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Rotating black holes with an anisotropic matter field

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based on
H.-C. Kim, B.-H. Lee, WL, Y. Lee, arXiv:1912.09709

The plan of this talk

1. Motivations

2. Static black hole with anisotropic fluid

3. Rotating black hole with anisotropic fluid

4. Thermodynamics of the black hole

5. Summary and discussions

1. Motivations

The observation of astronomical objects shows that most of them are rotating.

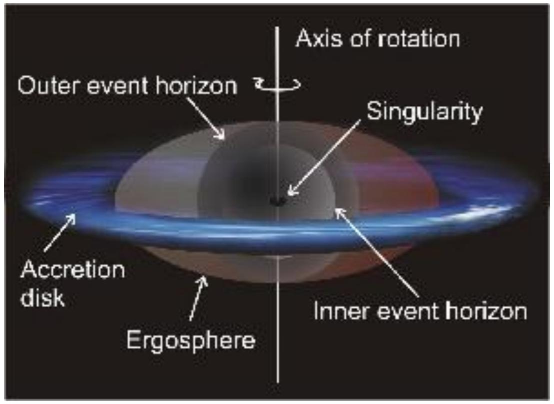
It is believed that the gravitational collapse of a super-massive star forms a rotating black hole eventually.

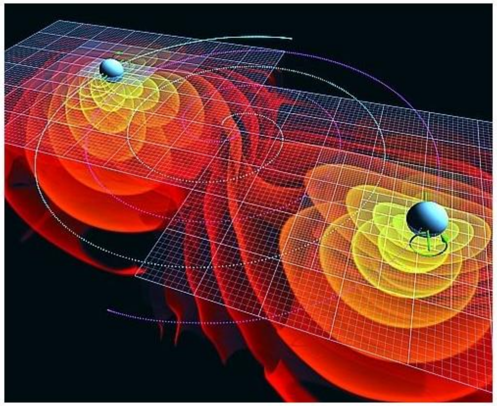
The energy extraction mechanism from black holes is a promising candidate describing astrophysical events such as active galactic nuclei, gamma-ray bursts and ultra-high-energy cosmic rays.

⟹ Thus, the spacetime geometry describing those rotational black holes has drawn our interest over time.

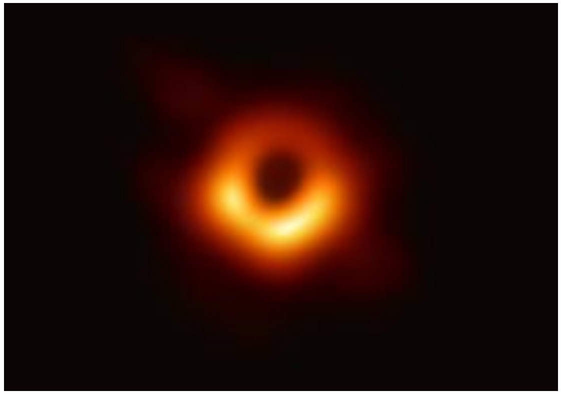
In addition to this aspect, observationally, black hole has recently gained most attention among astrophysical objects, thanks to the observational reports on the shadow of the black hole by the Event Horizon Telescope.

The recent detections of the gravitational waves coming from binary black hole collisions have also open a new horizon on the studies of astrophysical phenomena and the gravitational theory itself.





Two inspiraling black holes on the path to merger. Credit: NASA



Furthermore, actual astrophysical black holes reside in the background of matters or fields. Therefore, we need to find a way of describing a realistic black hole that coexists with matter field!

2. Static black hole with anisotropic matter field

Reissner (1916) and G. Nordström(1918) black hole

$$I = \int_{\mathcal{M}} \sqrt{-g} d^4x \left[\frac{1}{16\pi}(R - F_{\mu\nu}F^{\mu\nu}) + \mathcal{L}_m\right] + I_b \qquad R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu}$$

geometry

$$\nabla_\mu F^{\mu\nu} = \frac{1}{\sqrt{-g}}[\partial_\mu(\sqrt{-g}F^{\mu\nu})] = 0$$

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega_2^2$$

Energy-momentum tensor

$$A_\mu = \left(-\frac{Q}{r}, 0, 0, 0\right) \quad F_{tr} = \partial_t A_r(r) - \partial_r A_t = \Phi'(r) = -E_r, \qquad (E_r = \frac{Q}{r^2})$$

$$T^\nu_\mu = g^{\nu\beta}T_{\mu\beta} = \frac{1}{4\pi}\left[F_{\mu\alpha}F^{\nu\alpha} - \frac{1}{4}\delta^\nu_\mu F_{\alpha\beta}F^{\alpha\beta}\right]$$

$$= \mathrm{diag}(-1, -1, +1, +1)\frac{Q^2}{8\pi r^4} = \mathrm{diag}(-\varepsilon\,, p_r, p_\theta, p_\phi)$$

$$p_r(r) = -\varepsilon(r)$$

$$p_\theta(r) = p_\phi(r) = \varepsilon(r)$$

Static black holes with an anisotropic matter field

Geometry

The black hole with anisotropic fluid obtained by Cho & Kim (Chin. Phys. C43 (2019) no.2, 025101).

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\psi^2)$$

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{K}{r^{2w}}$$

Energy-momentum tensor

$$T^\mu_\nu = \begin{pmatrix} -\varepsilon(r) & 0 & 0 & 0 \\ 0 & p_r(r) & 0 & 0 \\ 0 & 0 & p_\theta(r) & 0 \\ 0 & 0 & 0 & p_\phi(r) \end{pmatrix}$$

$$\varepsilon(r) = \frac{Q^2}{8\pi r^4} + \frac{r_o^{2w}}{8\pi r^{2w+2}}$$

$$r_o^{2w} = (1-2w)K$$

$$Q^2 + r_o^{2w} r^{2(1-w)} \geq 0$$

where $p_r(r) = -\varepsilon(r)$ **and** $p_\theta(r) = p_\psi(r) = w\varepsilon(r)$

3. Rotating black hole with an anisotropic matter field

How can we obtain the solution describing the rotating black hole ?

1) Solve the Ernst equation [Ernst, PR 167, 1175 (1968)]

2) Employ the Newman-Janis algorithm

[Newman & Janis, JMP 6, 915 (1965)]

Einstein equations

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

i) vacuum? or some matter?

ii) interior solution? or proper collapsing matter?

3-1 Newman-Janis algorithm I

We take three steps!

null coordinate of the static BH

↓

Newman-Janis algorithm

↓

null coordinate of the rotating BH

Schwarzschild (1916)

Kerr (1963)

Newman and Janis, JMP 6, 915 (1965)

Reissner – Nordström (1916, 1918)

Kerr-Newman (1965)

Newman, Chinnapared, Exton, Prakash and Torrence, JMP 6, 918 (1965)

3-1 Newman-Janis algorithm II

We consider the black hole with anisotropic fluid

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\psi^2)$$

where

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{K}{r^{2w}}$$

We take the advanced Eddington-Finkelstein coordinate

$$dv = dt + \frac{dr}{f(r)}$$

then

$$ds^2 = -f(r)dv^2 + 2dvdr + r^2(d\theta^2 + \sin^2\theta d\psi^2)$$

In the NJ algorithm, we formally perform the complex coordinate transformations

- $v \to v' = v + ia\cos\theta, \quad r \to r' = r + ia\cos\theta\,,$
- $\dfrac{2M}{r} \to \dfrac{M}{r} + \dfrac{M}{\bar{r}} = \dfrac{2Mr'}{r'^2 + a^2\cos^2\theta'}\,,$
- $r^2 \to r\bar{r} = r'^2 + a^2\cos^2\theta'\,.$

Then, we get

$$f(r) \Rightarrow F(r', \theta'); \quad 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{K}{r^{2w}} \Rightarrow$$

$$1 - \frac{2Mr' - Q^2}{r'^2 + a^2\cos^2\theta'} - \frac{Kr'^{2(1-w)}}{(r'^2 + a^2\cos^2\theta')}\,.$$

3-3 Three classical representations

(1) We obtain Eddington-Finkelstein form

$$ds^2 = -F(r,\theta)dv^2 + 2dvdr - 2a\sin^2\theta drd\psi - 2(1-F(r,\theta))a\sin^2\theta dvd\psi + \rho^2 d\theta^2 + \frac{\Sigma}{\rho^2}\sin^2\theta d\psi^2$$

where $\rho^2 = r^2 + a^2\cos^2\theta, \quad \Delta = \rho^2 F(r,\theta) + a^2\sin^2\theta$

$$\Sigma = (r^2+a^2)^2 - a^2 \Delta \sin^2\theta$$

To obtain Boyer-Lindquist coordinates, one takes the coordinate transformation

$$dv = dt + \frac{r^2+a^2}{\Delta}dr, \quad d\psi = d\phi + \frac{a}{\Delta}dr$$

We note that the function △ does not depend on angle coordinates to ensure the integrability.

(2) The Boyer-Lindquist form is given by

$$\begin{aligned} ds^2 &= -F(r,\theta)dt^2 - 2[1-F(r,\theta)]a\sin^2\theta dt d\phi + \frac{\Sigma}{\rho^2}\sin^2\theta d\phi^2 \\ &+ \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2\,, \\ &= -\frac{\rho^2\,\Delta}{\Sigma}dt^2 + \frac{\Sigma}{\rho^2}\sin^2\theta(d\phi - \Omega dt)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2\,. \end{aligned}$$

where $\Omega \equiv -\frac{g_{t\phi}}{g_{\phi\phi}} = \frac{[1-F(r,\theta)]\rho^2 a}{\Sigma}$

(3) We present Kerr-Schild form

$$ds^2 \;=\; -d\bar{t}^2 + dx^2 + dy^2 + dz^2 + \mathcal{F}(dg)^2$$

where

$$\mathcal{F} = \frac{2Mr^3 - Qr^2 + Kr^{2(2-w)}}{r^4 + a^2 z^2}$$

and

$$dg = d\bar{t} + \frac{z}{r}dz + \frac{r(xdx + ydy) + a(ydx - xdy)}{r^2 + a^2}$$

We used the coordinate transformations

$$\begin{aligned} x &= (r\cos\psi - a\sin\psi)\sin\theta, \quad y = (r\sin\psi + a\cos\psi)\sin\theta \\ z &= r\cos\theta, \quad \bar{t} = v - r\,. \end{aligned}$$

The surfaces, r = const., are confocal ellipsoids of rotation about the z-axis and given by

$$\frac{x^2 + y^2}{r^2 + a^2} + \frac{z^2}{r^2} = 1$$

3-4 Einstein and energy-momentum tensors

Using $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ **, the non-vanishing components of the Einstein tensor are given by**

$$
\begin{aligned}
G_{tt} &= \frac{2[r^4 - 2r^3 b + a^2 r^2 - a^4 \sin^2\theta \cos^2\theta] b'}{\rho^6} - \frac{r a^2 \sin^2\theta b''}{\rho^4}, \\
G_{rr} &= -\frac{2r^2 b'}{\Delta\ \rho^2}, \quad G_{\theta\theta} = -\frac{2a^2 \cos^2\theta b'}{\rho^2} - r b'', \\
G_{t\phi} &= \frac{2a \sin^2\theta [(r^2 + a^2)(a^2 \cos^2\theta - r^2) + 2r^3 b] b'}{\rho^6} + \frac{r a \sin^2\theta (r^2 + a^2) b''}{\rho^4} \\
G_{\phi\phi} &= -\frac{a^2 \sin^2\theta [(r^2 + a^2)(a^2 + (2r^2 + a^2)\cos 2\theta) + 4r^3 \sin^2\theta b] b'}{\rho^6} \\
&- \frac{r \sin^2\theta (r^2 + a^2)^2 b''}{\rho^4},
\end{aligned}
$$

where a prime denotes differentiation with respect to r.

$$
\begin{aligned}
2b &= 2M - Q^2 r^{-1} + K r^{1-2w}, \quad 2b' = Q^2 r^{-2} + (1-2w) K r^{-2w} \\
2b'' &= -2Q^2 r^{-3} - 2(1-2w) w K r^{-2w-1}.
\end{aligned}
$$

Let us consider physical quantities in an orthonormal frame, $(e_{\hat{t}}, e_{\hat{r}}, e_{\hat{\theta}}, e_{\hat{\phi}})$ in which the stress-energy tensor for the anisotropic matter field is diagonal,

$$
\begin{aligned}
e^{\mu}_{\hat{t}} &= \frac{(r^2+a^2,0,0,a)}{\rho\sqrt{\Delta}}, \quad e^{\mu}_{\hat{r}} = \frac{\sqrt{\Delta}(0,1,0,0)}{\rho} \\
e^{\mu}_{\hat{\theta}} &= \frac{(0,0,1,0)}{\rho}, \quad e^{\mu}_{\hat{\phi}} = \frac{(a\sin^2\theta,0,0,1)}{\rho\sin\theta}
\end{aligned}
$$

The components of the energy-momentum tensor are expressed in terms of $G_{\mu\nu}$ as

$$8\pi\varepsilon = e^{\mu}_{\hat{t}} e^{\nu}_{\hat{t}} G_{\mu\nu}, \quad 8\pi p_{\hat{r}} = e^{\mu}_{\hat{r}} e^{\nu}_{\hat{r}} G_{\mu\nu}, \, 8\pi p_{\hat{\theta}} = e^{\mu}_{\hat{\theta}} e^{\nu}_{\hat{\theta}} G_{\mu\nu}, \, 8\pi p_{\hat{\phi}} = e^{\mu}_{\hat{\phi}} e^{\nu}_{\hat{\phi}} G_{\mu\nu}.$$

We obtain

$$\varepsilon = \frac{Q^2 + r_o^{2w} r^{2(1-w)}}{8\pi\rho^4}, \quad p_{\hat{r}} = (-\varepsilon) = -\frac{Q^2 + r_o^{2w} r^{2(1-w)}}{8\pi\rho^4},$$

$$p_{\hat{\theta}} = (p_{\hat{\phi}}) = [(r^2 + a^2\cos^2\theta)w - a^2\cos^2\theta]\frac{\varepsilon}{r^2} + (1-w)\frac{Q^2}{8\pi\rho^2 r^2}$$

When the rotation is vanishing, the components of the energy-momentum tensor correspond to those for the static black hole with $p_r(r) = -\varepsilon(r)$ and $p_\theta(r) = p_\phi(r) = w\varepsilon(r)$.

The Kretschmann invariant is given by

$$R^{\alpha\beta\mu\nu}R_{\alpha\beta\mu\nu} \;=\; R_{KN} + AK + BK^2$$

where

$$\begin{aligned} R_{KN} &= \frac{8}{\rho^{12}}\left[6M^2(r^2-a^2\cos^2\theta)(\rho^4-16^2a^2\cos^2\theta)-12MQ^2r(r^4-10a^2r^2\cos^2\theta+5a^4\cos^4\theta)\right. \\ &+ \left. Q^4(7r^4-34a^2r^2\cos^2\theta+7a^4\cos^4\theta)\right] , \qquad (23) \end{aligned}$$

$$\begin{aligned} A &= \frac{-8r^{-2w}}{\rho^{12}}\left\{r^6(1+w)[-2Mr(1+2w)+Q^2(1+6w)]+a^2r^4[2Mr(31+27w+2w^2)\right. \\ &+ Q^2(-47-31w+10w^2)]\cos^2\theta+a^4r^2[Q^2(47-35w+2w^2)+10Mr(-11+3w+2w^2)]\cos^4\theta \\ &+ \left. a^6(w-1)[Q^2(1-2w)+6Mr(2w-3)]\cos^6\theta\right\} , \qquad (24) \end{aligned}$$

$$\begin{aligned} B &= \frac{4r^{-4w}}{\rho^{12}}\left[r^8(1+5w^2+4w^3+4w^4)+2a^2r^6(8w^4-10w^2-23w-9)\cos^2\theta\right. \\ &+ 2a^4r^4(12w^4-12w^3-21w^2-w+29)\cos^4\theta+2a^6r^2(8w^4-16w^3-2w^2+19w-9)\cos^6\theta \\ &+ \left. a^8(2w^2-3w+1)^2\cos^8\theta\right] , \qquad (25) \end{aligned}$$

The R_{KN} part diverges at $\rho = \sqrt{r^2+a^2\cos^2\theta} = 0.$ **Therefore, a ring singularity appears at r = 0 and θ = π/2. In addition, for positive w≠1/2, 1, we find that an additional sphere-like singularity exists at r = 0.**

3-5 Ergosphere and event horizons

The static limit surfaces (boundaries of the ergosphere), corresponding to the timelike Killling vectoc ξ_t^μ being null, are obtained from $g_{tt} = 0.$

The ergosphere is a region located between the static limit surface and the event horizon.

R. Ruffini & J. A. Wheeler, PRINT-70-2077

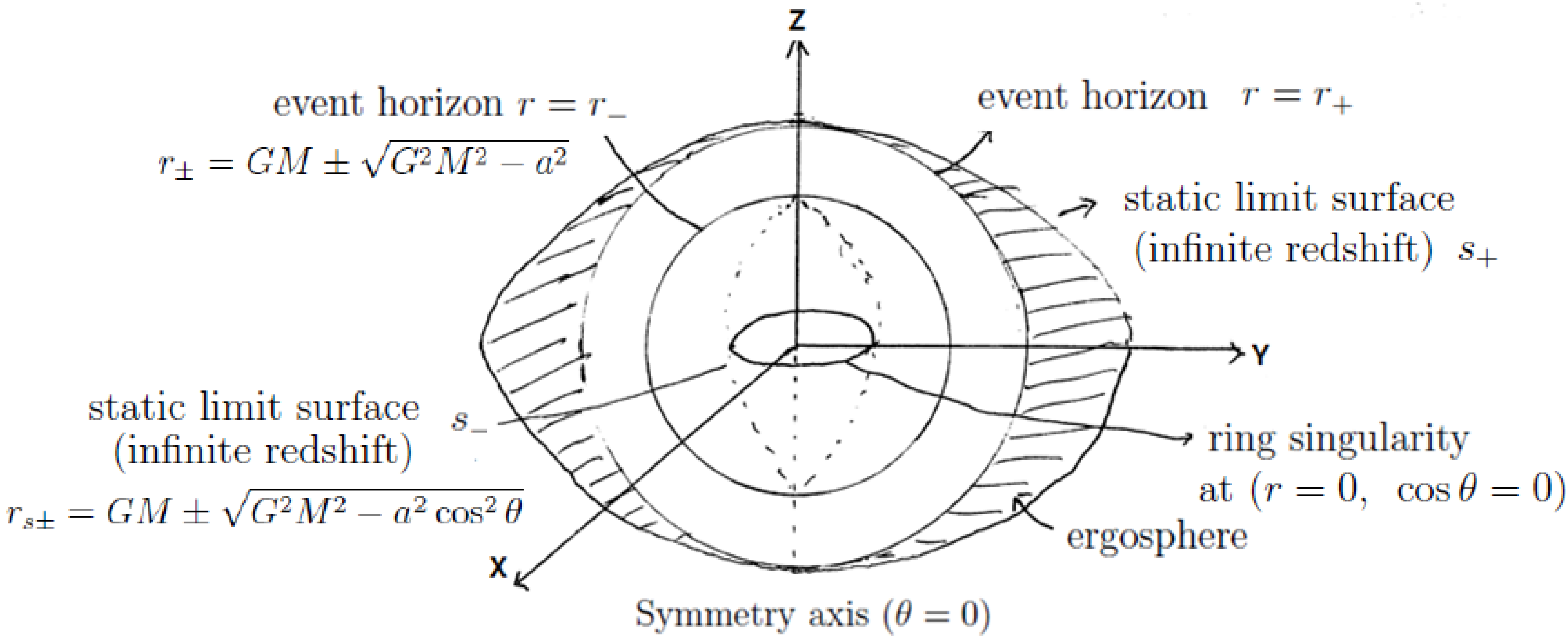
The event horizon corresponds to the Killing horizon and reduces to $\triangle$= 0. The locations of the event horizon do not depend on the angle coordinates.

	Ergosphere	Event horizon
$w \neq \frac{1}{2}$	$\rho^2 - 2Mr + Q^2 - Kr^{2(1-w)} = 0$	$(r_H^2 + a^2) - 2Mr_H + Q^2 - Kr_H^{2(1-w)} = 0$
$w = \frac{1}{2}$	$\rho^2 + Q^2 - 2\bar{M}r\left(1 + \frac{r_o}{2M}\log\frac{r}{r_o}\right) = 0$	$(r_H^2 + a^2) + Q^2 - 2\bar{M}r_H\left(1 + \frac{r_o}{2M}\log\frac{r_H}{r_o}\right) = 0$

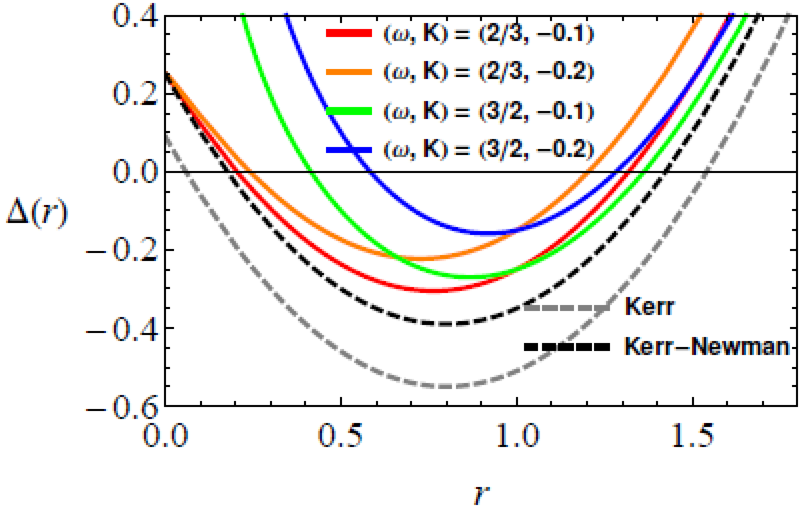
Table 1: Ergosphere and Event horizon

Where $\bar{M} = M + K$ **.**

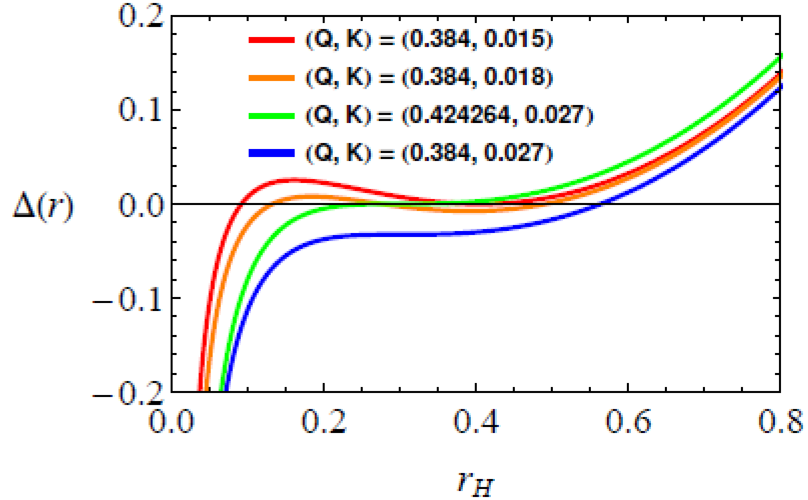
For a Kerr black hole



The domain between the static limit and the horizon r_+ **is traditionally called the ergosphere (ergoregion).**



(a) The shape of the function $\triangle(r)$ with $K < 0$. We take $M = 0.8$, $a = 0.3$ and $Q = 0.4$. The black dashed curve corresponds to a Kerr-Newman black hole, the gray dashed curve to a Kerr one. The red curve, orange, green, and blue curves correspond to a black hole with $(w, K) = (2/3, -0.1)$, $(2/3, -0.2)$, $(3/2, -0.1)$ and $(3/2, -0.2)$, respectively.



(b) The shape of the function $\triangle(r)$ with $K > 0$ and $w = 3/2$. We take $M = 0.45$ and $a = 0.3$. The red, orange, green, and blue curves correspond to a black hole with $(Q, K) = (0.384, 0.015)$, $(0.384, 0.018)$, $(0.424264, 0.027)$ and $(0.384, 0.027)$, respectively.

Figure 1: (color online.) The shape of the function $\triangle(r)$ in terms of r.

4. Thermodynamics of the black hole

4-1 Temperature

The surface gravity is given by

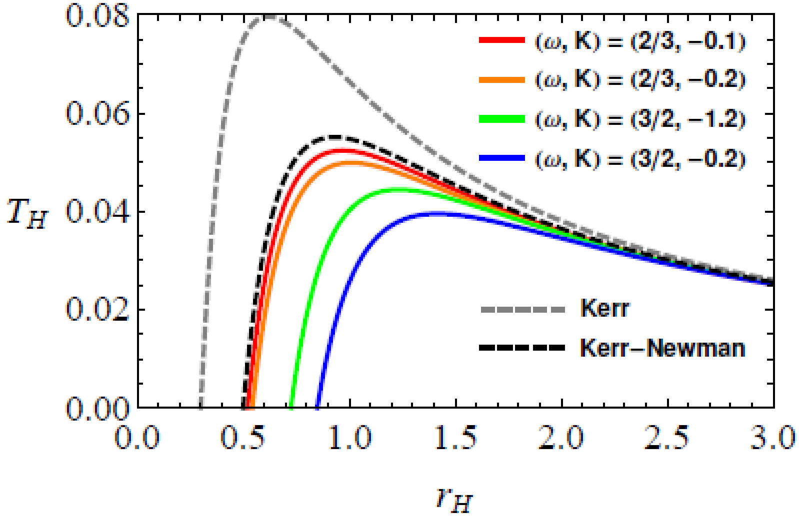
$$\kappa = \frac{r_h - M - (1-w)Kr_h^{1-2w}}{r_h^2 + a^2}$$

The temperature is

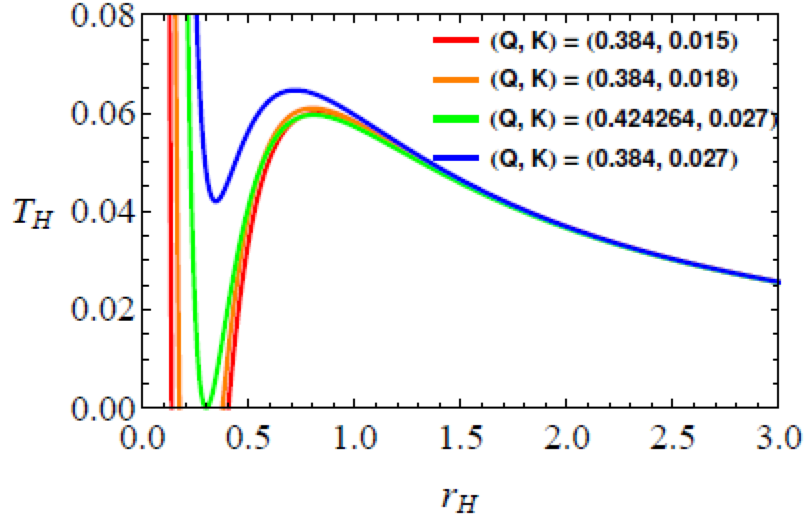
$$T_H = \frac{\kappa}{2\pi} = \frac{r_h - M - (1-w)Kr_h^{1-2w}}{2\pi(r_h^2 + a^2)} = \frac{r_H^2 - (a^2 + Q^2) - (1-2w)Kr_H^{2(1-w)}}{4\pi r_H(r_H^2 + a^2)}$$

The entropy is

$$S = \frac{A}{4} = \pi(r_h^2 + a^2)$$



(a) Temperature for $K < 0$ and with $a = 0.3$ and $Q = 0.4$, in which the parameter values are the same as those in Fig. 1(a)



(b) Temperature for $K > 0$ and $w = 3/2$ and with $a = 0.3$, in which the parameter values are the same as those in Fig. 1(b).

Figure 2: (color online). Temperature as a function of r_H.

4-2 Smarr relation and the first law

Smarr relation : From the horizon information

$$\triangle = r_H^2 + a^2 + Q^2 - 2Mr_H - Kr_H^{2(1-w)} = 0$$

The mass formula for our rotating black hole can be represented as follows:

$$M = \frac{(K_N + \Phi_o r_o + \Omega_H J) + \sqrt{(K_N + \Phi_o r_o - \Omega_H J)^2 - \frac{4(1-w)}{w}\Phi_o r_o \Omega_H J}}{2}$$

where $K_N = 2T_H S + 2\Omega_H J + \Phi_H Q,\ \Phi_H = \frac{r_H Q}{r_H^2 + a^2},\ \Phi_o = \frac{w}{2w-1}\left(\frac{r_H^2}{r_H^2+a^2}\right)\left(\frac{r_o}{r_H}\right)^{2w-1}$

Let us present the first law of the black hole mechanics. This law represents a differential relationship between the mass, the entropy, the charge and the angular momentum of the black hole.

$$\delta M = \frac{M(M-\Omega_H J)}{M(M-\Omega_H J) + \frac{w-1}{w}\Phi_o r_o \Omega_H J}\left[\delta M_{KN} + \Phi_o \delta r_o + \frac{(w-1)}{w}\frac{\Phi_o r_o \Omega_H}{M - \Omega_H J}\delta J\right]$$

where $\delta M_{KN} = T_H \delta S + \Omega_H \delta J + \Phi_H Q$

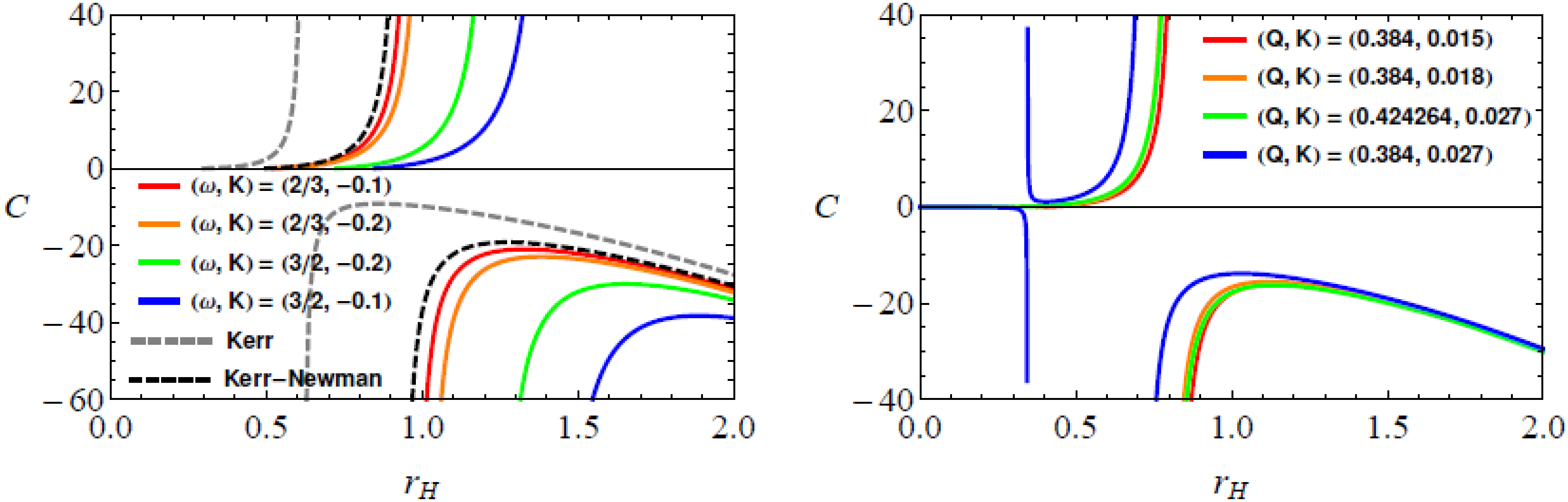
4-3 Specific heat

The specific heat (heat capacity), $C = T_H \frac{\partial S}{\partial T_H}$, calculated at constant angular momentum and charge in the canonical ensemble, determining local thermodynamic stability can be obtained as follows:

$$C = \frac{2\pi r_H^2 (r_H^2 + a^2)[r_H^2 - (a^2 + Q^2) + (2w-1)K r_H^{2(1-w)}]}{a^4 + a^2(Q^2 + 4r_H^2 - (2w-1)^2 K r_H^{2(1-w)}) - r_H^4 + 3Q^2 r_H^2 - (4w^2-1)K r_H^{2(2-w)}}$$

If K vanishes, this reduces to that for Kerr-Newman black hole.

$$C = \frac{2\pi r_H^2 (r_H^2 + a^2)[r_H^2 - (a^2 + Q^2)]}{a^4 + a^2(Q^2 + 4r_H^2) - r_H^4 + 3Q^2 r_H^2}$$



(a) Heat capacity for $K < 0$ and with $a = 0.3$ and $Q = 0.4$, in which the parameter values are the same as those in Fig. 1(a)

(b) Heat capacity for $K > 0$ and $w = 3/2$ and with $a = 0.3$, in which the parameter values are the same as those in Fig. 1(b).

Figure 3: (color online). Heat capacity as a function of the horizon radius r_H.

4-3 Energy extraction

Penrose process

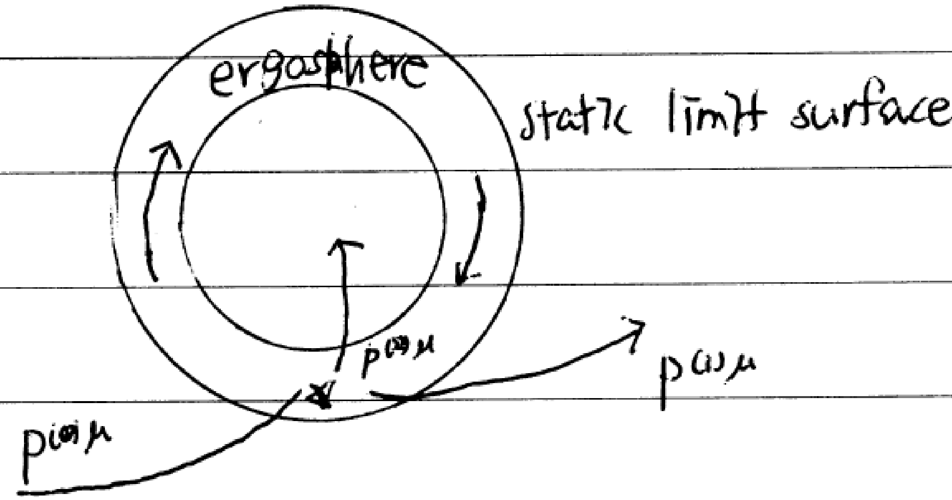
In 1969, Roger Penrose, published a paper on the role of general relativity in gravitational collapse. In it, he presented a mechanism by which the rotational energy of a black hole might be extracted.

R. Penrose, Rivista del Nuovo Cimento (1969)

$$E^{(0)} = E^{(1)} + E^{(2)}$$

$$E^{(2)} < 0$$

$$E^{(1)} > E^{(0)}$$



The Penrose process (top view)

(i) A particle 0 enters the ergosphere and decays there into two particles, 1 and 2.

(ii) One of them with a negative energy (2) falls into the black hole.

(iii) The other one (1) escapes the ergosphere with an energy exceeding an energy of the original particle.

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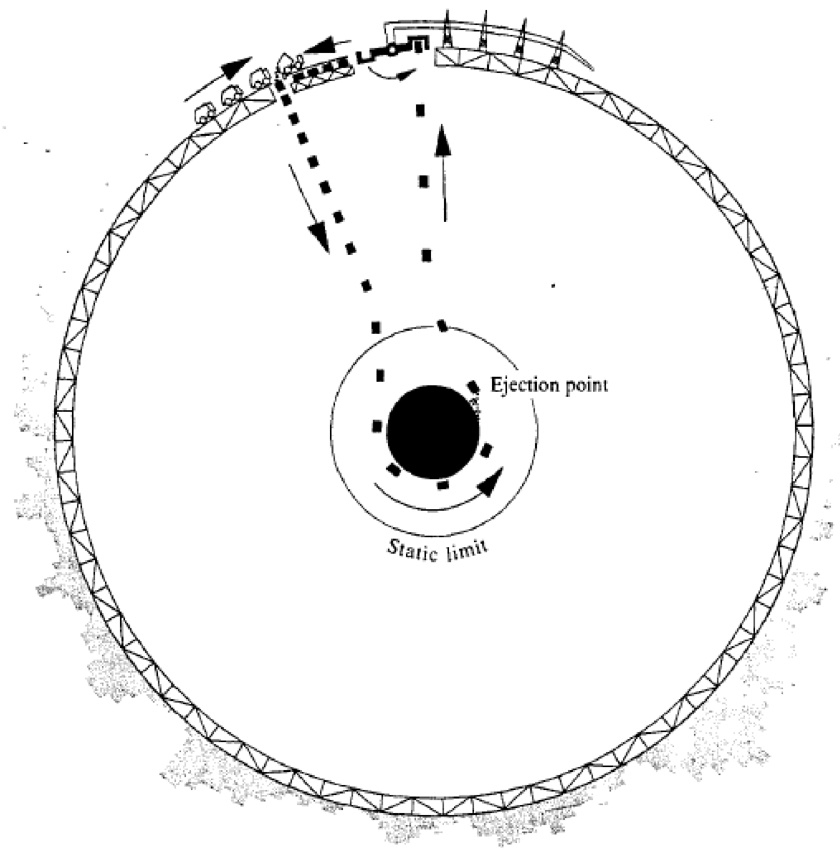

Gravitation, Charles W. Misner, Kip S. Thorne, John Archibald Wheeler, W. H. Freeman and Company (1973).

Irreducible mass

D. Christodoulou, PRL 25, 1596 (1970)
D. Christodoulou & R. Ruffini, PRD 4, 3552 (1971)

Let us examine the efficiency of an engine which extracts energy from the rotating black holes.

The irreducible mass represents the minimum mass which cannot be lowered through classical reversible processes.

$$M^2 = \left[M_I + \frac{Q^2}{4M_I} - \frac{K r_H^{2(1-w)}}{4M_I} \right]^2 + \frac{J^2}{4M_I^2} \,,$$

where $M_I \equiv \frac{\sqrt{r_H^2 + a^2}}{2}$

As a special case of the second law of black hole thermodynamics, this is related to the area of the horizon by

$$\boxed{S = \frac{A}{4} = \pi(r_H^2 + a^2) = 4\pi M_I}$$

The ratio of the extracted mass energy relative to the black hole mass is given by

$$\frac{\Delta M}{M} \equiv 1 - \frac{M_I}{M}$$

Through this procedure, 29% of the mass energy for a Kerr black hole and 50% for a Kerr-Newman black hole can be extracted by reversible transformations.

It is natural to ask whether or not the mass energy for a rotating black hole with an anisotropic matter field can be more extracted than the case of a Kerr-Newman black hole.

We examine the black hole with w = 3/2 and K $>$ 0.

$$r_H^2 - 2Mr_H + (a^2 + Q^2) - \frac{K}{r_H} = 0$$

We are interested in the positive $\varepsilon = \frac{Q^2 + r_o^{2w} r^{2(1-w)}}{8\pi\rho^4}$**.**

If $Q^2 \dot{\geqq} 2a^2$**, then ε is non-negative.**

For $4M^2 = 3(a^2 + Q^2)$ **and** $K = \frac{8M^3}{27}$ **,** $r_c = \frac{2M}{3}$

Then 66.7% of the mass energy can be extracted by reversible transformations for a=0.

And 33.3% of the mass energy can be extracted by reversible transformations for Q=0.

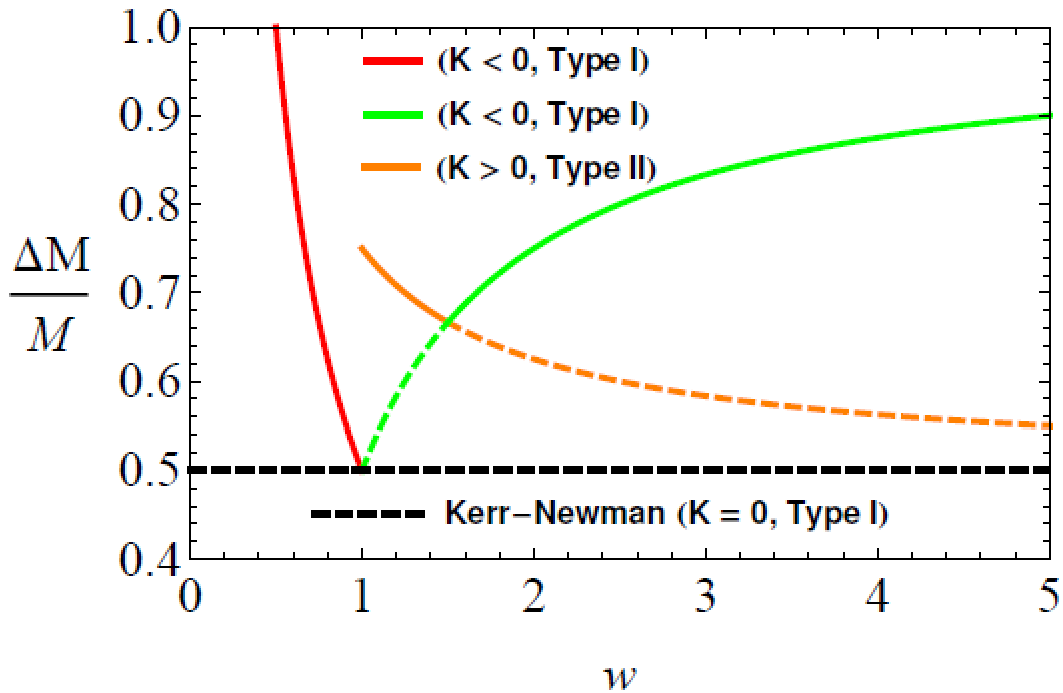

Figure 4: (color online). The best efficiency of mass extraction with respect to the equation of state w.

5. Summary and discussions

We have presented a family of new rotating black hole solutions to Einstein's equations with the anisotropic matter field. The rotating geometry was obtained from the known static solution by employing the Newman-Janis algorithm.

We have considered a set of successive reversible processes extracting energy from the black hole and calculated the efficiency. At the present solutions having anisotropic matter, we have found that the efficiency is better than that of the Kerr-Newman.

We have investigated the thermodynamic properties associated with the event horizon for black hole solutions, which means that the contributions from the matter fields are also added to the physical quantities associated with the event horizon.

Are black holes particles (mass, charge, spin)?

Additional global charge? We may have open mind because we only know a few percent of the energy in our Universe.

Stability!

Astrophysical phenomena!

Thank you for your attention!