Gravitational Wave Reflectivity from an Ultralight Bosonic Cloud

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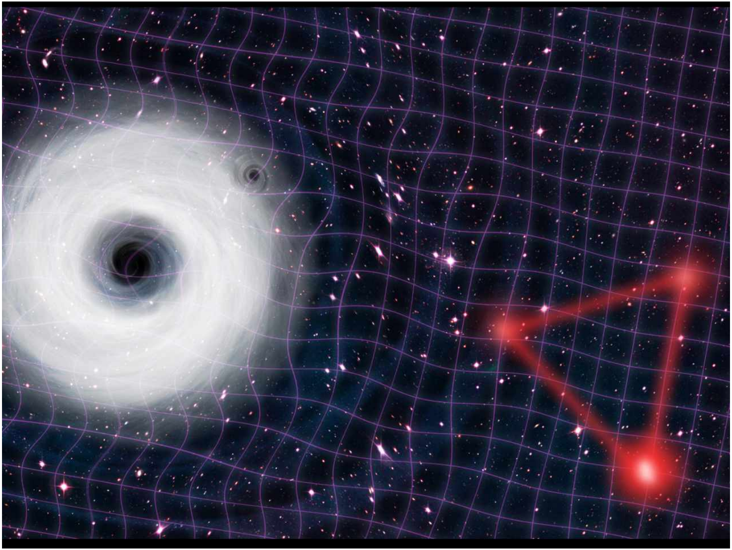
Content

- Ultralight bosons/bosonic cloud formation
- Probing ultralight boson mass
- Echo signatures of bosonic cloud due to reflectivity
- Concluding thoughts

Ultralight Bosons and Cloud Formation

- Ultralight boson: dark matter candidate, scalar
- mass $\approx O(10^{-22})$eV
- Superradiant condition for Kerr BH $0 < \omega_B < m\Omega_H$
- Negative energy and angular momentum flux at horizon
- Energy and angular momentum of BH goes to bosonic cloud
- For massive boson $\rightarrow$ bound state
- "Gravitational atom"



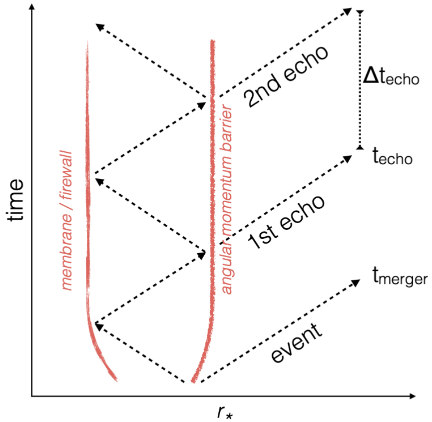
How can we probe Bosonic cloud

- Stochastic background[1]
- "Holes" in spin-mass plane[2]
- Our focus: echoes from bosonic cloud

[1] Leo Tsukada et al. "First search for a stochastic gravitational-wave background from ultralight bosons". In: *Physical Review D* 99.10 (2019), p. 103015.

[2] Richard Brito, Vitor Cardoso, and Paolo Pani. "Superradiance". In: *Lect. Notes Phys* 906.1 (2015), pp. 1501–06570.

Echoes in GW searches



[3]

- Can bosonic cloud produce a similar effect?

[3] Jahed Abedi, Hannah Dykaar, and Niayesh Afshordi. "Echoes from the Abyss: Tentative evidence for Planck-scale structure at black hole horizons". In: *Physical Review D* 96.8 (2017), p. 082004.

Calculating the Bosonic Cloud Reflectivity

Bosonic cloud wavefunction

Bosonic cloud wavefunction for $l = m = 1$ mode

$$\varphi(t, r, \theta, \phi) = \sqrt{\frac{3M_s}{4\pi i M}}(M\mu)^3 \mu r e^{-M\mu^2 r/2} \sin\theta \cos(\phi - \mu t), \tag{1}$$

- M: BH mass
- M_s: Total mass of bosonic cloud extracted in superradiance from BH
- $\mu = \frac{m_s}{\hbar}$: boson frequency for boson mass m_s

Stress tensor of wavefunction given by

$$T_{\mu\nu} = \nabla_\mu \varphi \nabla_\nu \varphi - \frac{1}{2} g_{\mu\nu} \mathcal{L} \tag{2}$$

- How does stress tensor modify gravitational waveforms?

Teukolsky Equation

- Black hole perturbations method: Teukolsky equation
- Curvature perturbation
- Master equation for GW waveform
- Separable

$$\left[\frac{(r^2+a^2)^2}{\Delta} - a^2 \sin^2\theta\right]\frac{\partial^2\psi}{\partial t^2} + \frac{4Mar}{\Delta}\frac{\partial^2\psi}{\partial t\partial\varphi} + \left[\frac{a^2}{\Delta} - \frac{1}{\sin^2\theta}\right]\frac{\partial^2\psi}{\partial\varphi^2}$$
$$- \Delta^{-s}\frac{\partial}{\partial r}\left(\Delta^{s+1}\frac{\partial\psi}{\partial r}\right) - \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) - 2s\left[\frac{a(r-M)}{\Delta} + \frac{i\cos\theta}{\sin^2\theta}\right]\frac{\partial\psi}{\partial\varphi}$$
$$- 2s\left[\frac{M(r^2-a^2)}{\Delta} - r - ia\cos\theta\right]\frac{\partial\psi}{\partial t} + (s^2\cot^2\theta - s)\psi = 4\pi\Sigma T\,.$$

- $\Delta = r^2 - 2Mr + a^2$
- $\rho = r - ia\cos\theta$

Getting Bosonic Cloud potential

- Project stress-tensor with null vectors, set $a = 0$

$$T_{\bar{m}\bar{m}} = T_{\mu\nu}\bar{m}^{\mu}\bar{m}^{\nu} \approx \frac{1}{2\omega^2}\psi_4[\partial_\alpha\varphi\partial^\alpha\varphi + \mu^2\varphi^2] \tag{3}$$

$$\bar{m}^{\mu} = \frac{1}{r\sqrt{2}}(0, 0, 1, -i\sin^{-1}\theta) \tag{4}$$

- $\Sigma = r^2 + a^2\cos^2\theta$

Eigenfunctions

$$\psi_4 = \rho^4 \sum_{l,m} \int \frac{\mathrm{d}\tilde{\omega}}{\sqrt{2\pi}} Y_{lm}(\theta,\phi) e^{-i\tilde{\omega}t} R_{lm\tilde{\omega}}(r) \tag{5}$$

- $l = m = 2$ mode
- Extract Teukolsky source term

$$\tilde{T}(t,r,\Omega) \sim -\frac{1}{4}\rho^8\bar{\rho}\Delta^2\hat{J}_+[\rho^{-4}\hat{J}_+(\rho^{-2}\bar{\rho}T_{\bar{m}\bar{m}})] \tag{6}$$

- $\hat{J}_+ = \partial_r - \Delta^{-1}(r^2\partial_t)$
- $R(r,\omega) \propto e^{i\omega r}$
- Ansatz: $\frac{\partial R}{\partial r} \sim i\omega R(r)$

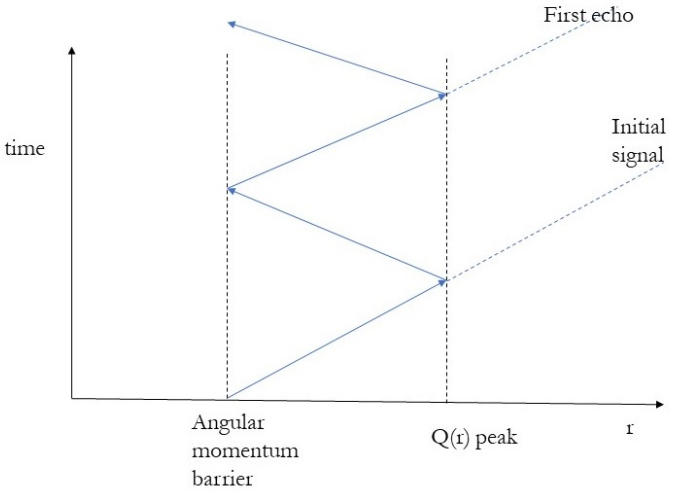
From Potentials to Reflectivity

- Radial and angular equation decouple
- Radial: modified wave equation with extra potential

$$\frac{\partial^2 R}{\partial r^2} + (\omega^2 - V(r) - Q(r))R(r,\omega) = 0 \tag{7}$$

- $V(r)$ usual Schwarzschild potential in Teukolsky equation
- $Q(r)$ extra contribution from bosonic cloud
- How does this potential affect waveforms?
- GW Echoes/scattering

Conceptual Idea



Dyson Series Approach

- Asymptotic solutions

$$\Psi(t,x) = \left\{ \begin{array}{ll} e^{-i\omega x} + \mathcal{R}(\omega) e^{i\omega x}, & \text{for } x \to \infty \\ e^{-i\omega x}, & \text{for } x \to -\infty \end{array} \right\} \tag{8}$$

- Full solution sum of free wave solution plus potential dependant term

$$\tilde{\Psi}(\omega, x) = \tilde{\Psi}_0(\omega, x) + \sum_{n=1}^{\infty} \tilde{\Psi}_n(\omega, x) \tag{9}$$

- Powers in $\frac{V}{\omega^2}$

Continuing Reflectivity

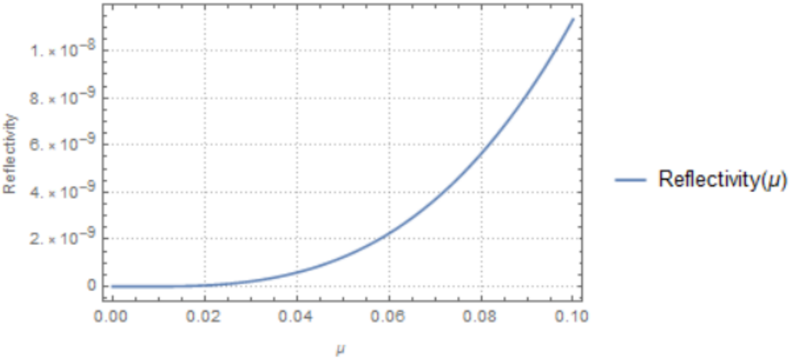
- To first order in boson cloud potential

$$\mathcal{R}(\omega) \sim \int \mathrm{d}x' \frac{e^{-i\omega x'}}{2i\omega} \frac{r'^2}{\Delta} Q(r') \tag{10}$$

- Integrate from light ring $r = 3M$ to $\frac{1}{M^2\mu^3}$
- Peak of Q at $r_{peak} \sim \frac{1}{M\mu^2}$
- Numerical calculation: vary μ, ω

Reflectivity

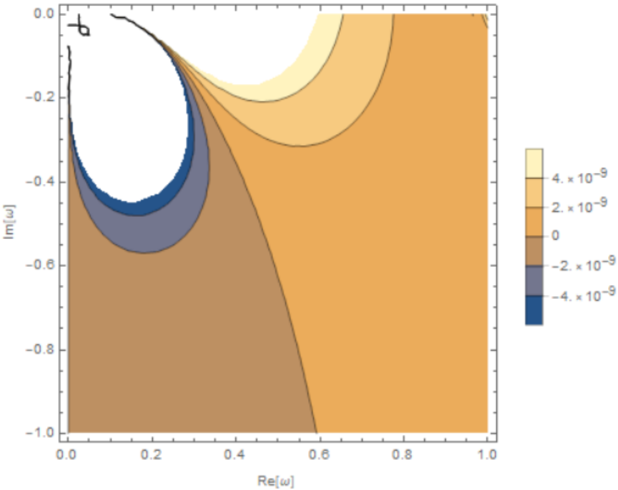
- Fixed QNM frequency



- μ^8 dominates

Reflectivity(ω)

- Re[$\mathcal{R}(\omega)$]
- Fixed $M\mu = 0.1$



Future work

- Invert into time-domain for waveform
- How does potential change echo model?
- Draw constraints from LIGO events
- Check degeneracy with other models
- Other signatures?

The End