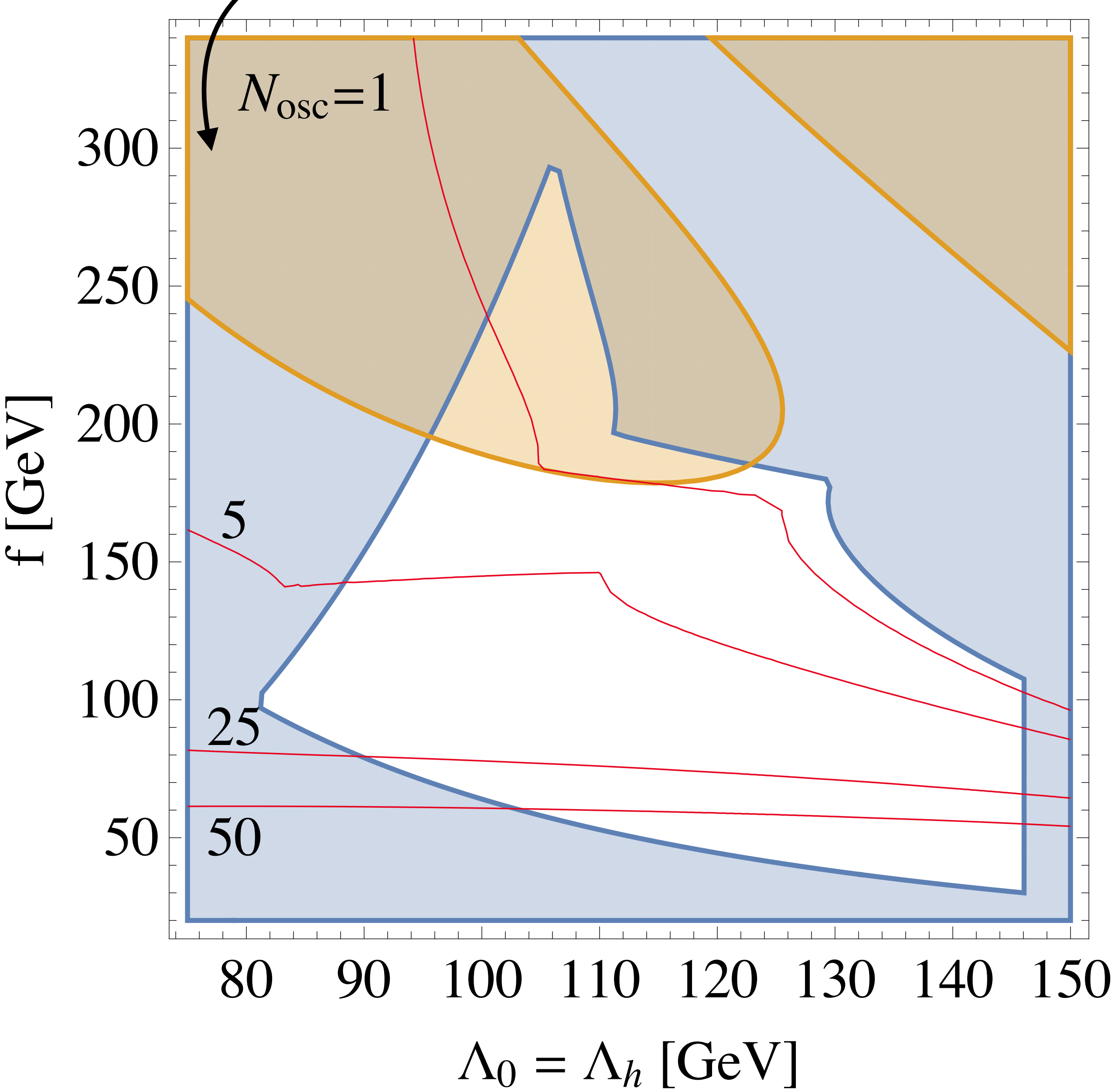


Each oscillation has a random phase

No exponential growth of inhomogeneous modes

The number of the Higgs oscillations

Approximated solution of the Higgs oscillation is not reliable



$N_{\rm osc}$

: the number of the Higgs oscillations

We require

$$N_{\rm osc} \lesssim 25\ ,$$

Collider constraints

Relaxion-Higgs mixing

Mass matrix

$$
\begin{array}{c}
\begin{array}{cc} \text{Higgs} & \text{Relaxion} \end{array} \\
\begin{pmatrix} \frac{\lambda v^2}{2} & \frac{\Lambda_h^2 v}{f} s_X \\ \frac{\Lambda_h^2 v}{f} s_X & \frac{\Lambda_0^4 + \Lambda_h^2 v^2}{2f^2} c_X \end{pmatrix}
\end{array}
$$

Large off-diagonal due to small f

$$
s_X \equiv -\sin\frac{\langle X \rangle}{f} = \frac{2fr\epsilon M^2}{\Lambda_0^4 + \Lambda_h^2 v^2} \ , \ c_X \equiv -\cos\frac{\langle X \rangle}{f} = \sqrt{1 - s_X^2} \ ,
$$

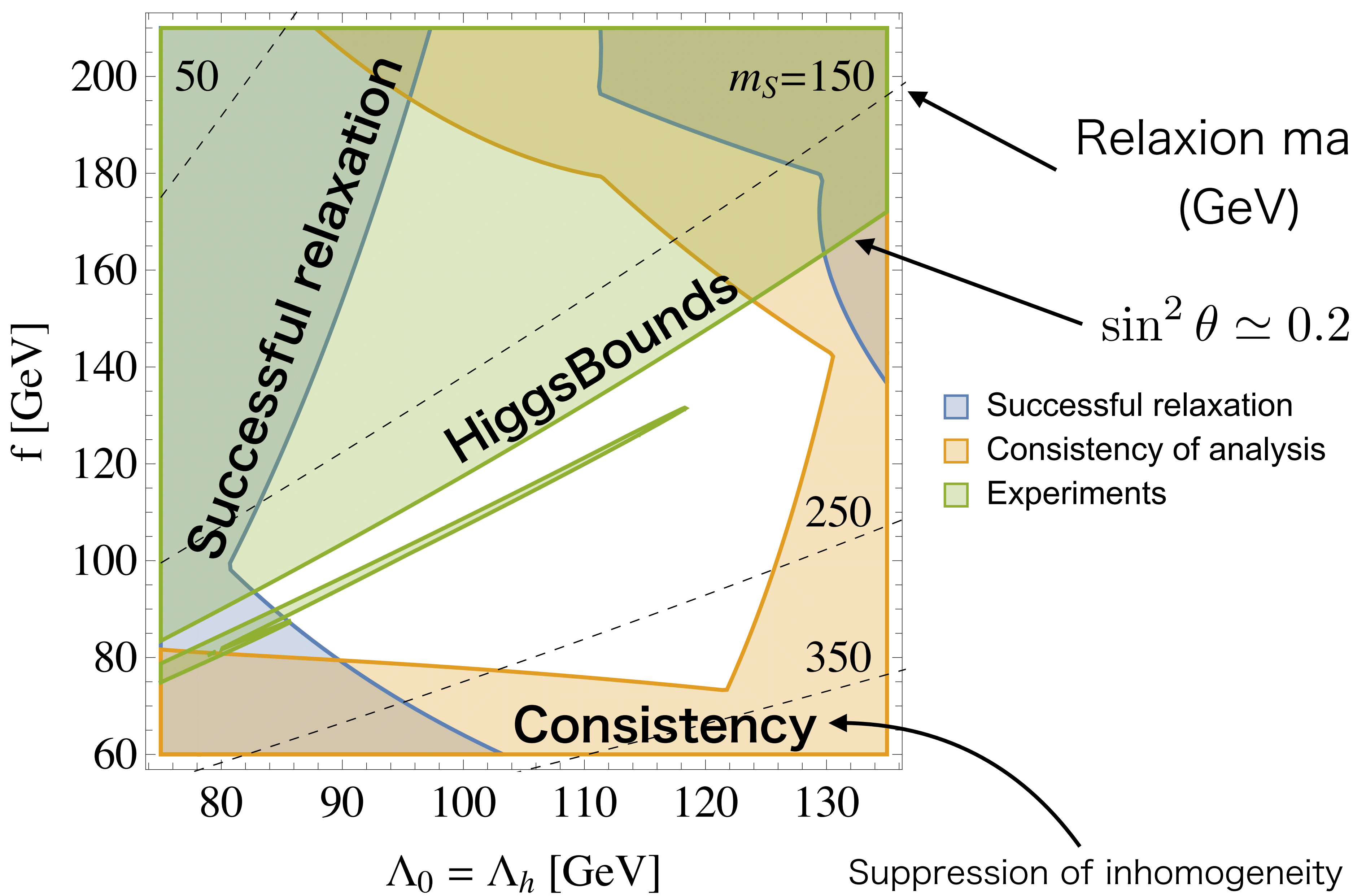
Mixing angle

$$
\theta \simeq \frac{-2fv\Lambda_h^2 s_X}{(\Lambda_0^4 + v^2\Lambda_h^2)c_X - \lambda f^2 v^2}
$$

Mixing angle is not suppressed
if the masses are comparable

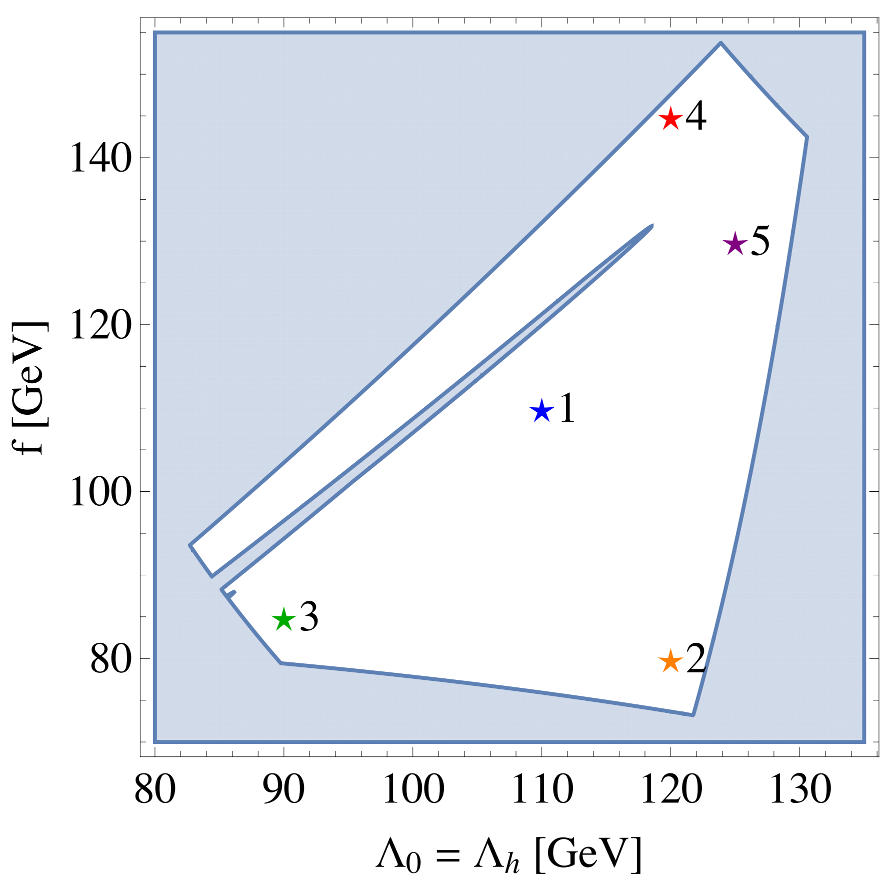
Collider constraints

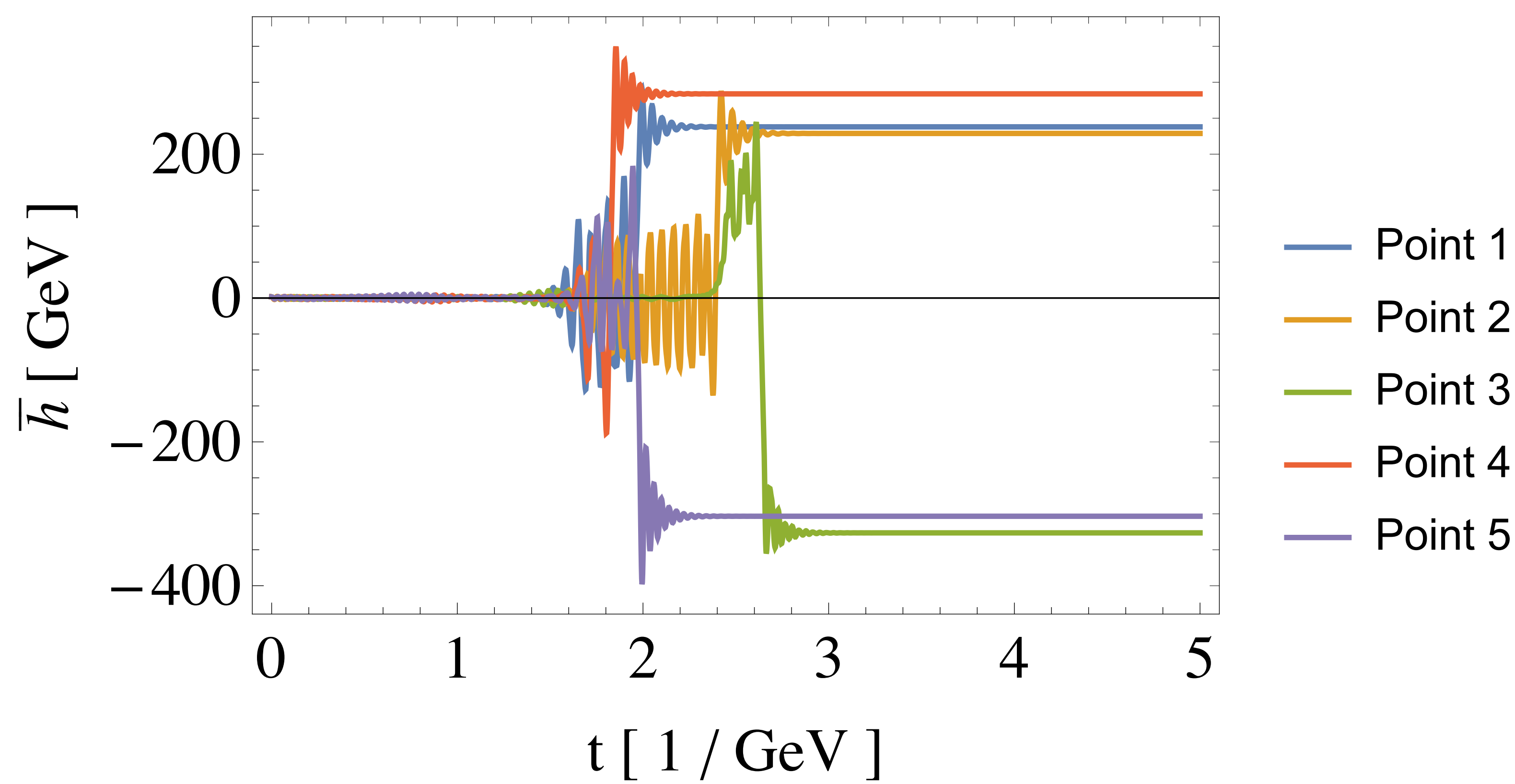
$$H = 10 \text{ GeV}, \ \epsilon = 0.8 \text{ GeV}, \ r = 0.002, \ M = 20 \text{ TeV}.$$



Existence of solutions

$$H = 10\ \text{GeV},\ \epsilon = 0.8\ \text{GeV},\ r = 0.002,\ M = 20\ \text{TeV}.$$





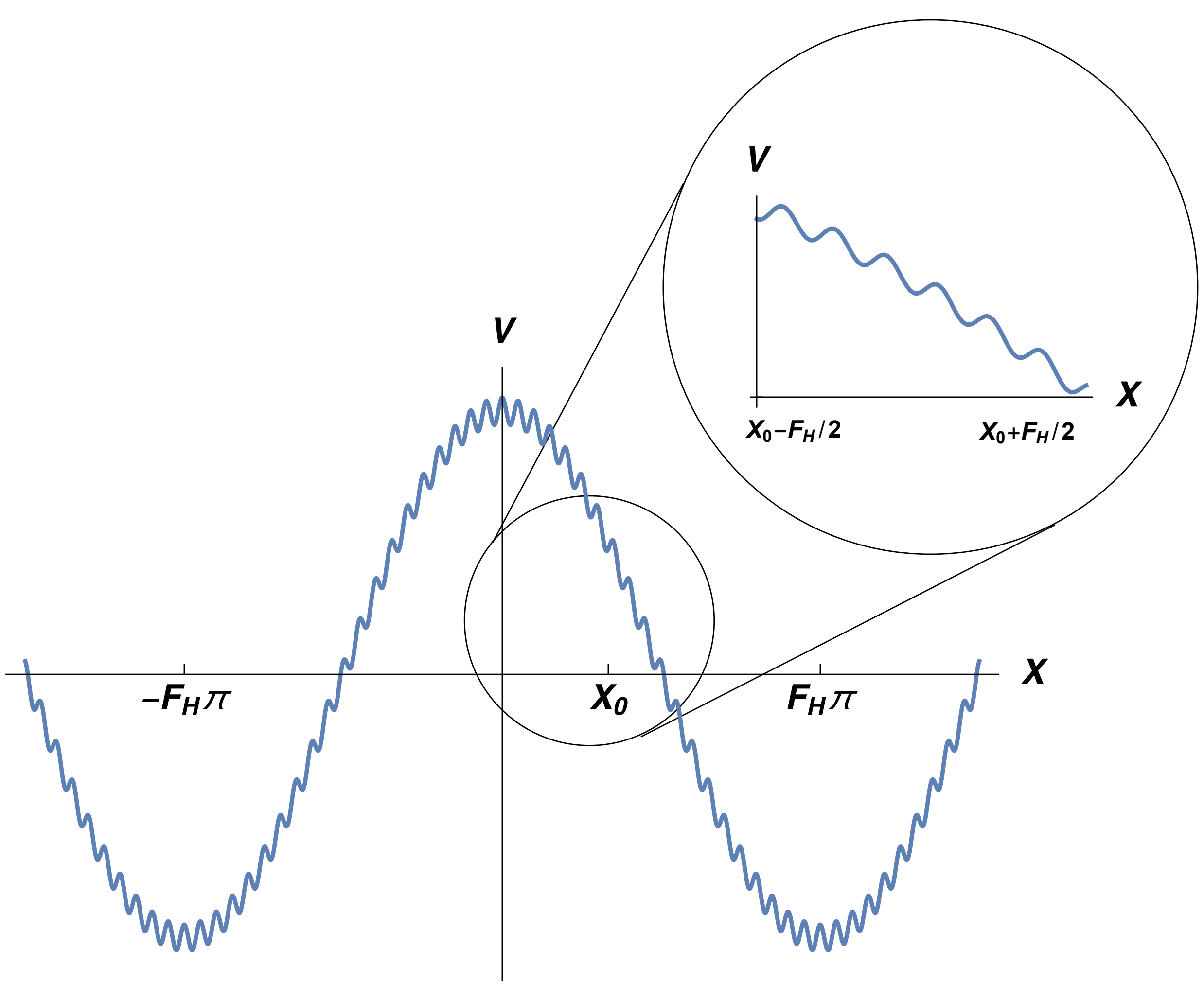
Relaxion as a pseudo NGB

Compact field space

$$V(X) = \kappa_H \left(rM^2 + |\Phi|^2\right) F_H^2 \cos\left(\frac{X}{F_H}\right) + \Lambda^4(|\Phi|^2) \cos\left(\frac{X}{F_L} + \delta\right) \ ,$$

Large structure

Small structure



Origin of the slope of the relaxion

Field space is sub-Planckian

Difficulties

$$V(X) = \kappa_H \left(rM^2 + |\Phi|^2\right) F_H^2 \cos\left(\frac{X}{F_H}\right) + \Lambda^4(|\Phi|^2) \cos\left(\frac{X}{F_L} + \delta\right) \ ,$$

1. Two hierarchical decay constants

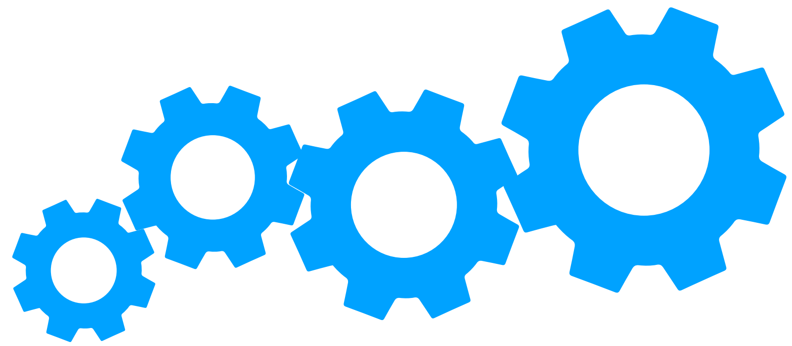
$F_H \gg F_L$ ➡ **Clockwork mechanism**

[D. E. Kaplan, R. Rattazzi, '16; K. Choi, S. H. Im, '16; G. F. Giudice, M. McCullough, '17; ⋯]

2. A very large explicit breaking

$M^2 \gg F_L^2$ ➡ **Clockwork mechanism with progressively increasing VEVs**

Clockwork with progressively increasing VEVs



Consider scalar clockwork with U(1) charges $Q(\phi_k) = 3^{-k}$

<u>Increasing VEVs</u>

$$V_{\rm self} = \sum_{k=0}^{N-1} \left(|\phi_k|^2 - \frac{f_k^2}{2} \right)^2 , \qquad f_k = a^k f, \ (1 < a \leq 3)$$

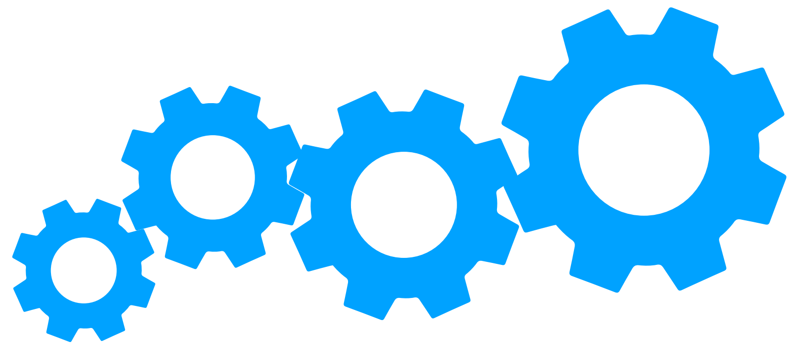
<u>Safe connections</u>

$$V_{\rm conn} = - \sum_{k=0}^{N-2} \kappa_k \phi_k^* \phi_{k+1}^3 + h.c. \ , \qquad \kappa_k \sim a^{-3}$$

<u>Safe explicit breaking</u>

$$V_{\rm br} = \mathcal{E}_0^* \phi_0 + \mathcal{E}_{N-1}^* \phi_{N-1} + h.c. \ , \quad \mathcal{E}_0 \ll f^3 \ , \quad \mathcal{E}_{N-1} \ll (a^{N-1} f)^3 \ .$$

Clockwork with progressively increasing VEVs



Eigenfunctions for $\kappa_{N-2} = 0$

$$\frac{\pi_L}{F_L} \equiv \frac{\pi_0}{f_0} = 3\frac{\pi_1}{f_1} = \cdots = 3^{N-2}\frac{\pi_{N-2}}{f_{N-2}}\,, \qquad \frac{\pi_H}{F_H} \equiv \frac{\pi_{N-1}}{f_{N-1}}\,,$$

$$\phi_k = f_k e^{i\pi_k/f_k}/\sqrt{2}$$

They are connected as

$$V = \sqrt{2}\mathcal{E}_0 f_0 \cos\left(\frac{\pi_L}{F_L}\right) + \sqrt{2}\mathcal{E}_H F_H \cos\left(\frac{\pi_H}{F_H}\right) - \underline{\frac{1}{2}\kappa_{N-2} f_{N-2} f_{N-1}^3} \cos\left(\frac{\pi_L}{3^{(N-2)}F_L} - 3\frac{\pi_H}{F_H}\right)\,,$$

Large enough

$\frac{\pi_L}{3^{(N-2)}F_L} = 3\frac{\pi_H}{F_H}$ **becomes can be identified as the relaxion**

Summary

- In the original relaxion model, the tunneling phase requires an unacceptably large number of e-folds.
- The fast-roll relaxion can easily solve this problem, but we can not use the original stopping mechanism.
- We propose a mechanism to stop the relaxion without extending the original model.
- The mechanism predicts a relaxion that has a mass of O(100) GeV and mixes with the Higgs boson. It improves the testability of our mechanism.
- The relaxion can be identified as a pseudo NGB and its potential is given by the clockwork mechanism with progressively increasing VEVs.