Reconstrunction of f(G) gravity models

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Based on arXiv:2001.07021, collaboration with G. Tumurtushaa

Reconstruction of f(G)-Gravity Theories

- Motivation of f(G)-gravity theories (*Refer Naruko & Buoninfante's talks for even more general models*)
- Cosmological constraints Reconstruction methods from observations
- Rewrite models with observed quantities
- Viable f(G)-gravity theories
- Conclusions

MOTIVATIONS OF f(G) GRAVITY

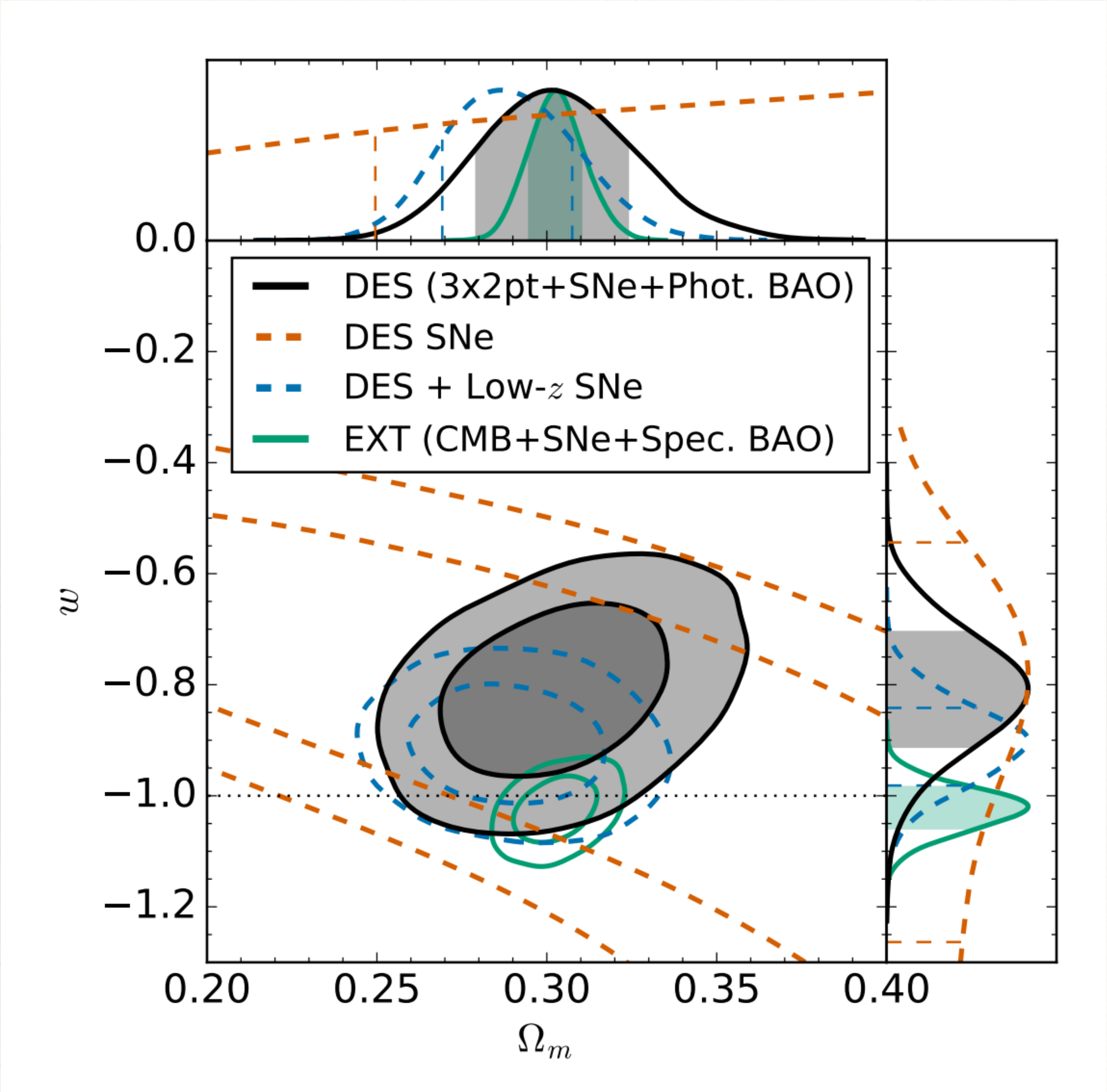
- Current observations indicate accelerating expansion of the Universe.
- Especially, accelerating universe prefers phantom dark energy (DE)
- Usual DE models can't explain phantom accelerating universe (e.o.s w < -1, energy creation)

$$\rho = \rho_0 a^{-3(1+\omega)}\,, \text{ if } \omega < -1\,, \text{ energy density increases as a does}$$

- Modified gravities can explain phantom region without violating energy conditions

COSMOLOGICAL CONSTRAINTS

- Cosmological observations prefer phantom DE (w <-1)
- (Refer 1811.02375)



PROBLEM OF DE PHANTOM

- DE with $w < -1$: creation of energy
- Modified gravities : $w_{eff} < -1$ doesn't mean creation of energy
- f(R) gravities are well studied
- Including Gauss-Bonnet term does not change Einstein field equations.
- f(G) gravities are not well known

RECONSTRUCTION OF f(G) FROM OBSERVATIONS

- Not assume any specific form of f(G)
- Reconstruct f(G) from observational observables
- Observables background (e.o.s, ω) and LSS (growth rate index, γ)

RECONSTRICTION f(G) GRAVITY THEORIES

- Model buildings

$$S=\frac{c^4}{16\pi G}\int d^4x\sqrt{-g}\left[R+f\left(\mathcal{G}\right)+\mathcal{L}_{\mathrm{m}}\right]$$

$$\mathcal{G}=R^2-4R_{\mu\nu}R^{\mu\nu}+R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

$$G_{\mu\nu}-\Sigma_{\mu\nu}=\frac{8\pi G}{c^4}T_{\mu\nu}\,,$$

$$\Sigma_{\mu\nu}=4\left[R_{\mu\rho\sigma\nu}+R_{\mu\rho}g_{\nu\sigma}+R_{\rho\nu}g_{\mu\sigma}-R_{\mu\nu}g_{\rho\sigma}-R_{\rho\sigma}g_{\mu\nu}-\frac{1}{2}R\left(g_{\mu\nu}g_{\rho\sigma}-g_{\mu\sigma}g_{\nu\rho}\right)\right]\nabla^\rho\nabla^\sigma F$$

$$-\frac{1}{2}\left(\mathcal{G}F-f\right)g_{\mu\nu}\,,\ \text{with}\ F=f_{,\mathcal{G}}=\frac{\partial f}{\partial\mathcal{G}}\,,$$

$$T_{\mu\nu}=-\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}_{\mathrm{m}}\right)}{\delta g^{\mu\nu}}\,.$$

$$R+2f-2\mathcal{G}F-2R\Box F+4R_{\mu\nu}\nabla^\mu\nabla^\nu F=-\frac{8\pi G}{c^4}\left(\rho_{\mathrm{m}}-3p_{\mathrm{m}}\right)$$

RECONSTRICTION f(G) GRAVITY THEORIES

- Background evolution equations

$$3H^2 = \frac{1}{2}\left(\mathcal{G}F - f - 24H^3\dot{F}\right) + \frac{8\pi G}{c^2}\rho_{\mathrm{m}}\,,$$

$$-2\dot{H} = 4H\dot{F}\left(2\dot{H} - H^2\right) + 4H^2\ddot{F} + \frac{8\pi G}{c^2}\left(1+\omega_{\mathrm{m}}\right)\rho_{\mathrm{m}}\,,$$

$$\dot{\rho}_{\mathrm{m}} = -3H(1+\omega_{\mathrm{m}})\rho_{\mathrm{m}}\,,$$

$$R = 6(2H^2 + \dot{H}) = 6H^2\left(2 + \frac{H'}{H}\right)\,,$$

$$\mathcal{G} = 24H^2(H^2 + \dot{H}) = 24H^4\left(1 + \frac{H'}{H}\right)\,,$$

prime denotes the derivatives w.r.t the N.

RECONSTRICTION f(G) GRAVITY THEORIES

- Model buildings

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\rho_m \left[\frac{A_1 + A_2\left(\frac{ck}{aH}\right)^2}{B_1 + B_2\left(\frac{ck}{aH}\right)^2}\right]\delta_m = 0\,,$$

$$A_1 = 1 + 4\ddot{F}\,,$$

$$A_2 = 64H^2 F_{,\mathcal{G}}\left(\dot{H} + H^2\right)^2 = 64H^2\frac{\dot{F}}{\dot{\mathcal{G}}}\left(\dot{H} + H^2\right)^2\,,$$

$$B_1 = \left(1 + 4H\dot{F}\right)^2\,,$$

$$B_2 = 16H^4 F_{,\mathcal{G}}\left[\left(4 + 16H\dot{F}\right)\left(\dot{H} + H^2\right) - H^2(1 + 4\ddot{F})\right]$$

$$= 16H^4\frac{\dot{F}}{\dot{\mathcal{G}}}\left[\left(4 + 16H\dot{F}\right)\left(\dot{H} + H^2\right) - H^2(1 + 4\ddot{F})\right]\,,$$

FOR NUMERICAL CALCULATION

- In order to perform numerical calculation, it's better to change the derivatives w.r.t e-folding N=ln a = - ln (1+z) and change functions as dimensionless quantities

$$f(\tilde{\mathcal{G}}) = f(\tilde{\mathcal{G}} + AH^2)\,,$$

$$\tilde{\mathcal{G}} = 24\frac{H^4}{H_0^4}\left(1 + \frac{H'}{H}\right)\,,$$

$$\tilde{f} = \frac{f}{H_0^2}\,,\ \tilde{\mathcal{G}} = \frac{\mathcal{G}}{H_0^4}\,,$$

$$F = \frac{\partial f}{\partial \mathcal{G}}\,,\ \tilde{F} = F H_0^2\,,$$

$$\tilde{F} = \frac{1}{\tilde{\mathcal{G}}'}\tilde{f}'\,,$$

$$\tilde{F}' = \frac{1}{\tilde{\mathcal{G}}'}\left(\tilde{f}'' - \frac{\tilde{\mathcal{G}}''}{\tilde{\mathcal{G}}'}\tilde{f}'\right)\,,$$

$$\tilde{F}'' = \frac{1}{\tilde{\mathcal{G}}'}\left(\tilde{f}''' - 2\frac{\tilde{\mathcal{G}}''}{\tilde{\mathcal{G}}'}\tilde{f}'' + \left(2\left(\frac{\tilde{\mathcal{G}}''}{\tilde{\mathcal{G}}'}\right)^2 - \frac{\tilde{\mathcal{G}}'''}{\tilde{\mathcal{G}}'}\right)\tilde{f}'\right)$$

OBSERVATIONAL f(G) GRAVITY THEORIES

- Model buildings from observables

$$\frac{H^2}{H_0^2} = \frac{\rho_{\rm m}}{\rho_{\rm cr0}}\left(1+\frac{\rho_{\rm DE}}{\rho_{\rm m}}\right) = \Omega_{m0}\left(1+g[\Omega_{\rm m0},\omega_0,\omega_a,N]\right)e^{-3N}\,,$$

$$\frac{H'}{H} = -\frac{3}{2}\left(1+\omega_{\rm DE}\Omega_{\rm DE}\right) \equiv -\frac{3}{2}\left(1+Q[\Omega_{\rm m0},\omega_0,\omega_a,N]\right)\,,$$

$$d\ln\delta_{\rm m}/d\ln a \equiv \Omega_{\rm m}^{\gamma}\,, \quad \text{where} \quad \gamma \equiv \gamma_0 + \gamma_a\left(1-e^N\right)\,,$$

$$\mathcal{P}[\Omega_{\rm m0},\omega_0,\omega_a,\gamma_0,\gamma_a,N] \equiv \frac{\delta''_{\rm m}}{\delta_{\rm m}} + \left(2+\frac{H'}{H}\right)\frac{\delta'_{\rm m}}{\delta_{\rm m}}$$
$$= (1+g)^{-\gamma}\left[(1+g)^{-\gamma} - \gamma'\ln(1+g) + 3\gamma Q\frac{1}{2}(1-3Q)\right]\,.$$

$$\Omega_{\rm DE} \simeq \Omega_{\rm eff} \longrightarrow \frac{g[\Omega_{\rm m0},\omega_0,\omega_a,N]}{1+g[\Omega_{\rm m0},\omega_0,\omega_a,N]} \simeq \frac{-4\frac{H^4}{H_0^4}\frac{1}{\tilde{\mathcal{G}}'}\left[\tilde{f}''-\left(\frac{\tilde{\mathcal{G}}''}{\tilde{\mathcal{G}}'}+\frac{H_0^4}{H^4}\frac{\tilde{\mathcal{G}}}{24}\right)\tilde{f}'+\frac{H_0^4}{H^4}\frac{\tilde{\mathcal{G}}'}{24}\tilde{f}\right]}{-4\frac{H^4}{H_0^4}\frac{1}{\tilde{\mathcal{G}}'}\left[\tilde{f}''-\left(\frac{\tilde{\mathcal{G}}''}{\tilde{\mathcal{G}}'}+\frac{H_0^4}{H^4}\frac{\tilde{\mathcal{G}}}{24}\right)\tilde{f}'+\frac{H_0^4}{H^4}\frac{\tilde{\mathcal{G}}'}{24}\tilde{f}\right]+\Omega_{\rm m0}e^{-3N}}\,,$$

$$\omega_{\rm DE} \simeq \omega_{\rm eff} \longrightarrow \omega_{\rm DE}[\omega_0,\omega_a,N] \simeq -1 - \frac{\tilde{f}'''-\left(2\frac{\tilde{\mathcal{G}}''}{\tilde{\mathcal{G}}'}-3\frac{H'}{H}+1\right)\tilde{f}''-\left(\frac{\tilde{\mathcal{G}}'''}{\tilde{\mathcal{G}}'}-\left(2\frac{\tilde{\mathcal{G}}''}{\tilde{\mathcal{G}}'}-3\frac{H'}{H}+1\right)\frac{\tilde{\mathcal{G}}''}{\tilde{\mathcal{G}}'}\right)\tilde{f}'}{3\tilde{f}''-\left(3\frac{\tilde{\mathcal{G}}''}{\tilde{\mathcal{G}}'}+\frac{H_0^4}{H^4}\frac{\tilde{\mathcal{G}}}{8}\right)\tilde{f}'+\frac{H_0^4}{H^4}\frac{\tilde{\mathcal{G}}'}{8}\tilde{f}}\,.$$

RECONSTRICTION f(G) GRAVITY THEORIES

- Model buildings

$$\frac{H^2}{H_0^2} = -4\frac{H^4}{H_0^4}\frac{1}{\tilde{\mathcal{G}}'}\left[\tilde{f}'' - \left(\frac{\tilde{\mathcal{G}}''}{\tilde{\mathcal{G}}'} + \frac{1}{24}\frac{H_0^4}{H^4}\tilde{\mathcal{G}}\right)\tilde{f}' + \frac{1}{24}\frac{H_0^4}{H^4}\tilde{\mathcal{G}}'\tilde{f}\right] + \Omega_{\mathrm{m}0}\exp[-3N]\,,$$

$$\frac{H'}{H} = -2\frac{H^2}{H_0^2}\frac{1}{\tilde{\mathcal{G}}'}\left[\tilde{f}''' + \left(3\frac{H'}{H} - 2\frac{\tilde{\mathcal{G}}''}{\tilde{\mathcal{G}}'} - 1\right)\tilde{f}'' - \left(\frac{\tilde{\mathcal{G}}'''}{\tilde{\mathcal{G}}'} + \left(3\frac{H'}{H} - 2\frac{\tilde{\mathcal{G}}''}{\tilde{\mathcal{G}}'} - 1\right)\frac{\tilde{\mathcal{G}}''}{\tilde{\mathcal{G}}'}\right)\tilde{f}'\right] - \frac{3}{2}\frac{H_0^2}{H^2}\Omega_{m0}\exp[-3N]\,.$$

$$\tilde{f}_0''' - \left(2\frac{\tilde{\mathcal{G}}_0''}{\tilde{\mathcal{G}}_0'} + \frac{H_0'}{H_0}\right)\tilde{f}_0'' - \left(\frac{\tilde{\mathcal{G}}_0'''}{\tilde{\mathcal{G}}_0'} - \frac{H_0'}{H_0}\frac{\tilde{\mathcal{G}}_0''}{\tilde{\mathcal{G}}_0'} - 2\left(\frac{\tilde{\mathcal{G}}_0''}{\tilde{\mathcal{G}}_0'}\right)^2\right)\tilde{f}_0' + \left(1 - 2\frac{H_0'}{H_0}\right)\tilde{f}_0 = -\frac{3}{4}\left(1 - \Omega_{\mathrm{m}0}\right)\left(1 + \omega_{\mathrm{DE}0}\right)\tilde{\mathcal{G}}_0'\,,$$

master equation

$$\frac{\delta_{\mathrm{m}}''}{\delta_{\mathrm{m}}} + \left(2 + \frac{H'}{H}\right)\frac{\delta_{\mathrm{m}}'}{\delta_{\mathrm{m}}} = \frac{3}{2}\Omega_m\left(\frac{A_1 + A_2\left(\frac{ck}{aH}\right)^2}{B_1 + B_2\left(\frac{ck}{aH}\right)^2}\right)$$

INITIAL CONDITIONS

$$1 = -4\frac{1}{\tilde{\mathcal{G}}_0'}\left[\tilde{f}_0'' - \left(\frac{\tilde{\mathcal{G}}_0''}{\tilde{\mathcal{G}}_0'} + \frac{1}{24}\tilde{\mathcal{G}}_0\right)\tilde{f}_0' + \frac{1}{24}\tilde{\mathcal{G}}_0'\tilde{f}_0\right] + \Omega_{\mathrm{m}0}\,, \tag{64}$$

$$\frac{H_0'}{H_0} = -2\frac{1}{\tilde{\mathcal{G}}_0'}\left[\tilde{f}_0''' + \left(3\frac{H_0'}{H_0} - 2\frac{\tilde{\mathcal{G}}_0''}{\tilde{\mathcal{G}}_0'} - 1\right)\tilde{f}_0'' - \left(\frac{\tilde{\mathcal{G}}_0'''}{\tilde{\mathcal{G}}_0'} + \left(3\frac{H_0'}{H_0} - 2\frac{\tilde{\mathcal{G}}_0''}{\tilde{\mathcal{G}}_0'} - 1\right)\frac{\tilde{\mathcal{G}}_0''}{\tilde{\mathcal{G}}_0'}\right)\tilde{f}_0'\right] - \frac{3}{2}\Omega_{\mathrm{m}0} \tag{65}$$

$$= -\frac{3}{2}\left(1 + Q_0\right) = -\frac{3}{2}\left(1 + \omega_{\mathrm{DE}0}\Omega_{\mathrm{DE}0}\right)\,,$$

$$\frac{2}{3}\frac{\mathcal{P}_0}{\Omega_{\mathrm{m}0}} - 1 = A_{10}^{(3)}\tilde{f}_0''' + \left[A_{10}^{(2)} - B_{10}^{(2)} + \left(A_{20}^{(2)} - B_{20}^{(2)}\right)\left(\frac{ck}{H_0}\right)^2\right]\tilde{f}_0'' + \left[A_{10}^{(1)} - B_{10}^{(1)} + \left(A_{20}^{(1)} - B_{20}^{(1)}\right)\left(\frac{ck}{H_0}\right)^2\right]\tilde{f}_0'$$

$$= \frac{2}{3}\Omega_{\mathrm{m}0}^{\gamma_0 - 1}\left[\Omega_{\mathrm{m}0}^{\gamma_0} - \gamma_a \ln\Omega_{\mathrm{m}0} + 3\gamma_0 Q_0 + \frac{1}{2}\left(1 - 3Q_0\right)\right] - 1\,, \tag{66}$$

MAIN NOTICE

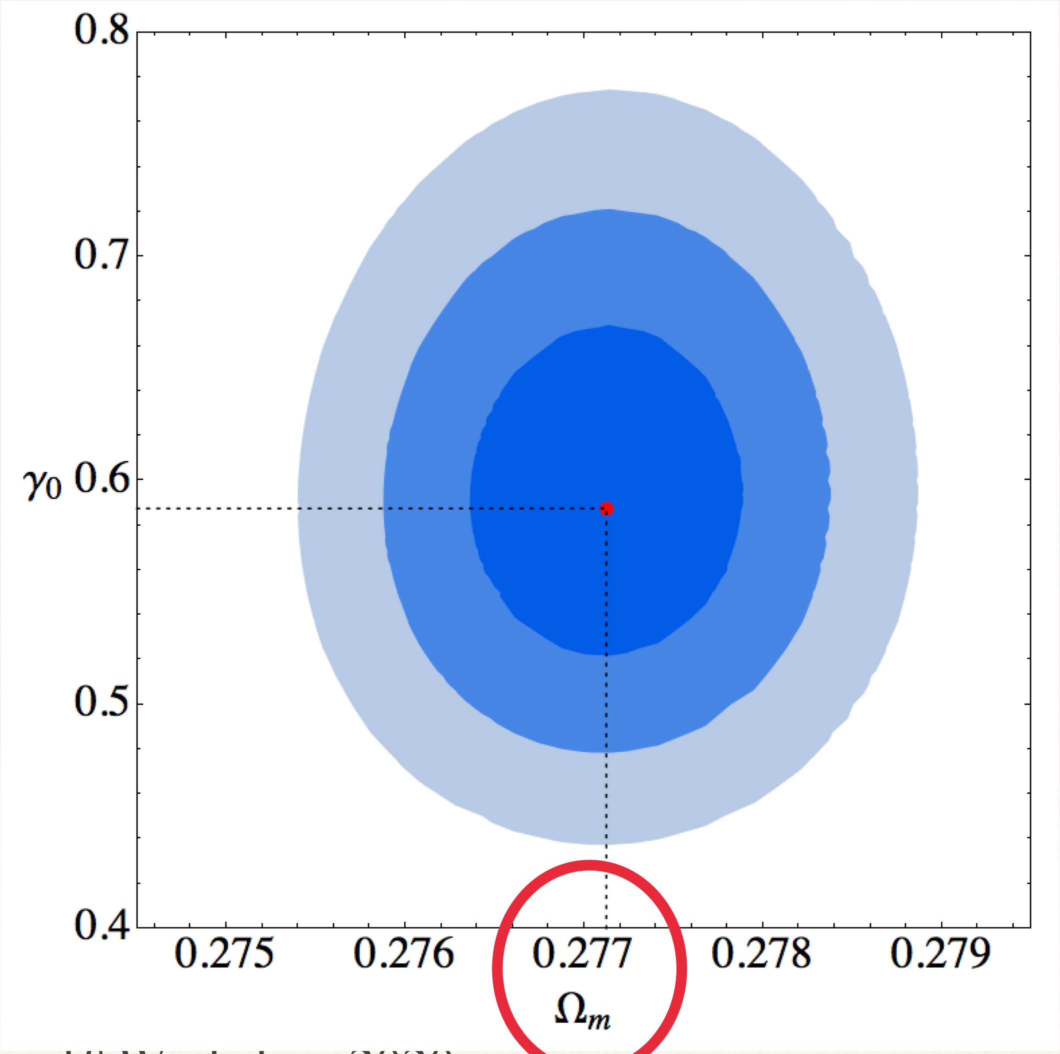
- One need three initial conditions. However, one can obtain only two initial conditions.
- One can have a further constraint from stability condition $f_{,\mathcal{G}\mathcal{G}} > 0$
- In order to avoid the singularity problem, one needs to extend the simple f(G) model. We assume

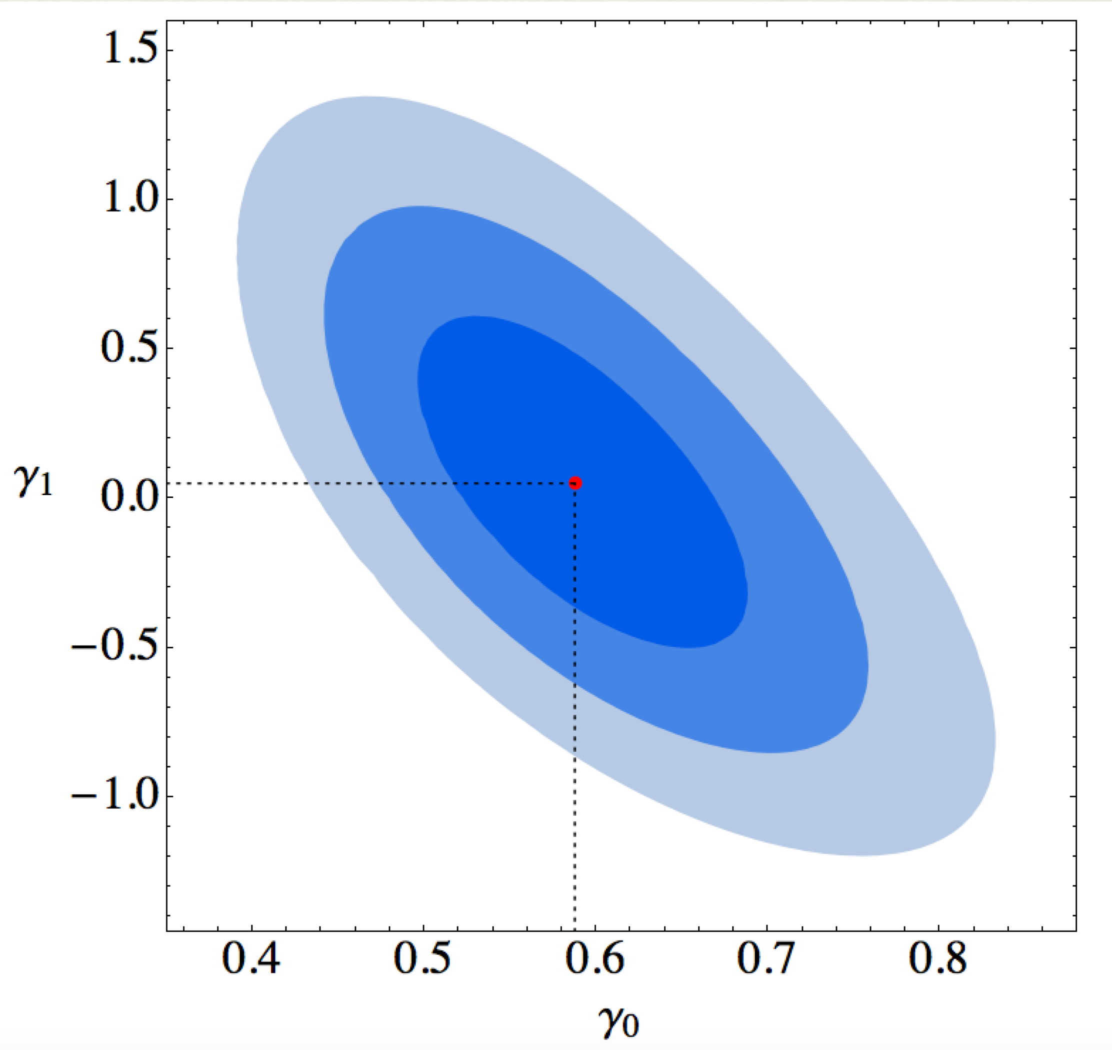
$$\mathcal{G}_{\mathcal{A}} \equiv \mathcal{G} + \mathcal{A}H^4 = 24H^4\left(\mathcal{A} + 1 + \frac{H'}{H}\right),$$

$$\tilde{\mathcal{G}}_{\mathcal{A}} \equiv \tilde{\mathcal{G}} + \mathcal{A}\left(\frac{H}{H_0}\right)^4 = 24\left(\frac{H}{H_0}\right)^4\left(\mathcal{A} + 1 + \frac{H'}{H}\right),$$

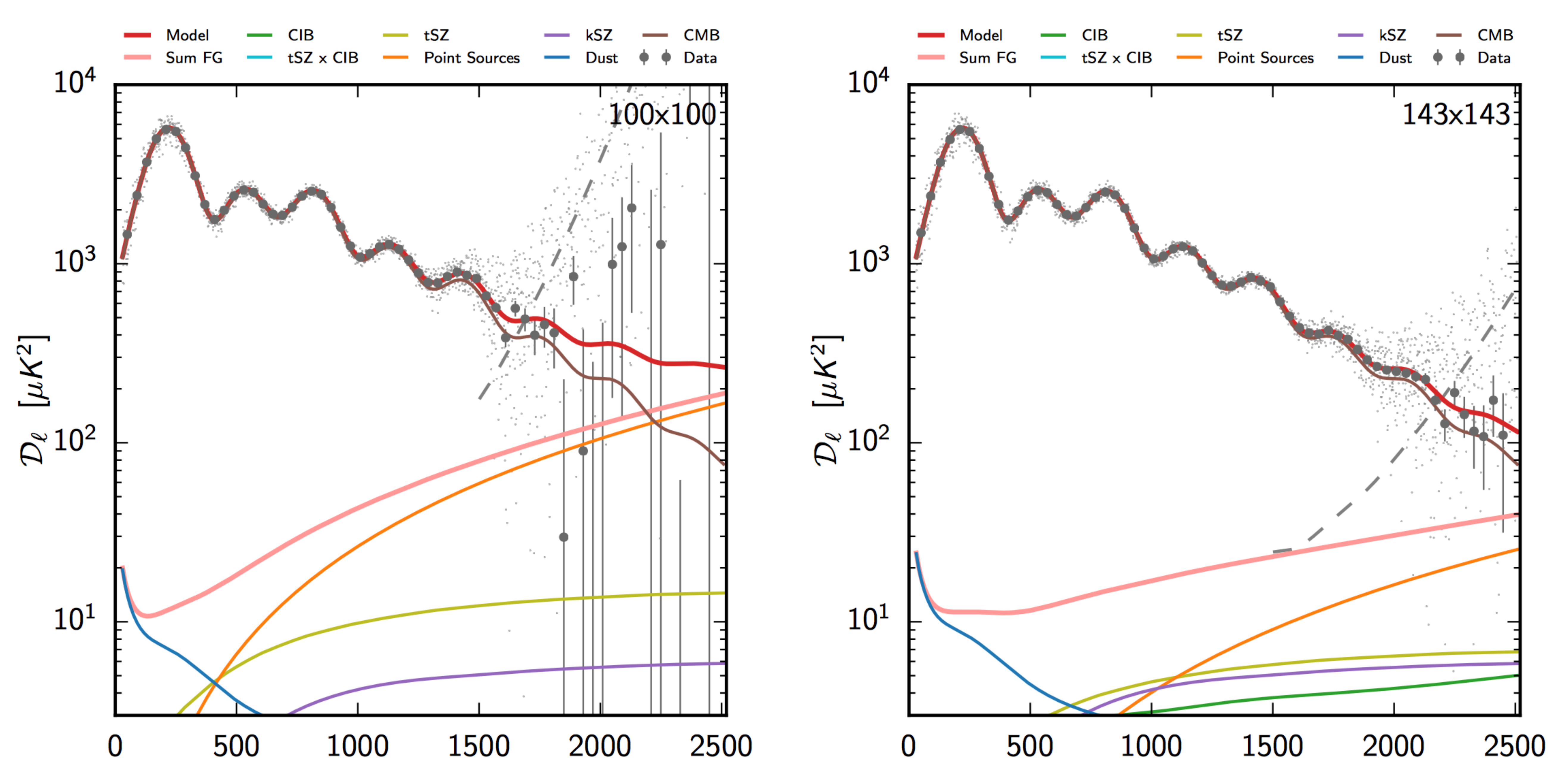
OBSERVATIONS

CMB[6, 43], Supernovae type Ia (SnIa) [44], BAO [45–49], Hubble expansion $H(z)$ [50, 51], and the growth-rate data [52, 53]. The total likelihood function $\mathcal{L}_{tot}$ can, therefore, be given as the product of the separate likelihoods of each data as follows: $\mathcal{L}_{tot} = \mathcal{L}_{CMB} \times \mathcal{L}_{SnIa} \times \mathcal{L}_{BAO} \times \mathcal{L}_{H(z)} \times \mathcal{L}_{growth}$, which is also related to the total χ^2 via $\chi^2_{tot} = -\log \mathcal{L}_{tot}$ or $\chi^2_{tot} = \chi^2_{CMB} + \chi^2_{SnIa} + \chi^2_{BAO} + \chi^2_{H(z)} + \chi^2_{growth}$. By employing the aforementioned cosmological data together with the statistical methods of minimizing the χ^2_{tot}, we can obtain the best-fit values of the cosmological parameters $\{\Omega_{m0}, \omega_0, \omega_a, \gamma_0, \gamma_a\}$ and their uncertainties.

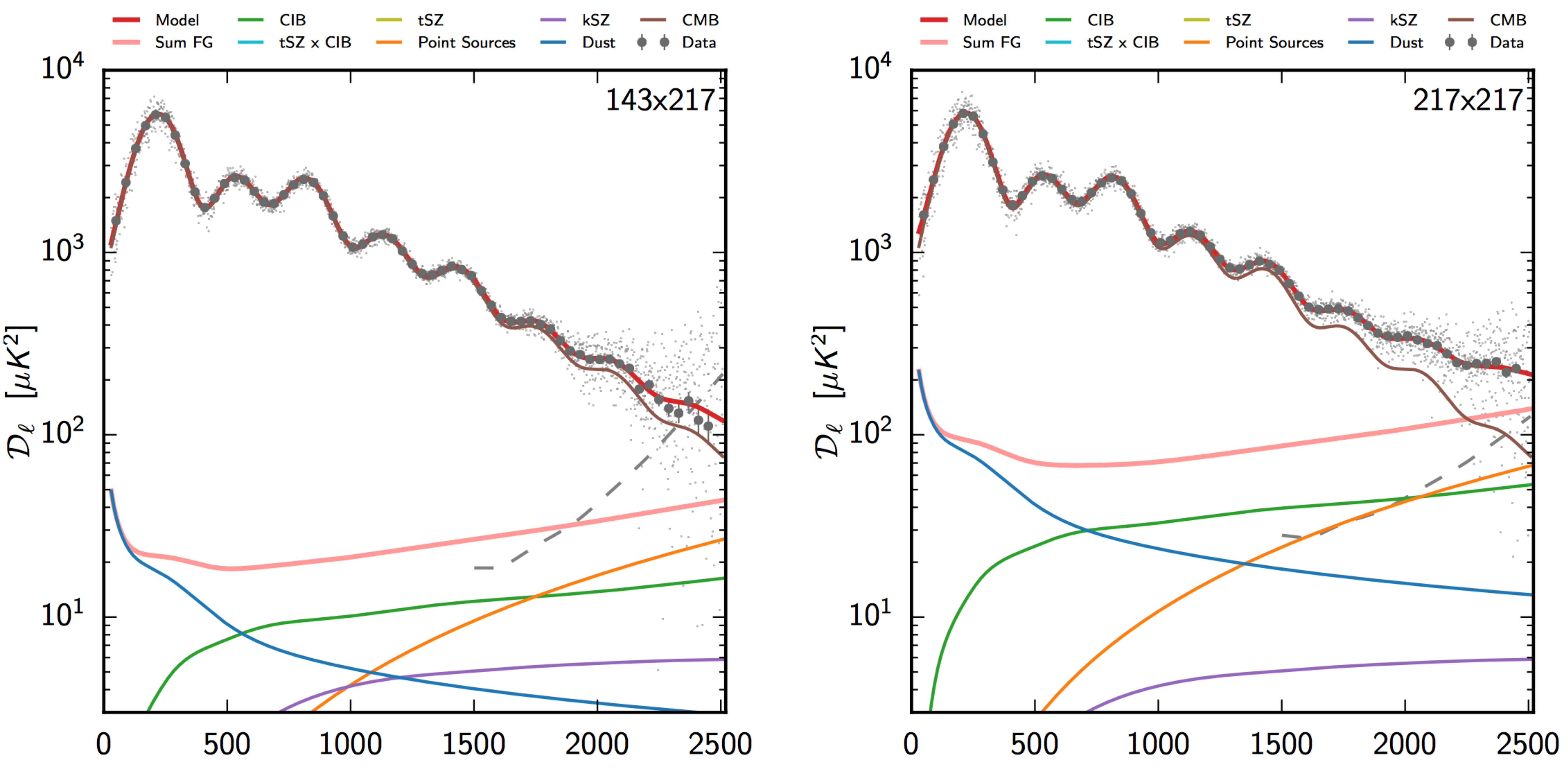




OBSERVATIONS (Planck)



OBSERVATIONS (Planck)



OBSERVATIONS (Planck)

Best-fit cosmological parameters from WMAP seven-year results[29]

Parameter	Symbol	Best fit (WMAP only)	Best fit (WMAP + BAO[30] + H_0[31])
Age of the universe (Ga)	t_0	13.75 ±0.13	13.75 ±0.11
Hubble's constant (km/Mpc·s)	H_0	71.0 ±2.5	$70.4^{+1.3}_{-1.4}$
Baryon density	Ω_b	0.0449 ±0.0028	0.0456 ±0.0016
Physical baryon density	$\Omega_b h^2$	$0.022\,58^{+0.000\,57}_{-0.000\,56}$	0.022 60 ±0.000 53
Dark matter density	Ω_c	0.222 ±0.026	0.227 ±0.014
Physical dark matter density	$\Omega_c h^2$	0.1109 ±0.0056	0.1123 ±0.0035
Dark energy density	Ω_Λ	0.734 ±0.029	$0.728^{+0.015}_{-0.016}$
Fluctuation amplitude at 8h^{-1} Mpc	σ_8	0.801 ±0.030	0.809 ±0.024
Scalar spectral index	n_s	0.963 ±0.014	0.963 ±0.012
Reionization optical depth	τ	0.088 ±0.015	0.087 ±0.014
*Total density of the universe	Ω_{tot}	$1.080^{+0.093}_{-0.071}$	$1.0023^{+0.0056}_{-0.0054}$
*Tensor-to-scalar ratio, k_0 = 0.002 Mpc^{-1}	r	< 0.36 (95% CL)	< 0.24 (95% CL)
*Running of spectral index, k_0 = 0.002 Mpc^{-1}	$dn_s/d\ln k$	−0.034 ±0.026	−0.022 ±0.020
Note: * = Parameters for extended models (parameters place limits on deviations from the Lambda-CDM model)[29]			

OBSERVATIONS (Planck)

Best-fit cosmological parameters from WMAP nine-year results[16]

Parameter	Symbol	Best fit (WMAP only)	Best fit (WMAP + eCMB + BAO + H_0)
Age of the universe (Ga)	t_0	13.74 ±0.11	13.772 ±0.059
Hubble's constant (km⁄Mpc·s)	H_0	70.0 ±2.2	69.32 ±0.80
Baryon density	Ω_b	0.0463 ±0.0024	0.046 28 ±0.000 93
Physical baryon density	$\Omega_b h^2$	0.022 64 ±0.000 50	0.022 23 ±0.000 33
Cold dark matter density	Ω_c	0.233 ±0.023	$0.2402^{+0.0088}_{-0.0087}$
Physical cold dark matter density	$\Omega_c h^2$	0.1138 ±0.0045	0.1153 ±0.0019
Dark energy density	Ω_Λ	0.721 ±0.025	$0.7135^{+0.0095}_{-0.0096}$
Density fluctuations at $8h^{-1}$ Mpc	σ_8	0.821 ±0.023	$0.820^{+0.013}_{-0.014}$
Scalar spectral index	n_s	0.972 ±0.013	0.9608 ±0.0080
Reionization optical depth	τ	0.089 ±0.014	0.081 ±0.012
Curvature	$1-\Omega_{\rm tot}$	$-0.037^{+0.044}_{-0.042}$	$-0.0027^{+0.0039}_{-0.0038}$
Tensor-to-scalar ratio (k_0 = 0.002 Mpc^{-1})	r	< 0.38 (95% CL)	< 0.13 (95% CL)
Running scalar spectral index	$dn_s/d\ln k$	−0.019 ±0.025	−0.023 ±0.011

OBSERVATIONS (Planck)

Cosmological parameters from 2013 Planck results[30][32]

Parameter	Symbol	*Planck* Best fit	*Planck* 68% limits	*Planck*+lensing Best fit	*Planck*+lensing 68% limits	*Planck*+WP Best fit	*Planck*+WP 68% limits	*Planck*+WP +HighL Best fit	*Planck*+WP +HighL 68% limits	*Planck*+lensing +WP+highL Best fit	*Planck*+lensing +WP+highL 68% limits	*Planck*+WP +highL+BAO Best fit	*Planck*+WP +highL+BAO 68% limits
Baryon density	$\Omega_b h^2$	0.022068	0.022 07 ±0.000 33	0.022242	0.022 17 ±0.000 33	0.022032	0.022 05 ±0.000 28	0.022069	0.022 07 ±0.000 27	0.022199	0.022 18 ±0.000 26	0.022161	0.022 14 ±0.000 24
Cold dark matter density	$\Omega_c h^2$	0.12029	0.1196 ±0.0031	0.11805	0.1186 ±0.0031	0.12038	0.1199 ±0.0027	0.12025	0.1198 ±0.0026	0.11847	0.1186 ±0.0022	0.11889	0.1187 ±0.0017
100x approximation to r_s / D_A (CosmoMC)	$100\,\theta_{MC}$	1.04122	1.041 32 ±0.000 68	1.04150	1.041 41 ±0.000 67	1.04119	1.041 31 ±0.000 63	1.04130	1.041 32 ±0.000 63	1.04146	1.041 44 ±0.000 61	1.04148	1.041 47 ±0.000 56
Thomson scattering optical depth due to reionization	τ	0.0925	0.097 ±0.038	0.0949	0.089 ±0.032	0.0925	$0.089^{+0.012}_{-0.014}$	0.0927	$0.091^{+0.013}_{-0.014}$	0.0943	$0.090^{+0.013}_{-0.014}$	0.0952	0.092 ±0.013
Power spectrum of curvature perturbations	$\ln(10^{10} A_s)$	3.098	3.103 ±0.072	3.098	3.085 ±0.057	3.0980	$3.089^{+0.024}_{-0.027}$	3.0959	3.090 ±0.025	3.0947	3.087 ±0.024	3.0973	3.091 ±0.025
Scalar spectral index	n_s	0.9624	0.9616 ±0.0094	0.9675	0.9635 ±0.0094	0.9619	0.9603 ±0.0073	0.9582	0.9585 ±0.0070	0.9624	0.9614 ±0.0063	0.9611	0.9608 ±0.0054
Hubble's constant (km Mpc^{-1} s^{-1})	H_0	67.11	67.4 ±1.4	68.14	67.9 ±1.5	67.04	67.3 ±1.2	67.15	67.3 ±1.2	67.94	67.9 ±1.0	67.77	67.80 ±0.77
Dark energy density	Ω_Λ	0.6825	0.686 ±0.020	0.6964	0.693 ±0.019	0.6817	$0.685^{+0.018}_{-0.016}$	0.6830	$0.685^{+0.017}_{-0.016}$	0.6939	0.693 ±0.013	0.6914	0.692 ±0.010
Density fluctuations at $8h^{-1}$ Mpc	σ_8	0.8344	0.834 ±0.027	0.8285	0.823 ±0.018	0.8347	0.829 ±0.012	0.8322	0.828 ±0.012	0.8271	0.8233 ±0.0097	0.8288	0.826 ±0.012
Redshift of reionization	z_{re}	11.35	$11.4^{+4.0}_{-2.8}$	11.45	$10.8^{+3.1}_{-2.5}$	11.37	11.1 ±1.1	11.38	11.1 ±1.1	11.42	11.1 ±1.1	11.52	11.3 ±1.1
Age of the Universe (Gy)	t_0	13.819	13.813 ±0.058	13.784	13.796 ±0.058	13.8242	13.817 ±0.048	13.8170	13.813 ±0.047	13.7914	13.794 ±0.044	13.7965	13.798 ±0.037
100x angular scale of sound horizon at last-scattering	$100\,\theta_*$	1.04139	1.041 48 ±0.000 66	1.04164	1.041 56 ±0.000 66	1.04136	1.041 47 ±0.000 62	1.04146	1.041 48 ±0.000 62	1.04161	1.041 59 ±0.000 60	1.04163	1.041 62 ±0.000 56
Comoving size of the sound	r_{drag}	147.34	147.53 ±0.64	147.74	147.70 ±0.63	147.36	147.49 ±0.59	147.35	147.47 ±0.59	147.68	147.67 ±0.50	147.611	147.68 ±0.45

Planck Collaboration Cosmological parameters[18]

	Description	Symbol	Value
Independent parameters	Physical baryon density parameter[a]	$\Omega_b h^2$	0.022 30 ±0.000 14
	Physical dark matter density parameter[a]	$\Omega_c h^2$	0.1188 ±0.0010
	Age of the universe	t_0	$13.799 \pm 0.021 \times 10^9$ years
	Scalar spectral index	n_s	0.9667 ±0.0040
	Curvature fluctuation amplitude, $k_0 = 0.002\ \mathrm{Mpc}^{-1}$	Δ^2_R	$2.441^{+0.088}_{-0.092} \times 10^{-9}$[21]
	Reionization optical depth	τ	0.066 ±0.012
Fixed parameters	Total density parameter[b]	Ω_{tot}	1
	Equation of state of dark energy	w	−1
	Tensor/scalar ratio	r	0
	Running of spectral index	$dn_s/d\ln k$	0
	Sum of three neutrino masses	$\sum m_\nu$	0.06 eV/c^2[c][17]:40
	Effective number of relativistic degrees of freedom	N_{eff}	3.046[d][17]:47
Calculated values	Hubble constant	H_0	$67.74 \pm 0.46\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$
	Baryon density parameter[b]	Ω_b	0.0486 ±0.0010[e]
	Dark matter density parameter[b]	Ω_c	0.2589 ±0.0057[f]
	Matter density parameter[b]	Ω_m	0.3089 ±0.0062
	Dark energy density parameter[b]	Ω_Λ	0.6911 ±0.0062
	Critical density	ρ_{crit}	$(8.62 \pm 0.12) \times 10^{-27}\ \mathrm{kg/m^3}$[g]
	The present root-mean-square matter fluctuation averaged over a sphere of radius $8h^{-1}$ Mpc	σ_8	0.8159 ±0.0086
	Redshift at decoupling	z_*	1 089.90 ±0.23
	Age at decoupling	t_*	377 700 ±3200 years[21]

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model. Our results indicate that cosmological model differences between *Planck* and *WMAP* do not arise from measurement differences, but from the high multipoles not measured by *WMAP*.

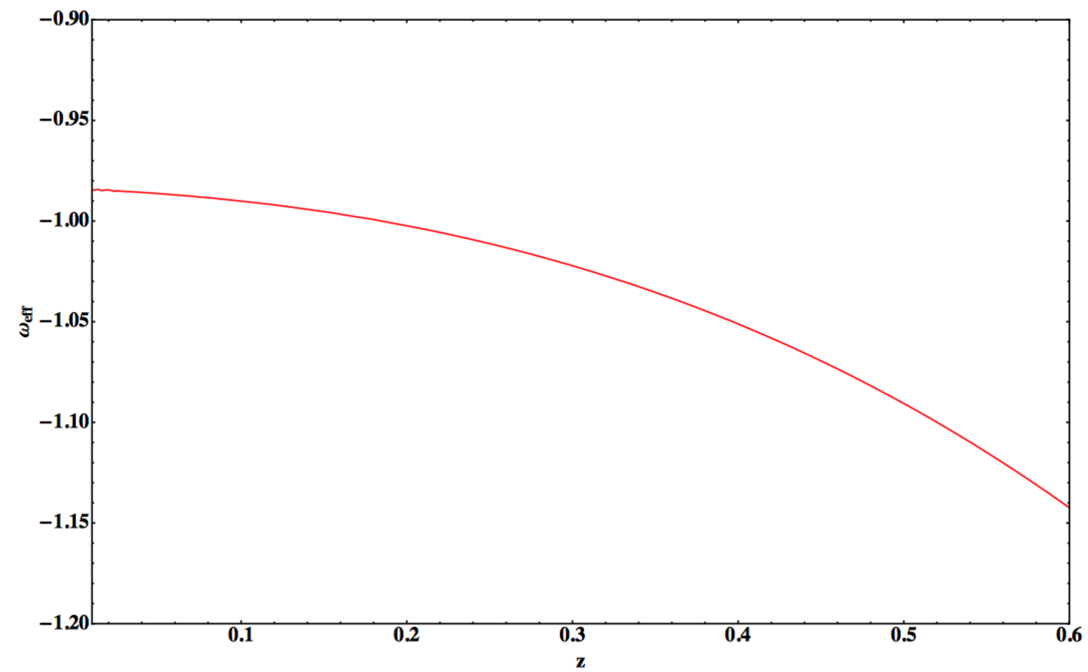
VIABLE f(G) GRAVITIES

$$\tilde{f}_1(\mathcal{G}_\mathcal{A}) = \left(\tilde{\mathcal{G}}_\mathcal{A}\right)^{m_1}\left[a_1 + b_1\left(\tilde{\mathcal{G}}_\mathcal{A}\right)^{n_1}\right],$$

$$\tilde{f}_2(\mathcal{G}_\mathcal{A}) = \frac{a_2 + b_2\left(\tilde{\mathcal{G}}_\mathcal{A}\right)^{m_2}}{c_2 + d_2\left(\tilde{\mathcal{G}}_\mathcal{A}\right)^{n_2}},$$

$$\{a_1, b_1, m_1, n_1\} = \{-3.6, 2.7\times 10^{-2}, 5.9\times 10^{-2}, 0.65\}$$

1. Model 1 : ΛCDM model



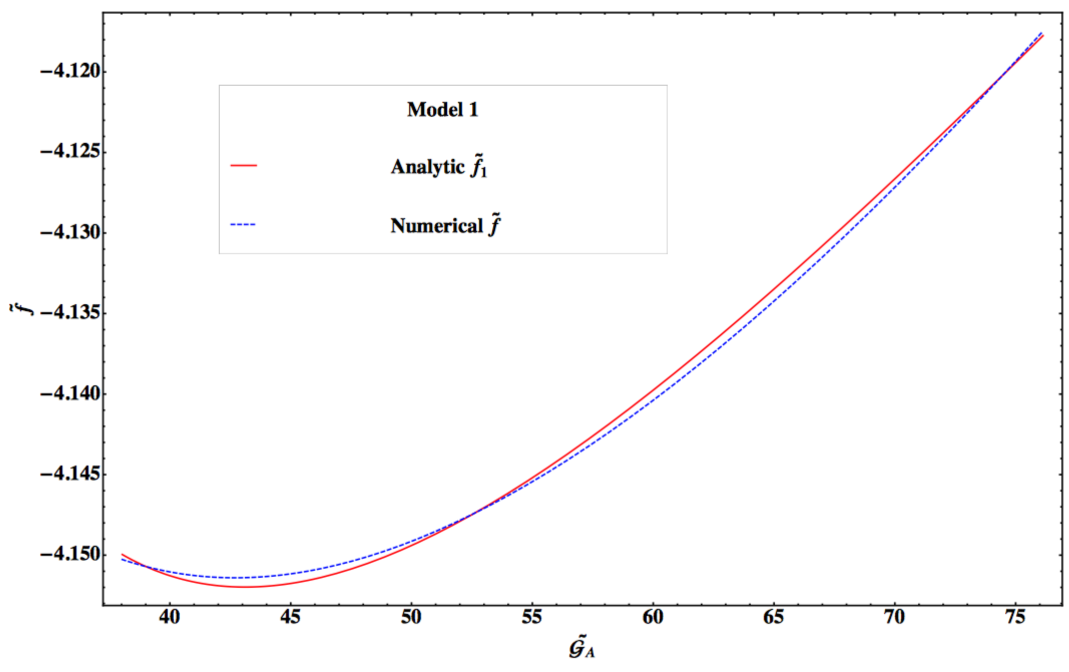


FIG. 1: For *Model 1*: a) The red-shift evolution of $\omega_{\rm eff}$. b) Evolutions of the numerical and the analytic solutions as a function $\tilde{\mathcal{G}}_\mathcal{A}$. The solid line indicates the analytic solution and the dashed one denotes the numerical one, respectively.

VIABLE f(G) GRAVITIES

$$\tilde{f}_1(\mathcal{G}_\mathcal{A}) = \left(\tilde{\mathcal{G}}_\mathcal{A}\right)^{m_1}\left[a_1 + b_1\left(\tilde{\mathcal{G}}_\mathcal{A}\right)^{n_1}\right],$$

$$\tilde{f}_2(\mathcal{G}_\mathcal{A}) = \frac{a_2 + b_2\left(\tilde{\mathcal{G}}_\mathcal{A}\right)^{m_2}}{c_2 + d_2\left(\tilde{\mathcal{G}}_\mathcal{A}\right)^{n_2}},$$

$$\{a_1, b_1, m_1, n_1\} = \{-4.0, 5.2\times10^{-3}, 3.3\times10^{-2}, 0.84\}$$

2. Model 2 : ωCDM model with $\omega_0 \neq -1$ and $\omega_a = 0$

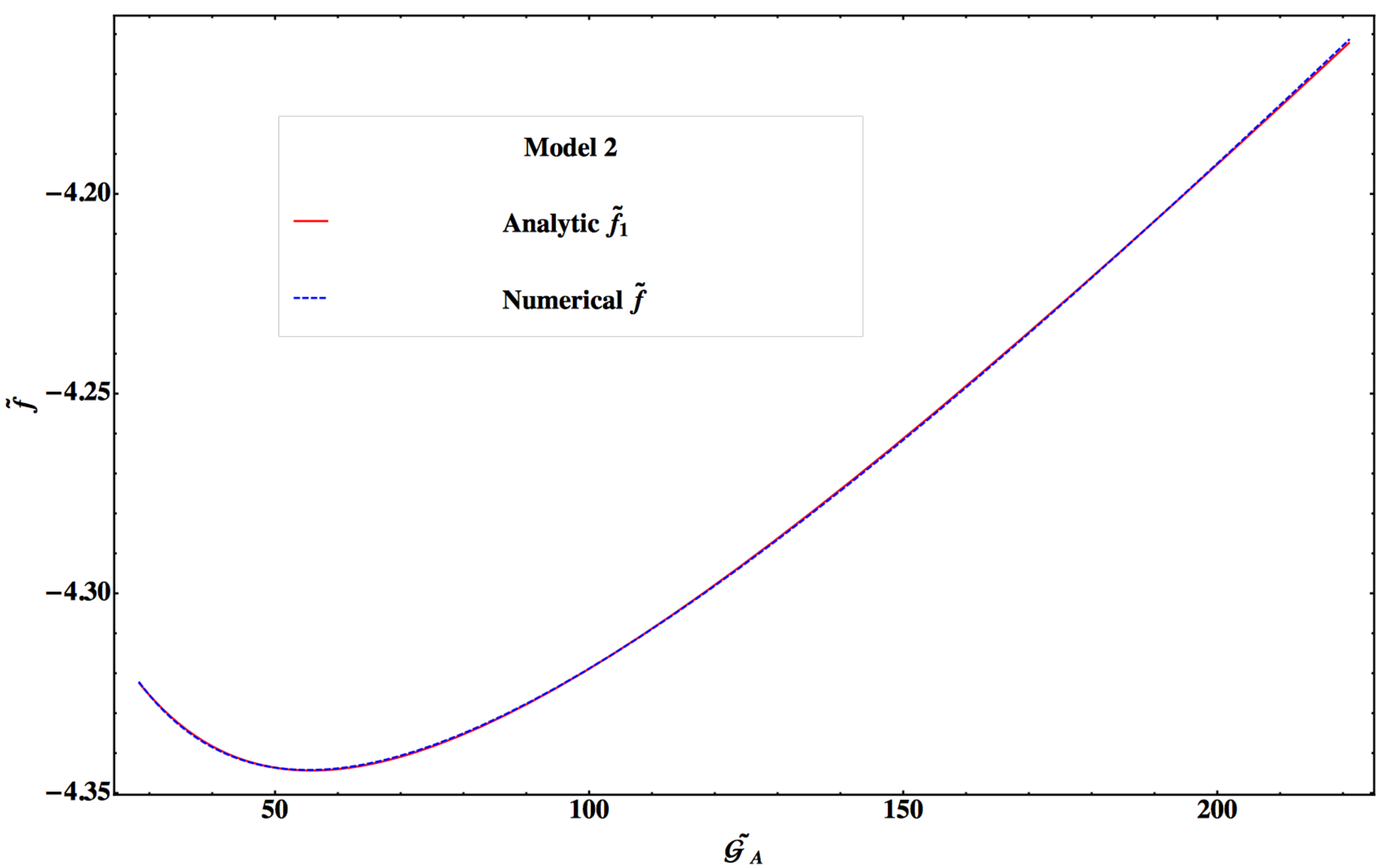


FIG. 3: The comparison of analytic forms $\tilde{f}_1$ with the numerical solution $\tilde{f}$ for *Model 2*.

VIABLE f(G) GRAVITIES

$$\tilde{f}_1(\mathcal{G}_\mathcal{A}) = \left(\tilde{\mathcal{G}}_\mathcal{A}\right)^{m_1}\left[a_1 + b_1\left(\tilde{\mathcal{G}}_\mathcal{A}\right)^{n_1}\right],$$

$$\tilde{f}_2(\mathcal{G}_\mathcal{A}) = \frac{a_2 + b_2\left(\tilde{\mathcal{G}}_\mathcal{A}\right)^{m_2}}{c_2 + d_2\left(\tilde{\mathcal{G}}_\mathcal{A}\right)^{n_2}},$$

3. Model 3: ΛCDM with $\Omega_{m0} = 0.32$

$$\{a_1, b_1, m_1, n_1\} = \{-4.0, 5.2\times10^{-3}, 3.3\times10^{-2}, 0.84\}$$

$$\{a_2, b_2, c_2, d_2, m_2, n_2\} = \{-314, -8.8, 82.9, 0.23, 0.41, 0.81\}.$$

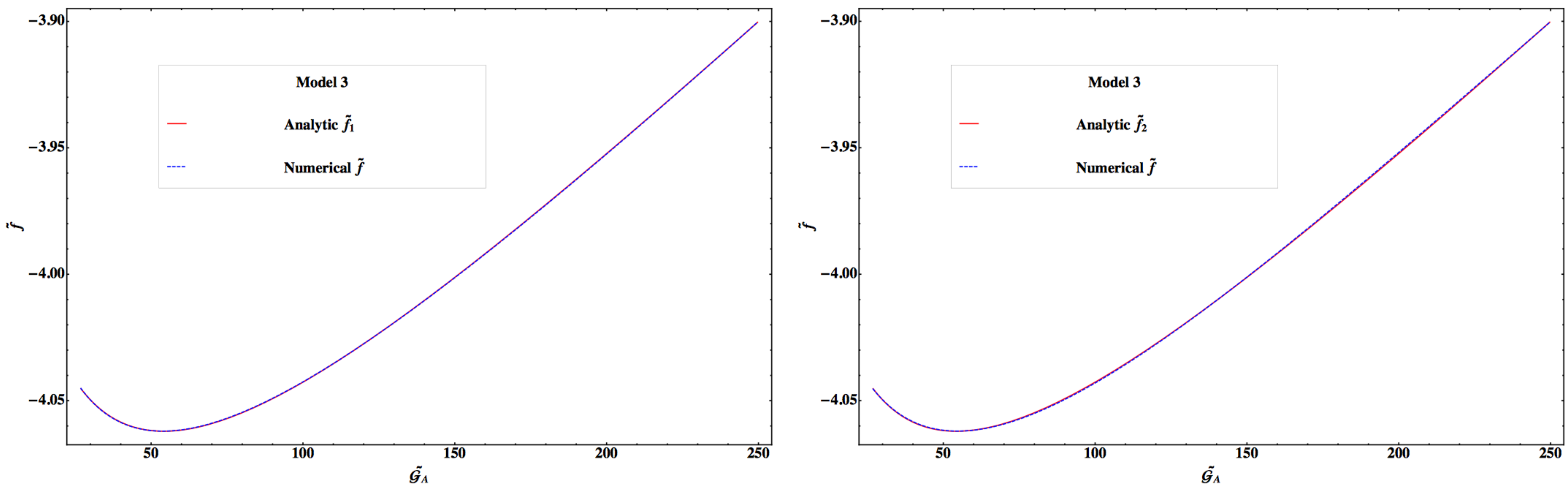


FIG. 5: The comparison of analytic forms $\tilde{f}_1$ and $\tilde{f}_2$ with the numerical solutions of $\tilde{f}$. a) The solid and dashed lines correspond $\tilde{f}_1$ and $\tilde{f}$, respectively. b) The solid and dashed lines correspond $\tilde{f}_2$ and $\tilde{f}$, respectively.

CONCLUSIONS

- Instead of assume any specific form of f(G) model, one can reconstruct viable f(G) models from observations.
- Constraint on G of viable f(G) in order to remove singularity can be generalized.
- They're first analytic forms of f(G) without any assumption.
- One needs one more eq for initial conditions (might be obtained from GW mass constraints). Stability conditions provide a constraint on ICs.
- Are there any physical origins of obtained analytic fitting forms.
- Extension to Horndeski's models are in progress.