Heating Neutron Star with light GeV Dark Matter

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Reference: 2001.09140, 1905.03401

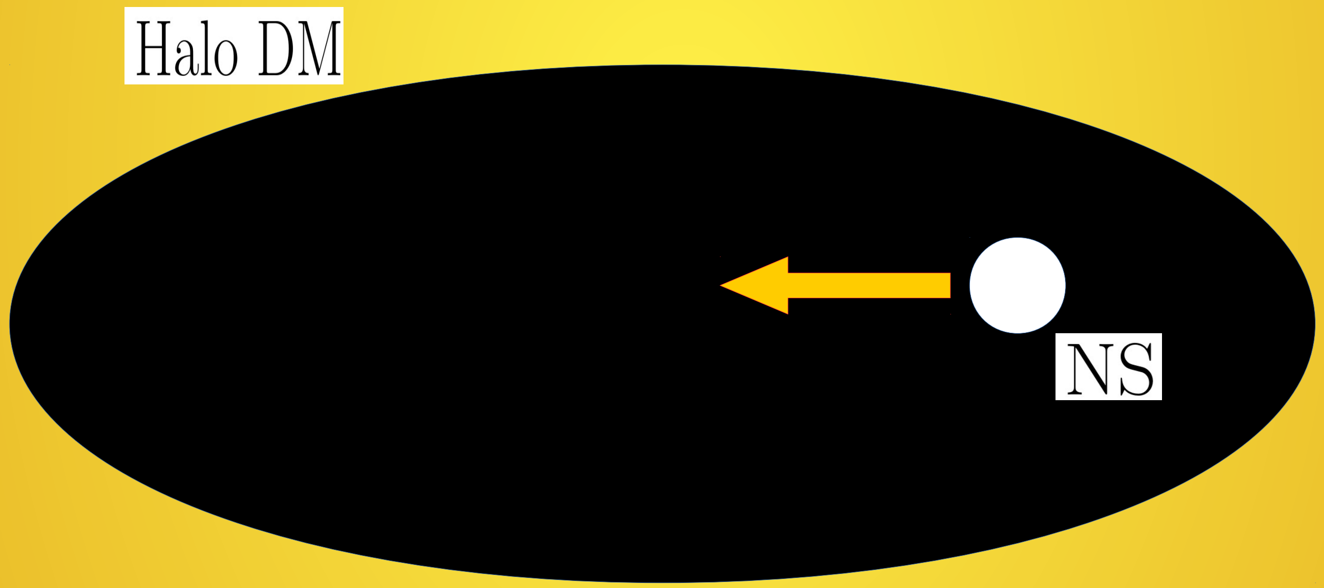
IBS-CTPU seminar, 6th May, 2020

Outline

- Neutron Star (NS) capture halo Dark Matter (DM)
- NS Temperature evolution
- Neutron Dark Decay Model
- Quark vector portal GeV DM

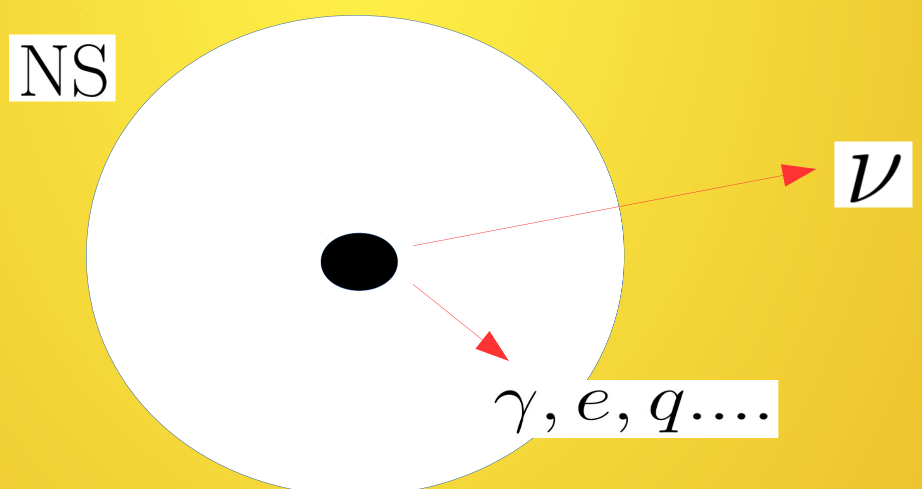
Introduction

- The **dark matter** be captured by **neutron star.**



Introduction

- What **DM** can do to **NS**, after be captured?
- After thermalization, **DM** accumulate at center of **NS**.
- DM-DM annihilate and emit neutrinos.

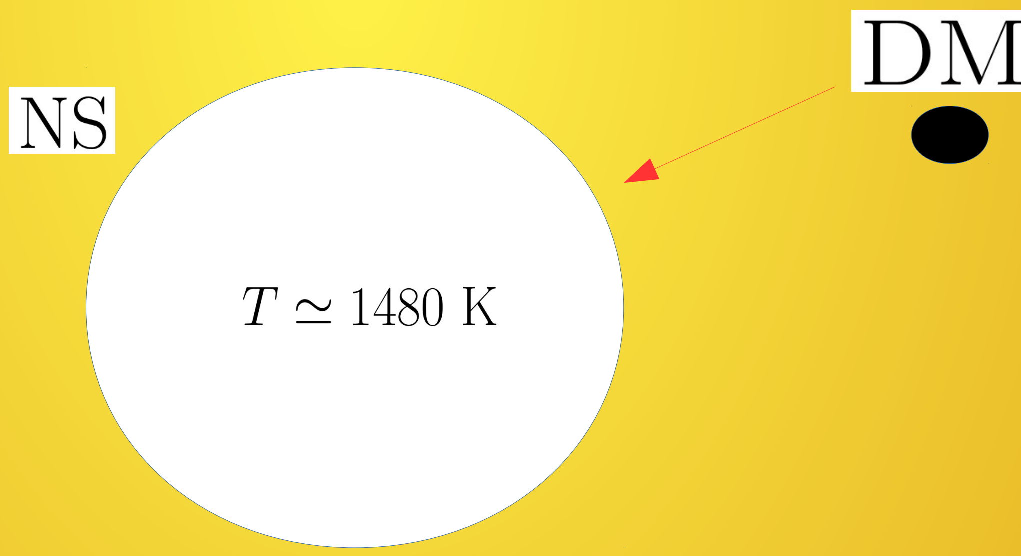


Introduction

- What **DM** can do to **NS**, after be captured?

- **DM** can kinematic heats **NS**, due to strong gravitational potential of **NS**, **DM** is accelerated to v~0.6 c.

$$\frac{1}{2} m_\chi v_{\rm esc}^2 = \frac{G M m_\chi}{R}$$



Introduction

- What **DM** can do to **NS**, after be captured?
- **DM** can kinematic heats **NS**, which increase **NS** temperature by **1480 K**.

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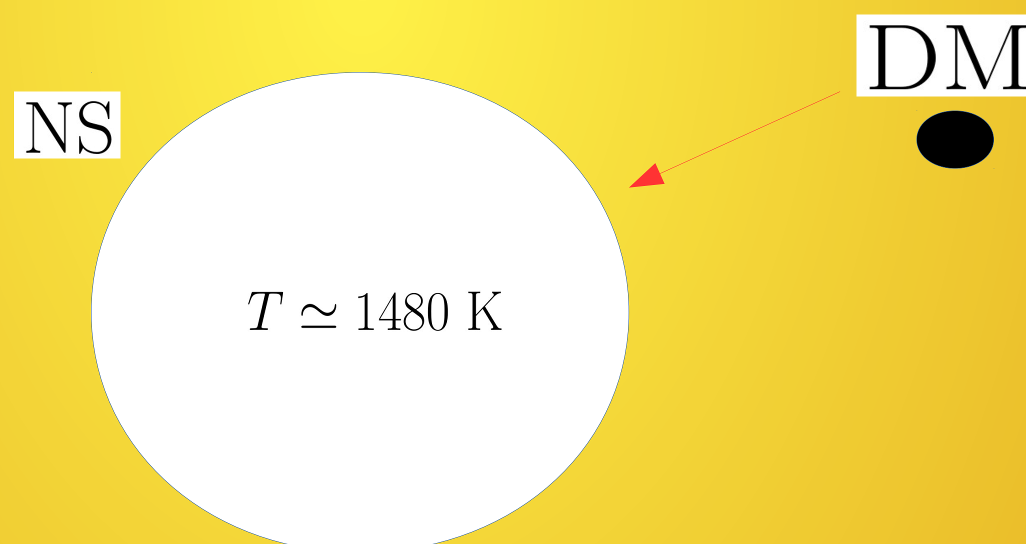

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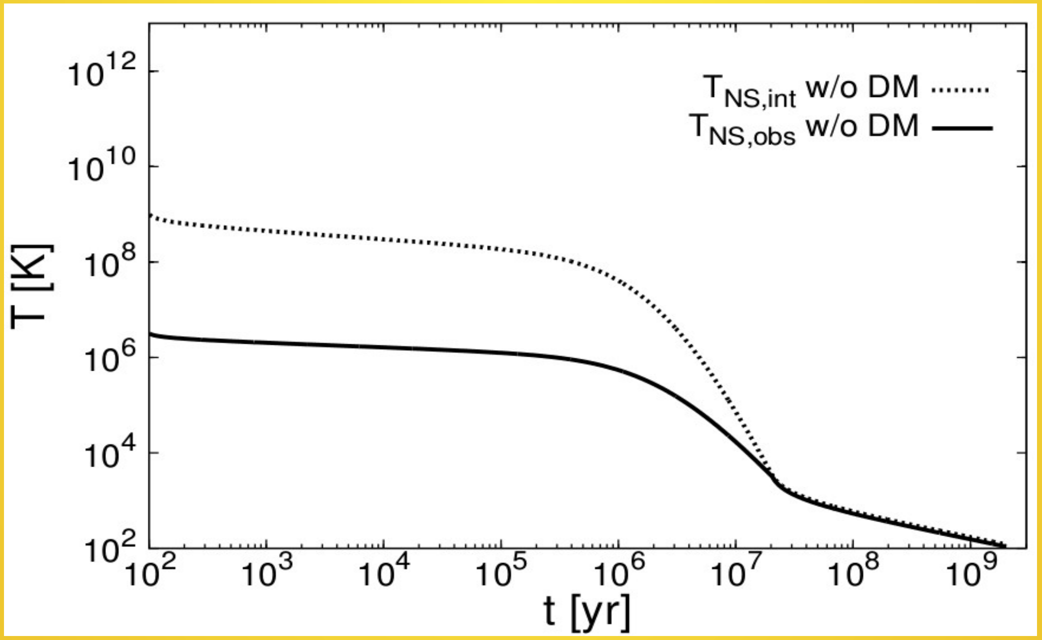

W.Y.Keung, D.Marfatia, P.Y.Tseng: 2001.09140

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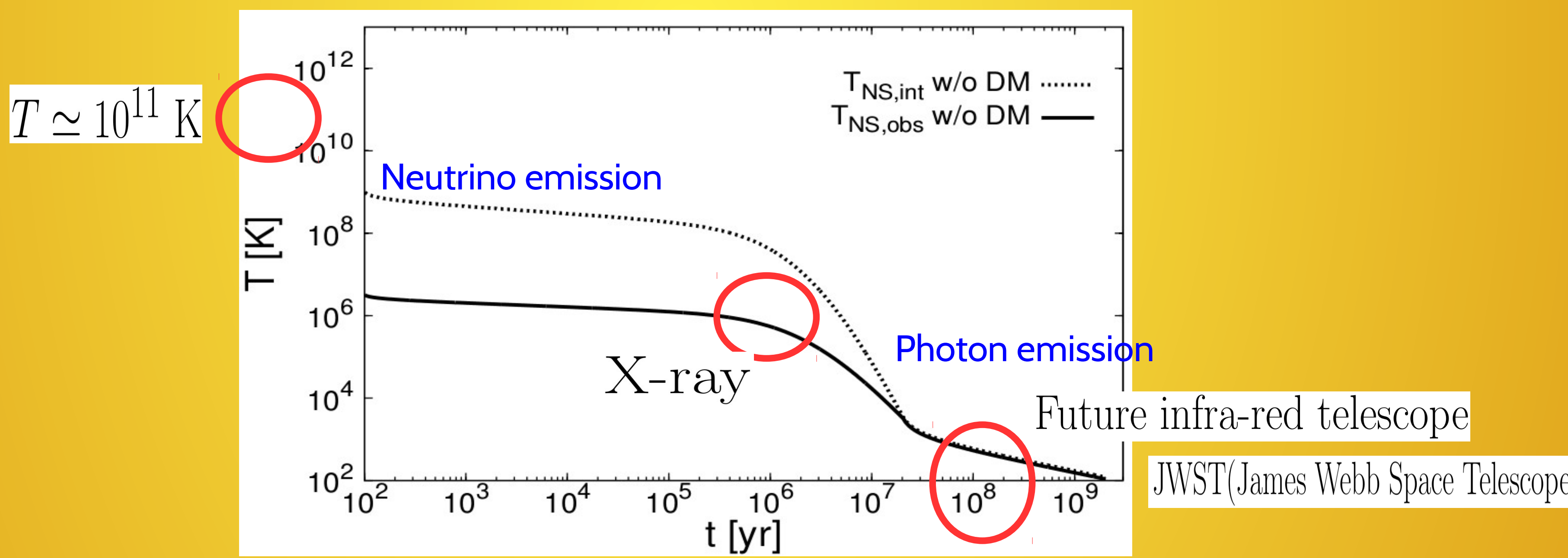

W.Y.Keung, D.Marfatia, P.Y.Tseng: 2001.09140

Introduction

- The evolution of NS temperature

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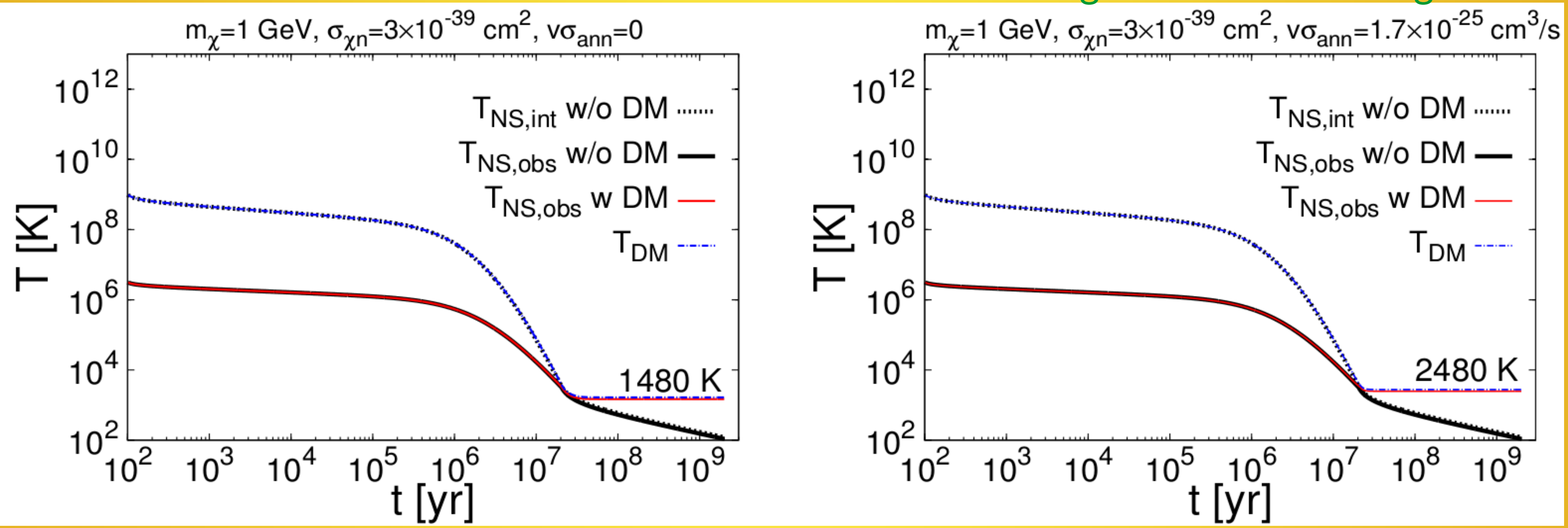
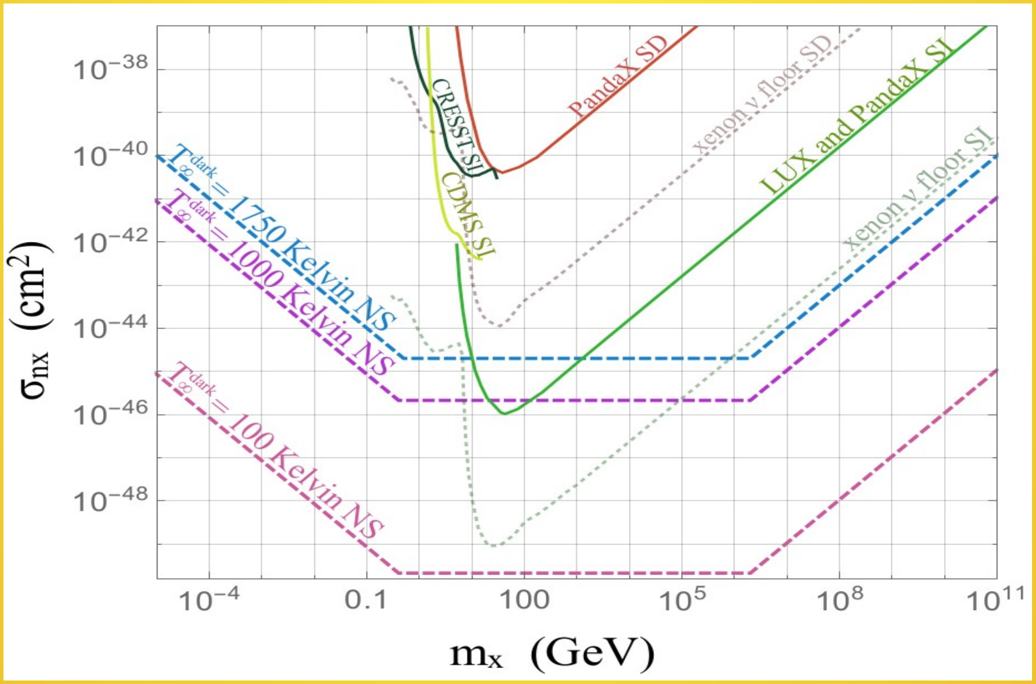


- **DM capture rate** reached *geometric limit* $C_c|_{\rm geom} \simeq 8.2 \times 10^{32}\,{\rm yr}^{-1}$, increasing $\sigma_{\chi n} \gtrsim \mathcal{O}(10^{-45})\ {\rm cm}^2$ does not increase NS temperature.

Introduction

- Quark vector portal DM model:

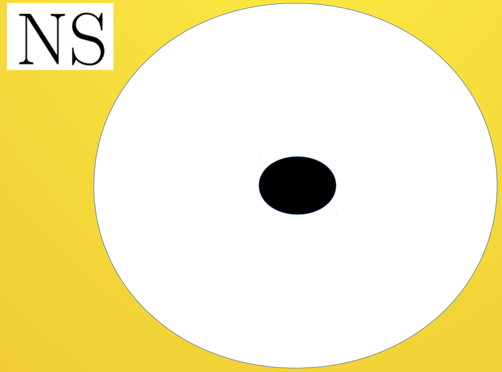
$$\mathcal{L}_{int} = \sum_{q=u,d,s} \frac{\alpha_q}{\Lambda^2} \bar{\chi}\gamma^\mu \chi \bar{q}\gamma_\mu q$$



M.Baryakhtar, J.Bramante, S.W. Li, T. Linden, and N. Raj: 1704.01577

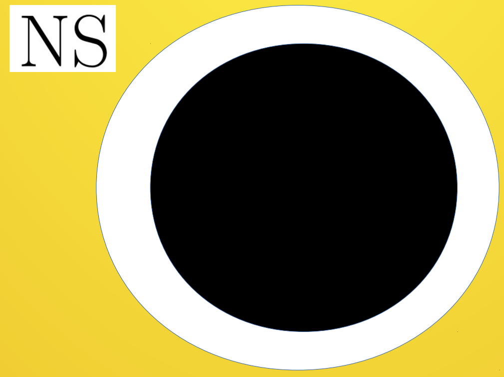
Introduction

- DM self-interaction help to increase the **DM capture rate**.
- There is **maximal** capture rate (*geometric limit*), due to the DM density~0.3 [GeV/cm^3]. It is about $\sigma_{\chi n} \simeq \mathcal{O}(10^{-45})\ \mathrm{cm}^2$
- For 10^8 year old **NS**, captured **DM** is 10^{-18} of total mass.



Introduction

- DM self-interaction help to increase the **DM capture rate**.
- However, in neutron dark decay model, **neutron** will convert into **DM** inside NS.
- More than 10% of **NS** could be **DM**. It helps to heat **NS**.



NS temperature evolution

- The **halo DM** captured rate by **NS** is

$$\frac{dN_{\rm DM}}{dt} = \begin{cases} C_c + C_s^{\chi\chi}(N_{\rm DM} + N_\chi)\,, & \text{If DM is } \chi \\ C_c + (C_s^{\bar\chi\bar\chi} N_{\rm DM} + C_s^{\bar\chi\chi} N_\chi) - C_a N_{\rm DM} N_\chi\,, & \text{If DM is } \bar\chi \end{cases}$$

Captured by neutron → $\bar\sigma^{\rm elatic}_{\chi n}$

DM-self captured → $\sigma_{\chi\chi\to\chi\chi}$

DM annihilation → $\langle \sigma^{\rm ann}_{\chi\bar\chi} v_\chi \rangle$

NS temperature evolution

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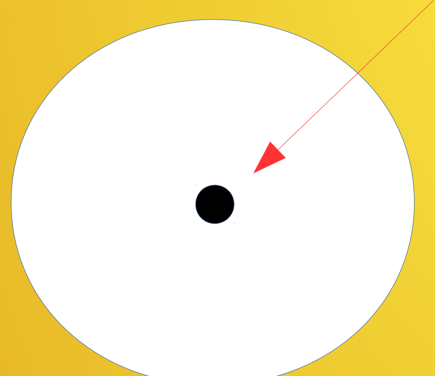
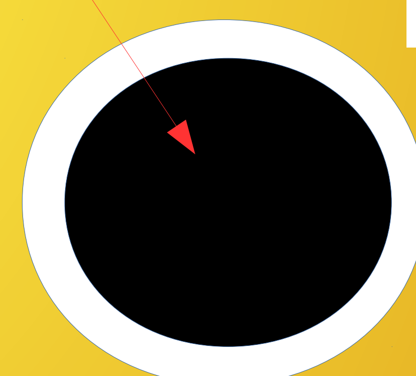
Captured halo DM

DM from neutron conversion

$n \to \chi\phi$





NS temperature evolution

- The evolution of NS temperature

$$\frac{dT_{\rm int}}{dt} = \frac{-\epsilon_\nu - \epsilon_\gamma + \epsilon_\chi}{c_V}$$

$$\epsilon_\nu \simeq 1.81 \times 10^{-27}\ {\rm GeV^4 yr^{-1}} \left(\frac{n_F}{n_0}\right)^{2/3} \left(\frac{T_{\rm int}}{10^7\ {\rm K}}\right)^8$$

NS temperature evolution

- The evolution of NS temperature

$$\frac{dT_{\rm int}}{dt} = \frac{-\epsilon_\nu - \epsilon_\gamma + \epsilon_\chi}{c_V}$$

$$L_\gamma = 4\pi R^2 \sigma_{\rm SB} T_{\rm sur}^4 \simeq 5.00 \times 10^{11}\ {\rm GeV\,s^{-1}} \left(\frac{T_{\rm sur}}{\rm K}\right)^4$$

Stefan-Boltzmann's law

NS temperature evolution

- The evolution of NS temperature

$$\frac{dT_{\rm int}}{dt} = \frac{-\epsilon_\nu - \epsilon_\gamma + \epsilon_\chi}{c_V}$$

$$\epsilon_\chi = \begin{cases} \text{DM annihilations} \\ \text{DM kinematic heating} \\ \text{DM-NS thermal transition} \end{cases}$$

NS temperature evolution

- The evolution of NS temperature

$$\frac{dT_{\rm int}}{dt} = \frac{-\epsilon_\nu - \epsilon_\gamma + \epsilon_\chi}{c_V}$$

Heat capacity of ideal Fermi gas

$$c_V = \frac{k_B^2 T_{\rm int}}{3} \sum_{i=\chi,n} p_{F,i}\sqrt{m_i^2 + p_{F,i}^2}$$

$$p_{F,\chi} = 0.34\ \mathrm{GeV}\left(\frac{n_F \tilde{r}_\chi}{n_0}\right)^{1/3},$$

$$p_{F,n} = 0.34\ \mathrm{GeV}\left(\frac{n_F(1-\tilde{r}_\chi)}{n_0}\right)^{1/3}$$

NS temperature evolution

- The evolution of NS temperature

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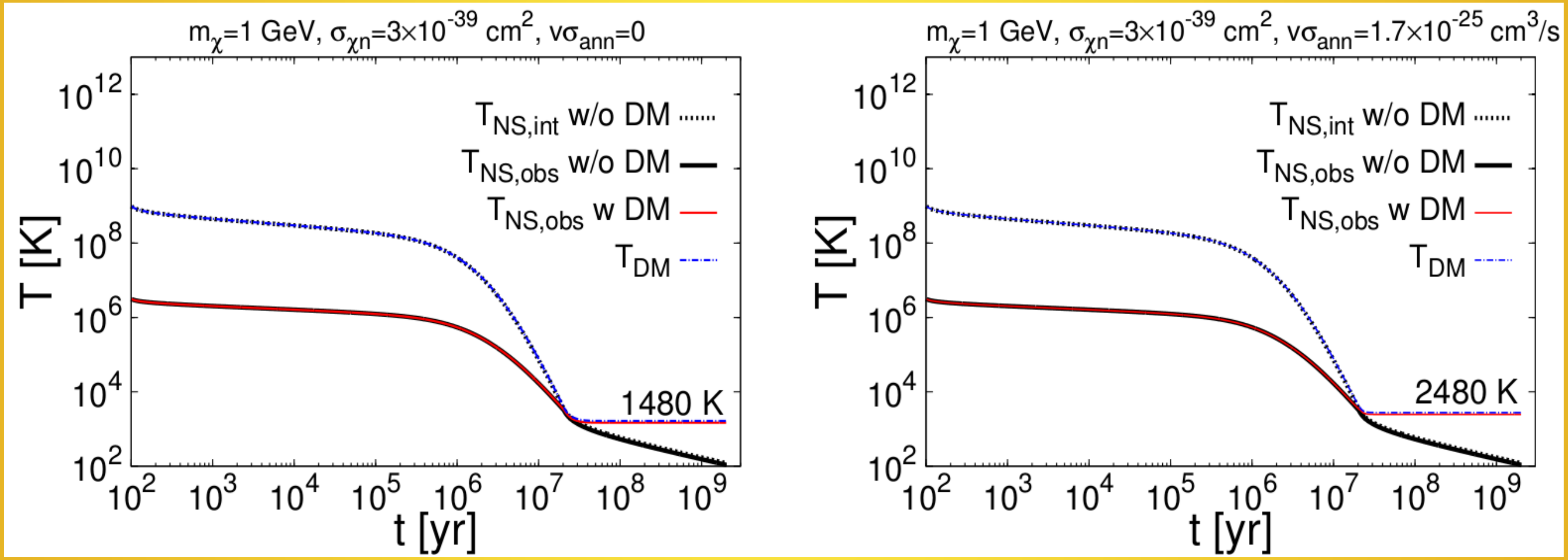


- **DM capture rate** had reached *geometric limit*, increase cross section do not increase NS temperature.

Neutron Dark Decay Model

Neutron dark decay model

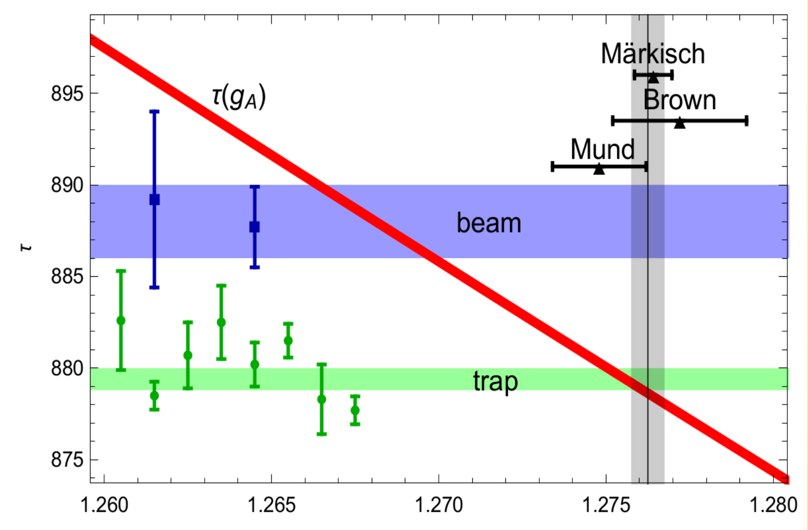
- The **neutron lifetime** is measured in ***bottle*** experiments and ***beam*** experiments.
- ***Bottle***: total lifetime is measured by counting the number of neutrons in a container.
- ***Beam***: count the number of protons from neutron decay.

$$\tau_n^{\text{beam}} = \frac{\tau_n^{\text{bottle}}}{\text{Br}(n \to p + \text{anything})}$$

Neutron dark decay model

- From **SM** prediction, ***bottle*** and ***beam*** experiments are almost equal.

- However, there is **4-sigma** tension between bottle and beam:



$$\tau_n^{\text{bottle}} = 879.6 \pm 0.6 \text{ s}$$

$$\tau_n^{\text{beam}} = 888.0 \pm 2.0 \text{ s}$$

B.Belfatto, R.Beradze, Z.Berezhiani, 1906.02714.

Particle Data Group, Chin.Phys.C40, 10, 100001 (2016),
G.L.Greene, P.Geltenbort, Sci.Am.314,36 (2016).

Neutron dark decay model

- From **SM** prediction, ***bottle*** and ***beam*** experiments are almost equal.
- However, there is **4-sigma** tension between bottle and beam:
- To explain the discrepancy, **1%** of neutron decay into channel without proton.

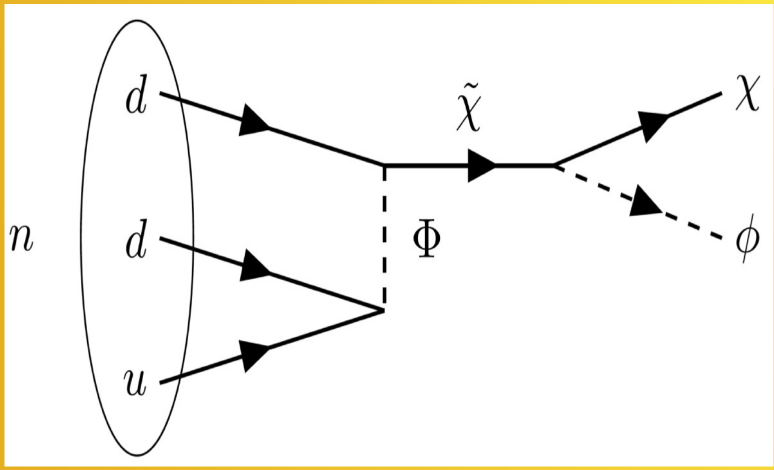
$$\Delta\Gamma(n \to \text{no proton}) \simeq 7.1 \times 10^{-30}\ \text{GeV}$$

Neutron dark decay model

- The model, invoking dark decays on neutron:

B.Fornal, B.Grinstein, PRL 120, 19, 191801 (2018), 1801.01124, 1810.00862.

$$n \rightarrow \chi + \phi$$



$$937.992 \text{ MeV} < m_\chi + m_\phi < 939.565 \text{ MeV}$$
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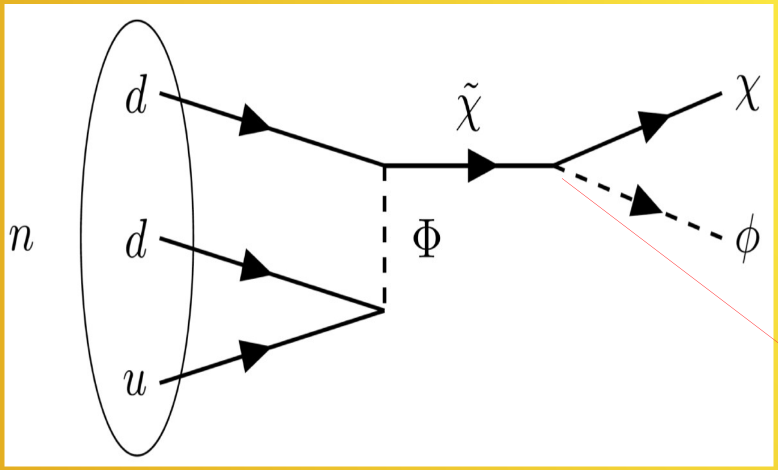

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$$\lambda_\phi \simeq 0.04$$

Other constraints

- Stability of neutron star (NS).

B.Grinstein, C. Kouvaris, N.G. Nielsen, 1811.06546.

$$\mathcal{L}_2 = \left(\lambda_q \, \epsilon^{ijk} \, \overline{u^c_{L}}_i \, d_{Rj} \Phi_k + \lambda_\chi \Phi^{*i} \bar{\tilde{\chi}} \, d_{Ri} + \lambda_\phi \, \bar{\tilde{\chi}} \, \chi \, \phi + \text{h.c.}\right) + M_\Phi^2 \, |\Phi|^2 + m_\phi^2 |\phi|^2 + m_\chi \, \bar{\chi} \, \chi + m_{\tilde{\chi}} \, \bar{\tilde{\chi}} \, \tilde{\chi} \,. \quad (38)$$

$$+ \mu H^\dagger H \phi + g_\chi \bar{\chi} \chi \, \phi$$

- Higgs portal and DM-self interactions:

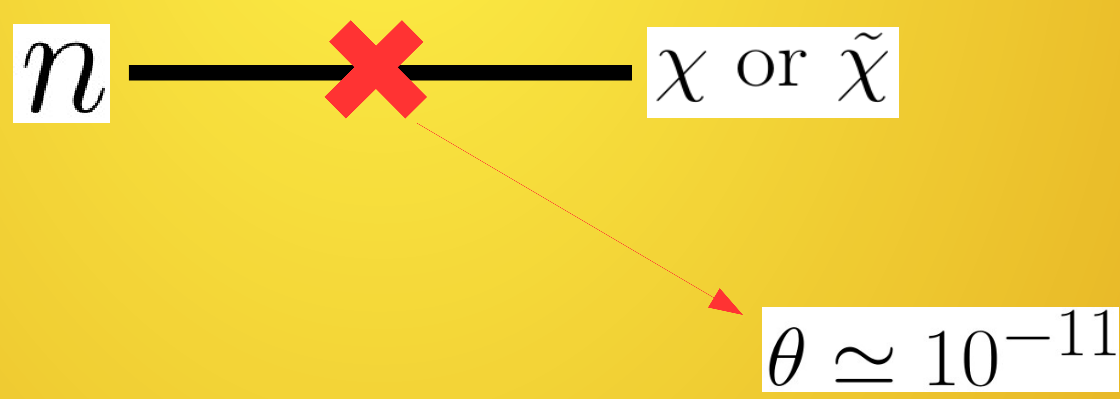
$$g_n \bar{n} n \phi \qquad z \equiv m_\phi / \sqrt{|g_\chi g_n|}$$

Neutron dark decay model

- The model, invoking dark decays on neutron

- **DM** mass is ~GeV, mixing with neutron, carries baryon number.

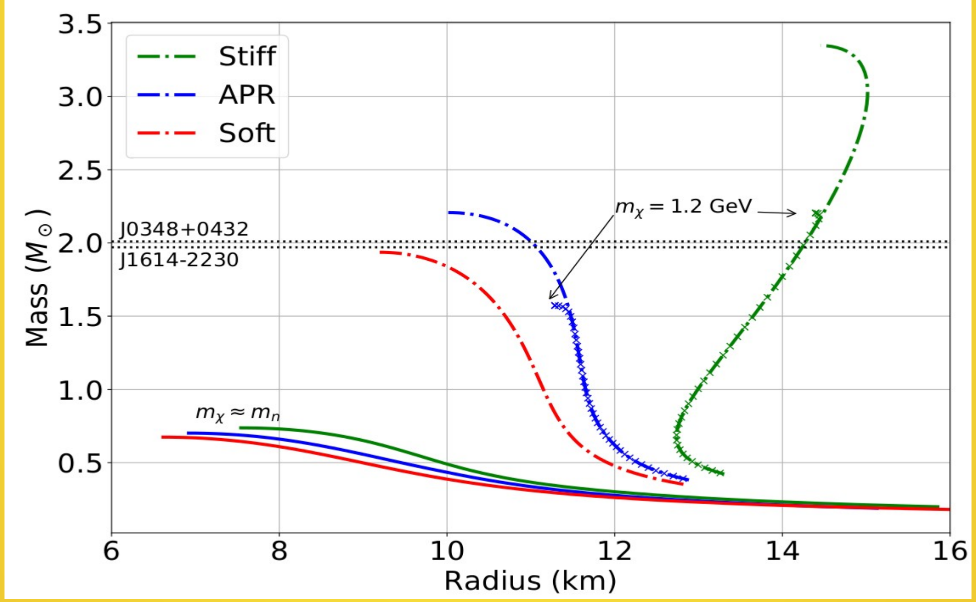
B.Fornal, B.Grinstein, PRL 120, 19, 191801 (2018), 1801.01124, 1810.00862.



Stability of neutron star

- **NS** becomes **unstable**: Equation of State (EoS) is too soft to maintain NS heavier than two solar mass.

$$n \to \chi + \phi$$



D.McKeen, A.E.Nelson, S.Reddy, and D.Zhou, 1802.08244.

Stability of neutron star

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- Cure by adding DM-neutron interaction, and repulsive DM-self interaction. B.Grinstein, C. Kouvaris, N.G. Nielsen, 1811.06546.

- The EoS and energy density are

$$\varepsilon(n_n, n_\chi) = \varepsilon_{\mathrm{nuc}}(n_n) + \varepsilon_\chi(n_\chi) + \frac{n_\chi n_n}{2z^2}$$

$$\varepsilon_\chi = \frac{m_\chi^4}{8\pi^2}\left[x\sqrt{1+x^2}(1+2x^2) - \ln(x+\sqrt{1+x^2})\right] \pm \frac{n_\chi^2}{2z'^2}$$

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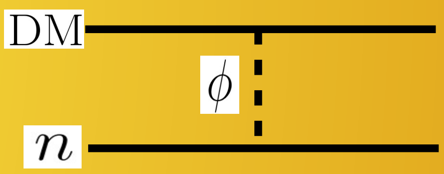

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$$\varepsilon(n_n, n_\chi) = \varepsilon_{\rm nuc}(n_n) + \varepsilon_\chi(n_\chi) + \frac{n_\chi n_n}{2z^2}$$

$$U = \pm \frac{g_\chi g_n}{4\pi} \frac{e^{-m_\phi r}}{r}$$

$$\varepsilon_\chi = \frac{m_\chi^4}{8\pi^2}\left[x\sqrt{1+x^2}(1+2x^2) - \ln(x+\sqrt{1+x^2})\right] \pm \frac{n_\chi^2}{2z'^2}$$

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- The amount of **DM** inside **NS** can be determined by

$$0 = \frac{\partial \varepsilon(n_F - n_\chi, n_\chi)}{\partial n_\chi} = \mu_\chi(n_\chi) - \mu_{\rm nuc}(n_n) + \frac{n_F - 2n_\chi}{2z^2}$$

TOV Eq.

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B.Grinstein, C. Kouvaris, N.G. Nielsen, 1811.06546.

- Tolman-Oppenheimer-Volkoff (TOV) equation:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{G\rho m}{r^2}\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi P r^3}{mc^2}\right)\left(1-\frac{2Gm}{rc^2}\right)^{-1},$$

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \rho \ , \qquad (11)$$

F.Douchin and P.Haensel ,astro-ph/0111092

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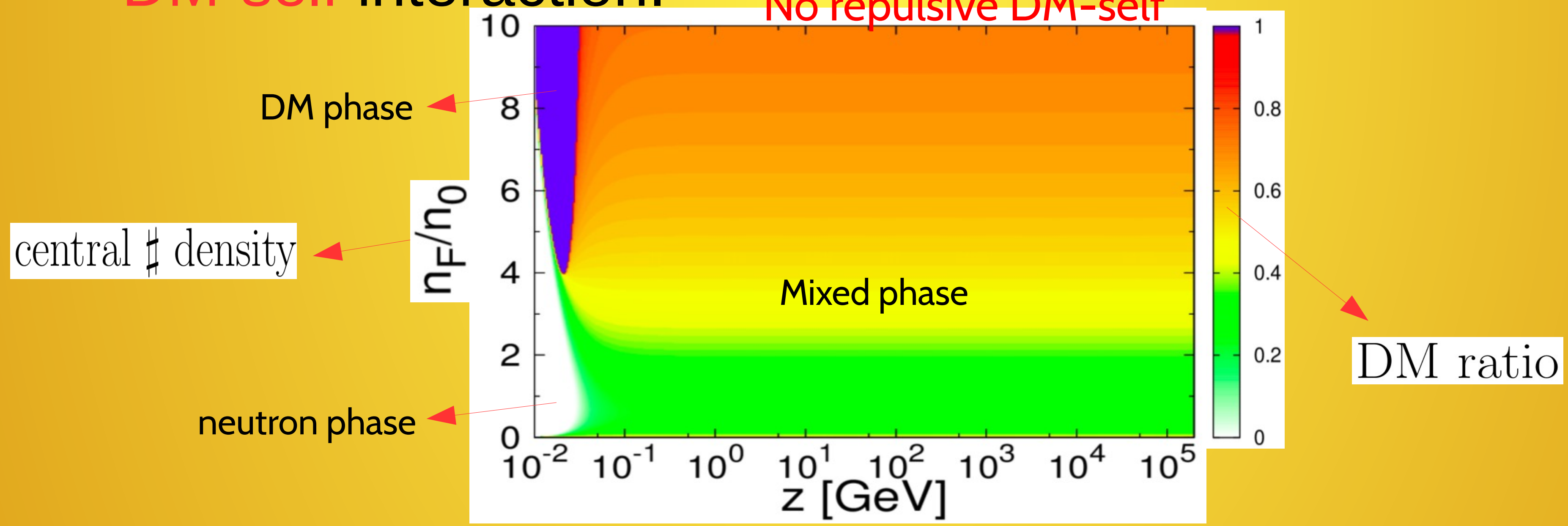

W.Y.Keung, D.Marfatia, P.Y.Tseng: 2001.09140

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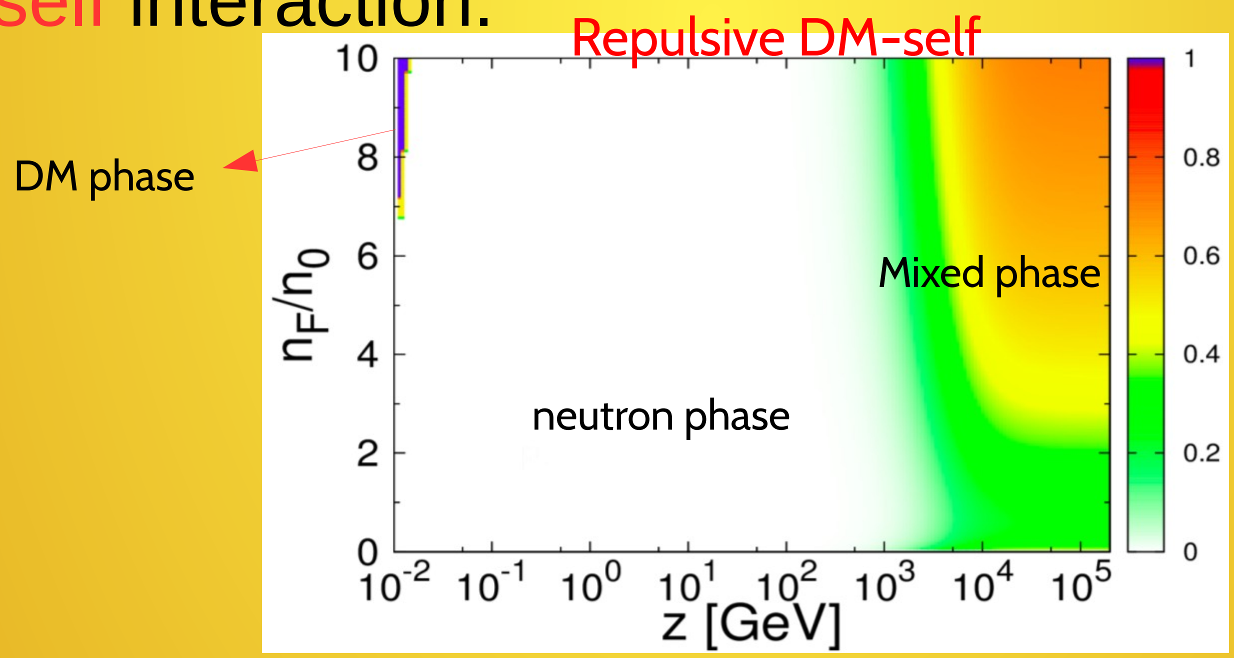

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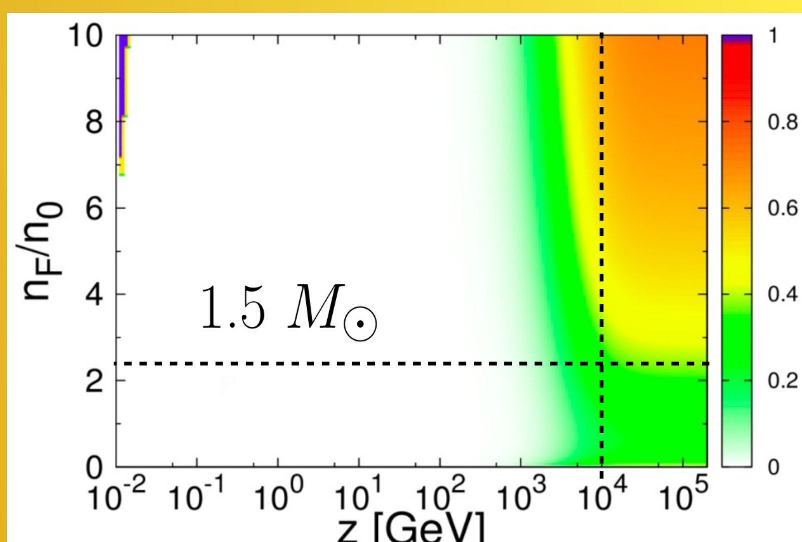
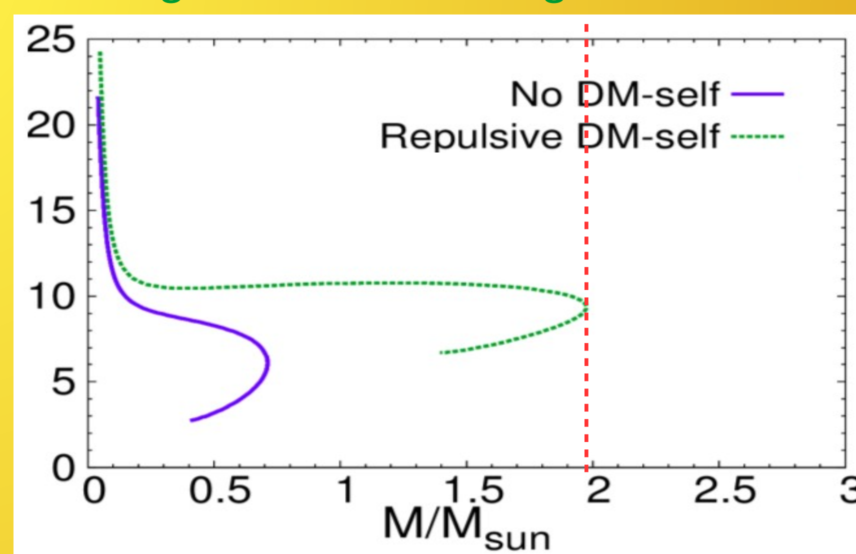


$z = 10^4$ GeV



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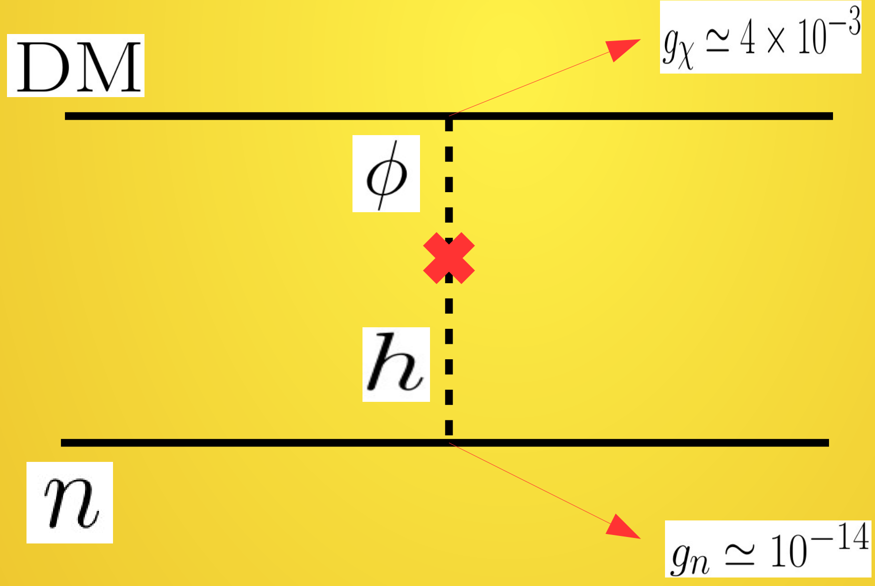
- **NS** can be composed by **30% of DM** and stable from neutron dark decay model.

$n \to \chi\phi$

Heating NS by neutron dark decay

- Neutron dark decay model: the DM-neutron cross section is $\mathcal{O}(10^{-60})\ \mathrm{cm}^2 \ll \mathcal{O}(10^{-45})\ \mathrm{cm}^2$, therefore the DM captured rate is much smaller than *geometric limit*.



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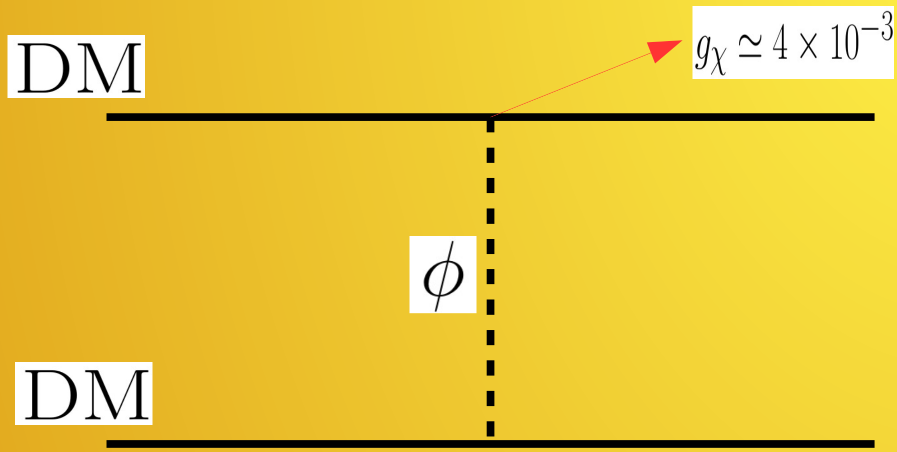
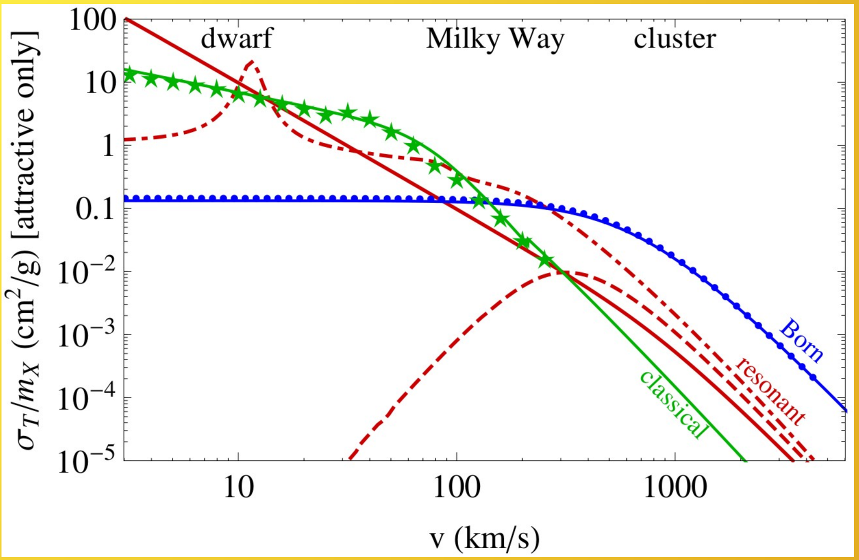
- However, the DM-self interactions help to increase the **DM capture** rate.



➢ S.Tulin, H.B.Yu, K.M.Zurek: PRL,110(2013),111301



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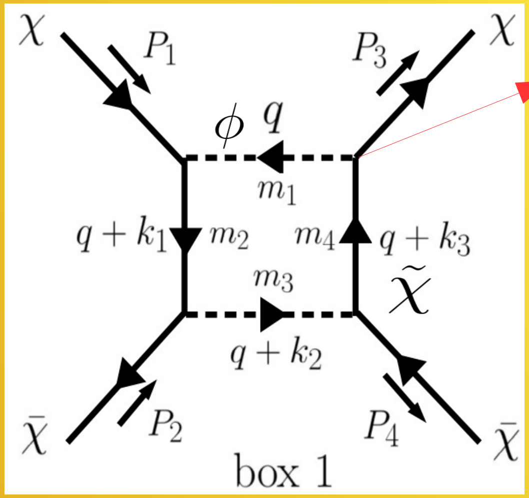
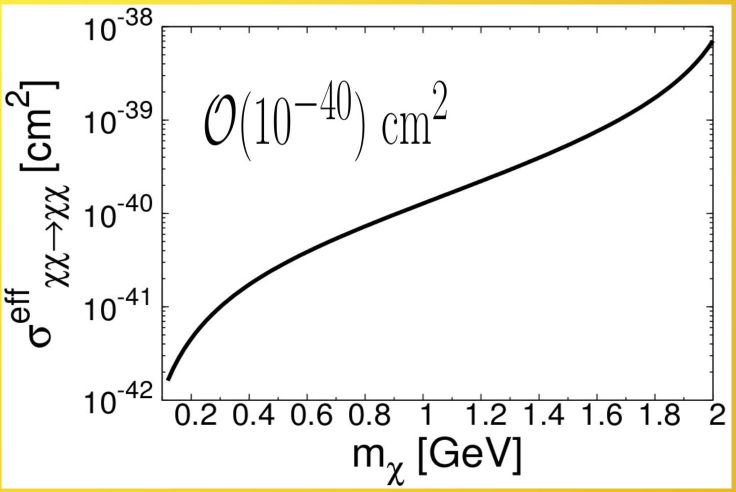
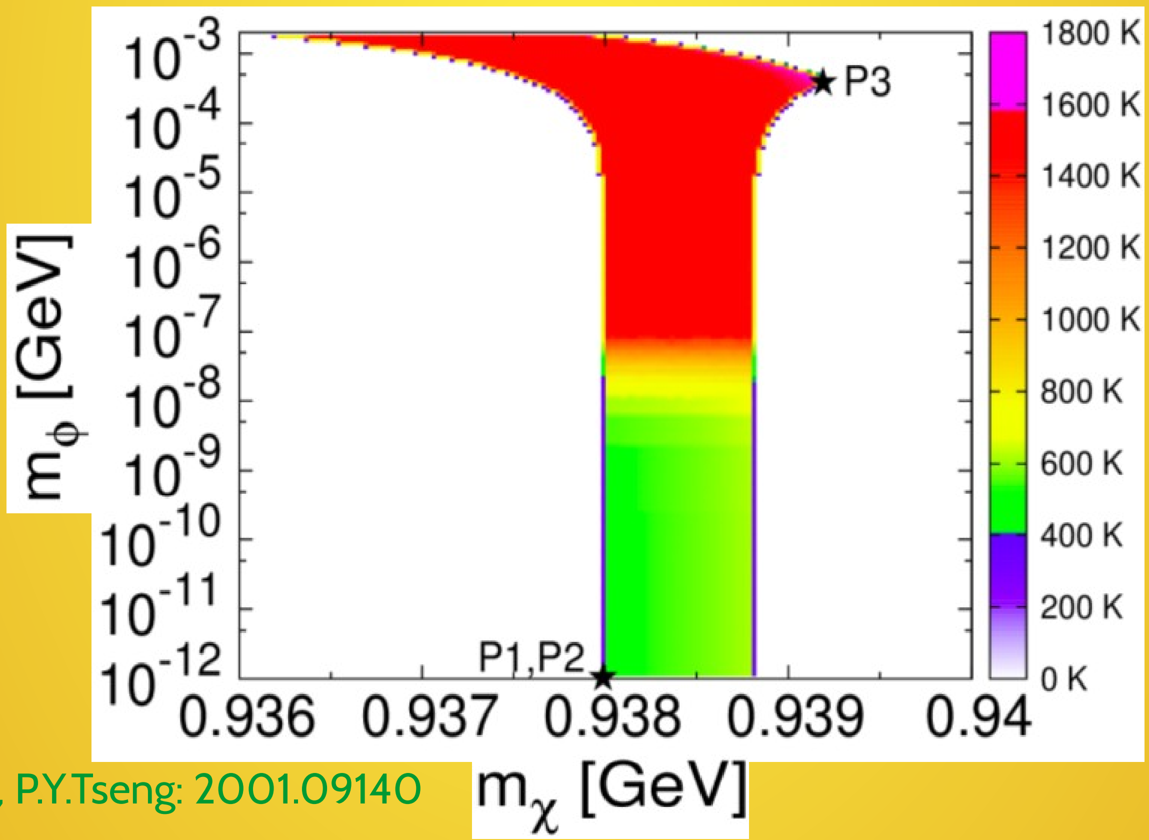




Heating NS by neutron dark decay

- Neutron dark decay model: can heat up NS more than 1500 K by i) **NS** is composed by *substantial* amount of **DM**. ii) DM-self cross section is *large enough*.



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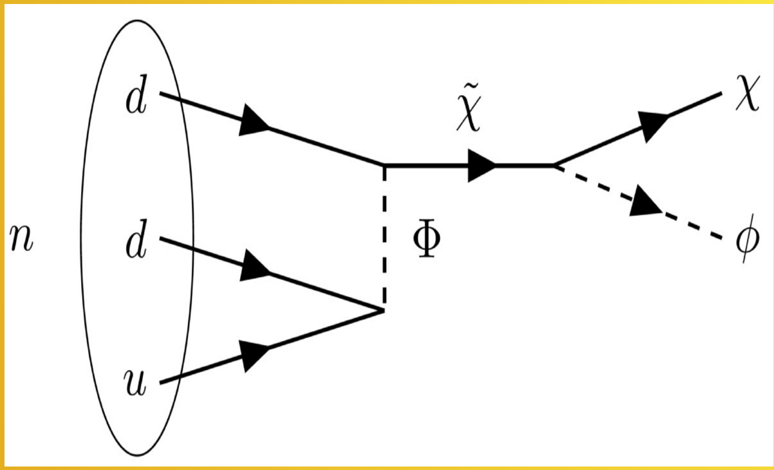

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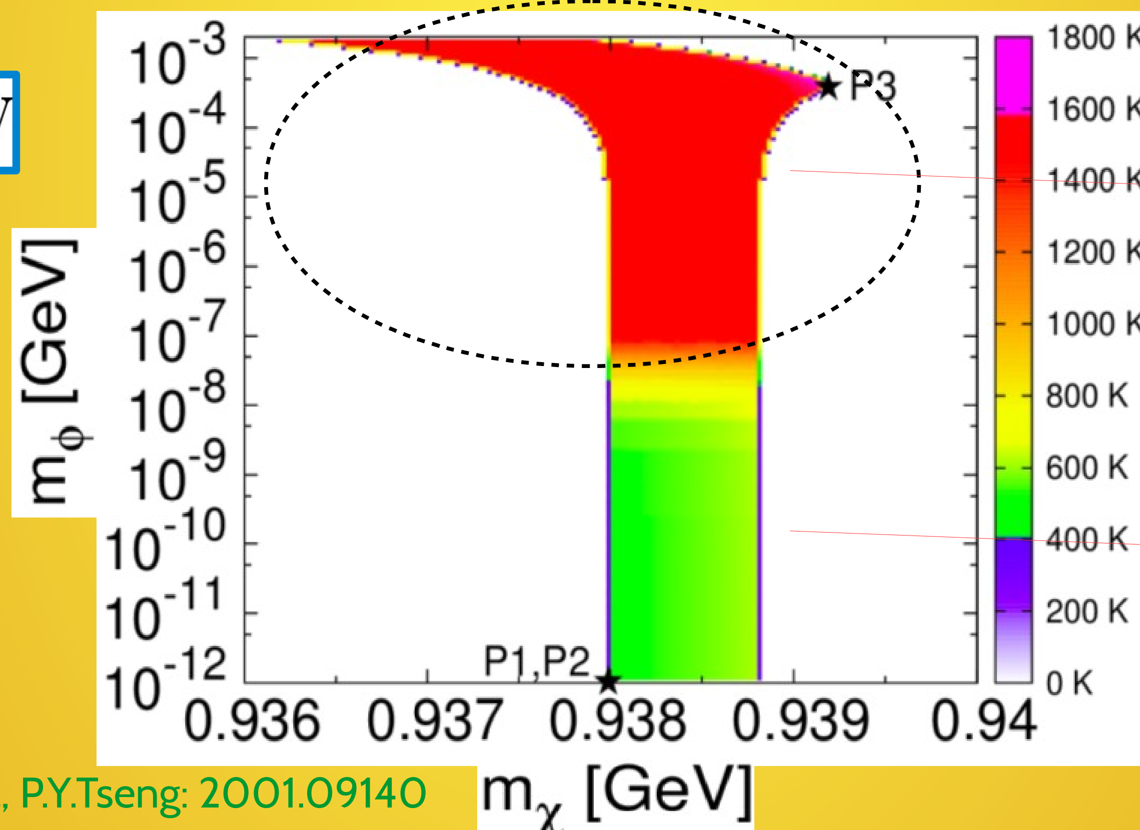
$m_\phi \gtrsim 100 \text{ eV}$



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Quark vector current portal GeV DM

Quark vector portal GeV DM

- Quark vector portal DM model:

$$\mathcal{L}_{int} = \sum_{q=u,d,s} \frac{\alpha_q}{\Lambda^2} \bar{\chi}\gamma^\mu \chi \bar{q}\gamma_\mu q$$

- Instead, DM-nucleon cross section need to be calculated in relativistic limit.

$$\frac{d\sigma_{\chi n,p}(s,t)}{d\cos\theta_{\rm cm}} = \left(\frac{c_{\chi n,p}}{\Lambda^4}\right) \frac{2(\bar{\mu}^2+1)^2 m_\chi^4 - 4(\bar{\mu}^2+1)\bar{\mu}^2 s m_\chi^2 + \bar{\mu}^4(2s^2+2st+t^2)}{16\pi\bar{\mu}^4 s} |F_n(E_R)|^2$$

N.F.Bell, G.Busoni, and S.Robles: 1807.02840

Quark vector portal GeV DM

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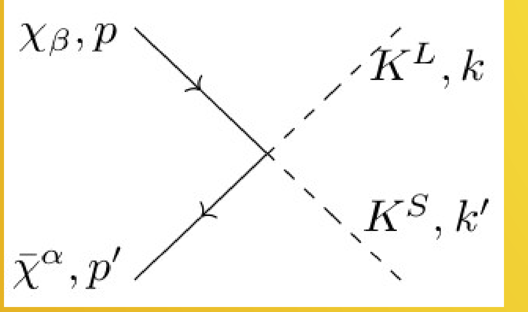
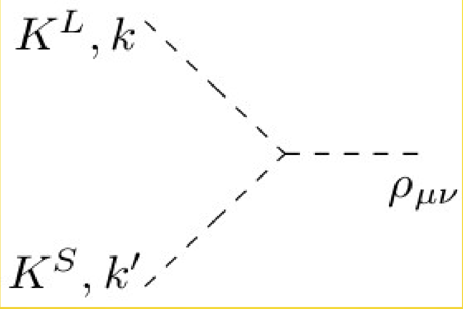
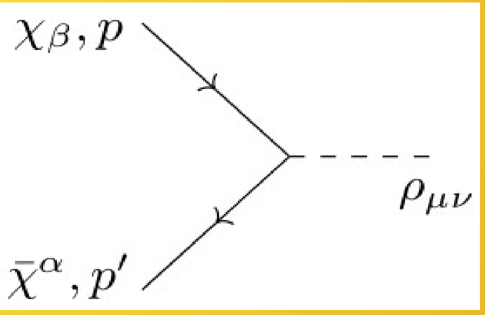
- At GeV scale, **chiral Lagrangian** is better description to calculate the DM-annihilation cross section.

D.Berger, A.Rajaraman, and J.Kumar: 1903.10632.
J.Kumar:1808.02579





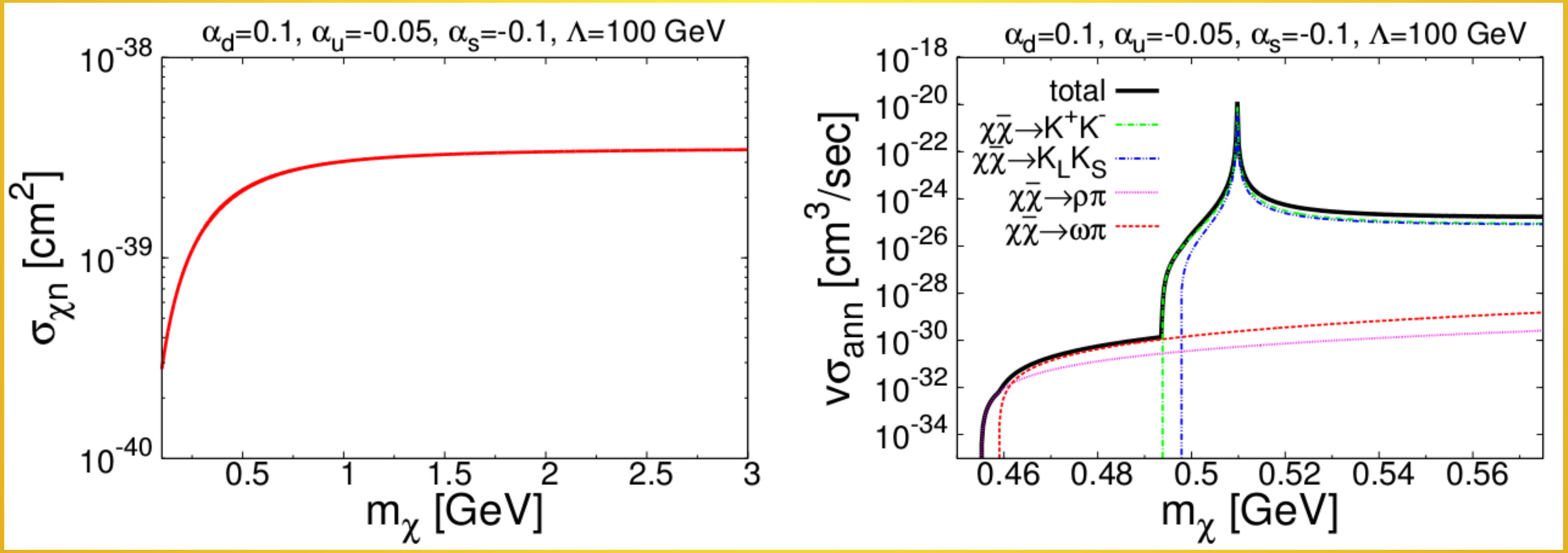


Quark vector portal GeV GeV DM

- Quark vector portal DM model:

$$\mathcal{L}_{int} = \sum_{q=u,d,s} \frac{\alpha_q}{\Lambda^2} \bar{\chi}\gamma^\mu \chi \bar{q}\gamma_\mu q$$

- The DM-neutron and DM-annihilation cross sections.



Quark vector portal GeV DM

- Quark vector portal DM model:

$$\mathcal{L}_{int} = \sum_{q=u,d,s} \frac{\alpha_q}{\Lambda^2} \bar{\chi}\gamma^\mu \chi \bar{q}\gamma_\mu q$$

- The DM-neutron and DM-annihilation cross sections.
- The couplings of $\alpha_q/\Lambda^2 \simeq \mathcal{O}(10^{-4})/(100\ \mathrm{GeV})^2$, the **capture rate** reaches *geometric limit*. This is about the sensitivity from heating NS up to 1500 K.

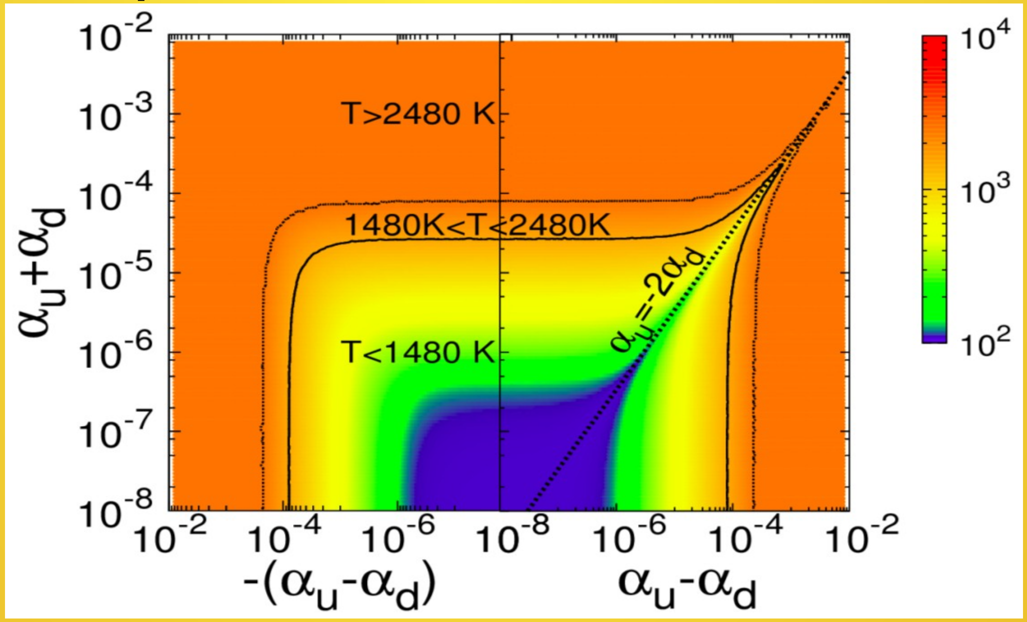
Heating NS by Quark vector portal DM

- Quark vector portal DM model:

$$\mathcal{L}_{int} = \sum_{q=u,d,s} \frac{\alpha_q}{\Lambda^2} \bar{\chi}\gamma^\mu \chi \bar{q}\gamma_\mu q$$

- Heating NS temperature:

$$\alpha_{u,d} \gtrsim \mathcal{O}(10^{-4})$$

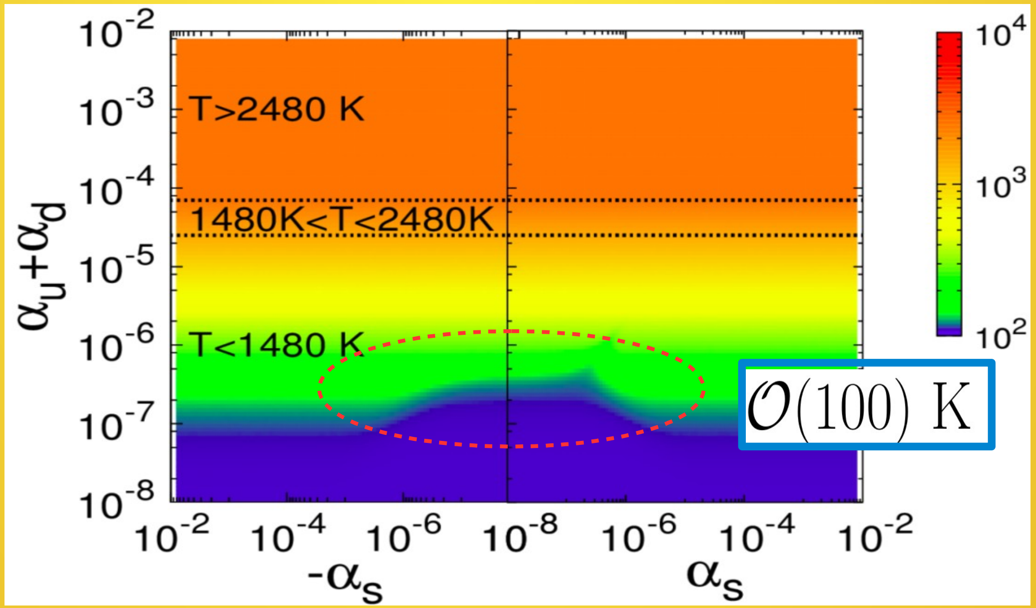


Heating NS by Quark vector portal DM

- Quark vector portal DM model:

$$\mathcal{L}_{int} = \sum_{q=u,d,s} \frac{\alpha_q}{\Lambda^2} \bar{\chi}\gamma^\mu \chi \bar{q}\gamma_\mu q$$

- Heating NS temperature varying α_s :



Summary

- We studied the GeV-mass **DM** captured by **NS.**

- I) Neutron dark decay model. ii) Quark vector portal GeV DM.

- In general, neutron can convert into **DM**, which becomes substantial portion of **NS**. DM-self interaction helps to enhance the DM captured rate,and **heating NS** up to 1500 K.

Summary

- Apply to quark vector portal GeV DM.
  The couplings down to $\alpha_q/\Lambda^2 \simeq \mathcal{O}(10^{-4})/(100\ \mathrm{GeV})^2$ can be probed.

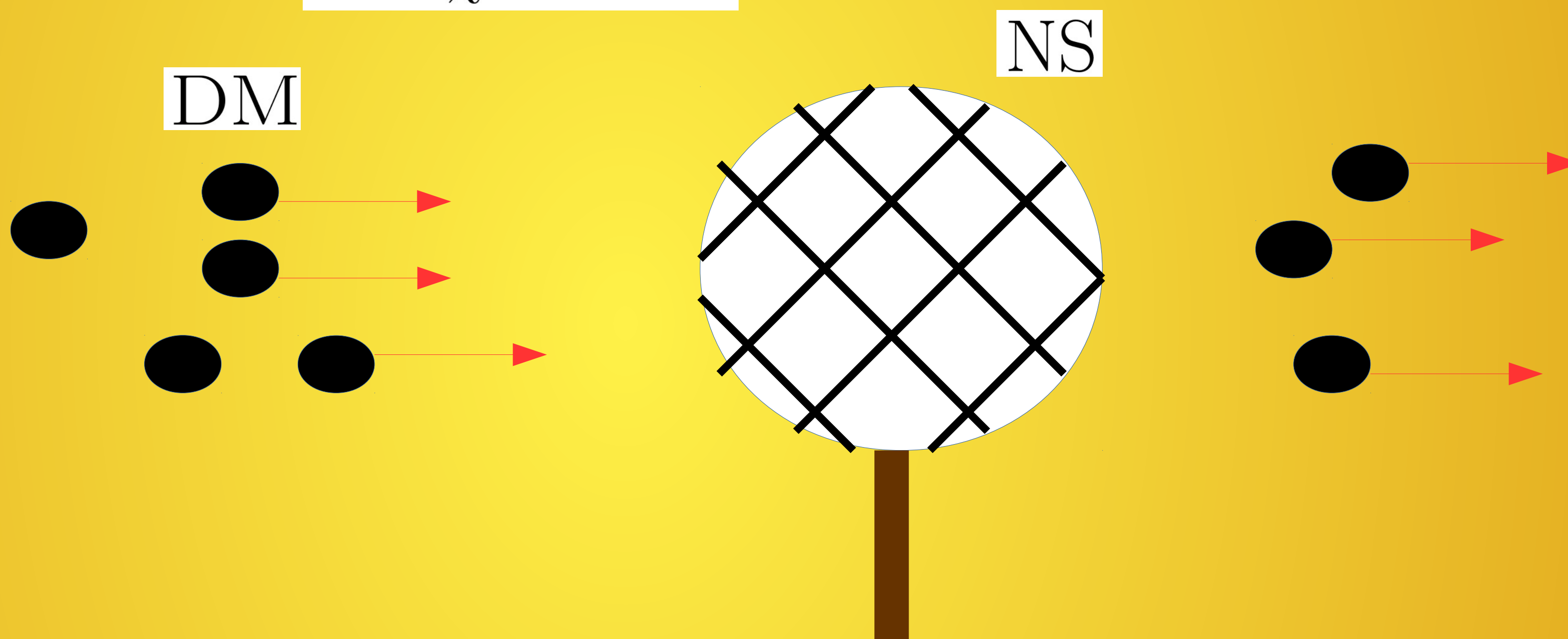
- Near future infra-red telescopes (James Webb Space Telescope) will be sensitive to 10^8 year old **NS** with 1500 K.

Thank You!

Back Up

Introduction

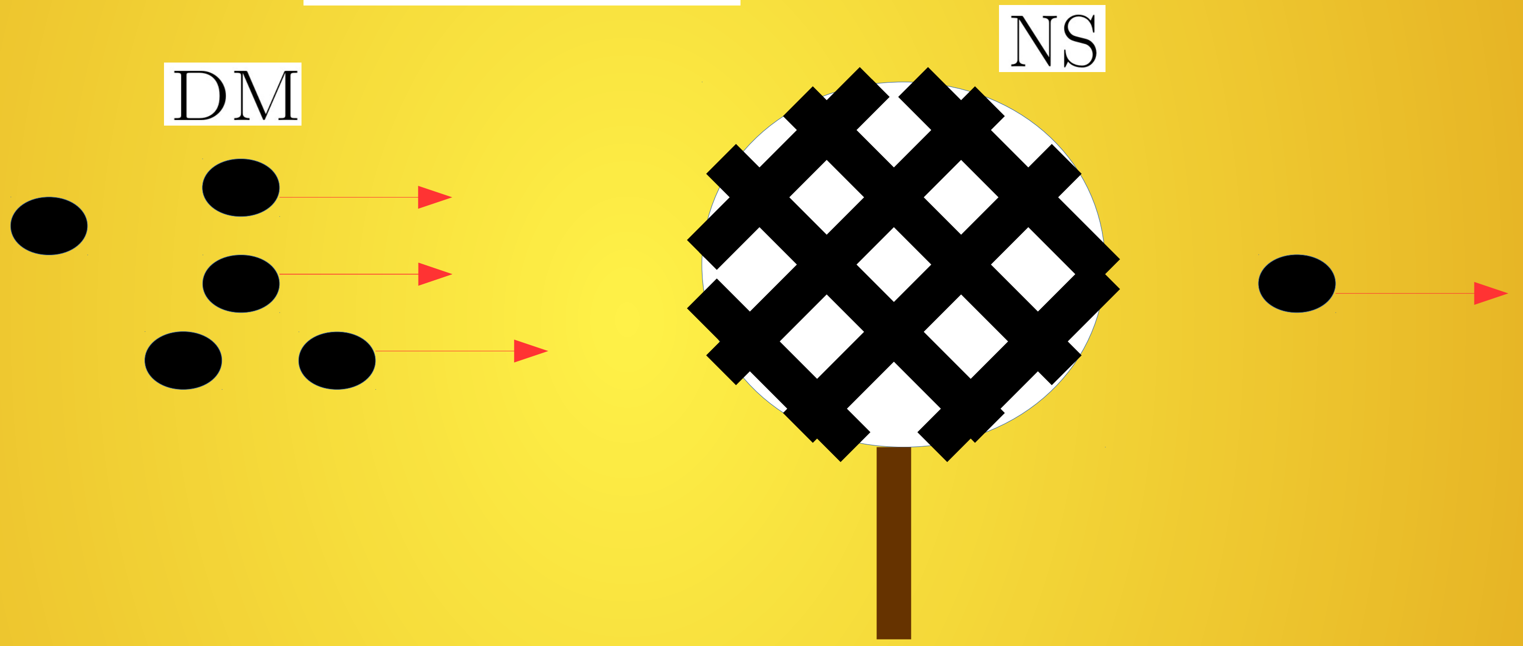
- *geometric limit* $N_n \sigma_{\chi n} \leq \pi R^2$



$$\sigma_{\chi n} \ll \mathcal{O}(10^{-45})\ \mathrm{cm}^2$$

Introduction

- *geometric limit* $N_n \sigma_{\chi n} \leq \pi R^2$



$\sigma_{\chi n} \simeq \mathcal{O}(10^{-45})\ \mathrm{cm}^2$

Introduction

- *geometric limit* for 1 GeV DM:

$$C_c\big|_{\rm geom} \simeq 8.2 \times 10^{32}\,{\rm yr}^{-1}$$

$$\sigma_{\rm crit} \simeq 10^{-45}\ {\rm cm}^2$$

Introduction

- What **DM** can do to **NS**, after be captured?
- **DM** can kinematic heats **NS**, which increase **NS** temperature by **1480 K**.

[1] T. Gver, A. E. Erkoca, M. Hall Reno and I. Sarcevic, JCAP **1405**, 013 (2014), [arXiv:1201.2400 [hep-ph]].

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[3] S. D. McDermott, H. B. Yu and K. M. Zurek, Phys. Rev. D **85**, 023519 (2012), [arXiv:1103.5472 [hep-ph]].

[4] R. Garani, Y. Genolini and T. Hambye, JCAP **1905**, 035 (2019), [arXiv:1812.08773 [hep-ph]].

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[6] R. Garani and J. Heeck, Phys. Rev. D **100**, no. 3, 035039 (2019), [arXiv:1906.10145 [hep-ph]].

[7] M. Baryakhtar, J. Bramante, S. W. Li, T. Linden and N. Raj, Phys. Rev. Lett. **119**, no. 13, 131801 (2017), [arXiv:1704.01577 [hep-ph]].

NS temperature evolution

- The **halo DM** captured rate by **NS** is

$$\frac{dN_{\rm DM}}{dt} = \begin{cases} C_c + C_s^{\chi\chi}(N_{\rm DM} + N_\chi)\,, & \text{If DM is } \chi \\ C_c + (C_s^{\bar\chi\bar\chi} N_{\rm DM} + C_s^{\bar\chi\chi} N_\chi) - C_a N_{\rm DM} N_\chi\,, & \text{If DM is } \bar\chi \end{cases}$$

$$C_c = \sqrt{\frac{6}{\pi}}\frac{\rho_{\rm DM}}{m_\chi}\frac{v_{\rm esc}^2(R)}{\bar v^2}(\bar v \xi \sigma^{\rm elastic}_{{\rm DM}-n})N_n\left(1 - \frac{1-e^{-B^2}}{B^2}\right)$$

NS temperature evolution

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$$C_s^{\bar\chi\chi} = \sqrt{\frac{3}{2}}\frac{\rho_{\rm DM}}{m_\chi}\sigma_{\bar\chi\chi\to\bar\chi\chi} v_{\rm esc}(R)\frac{v_{\rm esc}(R)}{\bar v}\frac{{\rm erf}(\eta)}{\eta}\frac{1}{1-\frac{2GM}{R}}\,,$$

NS temperature evolution

- The **halo DM** captured rate by **NS** is

$$\frac{dN_{\rm DM}}{dt} = \begin{cases} C_c + C_s^{\chi\chi}(N_{\rm DM} + N_\chi)\,, & \text{If DM is } \chi \\ C_c + (C_s^{\bar\chi\bar\chi} N_{\rm DM} + C_s^{\bar\chi\chi} N_\chi) - C_a N_{\rm DM} N_\chi\,, & \text{If DM is } \bar\chi \end{cases}$$

$$C_a \simeq \frac{\langle \sigma^{\rm ann}_{\bar\chi\chi} v_{\rm DM} \rangle}{4\pi R^3/3}\,,$$