Gravitational lensing by slowly accelerating black holes

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Accelerating black holes

The creation

Some gauge theories allow the possibility of topological defects, such as cosmic strings.

Vilenkin, *Phys.Rep.* 121, 263 (1985).

Cosmic string could break/fray to produce pair of accelerating black holes.

Ashoorioon and MBJP, *Phys.Lett.B* 816, 136224, (2021).

Also, primordial black holes could be formed in the early Universe and get attached to cosmic strings.

Vilenkin, Levin, and Gruzinov, *JCAP* 2018, 008, (2018).

Accelerating black holes

The evolution

Black holes connected to cosmic string could evolve to supermassive black holes. Their velocity, however, should be small ($\lesssim 100\,\mathrm{km/s}$), so that they could be captured by galaxies.

Vilenkin, Levin, and Gruzinov, *JCAP* 2018, 008, (2018).

Therefore, the acceleration of these black holes have to be very small.

We take the acceleration so small that a ray of light, passing the black hole to the Earth, lie on the equatorial plane of the black hole in the whole way.
We will show that such *slowly* accelerating black holes have features which could be observed.

Accelerating black holes

The metric

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Accelerating black hole of mass m is described by C metric which can be written in the form

$$ds^2 = \frac{1}{(1+\alpha r\cos\theta)^2}\left[-Q(r)dt^2 + \frac{dr^2}{Q(r)} + \frac{r^2 d\theta^2}{P(\theta)} + P(\theta)r^2\sin^2\theta d\phi^2\right],$$

where

$$Q(r) = (1-\alpha^2 r^2)\left(1-\frac{2m}{r}\right), \qquad P(\theta) = 1 + 2\alpha m\cos\theta.$$

α is interpreted as the acceleration of the black hole and we have Schwarzschild solution for $\alpha \to 0$.

Griffiths, Krtouš, and Podolský, *Class.Quant.Grav.* 23, 6745 (2006).

Gravitational lensing

The lens diagram

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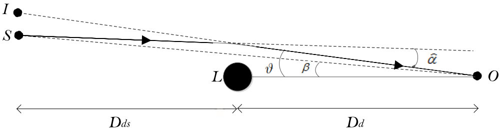


Due to the gravitational lensing by the black hole L, the observer O sees an image I of the source S. The observer cannot see the source itself.

Gravitational lensing

Basic equations

The equations governing the geodesics can be obtained using the Lagrangian

$$\mathcal{L} = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu = \frac{1}{2} \left(-Q\dot{t}^2 + \frac{\dot{r}^2}{Q} + r^2 \dot{\phi}^2 \right).$$

On the equatorial plane $\dot{\theta} = 0$. The constants of motion are

$$E = -\frac{\partial \mathcal{L}}{\partial \dot{t}} = Q\dot{t}, \qquad L_z = -\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = -r^2 \dot{\phi}.$$

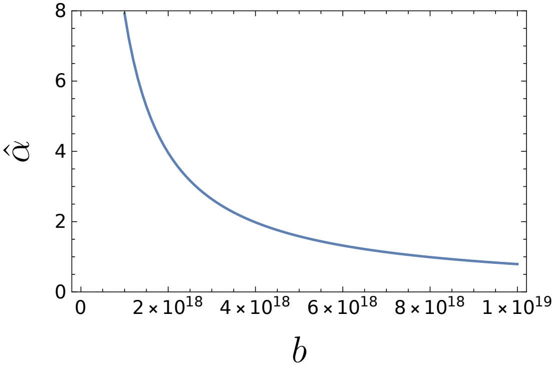
At the point of closest approach to the black hole, $r = b$, we have $\frac{dr}{d\phi} = 0$ and $\frac{dr}{dt} = 0$. The deflection angle would then be found as

$$\hat{\alpha}(b) = 2 \int_b^\infty \frac{dr}{r\sqrt{\left(\frac{r}{b}\right)^2 Q_b - Q}} - \pi.$$

Weinberg, (1972).

Gravitational lensing

Basic equations



The deflection angle $\hat{\alpha}$ is in *arcseconds* and the impact parameter b is in *meters*. We have set $M_{\mathrm{M87}^*} = 9.6 \times 10^{12}\,\mathrm{m}$ and $\alpha = 10^{-25}\,\mathrm{m}^{-1}$.

Gravitational lensing

Basic equations

The time delay is the difference between the time it takes for the light to travel the physical path from the source to the observer and the time it takes to travel the path from the source to the observer when there is no black hole. It can be found by the integral

$$\tau(b) = \left[\int_b^{r_s} dr + \int_b^{D_d} dr\right] \frac{1}{Q\sqrt{1-\left(\frac{b}{r}\right)^2 \frac{Q}{Q_b}}} - D_s \sec\beta .$$

Gravitational lensing

Basic equations

The lens equation is

$$\tan\beta = \tan\vartheta - \mathcal{D}\left[\tan\vartheta + \tan(\hat{\alpha} - \vartheta)\right],$$

where $\mathcal{D} = D_{ds}/D_s$.

Virbhadra and Ellis, *Phys.Rev.D* 62, 084003 (2000).

Also the image magnification is

$$\mu = \left(\frac{\sin\beta}{\sin\vartheta}\frac{d\beta}{d\vartheta}\right)^{-1}.$$

MBJP and Mann, *Phys.Rev.D* 99, 024035 (2019).

Gravitational lensing by M87*

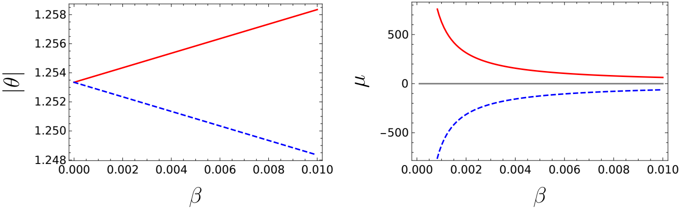
Image position

Table: Image positions θ and magnifications μ of primary and secondary images due to lensing by M87*. These results are the same in the non-accelerating and slowly accelerating cases. (a) p and s refer to primary and secondary images, respectively. (b) All angles are in *arcseconds*. (c) We have used $M_{\mathrm{M87^*}} = 9.6 \times 10^{12}\,\mathrm{m}$, $D_d = 5.2 \times 10^{23}\,\mathrm{m}$, $\mathcal{D} = 0.5$, and $\alpha = 10^{-25}\,\mathrm{m}^{-1}$.

β	θ_p	μ_p	θ_s	μ_s
0	1.25334	$\times$	-1.25334	$\times$
0.1	1.30436	6.79185	-1.20436	-5.78173
0.5	1.52806	1.82812	-1.02805	-0.826954
1	1.84943	1.26694	-0.849483	-0.267388
2	2.60342	1.05575	-0.603448	-0.0567785
3	3.45471	1.01705	-0.454745	-0.0176322
4	4.36033	1.00676	-0.360286	-0.00687431

Gravitational lensing by M87*

Image position



(Left) Angular position of primary images θ_p (red line) and the absolute value of angular position of secondary images $|\theta_s|$ (blue line). (Right) Magnification of primary (red line) and secondary (blue line) images. Angles are in *arcseconds*. These results are nearly the same for non-accelerating and slowly accelerating case.

Gravitational lensing by M87*

Differential time delay

Table: Time delays τ and the differential time delay $t_d = \tau_s - \tau_p$ which is of observational importance. (a) β is in *arcseconds* and the (differential) time delays are in *seconds*. (b) Barred quantities refer to values of the case that the black hole is not accelerating and $\Delta t_d = \bar{t}_d - t_d$. (c) We have taken $\alpha = 10^{-25}\,\mathrm{m}^{-1}$.

β	τ_p	t_d	$\bar{\tau}_p$	Δt_d
0	3.13×10^{12}	0	1.63×10^{6}	0
0.1	3.13×10^{12}	20445	1.62×10^{6}	1.5×10^{-9}
0.5	3.13×10^{12}	102871	1.58×10^{6}	8.4×10^{-9}
1	3.13×10^{12}	209681	1.54×10^{6}	2.2×10^{-8}
2	3.13×10^{12}	448736	1.48×10^{6}	8.9×10^{-8}
3	3.13×10^{12}	737888	1.44×10^{6}	2.8×10^{-7}
4	3.13×10^{12}	1089214	1.41×10^{6}	7.0×10^{-7}

Gravitational lensing by M87*

Differential time delay

Therefore, we conclude that, if in a future observation the images position match the prediction of general relativity, a possible deviation in the differential time delay can be due to the acceleration.

Thank you