Exploring Inflationary Magnetogenesis

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Outline

- The generation of large-scale magnetic fields
- Axion magnetogenesis during inflation
- The dilaton-axion PMF
- Investigating the back-reaction
- Inflationary magnetogenesis with back-reaction

The origin of large-scale magnetic fields

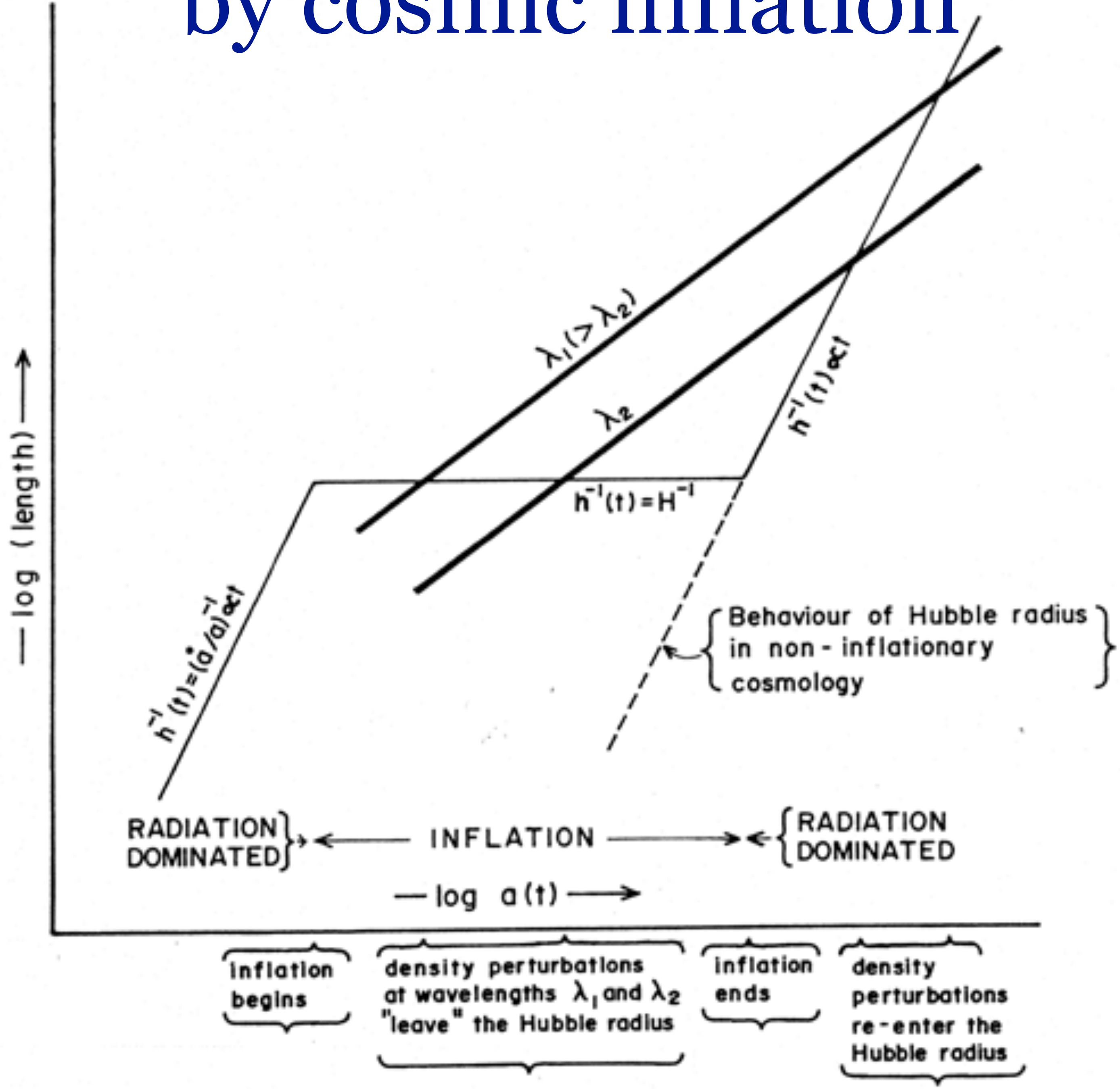
- The observed cluster and galactic magnetic fields of about a few μG may be resulted from the amplification of a seed field of $B_* \sim 10^{-23}$ G via the so-called galactic dynamo effect.

- Through most of its history, the universe has been a good conductor which preserves the magnetic flux:

$$r \equiv \frac{\rho_B}{\rho_\gamma} = \text{constant} \qquad \Rightarrow \qquad \rho_B \simeq 10^{-34} \rho_\gamma .$$

- The seed fields may be generated inside or outside the Hubble horizon.

Principle of constructing structures by cosmic inflation



The Weyl (Conformal) Invariance

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = a^2(\tau)[d\tau^2 - dx^2]$$

$$S_{\text{em}} = -\frac{1}{4} \int d^4x \sqrt{-g} F_{\mu\nu} F^{\mu\nu}$$

$$\Rightarrow \quad \partial_\mu (\sqrt{-g} F^{\mu\nu}) = 0$$

$$\sqrt{-g} F^{\mu\nu} = a^4(\tau) \frac{\eta^{\mu\alpha}}{a^2(\tau)} \frac{\eta^{\nu\beta}}{a^2(\tau)} F_{\alpha\beta} = F^{\mu\nu}$$

$\Rightarrow$ the evolution equations of Abelian gauge fields are the same in flat space-time and in a conformally flat FRW space-time

Breaking the Conformal Invariance

Turner & Widrow PRD (1988):

$$\frac{1}{m^2}F_{\mu\nu}F_{\alpha\beta}R^{\mu\nu\alpha\beta}, \qquad \frac{1}{m^2}R_{\mu\nu}F^{\mu\beta}F^{\nu\alpha}g_{\alpha\beta}, \qquad \frac{1}{m^2}F_{\alpha\beta}F^{\alpha\beta}R$$

Ratra ApJ (1992):

$$RA_{\mu}A^{\mu}, \qquad R_{\mu\nu}A^{\mu}A^{\nu}.$$

Carroll & Field (1990); Garretson et. al. (1992):

$$\sqrt{-g}c_{\psi\gamma}\alpha_{\rm em}\frac{\psi}{8\pi M}F_{\alpha\beta}\tilde{F}^{\alpha\beta}$$

Axial coupling magnetogenesis

- In a spatially flat universe, we consider an evolving pseudo-scalar field ϕ characterized by the action
$$S = \int d^4x \sqrt{g} \left\{ \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] - \frac{1}{4} g^{\alpha\mu} g^{\beta\nu} F_{\alpha\beta} F_{\mu\nu} + \frac{1}{\sqrt{g}} L_{\phi\gamma} \right\},$$
where $L_{\phi\gamma}$ is the ϕ-photon coupling given by
$$L_{\phi\gamma} = \frac{c}{M_{\mathrm{Pl}}} \phi \, \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu}$$

- The comoving magnetic field can be expressed in terms of the comoving coordinates $(\mathbf{x}, \eta)$ as
$$\left(\nabla^2 - \frac{\partial^2}{\partial \eta^2} \right) \mathbf{B} = 4c \frac{d\phi}{d\eta} \nabla \times \mathbf{B}$$

Magnetic mode equation

- The comoving magnetic field can be expressed in terms of the comoving coordinates $(\mathbf{x}, \eta)$ as

$$\left(\nabla^2 - \frac{\partial^2}{\partial \eta^2}\right) \mathbf{B} = 4c \frac{d\phi}{d\eta} \nabla \times \mathbf{B}$$

- The magnetic field can be recast as $\mathbf{B} = \nabla \times \mathbf{A}_T$, in which the transverse field $\mathbf{A}_T(\eta, \mathbf{x})$ can be further decomposed into Fourier modes such that

$$\mathbf{A}_T = \int \frac{d^3\mathbf{k}}{\sqrt{2(2\pi)^3 k}} \left[e^{i\mathbf{k}\cdot\mathbf{x}} \sum_{\lambda=\pm} b_{\lambda\mathbf{k}} V_{\lambda\mathbf{k}}(\eta) \epsilon_{\lambda\mathbf{k}} + \text{h.c.} \right],$$

where $b_{\pm\mathbf{k}}$ are destruction operators, and $\epsilon_{\pm\mathbf{k}}$ are circular polarization unit vectors. Then, it is straightforward to deriving the mode equations

$$\frac{d^2}{d\eta^2} V_{\pm q} + \left(q^2 \mp 4cq \frac{d\phi}{d\eta}\right) V_{\pm q} = 0,$$

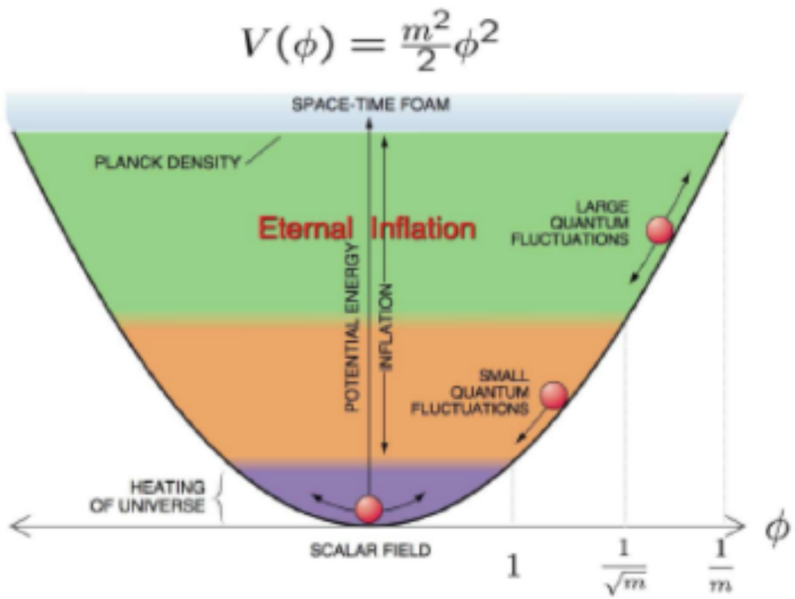
where the dimensionless comoving wavenumber $q = k/H$.

Magnetogenesis during inflation

$$\frac{d^2V_\pm}{d\eta^2} = \left(-q^2 \pm 4cq\cdot\frac{d\theta}{d\eta}\right)\cdot V_\pm \Rightarrow V_\pm \propto \exp\left\{\left(\sqrt{\pm 4cq\frac{d\theta}{d\eta} - q^2}\right)\cdot\eta\right\}$$

Hence the growing mode can be identified with $V_{+q} \propto \exp(\omega\eta)$ with

$$\omega \equiv \sqrt{4cq\left|\frac{d\theta}{d\eta}\right| - q^2}$$



Magnetogenesis during inflation

The energy density of electromagnetic field

$$\rho_{\mathrm{EM}} = \frac{1}{2}\left\langle \vec{E}^2 + \vec{B}^2 \right\rangle = \frac{1}{4\pi^2 a^4}\int dk \cdot k^2 \sum_{\lambda=\pm}\left(\left|A'_\lambda\right|^2 + k^2 \left|A_\lambda\right|^2 \right)$$

The energy density of electric field

$$\rho_{\mathrm{E}} = \frac{1}{4\pi^2 a^4}\int dk \cdot k^2 \sum_{\lambda=\pm}\left|A'_\lambda\right|^2 = \frac{1}{4\pi^2 a^2}\int dk \cdot k^2 \sum_{\lambda=\pm}\left|\dot{A}_\lambda\right|^2$$

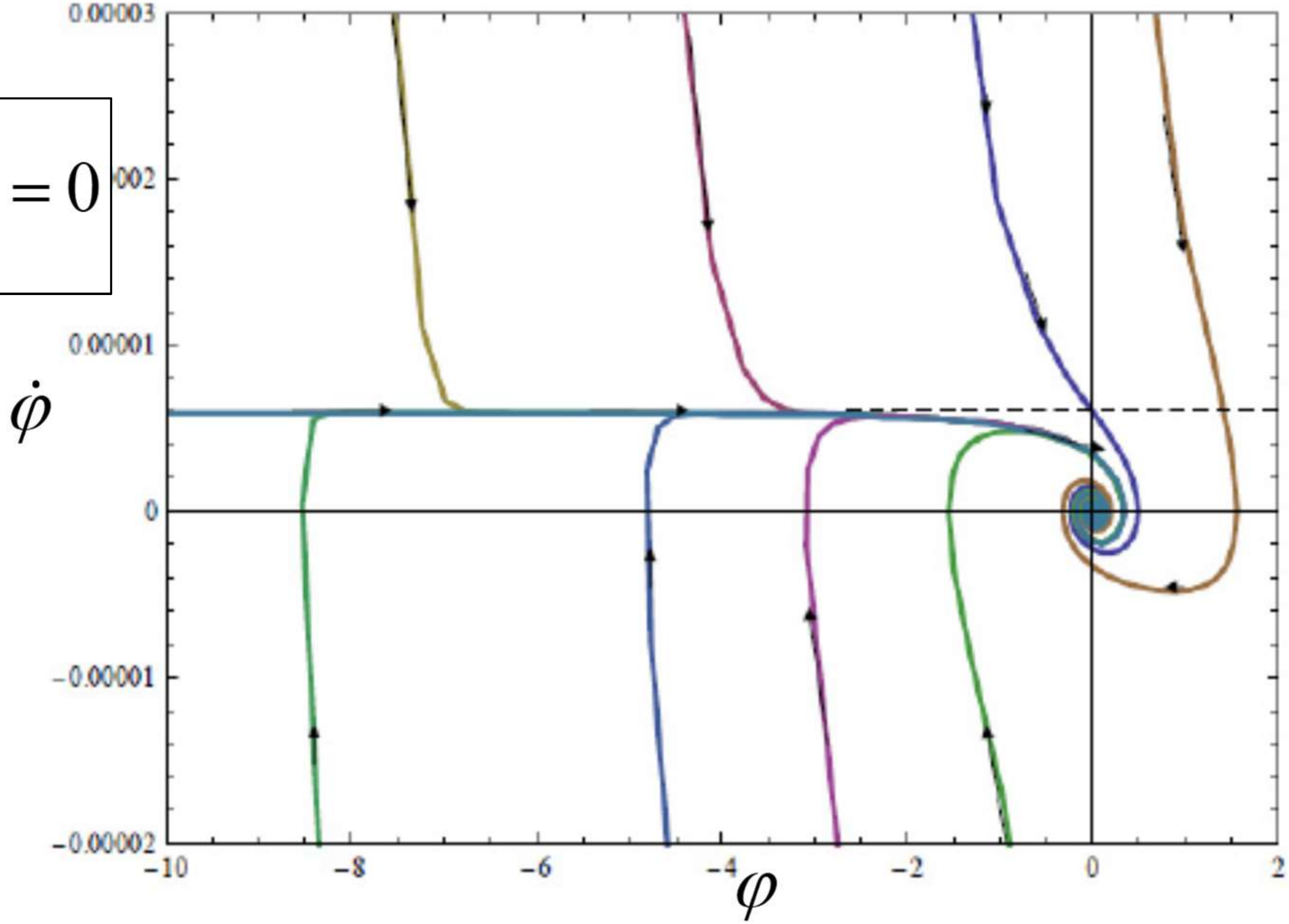
The energy density of magnetic field

$$\rho_{\mathrm{B}} = \frac{1}{4\pi^2 a^4}\int dk \cdot k^4 \sum_{\lambda=\pm}\left|A_\lambda\right|^2$$

a. Axial coupling magnetogenesis

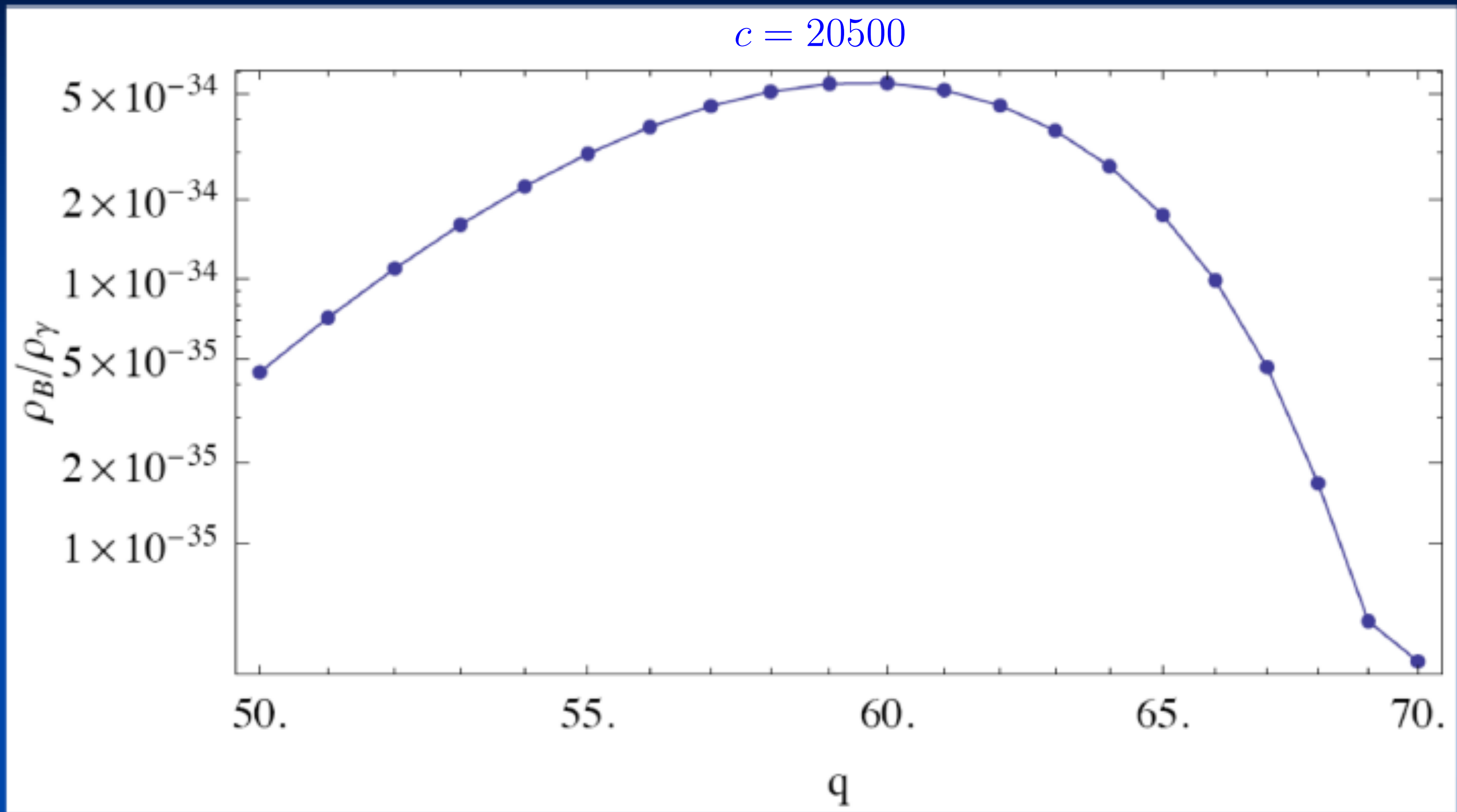
- **Evolution of inflation without backreaction**

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{dV}{d\varphi} = 0$$

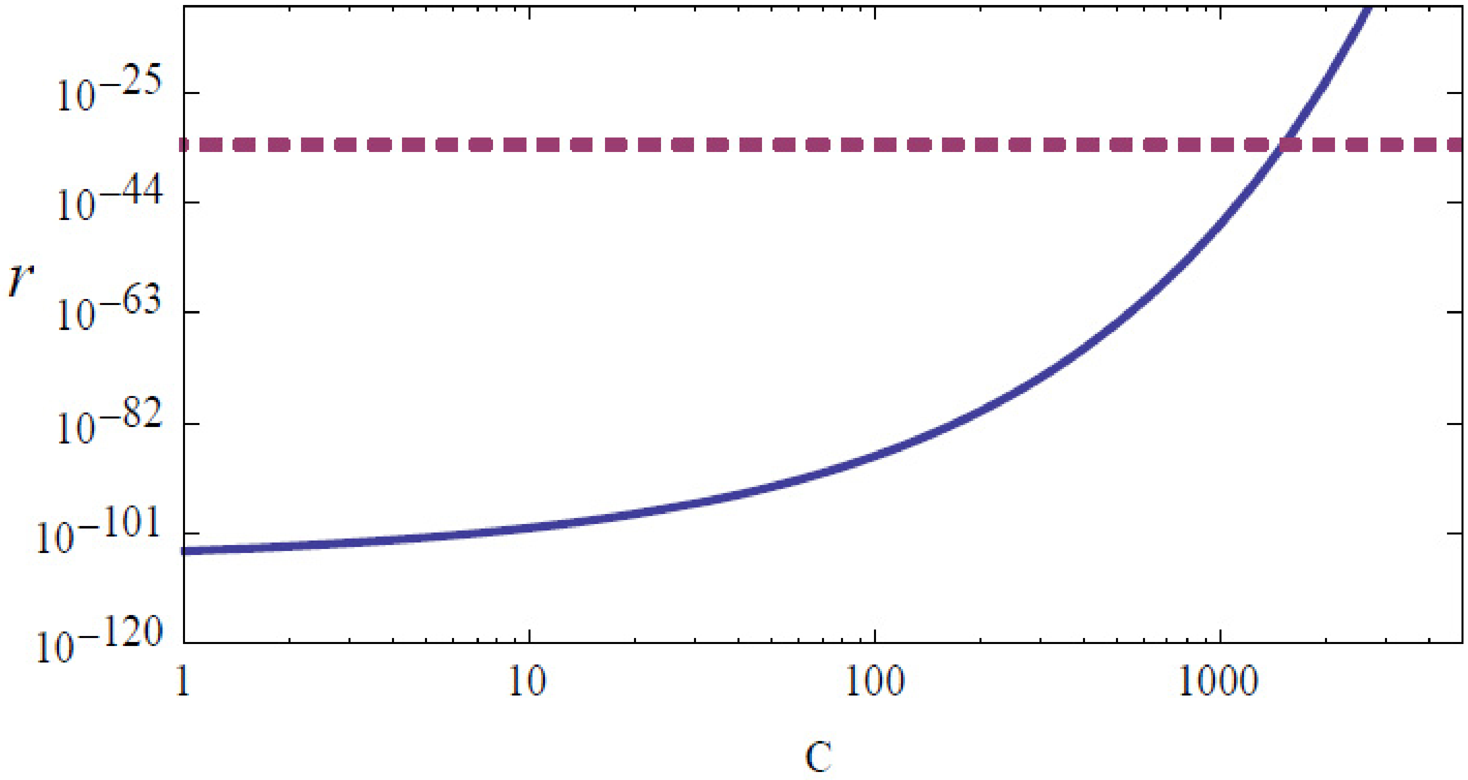


Inflationary PMF by axial coupling

$$\frac{d\rho_B}{dq} = \frac{q^3 H^4}{32\pi^3 a_{\rm end}^4} \sum_{i=\pm} |V_{iq}|^2$$



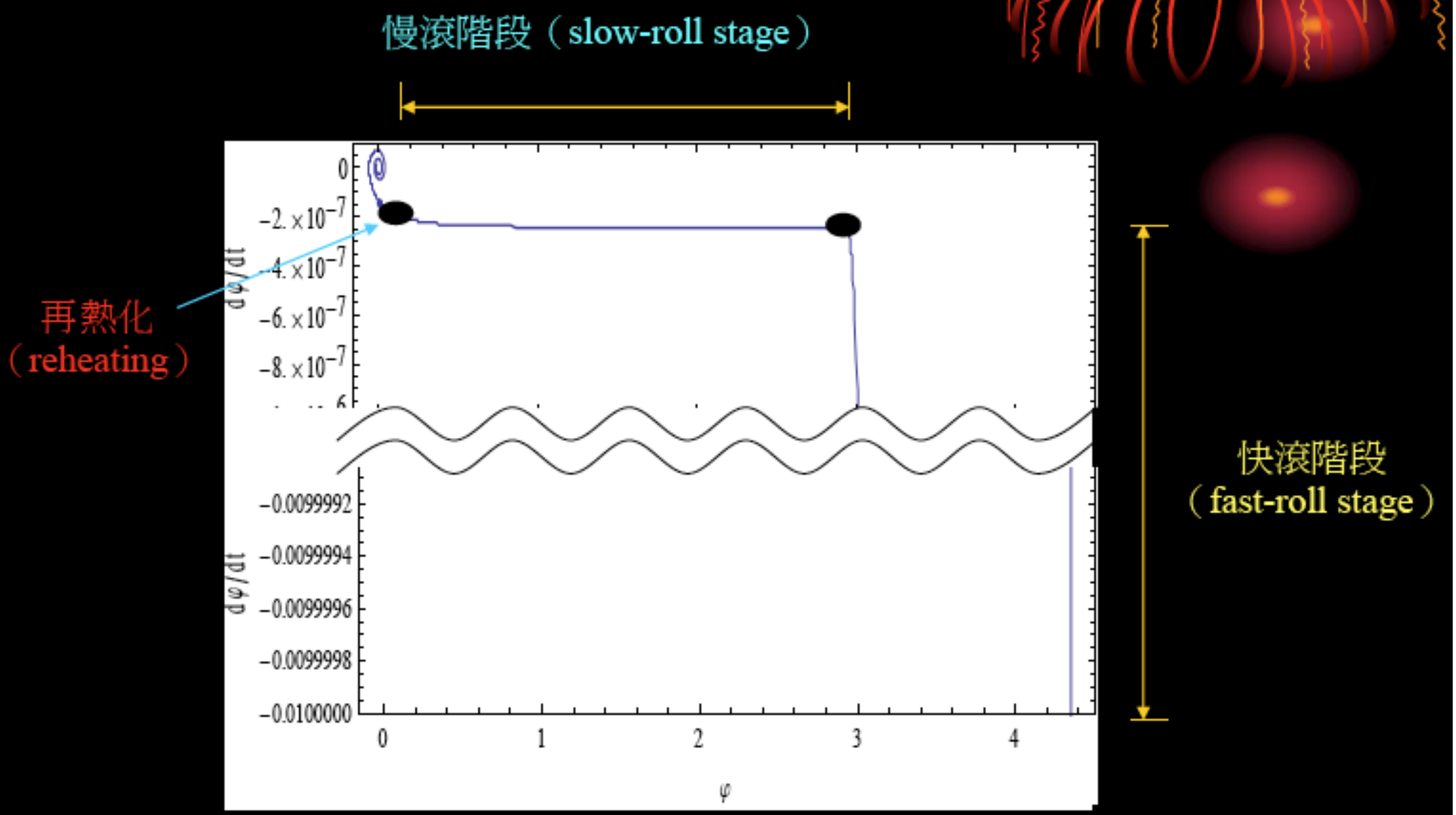
Slow-roll is not enough!



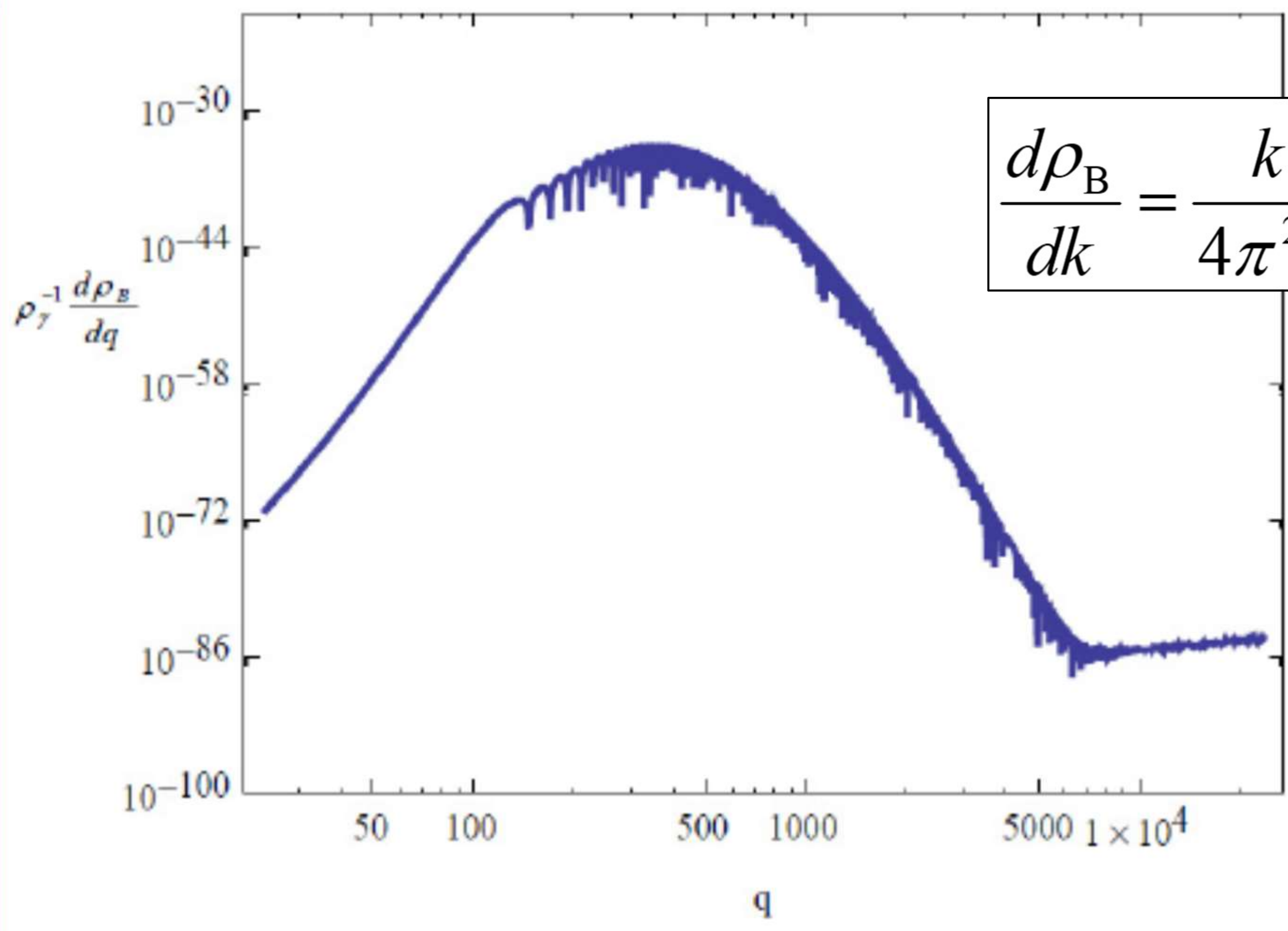
The r vs c diagram suggests that the megnetogenesis induced by the spinodal instability in the slow roll regime is not efficient enough. The dashed line marks the line of $r = 10^{-34}$.

This is for the magnetic mode of horizon size $q = 1$. For magnetic modes of comoving size about 100 Mpc, $q \simeq 400$, we find that $c \simeq 800$ is required.

Invoking a fast-roll stage



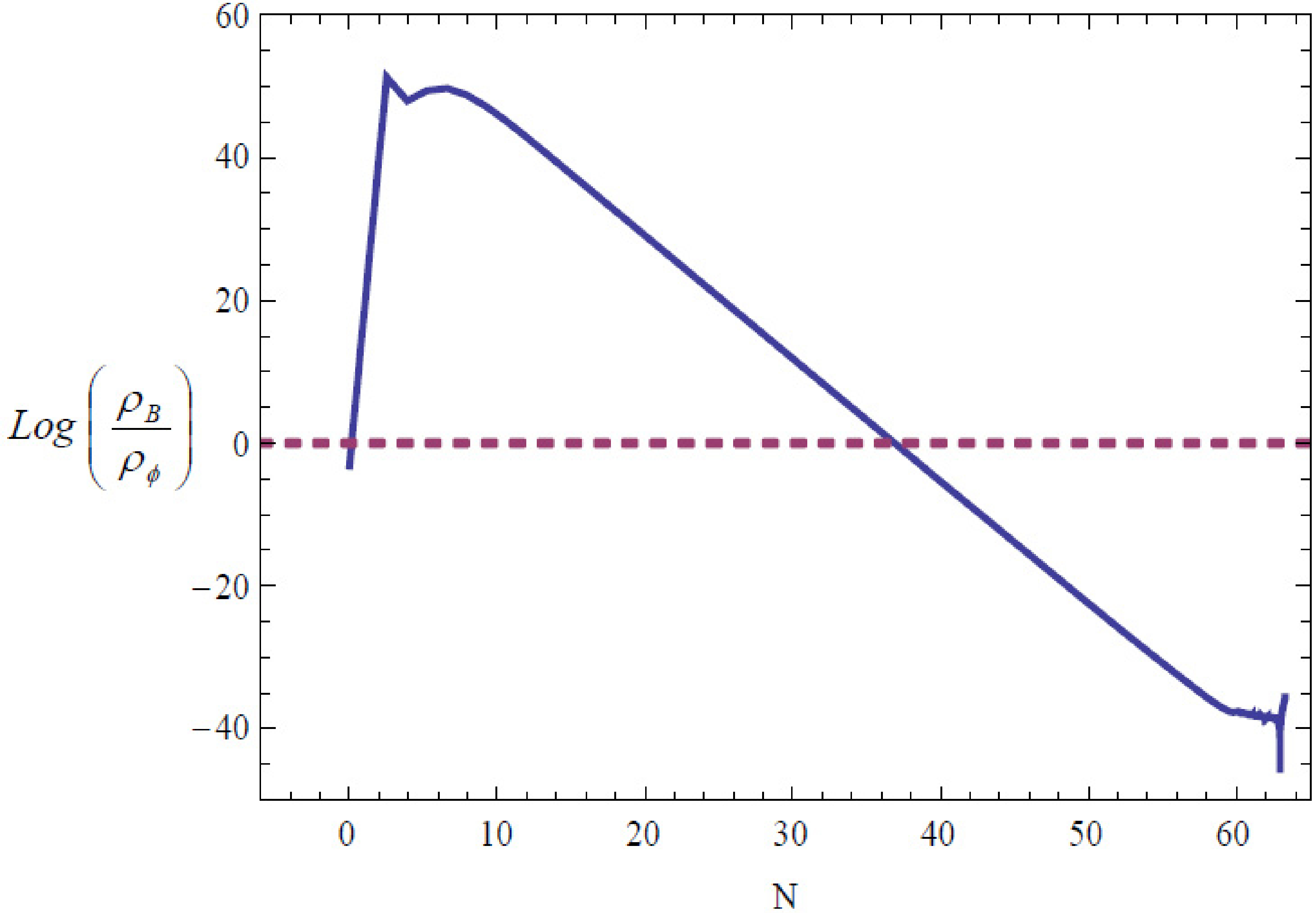
A fast-rolling inflationary PMF



$$\frac{d\rho_B}{dk} = \frac{k^4}{4\pi^2 a^4}\left(|A_+|^2 + |A_-|^2\right)$$

Cheng & Lee (2014)

The energy constraint



The evolution of the energy density ratio $\rho_{\rm B}/\rho_\phi$ within the inflationary epoch. The peak mode crossing out the horizon at $q_{\rm peak} \simeq 425$ corresponds to a comoving distance of about 10 Mpc. Apparently, $\rho_{\rm B} \gg \rho_\phi$ for the most time during the inflation.

Cheng & Lee (2014)

Dilton-axion magnetogenesis

The action of the system in a flat universe

$$S=\int d^4x\sqrt{-g}\times\left[\frac{M_P^2}{2}R-\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi-V(\varphi)-\frac{I}{4}F^{\mu\nu}F_{\mu\nu}-\frac{J}{4}\widetilde{F}^{\mu\nu}F_{\mu\nu}\right]$$

The evolution of the inflation and the thermal background

$$\ddot{\varphi}+(3H+\gamma)\dot{\varphi}+\frac{dV}{d\varphi}=0$$

$$H^2=\frac{1}{3}\left(\frac{1}{2}\dot{\varphi}^2+V(\varphi)+\rho_\gamma\right)$$

$$\dot{\rho}_\gamma+4H\rho_\gamma=\gamma\dot{\varphi}^2$$

Kin-Wang Ng, Shu-Lin Cheng, Wolung Lee(2015)
Inflationary Dilaton-Axion Magnetogenesis

Dilton-axion magnetogenesis

The action of the system in a flat universe

$$S=\int d^4x\sqrt{-g}\times\left[\frac{M_P^2}{2}R-\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi-V(\varphi)-\frac{I}{4}F^{\mu\nu}F_{\mu\nu}-\frac{J}{4}\tilde{F}^{\mu\nu}F_{\mu\nu}\right]$$

Mode function of EM fields

$$\frac{\partial^2}{\partial\eta^2}\vec{A}-\vec{\nabla}^2\vec{A}+\frac{I'}{I}\frac{\partial}{\partial\eta}\vec{A}-\frac{J'}{I}\vec{\nabla}\times\vec{A}=0$$

$$\left[\frac{\partial^2}{\partial\eta^2}+\frac{I'}{I}\frac{\partial}{\partial\eta}+k^2\mp k\frac{J'}{I}\right]A_{\pm}(\eta,k)=0$$

Kin-Wang Ng, Shu-Lin Cheng, Wolung Lee(2015)
Inflationary Dilaton-Axion Magnetogenesis

Dilton-axion magnetogenesis

$$\left[\frac{\partial^2}{\partial\tau^2} + \frac{I'}{I}\frac{\partial}{\partial\tau} + k^2 \mp k\frac{J'}{I}\right] A_{\pm}(\tau, k) = 0.$$

$$\rho_{\rm EM} \equiv \frac{I}{2}\langle \vec{E}^2 + \vec{B}^2 \rangle = \frac{I}{4\pi^2 a^4}\int dk\, k^2 \sum_{\lambda=\pm} \left(|A'_\lambda|^2 + k^2|A_\lambda|^2\right),$$

$$\rho_B = \frac{I}{4\pi^2 a^4}\int dk\, k^4 \left(|A_+|^2 + |A_-|^2\right).$$

- **Dilaton EM:** $I = I(\tau)$ and $J(\tau) = 0$
  - **strong** coupling $I(\tau) < 1$ $\Rightarrow$ a strong enough PMF?
  - **weak** coupling $I(\tau) > 1$ $\Rightarrow$ no sizable PMF can be produced
- **Axion EM:** $I = 1$ and $J(\tau) = \alpha\phi/f$ with the axion ϕ as the inflaton
  - the **spinodal instability** seems to be not effective

Dilton-axion magnetogenesis

Mode function of EM fields

$$\left[\frac{\partial^2}{\partial\eta^2} + \frac{I'}{I}\frac{\partial}{\partial\eta} + k^2 \mp k\frac{J'}{I}\right]A_\pm(\eta,k) = 0$$

$$I(\eta) = \left(\frac{a(\eta_f)}{a(\eta)}\right)^n$$

$$J'(\eta) = cH(\eta)a^p(\eta)$$

The comoving energy density of the magnetic field

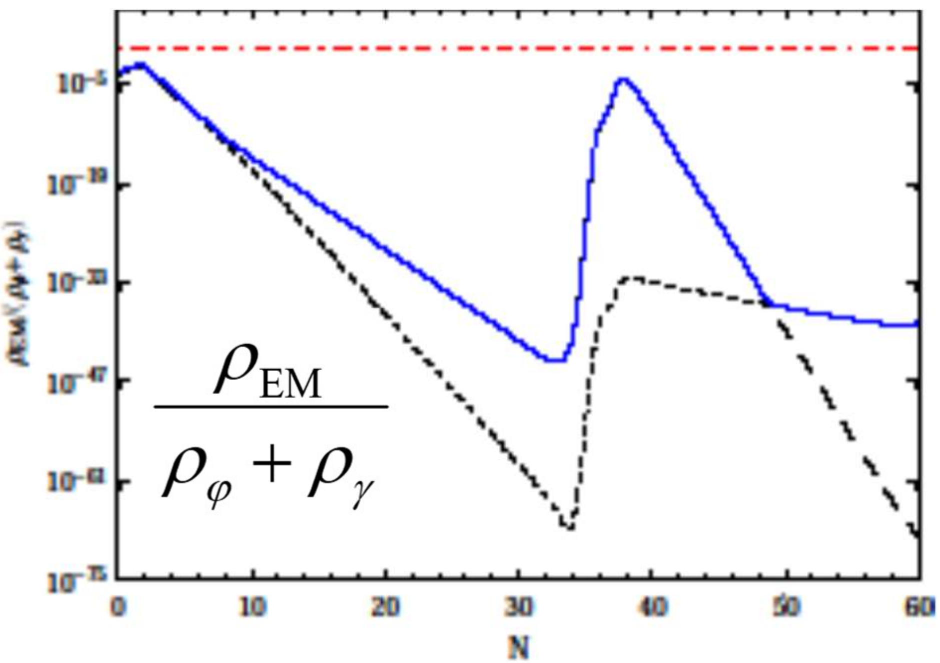
$$\frac{d\rho_B}{d\ln k} = \frac{k^5 I}{4\pi^2 a^4}\left(|A_+|^2 + |A_-|^2\right)$$

$$r \equiv \left[\rho_\gamma^{-1}\frac{d\rho_B}{d\ln k}\right]_{\eta=\eta_f}$$

Kin-Wang Ng, Shu-Lin Cheng, Wolung Lee(2015)
Inflationary Dilaton-Axion Magnetogenesis

Dilton-axion magnetogenesis

the electromagnetic energy density over the total energy density during inflation

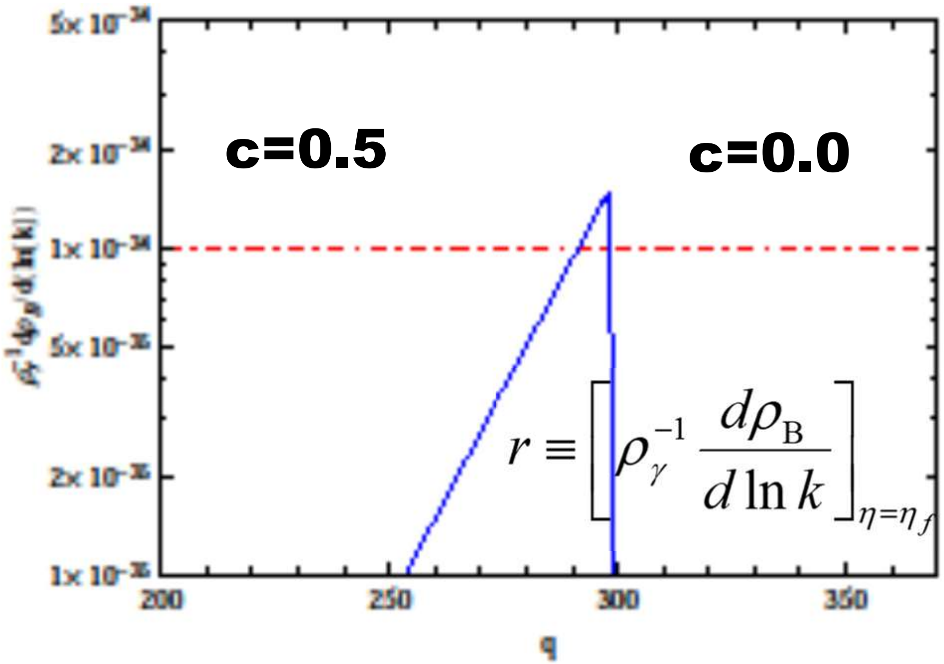


n=1.0

n=-3.1

p=0.064

Ratio of the spectral energy density of primordial magnetic fields to the thermal background

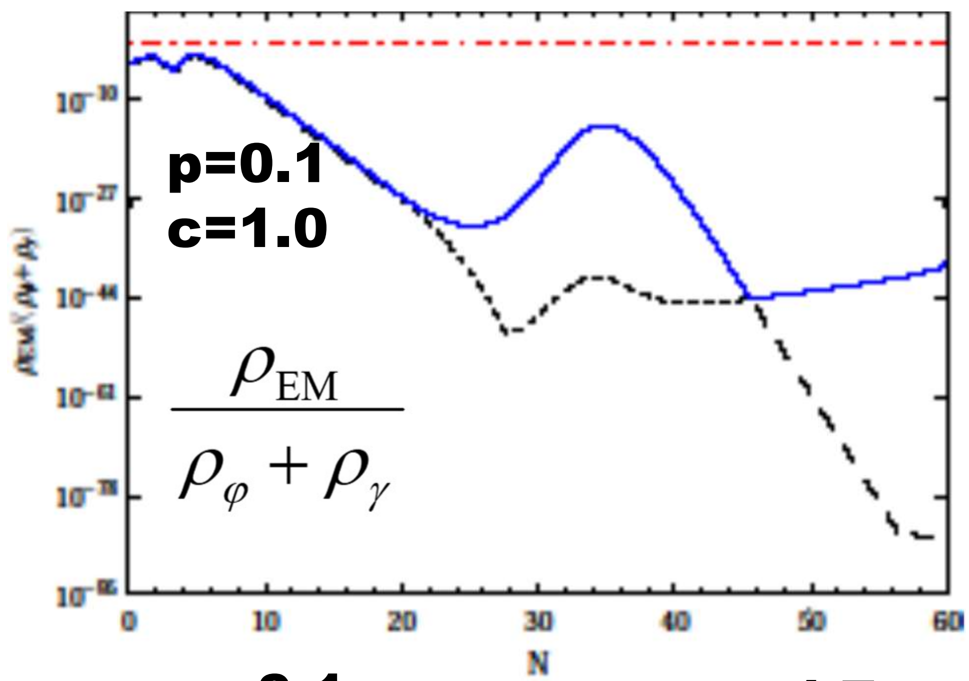


Kin-Wang Ng, Shu-Lin Cheng, Wolung Lee(2015)
Inflationary Dilaton-Axion Magnetogenesis

Dilton-axion magnetogenesis

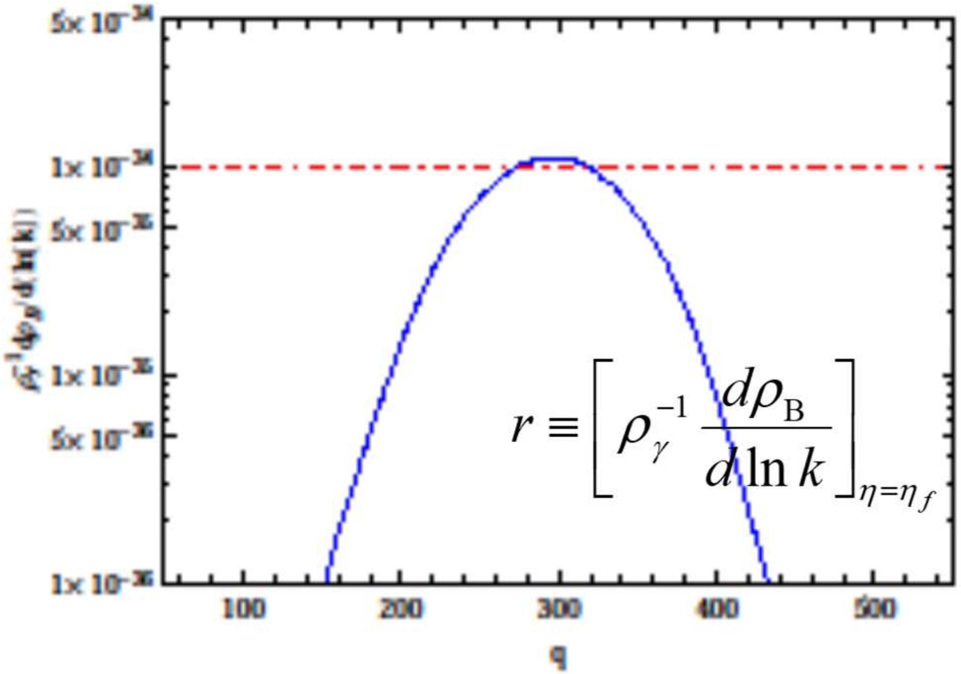
strong EM coupling

the electromagnetic energy density over the total energy density during inflation



n=-0.1 **n=-4.7**

Ratio of the spectral energy density of primordial magnetic fields to the thermal background

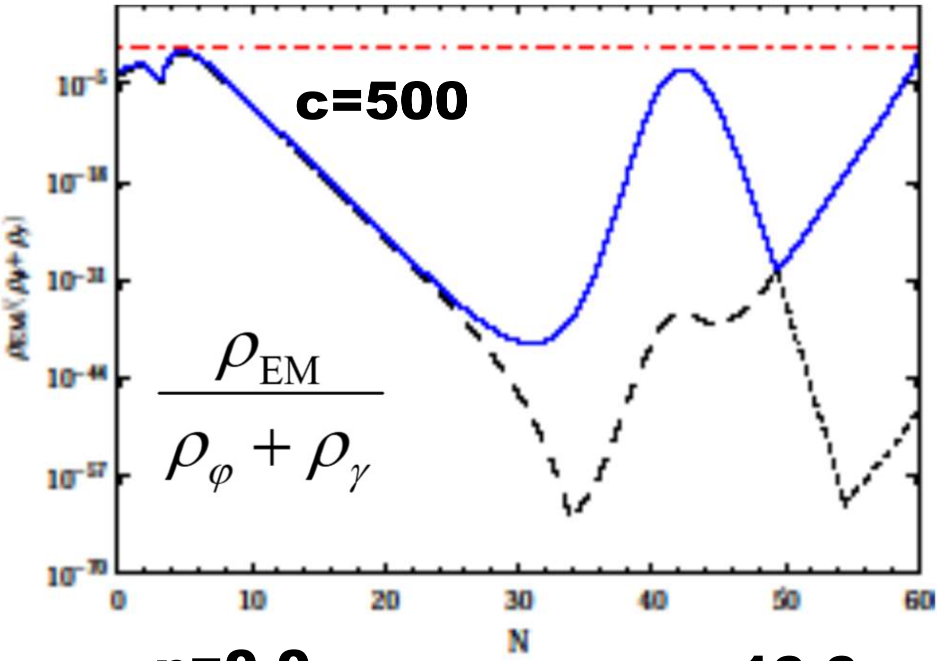


Kin-Wang Ng, Shu-Lin Cheng, Wolung Lee(2015)
Inflationary Dilaton-Axion Magnetogenesis

Dilton-axion magnetogenesis

weak EM coupling

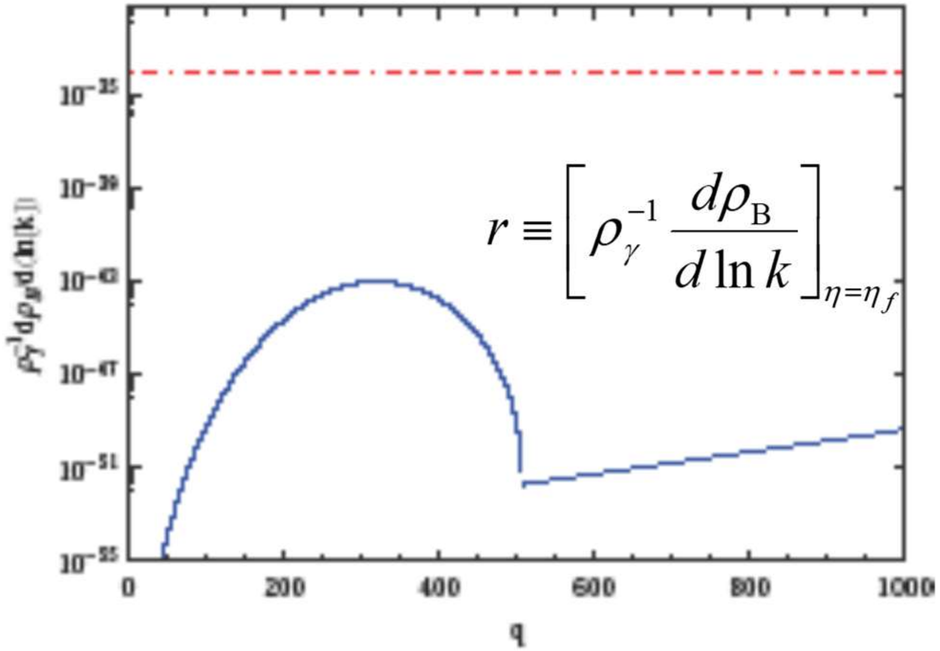
the electromagnetic energy density over the total energy density during inflation



n=0.0 **n=10.0**

p=0.0

Ratio of the spectral energy density of primordial magnetic fields to the thermal background



Kin-Wang Ng, Shu-Lin Cheng, Wolung Lee(2015)
Inflationary Dilaton-Axion Magnetogenesis

A brief summary on d-a PMF

- It is manageable to produce a strong enough PMF at the end of inflation in a generalized scenario of axion dynamics with a time dependent coupling $J(a)F\tilde{F}/4$ provided that the $J(a)$ function **delays the growth of the magnetic field to a later stage of inflation**.

- The effect of back reactions due to the rapid production of high k-modes seems inevitable. It would be interesting to take into account such a back action in the mode equations to study how the **production of high k-modes can be self-regulated by the back reaction**.

Retrieving back actions

Axial coupling magnetogenesis

Characterize the evolving pseudo-scalar field by the action

$$S=\int d^4x\sqrt{-g}\times\left[\frac{M_P^2}{2}R-\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi-V(\varphi)-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}-\frac{\alpha}{4f}\varphi\widetilde{F}^{\mu\nu}F_{\mu\nu}\right]$$

The evolution of the inflation and the thermal background

$$\ddot{\varphi}+3H\dot{\varphi}+\frac{dV}{d\varphi}=\frac{\alpha}{f}\left\langle\vec{E}\cdot\vec{B}\right\rangle$$

$$H^2=\frac{1}{3}\left(\frac{1}{2}\dot{\varphi}^2+V(\varphi)+\frac{1}{2}\left\langle E^2+B^2\right\rangle\right)$$

A. Linde, S. Mooij, and E. Pajer, Phys. Rev. D 87, 103506 (2013)

Mode Equations

$$\frac{\partial^2 \vec{A}}{\partial \eta^2} - \vec{\nabla}^2 \vec{A} - \frac{\alpha}{f} \phi' \vec{\nabla} \times \vec{A} = 0.$$

To proceed, we decompose the gauge field $\vec{A}(\eta, \vec{x})$ as

$$\vec{A}(\eta, \vec{x}) = \sum_{\lambda=\pm} \int \frac{d^3 k}{(2\pi)^{3/2}} [\vec{\epsilon}_\lambda(\vec{k}) a_\lambda(\vec{k}) A_\lambda(\eta, \vec{k}) e^{i\vec{k}\cdot\vec{x}} + \text{H.c.}],$$

where the annihilation and creation operators obey

$$[a_\lambda(\vec{k}), a^\dagger_{\lambda'}(\vec{k}')] = \delta_{\lambda\lambda'} \delta(\vec{k} - \vec{k}'),$$

$\vec{\epsilon}_\lambda$ are normalized circular polarization vectors satisfying $\vec{k} \cdot \vec{\epsilon}_\pm(\vec{k}) = 0$, $\vec{k} \times \vec{\epsilon}_\pm(\vec{k}) = \mp i k \vec{\epsilon}_\pm(\vec{k})$, $\vec{\epsilon}_\pm(-\vec{k}) = \vec{\epsilon}_\pm(\vec{k})^*$, and $\vec{\epsilon}_\lambda(\vec{k})^* \cdot \vec{\epsilon}_{\lambda'}(\vec{k}) = \delta_{\lambda\lambda'}$. Inserting the decomposition

$$\left[\frac{\partial^2}{\partial \eta^2} + k^2 \mp 2aHk\xi\right] A_\pm(\eta, k) = 0, \qquad \xi \equiv \frac{\alpha \dot{\phi}}{2fH}.$$

Energy density of gauge quanta

Axial coupling magnetogenesis

$$\frac{1}{2}\left\langle \vec{E}^2 + \vec{B}^2 \right\rangle = \frac{1}{4\pi^2 a^4}\int dk \cdot k^2 \sum_{\lambda=\pm}\left(\left|A'_\lambda\right|^2 + k^2\left|A_\lambda\right|^2\right)$$

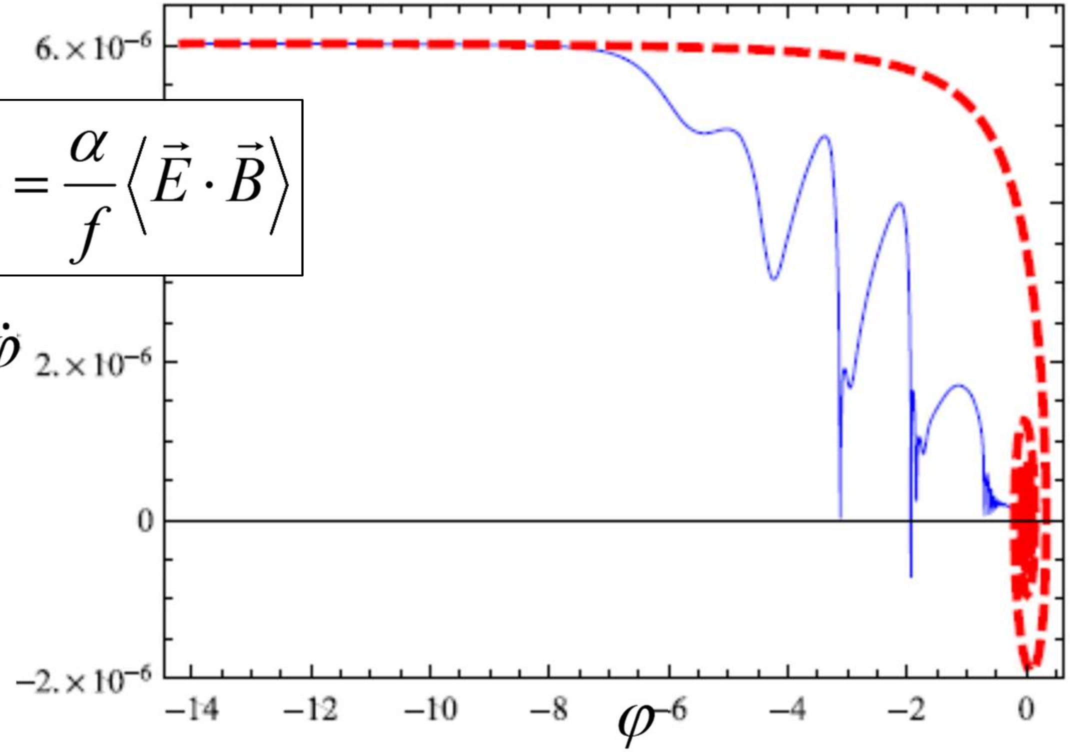
$$\left\langle \vec{E} \cdot \vec{B} \right\rangle = -\frac{1}{4\pi^2 a^4}\int dk \cdot k^3 \frac{d}{d\eta}\left(\left|A_+\right|^2 - \left|A_-\right|^2\right)$$

Shu-Lin Cheng, Wolung Lee, and Kin-Wang Ng,
Phys. Rev. D 93, 063510 (2016).

Background evolutions

- **Evolution of inflation with backreaction**

$$\ddot{\varphi}+(3H+\gamma)\dot{\varphi}+\frac{dV}{d\varphi}=\frac{\alpha}{f}\left\langle\vec{E}\cdot\vec{B}\right\rangle$$

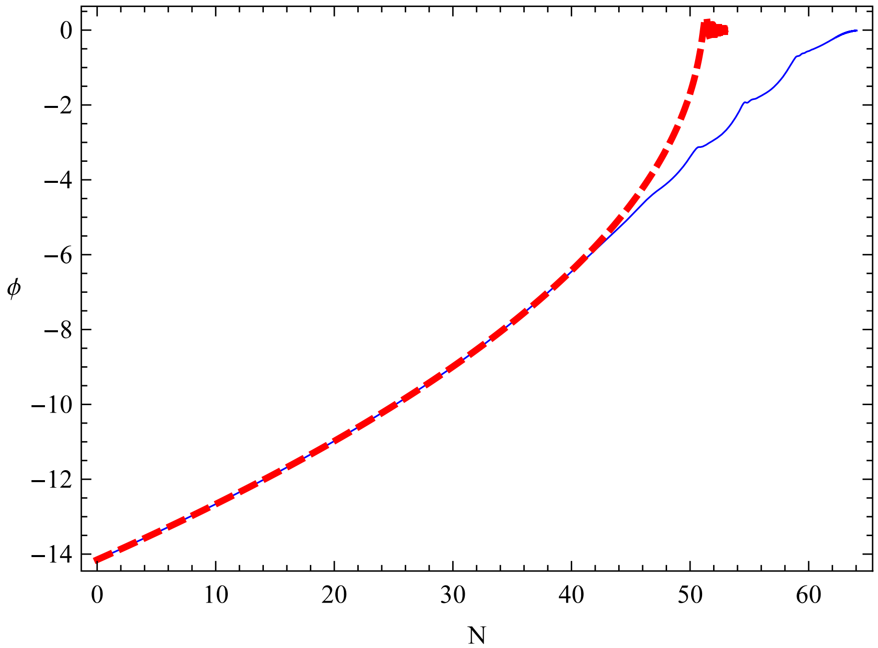


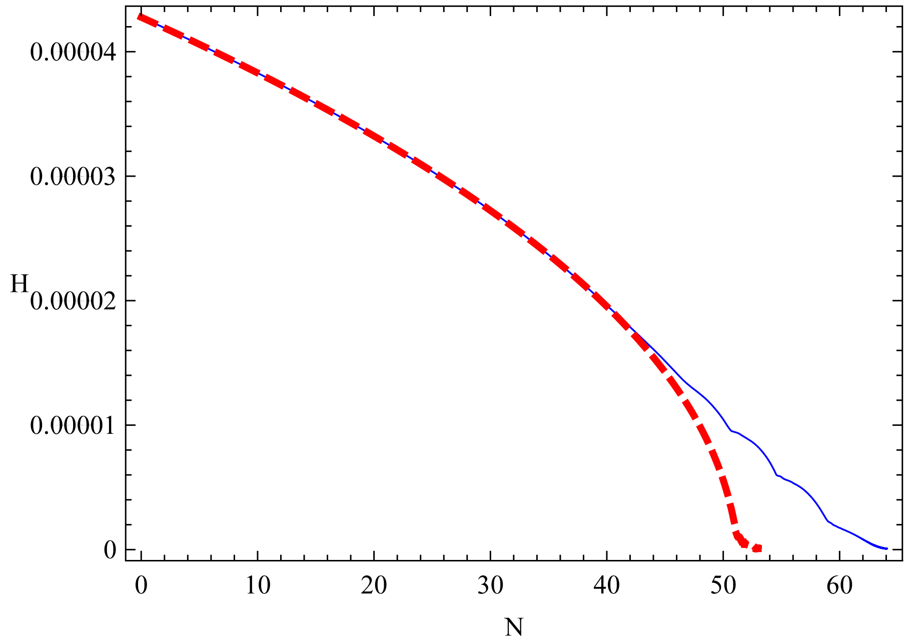
$\alpha = 32.$

$\xi = 2.2$

Shu-Lin Cheng, Wolung Lee, and Kin-Wang Ng, Phys. Rev. D 93, 063510 (2016).

Background evolutions





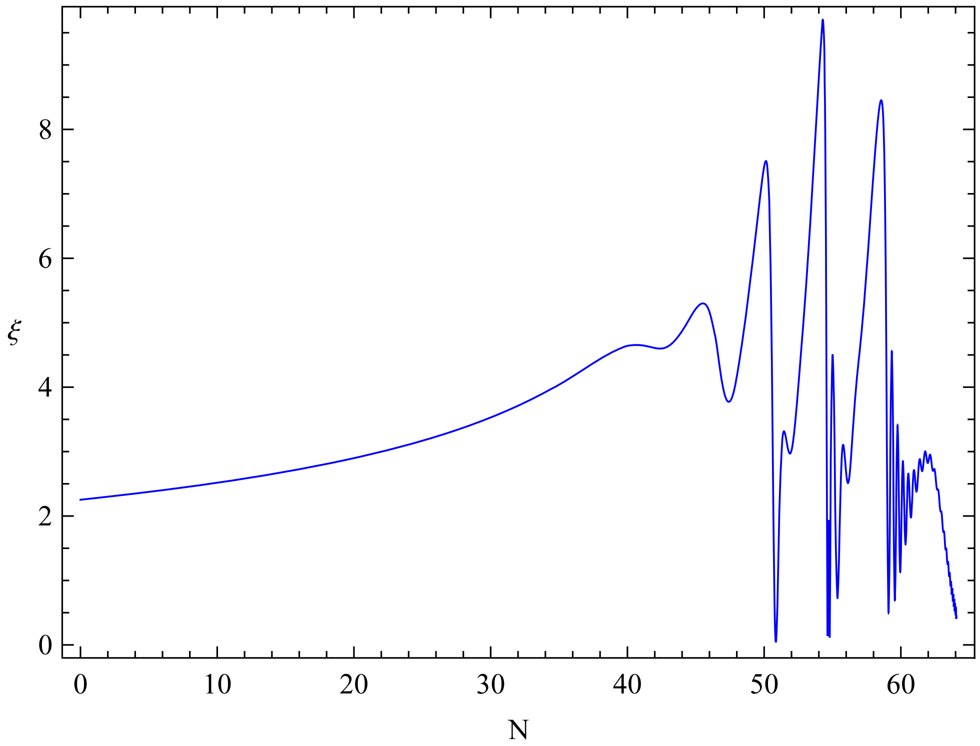
$\alpha = 32.$

$\xi = 2.2$

Shu-Lin Cheng, Wolung Lee, and Kin-Wang Ng,
Phys. Rev. D 93, 063510 (2016).

Evolution in ξ

$$\left[\frac{\partial^2}{\partial\eta^2} + k^2 \mp 2aHk\xi\right] A_\pm(\eta, k) = 0, \qquad \xi \equiv \frac{\alpha\dot{\phi}}{2fH}.$$

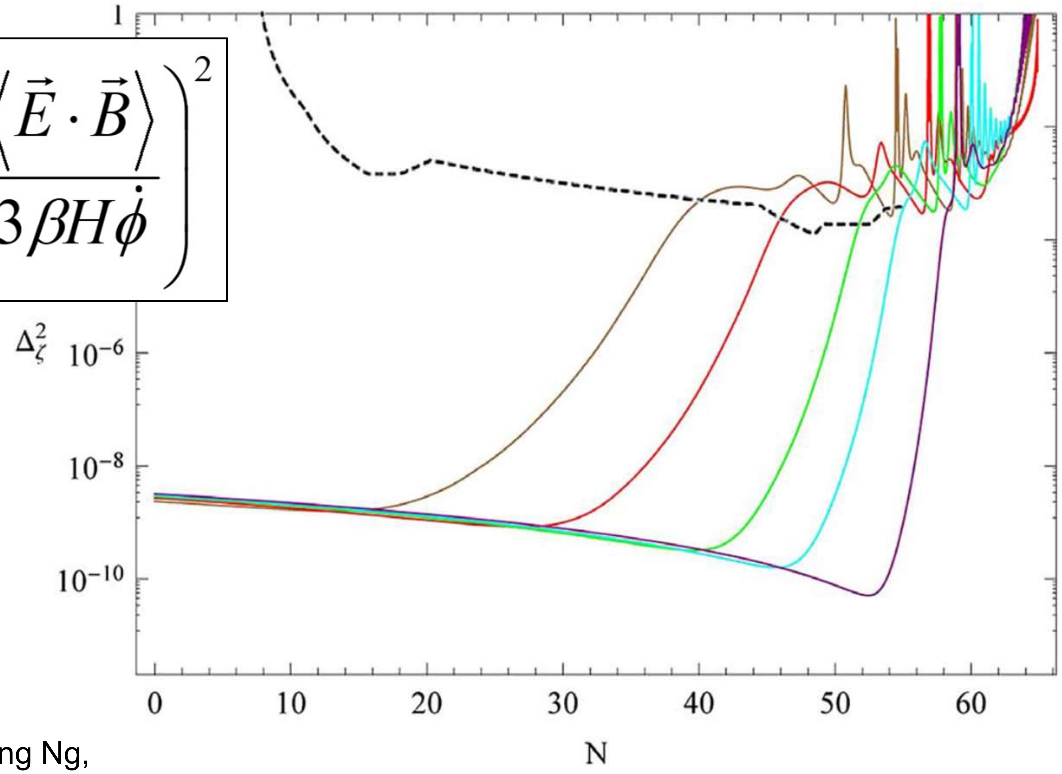


Power spectrum with back-actions

Axial coupling magnetogenesis

- **The power spectrum of the curvature perturbation**

$$\Delta_\zeta^2(k) = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2 = \left(\frac{\alpha}{f}\frac{\left\langle \vec{E}\cdot\vec{B}\right\rangle}{3\beta H\dot{\phi}}\right)^2$$



Shu-Lin Cheng, Wolung Lee, and Kin-Wang Ng,
Phys. Rev. D 93, 063510 (2016).

The energy constraint

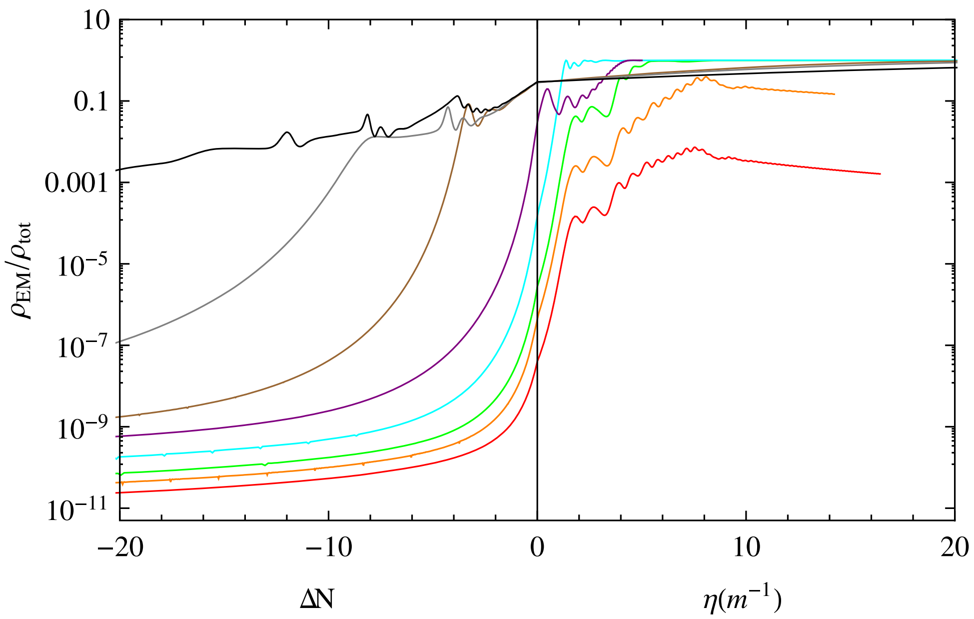


FIG. 8. Time evolution of the ratio of the energy density of gauge quanta, $\rho_{\rm EM} \equiv \langle \vec{E}^2 + \vec{B}^2 \rangle /2$, and the total energy density, $\rho_{\rm tot} \equiv 3M_p^2 H^2$. Inflation ends at $\Delta N = 0$ or $\eta = 0$. Before inflation, the time is reckoned by how many e-foldings there are prior to the end of inflation. After inflation, the scale factor is rescaled to unity, and the conformal time η is in units of m^{-1}. The lines shown in the figure, in order from bottom to top, are for $\alpha = 7, 8, 9, 11, 14, 16, 23, 32$, respectively.

Implications 1. Realization of warm inflation

$$T_{\rm per} \sim \frac{\alpha}{8\pi}\left(\frac{10}{g_{\rm eff}}\right)^{1/4}\left(\frac{m}{M_p}\right)^{3/2}\left(\frac{M_p}{f}\right)M_p \sim 4.42\times 10^{-10}\alpha$$

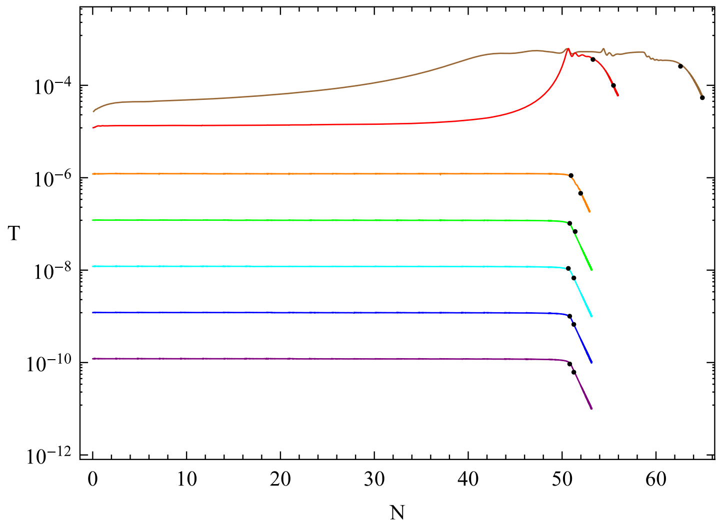


FIG. 7. Time evolution of the gauge quanta temperature T defined by $\langle \vec{E}^2 + \vec{B}^2 \rangle/2 \equiv (\pi^2/30) g_{\rm eff} T^4$, where $g_{\rm eff} = 106.75$. The lines shown in the figure, in order from top to bottom, are for $\alpha = 32$, 14.5, 1.45, 0.145, 1.45×10^{-2}, 1.45×10^{-3}, 1.45×10^{-4}, respectively. The left black dot on each line indicates the end of inflation timed by $-\dot{H}/H^2 = 1$. The right black dot denotes the beginning of the radiation-dominated era, after which $T \propto a^{-1}$.

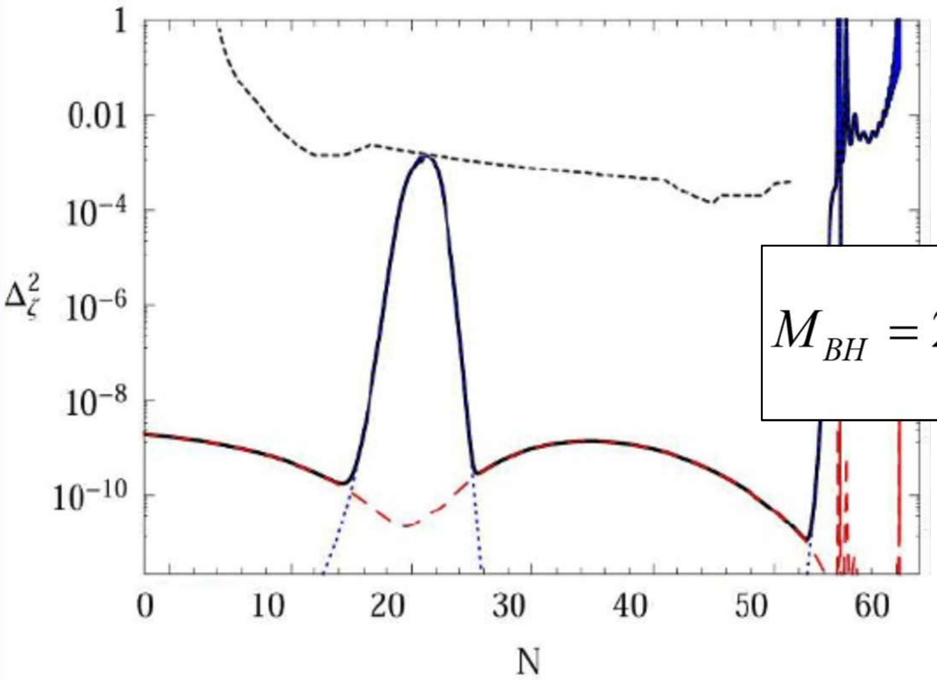
Large back reactions heat up the Early Universe without involving a post-inflationary **reheating** stage!

Shu-Lin Cheng, Wolung Lee, and Kin-Wang Ng, Phys. Rev. D 93, 063510 (2016).

Implications 2. PBH with high stellar mass

Axial coupling magnetogenesis

- **Power spectrum of the curvature perturbation**



$$M_{BH} = 2.74\times10^{-38} e^{2(N_e-N)}\left(\frac{M_p H_{end}}{H^2}\right) M_{sun}$$

Figure 4. Solid line is the total power spectrum of the curvature perturbation. The contribution induced by photon production is denoted by the dotted line and the vacuum contribution by the dashed line. The e-folding N denotes the time when the k-mode leaves the horizon. The primordial black hole bound is the short-dashed line.

Shu-Lin Cheng, Wolung Lee, and Kin-Wang Ng, JHEP02 (2017) 008

Implications 3. PMF generations

Consider the action including dilaton EM such that

$$S = \int d^4x\sqrt{g} \times \left[\frac{M_p^2}{2}R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V\left(\varphi\right) - \frac{1}{4}I(\tau)F_{\mu\nu}F^{\mu\nu}\right].$$

Expressing the physical electric and magnetic fields by

$$\vec{E} = -\frac{1}{a^2}\frac{\partial\vec{A}}{\partial\tau}, \qquad \vec{B} = \frac{1}{a^2}\vec{\nabla}\times\vec{A}.$$

we obtain the wave equation for the vector potential

$$\frac{\partial^2\vec{A}}{\partial\tau^2} - \vec{\nabla}^2\vec{A} + \frac{I'}{I}\frac{\partial\vec{A}}{\partial\tau} = 0.$$

Decomposing the gauge field $\vec{A}(\tau,\vec{x})$ as

$$\vec{A}\left(\tau,\vec{x}\right) = \sum_{\lambda=\pm}\int\frac{d^3k}{\left(2\pi\right)^{3/2}}\left[\vec{\epsilon}_\lambda\left(\vec{k}\right)\vec{a}_\lambda\left(\vec{k}\right)A_\lambda\left(\tau,\vec{k}\right)e^{i\vec{k}\cdot\vec{x}} + h.c.\right]$$

the equation of motion for the mode functions can be written as

$$\left[\frac{\partial^2}{\partial\tau^2} + \frac{I'}{I}\frac{\partial}{\partial\tau} + k^2\right]A_\pm\left(\tau,\vec{k}\right) = 0.$$

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An inflationary dilaton PMF

$$\left[\frac{\partial^2}{\partial\tau^2} + \frac{I'}{I}\frac{\partial}{\partial\tau} + k^2\right] A_{\pm}\left(\tau, \vec{k}\right) = 0.$$

The energy density of the produced EM fields is determined by

$$\rho_{EM} = \frac{I}{2}\left\langle \vec{E}^2 + \vec{B}^2 \right\rangle = \frac{I}{4\pi^2 a^4}\int dk k^2 \sum_{\lambda=\pm}\left(\left|A'_\lambda\right|^2 + k^2\left|A_\lambda\right|^2\right),$$

and the magnetic energy density of the PMF is governed by

$$\rho_B = \frac{I}{4\pi^2 a^2}\int dk k^4 \sum_{\lambda=\pm}\left|A_\lambda\right|^2.$$

Employing $I\left(\varphi\left(t\right)\right) = \exp(b\varphi^2/M_p^2)$, the equation of motion for the mode function becomes

$$\left[\frac{\partial^2}{\partial\tau^2} + 2b\varphi\frac{\partial}{\partial\tau} + k^2\right] A_{\pm}\left(\tau, \vec{k}\right) = 0.$$

- $b < 0$ for strong coupling cases
- $b > 0$ for weak coupling cases

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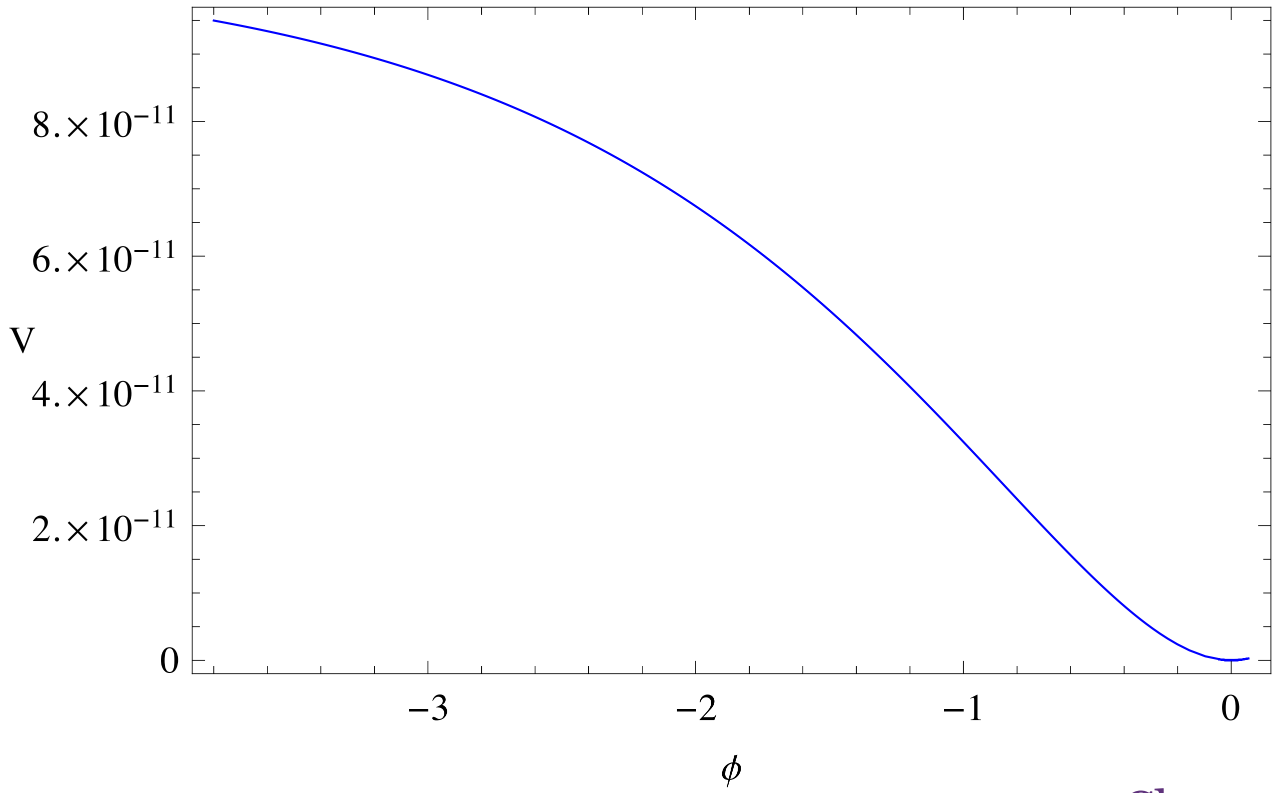
The Higgs potential

$$V(\varphi) = \frac{1}{\Omega^4(\varphi)} \frac{\lambda}{4} \left(h^2(\varphi) - \nu^2\right)$$

with $\Omega^2(\varphi) = 1 + \xi h^2/M_p^2$, $d\varphi/dh = \sqrt{(\Omega^2 + 6\xi^2 h^2/M_p^2)/\Omega^4}$, and $\nu = 246\text{GeV}$, $\lambda = 0.1291$, $\xi = 17606$.



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Higgs inflation with backactions

Taking into account the back reaction given rise by the production of gauge quanta, the background dynamics is governed by

$$\ddot{\varphi} + (3H + \gamma)\,\dot{\varphi} + \frac{dV}{d\varphi} = \frac{1}{2}\frac{dI}{d\varphi}\left\langle \vec{E}^2 - \vec{B}^2 \right\rangle,$$

$$\dot{\rho_\gamma} + 4H\rho_\gamma = \gamma\dot{\varphi}^2,$$

$$H^2 = \frac{1}{3}\left[\frac{\dot{\varphi}^2}{2} + V(\varphi) + \rho_\gamma + \rho_{EM}\right],$$

where

$$\frac{1}{2}\left\langle \vec{E}^2 - \vec{B}^2 \right\rangle = \frac{1}{4\pi^2 a^4}\int dk k^2 \sum_{\lambda=\pm}\left(\left|A'_\lambda\right|^2 - k^2\left|A_\lambda\right|^2\right).$$

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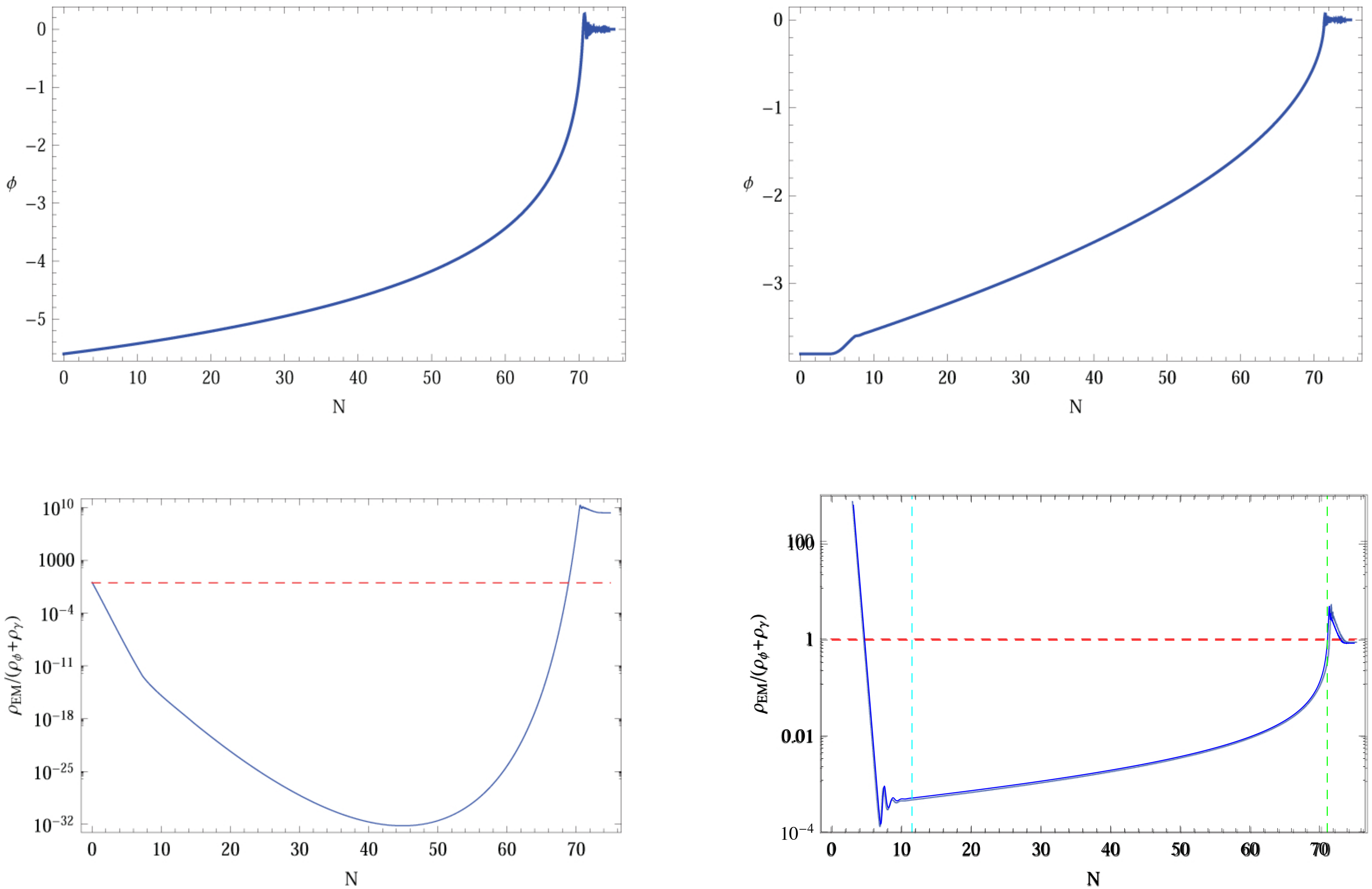
An inflationary dilaton PMF

w/o back actions $b = 20$ w. back actions



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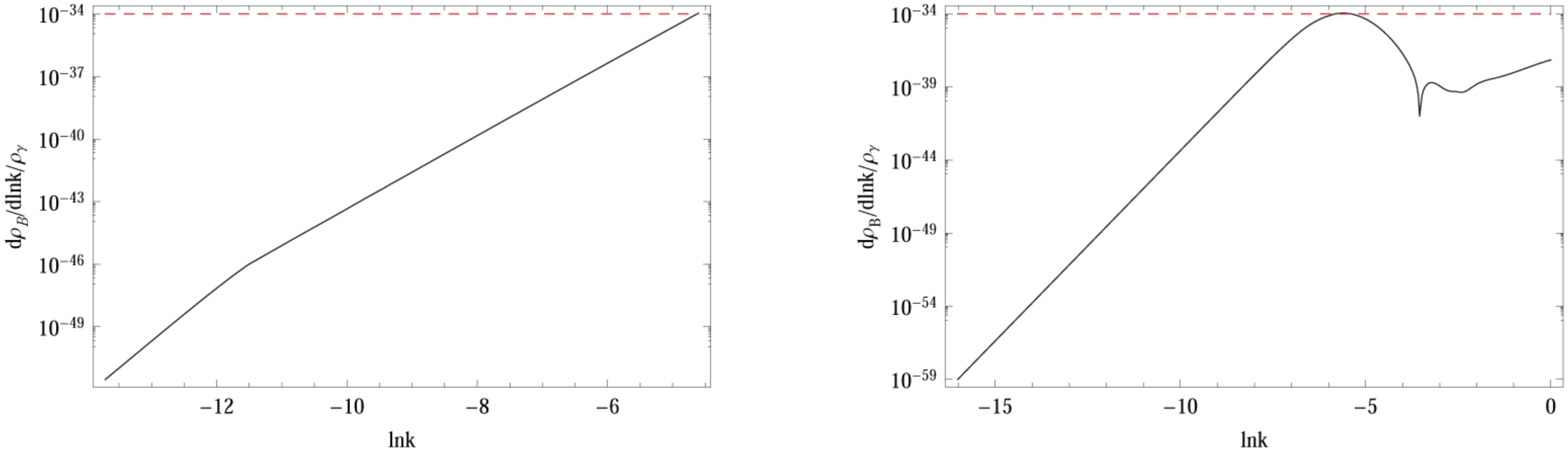
An inflationary dilaton PMF

w/o back actions $\quad b = 20 \quad$ w. back actions



With back reactions, a strong enough PMF at a proper scale can be achieved without violating the energy constraint!

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Summary

- Contrary to the conventional wisdom, it seems possible to generate a large enough PMF through the coupling of the EM field to the inflaton provided that a fast-roll stage is involved. However, due to the energy constraint, this scenario may not work.

- Though it seems rather contrived, the dilator-axion magnetogensesis works well provided that the time dependent coupling $J(a)$ would postpone the growth of magnetic fields to a later stage of inflation.

- The inflationary magnetogenesis looks very promising when the back reaction is taken into account. We are working to refine our numerical scheme and to increase the ratio r to fit the new observational constraint reported in literature.