Schwinger Effect from Charged Black Holes in (A)dS Space

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CMC, Sang Pyo Kim, arXiv:2002.00394 [hep-th]

Outline

- Introduction
- Emission from Near Horizon of Near Extremal Black Holes
- Emission from (Non)Extremal Black Holes: two special cases
- Discussion

Introduction

- spontaneous pair production: two important processes
  - Schwinger mechanism by electromagnetic force

$$N_S \sim \mathrm{e}^{-\frac{m}{T_S}}, \qquad T_S = 2T_U = 2\frac{qE/m}{2\pi} \qquad \text{(in flat spacetime)}$$

    - The factor 2 is essential.
  - Hawking radiation by tunneling through horizon

$$N_H \sim \mathrm{e}^{-\frac{\omega}{T_H}}, \qquad T_H = \frac{\kappa}{2\pi}$$

Introduction

- Schwinger effect in curved spacetimes: dS_2 or AdS_2
  - in-out formalism (one-loop effective action)
    Cai, Kim, JHEP 1409 (2014) 072 [arXiv:1407.4569 [hep-th]]

$$T_S = T_U + \sqrt{T_U^2 + \frac{\mathcal{R}}{8\pi^2}}$$

  - $\mathcal{R}$: curvature of dS_2/AdS_2
- Schwinger effect by instanton method: dS_2
  Samantray, JHEP 1604 (2016) 060 [arXiv:1601.01406 [hep-th]]
  - on-shell action $S = m \int ds + \frac{1}{4} \int F_{\mu\nu} F^{\mu\nu} dV$
- Spontaneous pair production from charged black holes should mixture both Schwinger and Hawking effects.

Emission from Near Horizon of Near Extremal BHs

- Schwinger effect in near horizon of near extremal charged black holes
  - technical simplicity: constant electric field (exactly solvable)
  - holographic description: anti de Sitter
- Reissner-Nordström (RN) Black Holes: near extremal
  - near horizon region: where the pair production occurs

$$AdS_2 \times S^2 + \text{constant electric field}$$

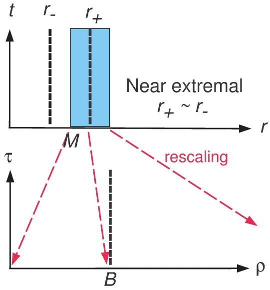
- extremal RN
  - vanishing HAWKING temperature: stable (thermal)
  - non-vanishing electric field: unstable (quantum)
- One can straightforwardly generalize the analysis to Kerr-Newman black holes.

Emission from Near Horizon of Near Extremal BHs

- Reissner-Nordström Black Holes: M, Q, L
  - Horizon radius: $r_-,\ r_+,\ r_c$ (dS)
  - Near extremal: upper/lower sign = AdS/dS

$$r_-,\ r_+ \ \rightarrow\ r_0 = L\sqrt{\delta}/\sqrt{6}, \quad \delta = \pm(\sqrt{1 \pm 12Q^2/L^2} - 1)$$

- Near horizon geometry: exact solution, AdS structure



Near horizon geometry

Emission from Near Horizon of Near Extremal BHs

- Reissner-Nordström (RN) black holes: $M, \ Q, \ L$

$$\begin{aligned} ds^2 &= -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2, \quad f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \pm \frac{r^2}{L^2} \\ A_{[1]} &= \frac{Q}{r} dt \end{aligned}$$

- Near horizon limits: $\epsilon \to 0, \ M_0 = L(3 \pm \delta)\sqrt{\delta}/3\sqrt{6}$

$$r \to r_0 + \epsilon\rho, \quad t \to \tau/\epsilon, \quad M \to M_0 + \epsilon^2 B^2 R_{\text{S}}/R_{\text{AdS}}^2$$

- Near horizon geometry of near-extremal RN

$$ds^2 = -\frac{\rho^2 - B^2}{R_{\text{AdS}}^2} d\tau^2 + \frac{R_{\text{AdS}}^2}{\rho^2 - B^2} d\rho^2 + R_{\text{S}}^2 d\Omega_2^2, \quad A_{[1]} = -\frac{Q}{R_{\text{S}}^2} \rho d\tau,$$

$$R_{\text{AdS}}^2 = \frac{L^2\delta}{6(1 \pm \delta)}, \quad R_{\text{S}}^2 = r_0^2 = \frac{L^2\delta}{6}, \quad T_H = \frac{B}{2\pi R_{\text{AdS}}^2}, \quad \Phi_H = -\frac{QB}{R_{\text{S}}^2}$$

Emission from Near Horizon of Near Extremal BHs

- Klein-Gordon equation

$$(\nabla_\mu - iqA_\mu)(\nabla^\mu - iqA^\mu)\Phi - m^2\Phi = 0$$

- Ansatz: $\Phi(\tau,\rho,\theta,\varphi) = \mathrm{e}^{-i\omega\tau}R(\rho)Y_l^n(\theta,\varphi)$

$$\partial_\rho[(\rho^2 - B^2)\partial_\rho R] + \left[\frac{R_{\mathrm{AdS}}^4}{R_{\mathrm{S}}^4}\frac{(qQ\rho - \omega R_{\mathrm{S}}^2)^2}{\rho^2 - B^2} - R_{\mathrm{AdS}}^2 m^2 - \frac{R_{\mathrm{AdS}}^2}{R_{\mathrm{S}}^2} l(l+1)\right] R = 0$$

- BF bound
  - effective mass: $m_{\mathrm{eff}}^2 = m^2 - \frac{R_{\mathrm{AdS}}^2 q^2 Q^2}{R_{\mathrm{S}}^4} + \frac{l(l+1)}{R_{\mathrm{S}}^2}$
  - instability: $m_{\mathrm{eff}}^2 < -\frac{1}{4R_{\mathrm{AdS}}^2}$

$$\mu^2 \equiv \frac{R_{\mathrm{AdS}}^4}{R_{\mathrm{S}}^4} q^2 Q^2 - R_{\mathrm{AdS}}^2 m^2 - \frac{R_{\mathrm{AdS}}^2}{R_{\mathrm{S}}^2} l(l+1) - \frac{1}{4} > 0$$

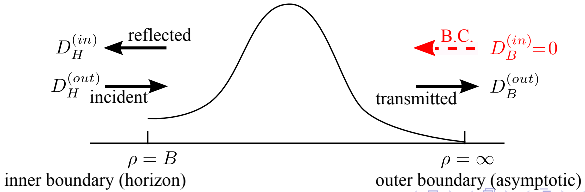
  - effective inertia mass: $\bar{m}^2 = m^2 + \frac{l(l+1)}{R_{\mathrm{S}}^2} + \frac{1/4}{R_{\mathrm{AdS}}^2}$

Emission from Near Horizon of Near Extremal BHs

- The Klein-Gordon (scalar) is exactly solvable in terms of hypergeometric function.

$$
\begin{aligned}
F(\alpha,\beta;\gamma;z) &= \frac{\Gamma(\gamma)\Gamma(\beta-\alpha)}{\Gamma(\beta)\Gamma(\gamma-\alpha)}(1-z)^{-\alpha}F\left(\alpha,\gamma-\beta;\alpha-\beta+1;\frac{1}{1-z}\right)\\
&+ \frac{\Gamma(\gamma)\Gamma(\alpha-\beta)}{\Gamma(\alpha)\Gamma(\gamma-\beta)}(1-z)^{-\beta}F\left(\beta,\gamma-\alpha;\beta-\alpha+1;\frac{1}{1-z}\right)
\end{aligned}
$$

- Boundary condition: no incoming flux at asymptotic



Emission from Near Horizon of Near Extremal BHs

- Mean number of scalar production: extremal $\tilde{\kappa} \to \infty$

$$\mathcal{N} = \left(\frac{\mathrm{e}^{-2\pi(\kappa-\mu)} - \mathrm{e}^{-2\pi(\kappa+\mu)}}{1 + \mathrm{e}^{-2\pi(\kappa+\mu)}} \right) \left(\frac{1 - \mathrm{e}^{-2\pi(\tilde{\kappa}-\kappa)}}{1 + \mathrm{e}^{-2\pi(\tilde{\kappa}-\mu)}} \right)$$

- three essential parameters: $\tilde{\kappa}, \kappa, \mu$

$$\tilde{\kappa} = \frac{\omega R_{\text{AdS}}^2}{B} = \frac{\omega}{2\pi T_H}, \quad \kappa = \frac{R_{\text{AdS}}^2}{R_{\text{S}}^2} qQ = -\frac{q\Phi_H}{2\pi T_H}$$

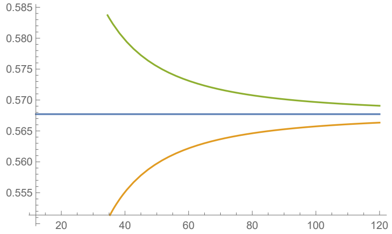
Reissner-Nordström (RN) black holes:
CMC, Kim, Lin, Sun, Wu, PRD 85 (2012) 124041 [arXiv:1202.3224 [hep-th]]
CMC, Sun, Tang, Tsai, CQG 32 (2015) 195003 [arXiv:1412.6876 [hep-th]]
Kerr-Newman (KN) black holes:
CMC, Kim, Sun, Tang, PRD 95 (2017) 044043 [arXiv:1607.02610 [hep-th]]
CMC, Kim, Sun, Tang, PLB 781 (2018) 129 [arXiv:1705.10629 [hep-th]]

Emission from Near Horizon of Near Extremal BHs

- Temperature for Schwinger effect:

$$T_S = \frac{\bar{m}}{2\pi(\kappa-\mu)} = T_U + \sqrt{T_U^2 - \frac{1}{4\pi^2 R_{\rm AdS}^2}}, \qquad T_U = \frac{\kappa/R_{\rm AdS}^2}{2\pi\bar{m}}$$

$$\bar{T}_S = \frac{\bar{m}}{2\pi(\kappa+\mu)} = T_U - \sqrt{T_U^2 - \frac{1}{4\pi^2 R_{\rm AdS}^2}}$$



- another factor: with $\tilde{\kappa}$

$$e^{-2\pi(\tilde{\kappa}-\kappa)} = e^{-(\omega+q\Phi_H)/T_H}$$

Emission from Near Horizon of Near Extremal BHs

- Mean number of produced pairs: $(\mathrm{AdS}_2)\{\mathrm{Rindler}_2\}$

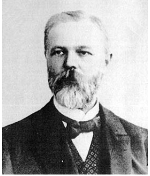
$$\mathcal{N} = \mathrm{e}^{\frac{\bar{m}}{T_S}} \left(\frac{\mathrm{e}^{-\frac{\bar{m}}{T_S}} - \mathrm{e}^{-\frac{\bar{m}}{T_S}}}{1 + \mathrm{e}^{-\frac{\bar{m}}{T_S}}} \right) \left\{ \frac{\mathrm{e}^{-\frac{\bar{m}}{T_S}} \left(1 - \mathrm{e}^{-\frac{\omega - q\Phi_H - n\Omega_H}{T_H}} \right)}{1 + \mathrm{e}^{-\frac{\omega - q\Phi_H - n\Omega_H}{T_H}} \mathrm{e}^{-\frac{\bar{m}}{T_S}}} \right\}$$

- For rotating black holes: Ω_H (angular velocity)

- Pair production in general case: non-extremal & full spacetime region?
- Solution: confluent Heun function (2nd order ODE with 4 poles)

Emission from (Non)Extremal BHs: two special cases

- Heun function: Karl Heun (1889)



  - Heun equation: four simple poles at $0, 1, a$ and ∞
  - confluent Heun equation: $a \to \infty$
- Heun Project: `https://www.theheunproject.org/`
- No relation of solution at $z = 1$ and $z = \infty$ is known yet!!
- Special cases: Heun to hypergeometric reduction (constraints on scalar field)

Emission from (Non)Extremal BHs: two special cases

- Non-extremal: $\boxed{m = \omega \text{ and } qQ = mM}$
  - Mean number of scalar production:

$$\mathcal{N} = \frac{\sinh(2\pi\mu)\sinh(\pi\beta_+ + \pi\beta_-)}{\cosh(\pi\beta_+ - \pi\mu)\cosh(\pi\beta_- - \pi\mu)}$$

where

$$\beta_\pm = \frac{\omega(r_+ \pm r_-)}{2}, \quad \mu = \sqrt{\omega^2(r_+ - r_-)^2/4 - (l+1/2)^2}$$

- Extremal ($M = Q$): $\boxed{q = \omega}$
  - Mean number of scalar production:

$$\mathcal{N} = \frac{\sin 2\pi\mu}{\cosh(\pi\kappa + \pi\mu)} e^{\pi\mu - \pi\kappa}$$

where

$$\kappa := -\sqrt{\omega^2 - m^2} r_0, \quad \mu := \sqrt{(\omega^2 - m^2) r_0^2 - (l+1/2)^2}$$

Discussion

- Spontaneous pair production of scalar in "near horizon" of near extremal charged black holes: exact Bogoliubov coefficients
- There is a remarkable thermal interpretation.
- Non-extremal/Extremal black holes:
  - hypergeometric equation (3 poles) $\rightarrow$ confluent/double confluent Heun equation (4 poles)
  - two special cases (with constraints on matter field): Heun to hypergeometric reduction

ICGAC14

- ICGAC14: The 14th International Conference on Gravitation, Astrophysics and Cosmology
  - Date: June 29 - July 3, 2020 $\rightarrow$ Late August 2020
  - Place: National Taiwan University, Taipei

  https://icgac14.phy.ncu.edu.tw/

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