

# Curing inflationary degeneracies using Reheating predictions and relic Gravitational Waves

Symposium talk @PASCOS 2021

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**Collaborators:** Prof. Varun Sahni (IUCAA) and  
Prof. Alexei Starobinsky (Landau Institute, Moscow)

18 June, 2021

*“Curing inflationary degeneracies using reheating predictions and relic gravitational waves”,*

**S. S. Mishra**, V. Sahni and A.A Starobinsky,

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[arXiv:2101:00271 [gr-qc]]

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  - ② Spatial flatness
  - ③ Almost scale invariant, nearly Gaussian and adiabatic initial density fluctuations
- **Cosmic Inflation** as the leading paradigm of the early universe

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  - ① Homogeneity and Isotropy on large length scales
  - ② Spatial flatness
  - ③ Almost scale invariant, nearly Gaussian and adiabatic initial density fluctuations
- **Cosmic Inflation** as the leading paradigm of the early universe  $\Rightarrow$  appropriate initial conditions for hot Big Bang phase.

# Inflationary Dynamics of a Scalar Field

Standard scenario – A single canonical scalar field minimally coupled to gravit

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

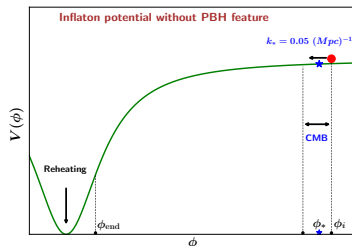
And Einstein's equations imply

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi G}{3}\right) \rho_\phi,$$

$$\dot{H} = -\frac{1}{2m_p^2}\dot{\phi}^2$$

$$\frac{\ddot{a}}{a} = -\left(\frac{4\pi G}{3}\right) (\rho_\phi + 3p_\phi)$$

$$\frac{\ddot{a}}{a} = H^2 (1 - \epsilon_H)$$



Condition for Inflation

$$\epsilon_H = -\frac{\dot{H}}{H^2} < 1 \Rightarrow \dot{\phi}^2 < V(\phi)$$

The dynamics of the scalar field is governed by

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

# Slow-roll Inflation

## Slow-roll Regime of Inflation

Under slow-roll approximation –

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During the slow-roll regime,  $\epsilon_H, \eta_H \ll 1$ ,

$$P_{\mathcal{R}} = \frac{1}{8\pi^2} \left( \frac{H}{m_p} \right)^2 \frac{1}{\epsilon_H} \Big|_{k=aH} = \left( \frac{H}{2\pi} \right)^2 \left( \frac{H}{m_p} \right)^2 \frac{m_p^2}{\dot{\phi}^2}$$

$$P_{\mathcal{T}} = \frac{2}{\pi^2} \left( \frac{H}{m_p} \right)^2$$

# Inflationary Power-spectrum

During slow-roll  $\epsilon_H, \eta_H \ll 1$

Primordial Power-spectra at  
least at large scales –

$$P_{\mathcal{R}} = A_s \left( \frac{k}{k_*} \right)^{n_s - 1}$$

$$P_t = A_t \left( \frac{k}{k_*} \right)^{n_t}$$

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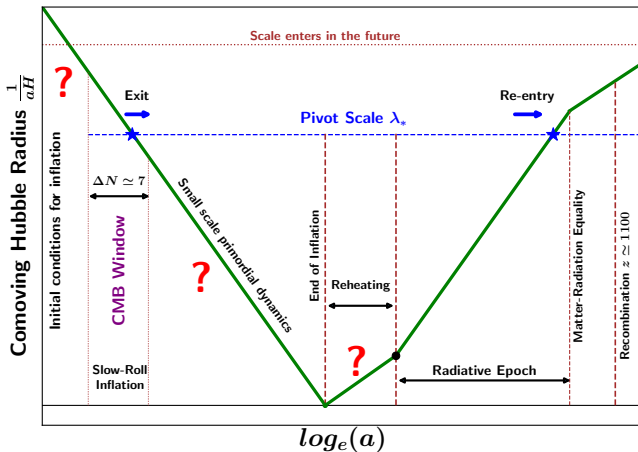
Scalar Spectral Index

$$n_s = 1 + 2\eta_H(k_*) - 4\epsilon_H(k_*)$$

Tensor to Scalar Ratio

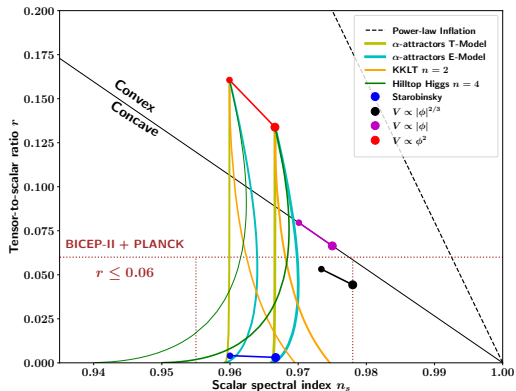
$$r = 16\epsilon_H(k_*)$$

# Behaviour of fluctuations

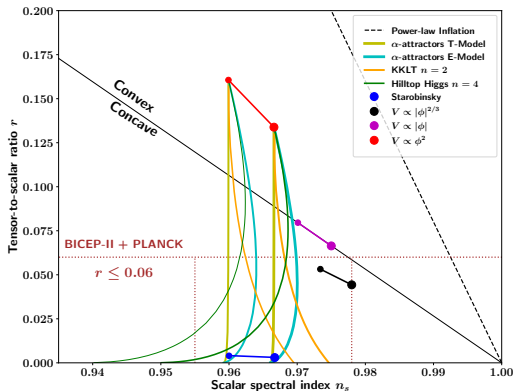


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$r \leq 0.06 \Rightarrow$  Asymptotically flat Concave potentials, nearly scale-invariant but slightly red tilted scalar power spectrum with  $n_s \simeq 0.967$  near the CMB pivot scale.

$$H_k^{\text{inf}} \leq 2.5 \times 10^{-5} m_p = 6.1 \times 10^{13} \text{ GeV}$$

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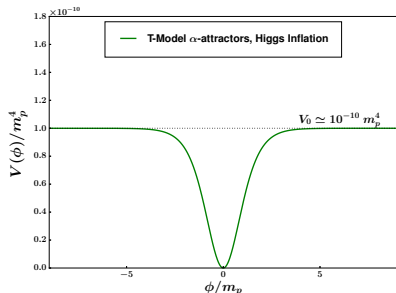
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- Either because several models predict same values of  $\{n_s, r\}$  or for a given model,  $\{n_s, r\}$  insensitive to some model parameter  $\lambda$ .
- $\Rightarrow$  CMB observations can not constitute a unique probe of the physics during inflation.
- Let's see a demonstration of such degeneracies.

# Degeneracy in the T-model

**T-model potential**  $V(\phi) = V_0 \tanh^{2p} (\lambda\phi/m_p)$  ;  $p = 1, 2, 3, \dots$

$$V(\phi) \simeq V_0, \quad |\lambda\phi| \gg m_p$$

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Mean EOS

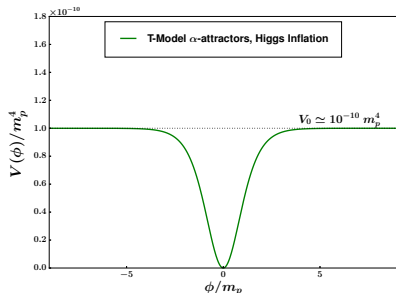
$$\langle w_\phi \rangle = \frac{p-1}{p+1}.$$

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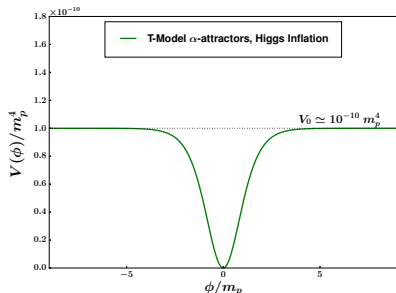
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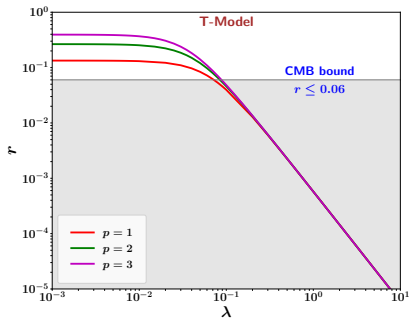
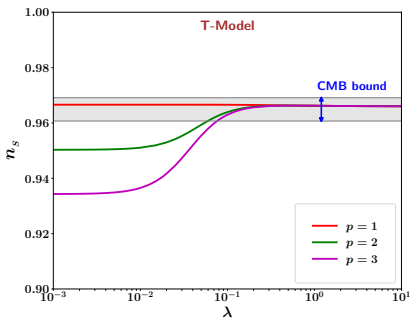
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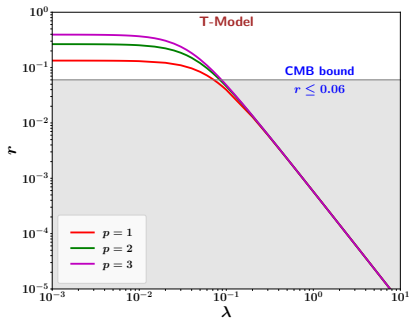
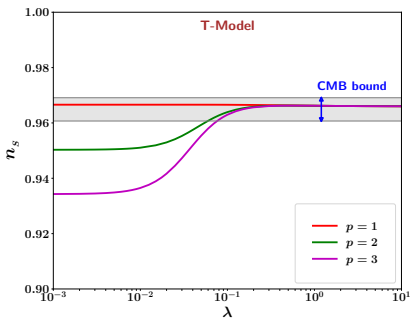
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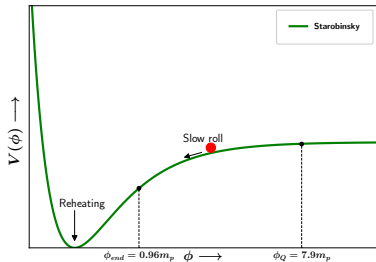
$\{n_s, r\}$  independent of 'p'

# Degeneracy in the E-model

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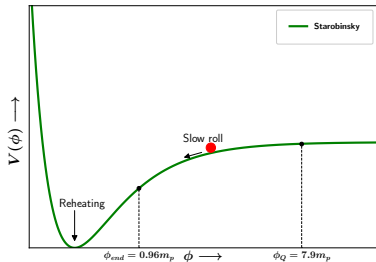
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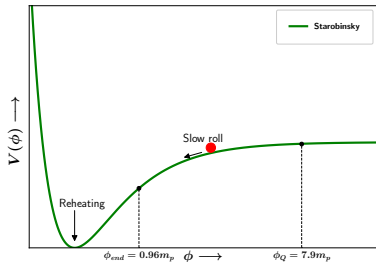
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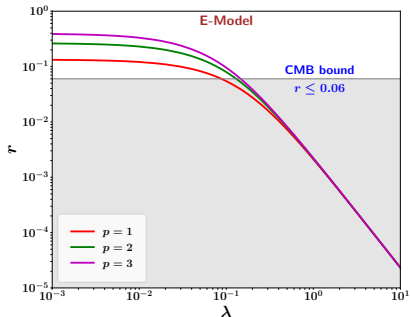
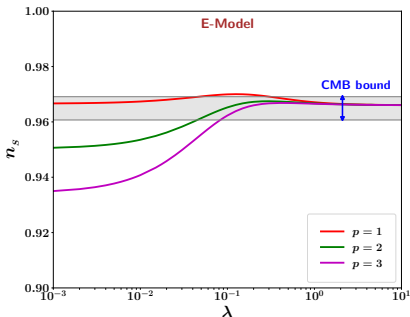
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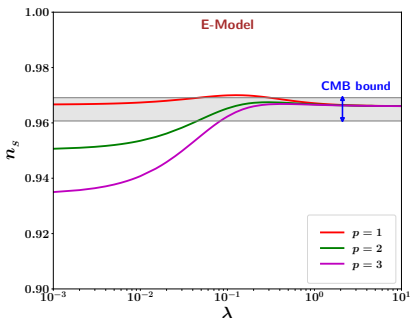
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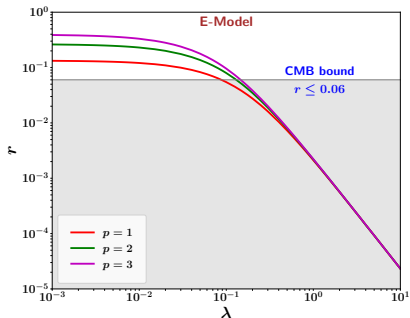


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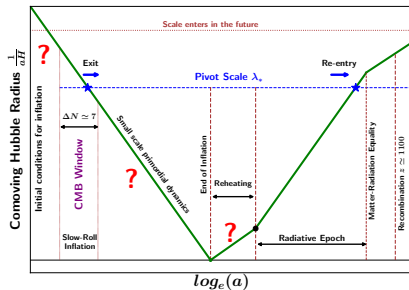
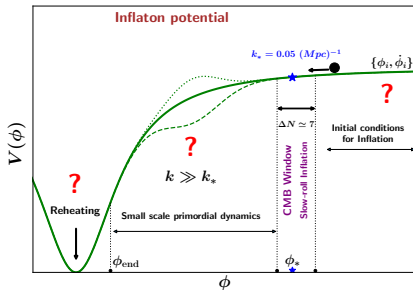
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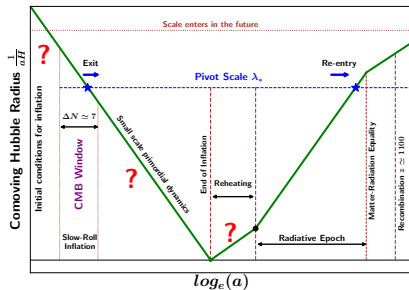
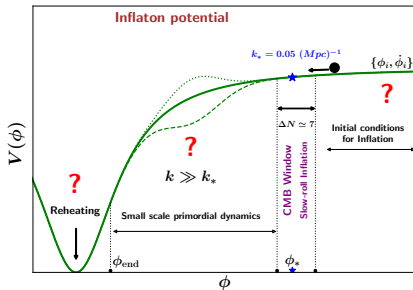
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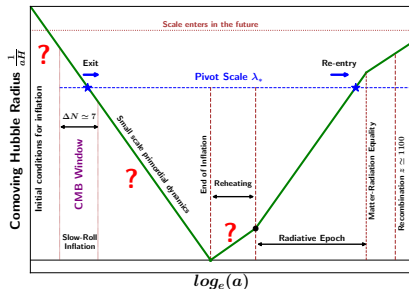
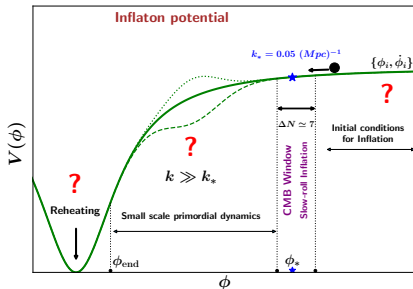


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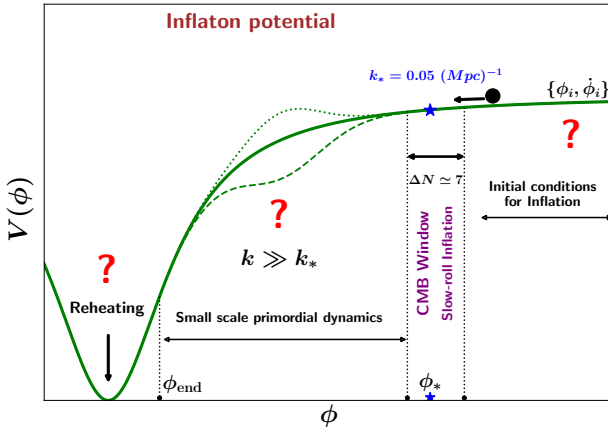
Inflation  $\rightarrow$  Reheating  $\rightarrow$  hot Big Bang

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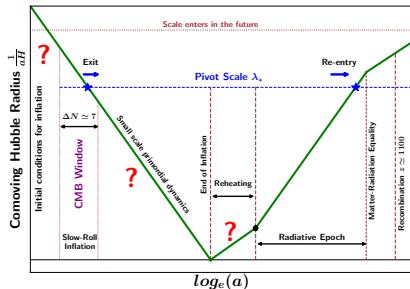
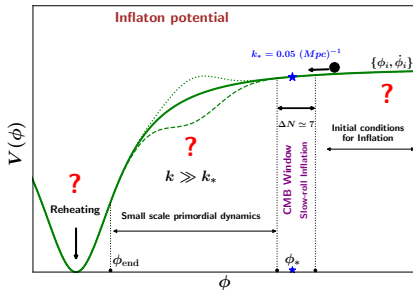
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Two implications

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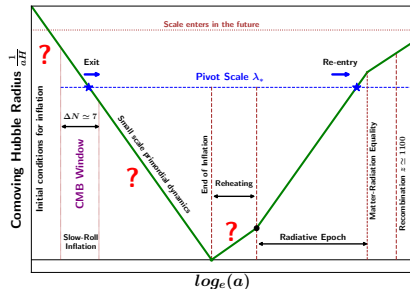
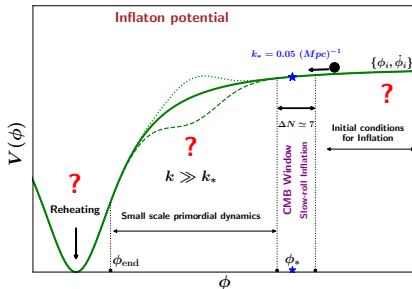


1) Where is  $\phi_*$

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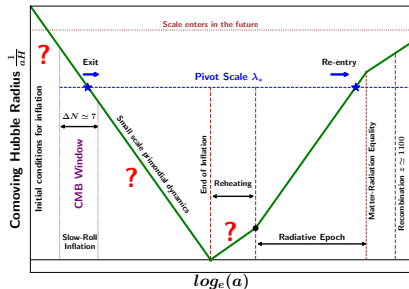
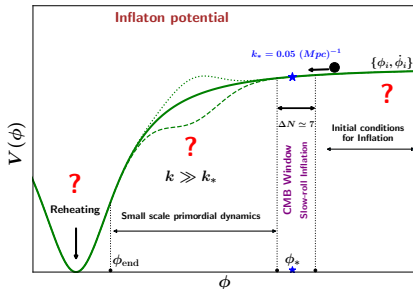


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Decay of inflaton field ultimately to the standard model d.o.f and beginning on 'Hot Big Bang' phase.

$\phi \longrightarrow \chi, \psi, \delta\phi$  etc.

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- **Non-perturbative Reheating**

Parametric Resonance,

Instant Preheating (Non-oscillatory potentials)

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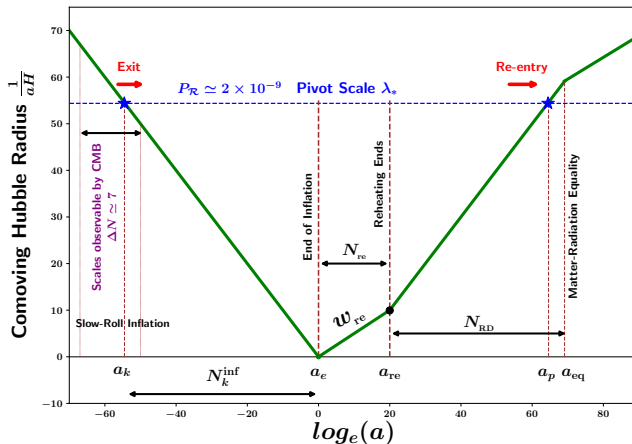
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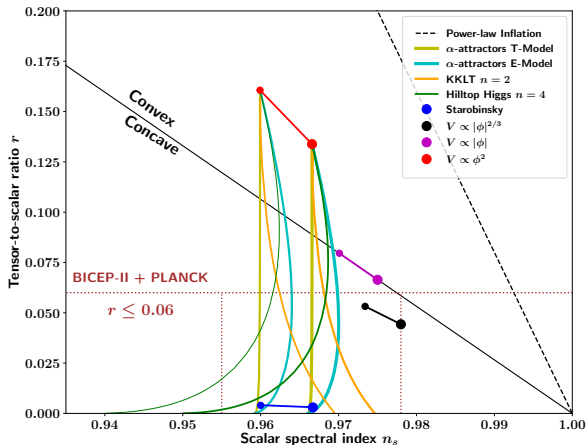
$w_{\text{re}} < 1/3$  if  $p < 2$  (**shallow**)       $w_{\text{re}} > 1/3$  if  $p > 2$  (**stiff**)

# Reheating



Reheating parameters  $\{w_{re}, N_{re}, T_{re}\}$

# The usual 50–60 e-folds



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If  $w_{\text{re}} \neq \frac{1}{3}$ , the **duration of reheating**

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$\Rightarrow$  **sharp predictions for  $\{n_s, r\}$**

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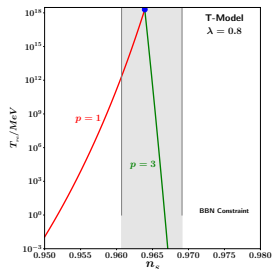
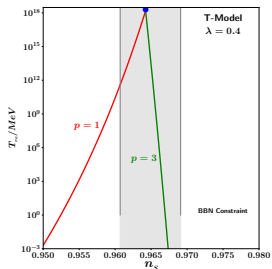
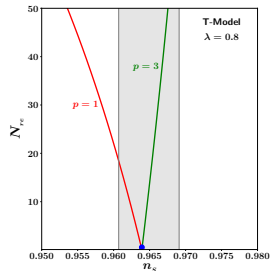
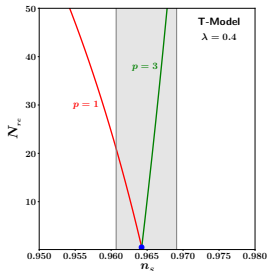
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**Good enough to break inflationary degeneracies**

# Breaking degeneracies in the T-Model



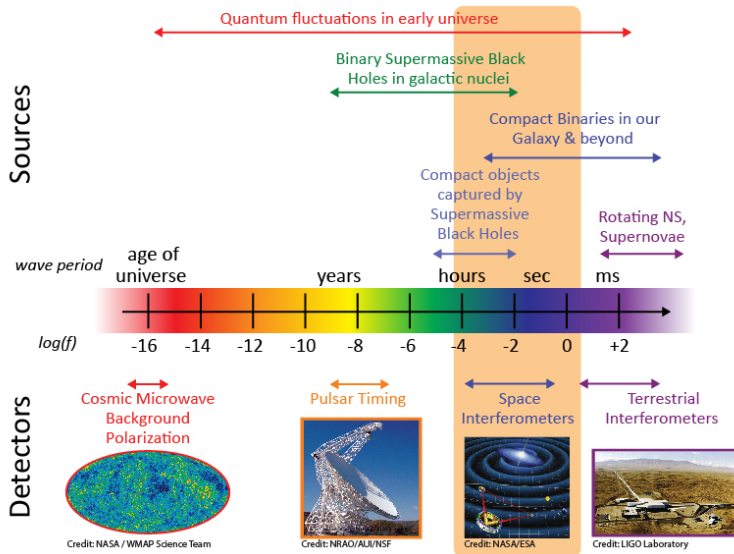
# Segregation of Inflationary Observables

| $\lambda$ | Observables        | $p = 1, w_{\text{re}} = 0$                  | $p = 2, w_{\text{re}} = 1/3$ | $p = 3, w_{\text{re}} = 1/2$        |
|-----------|--------------------|---------------------------------------------|------------------------------|-------------------------------------|
| 0.4       | $N_k^{\text{inf}}$ | [50.4931, 55.686]                           | 55.72                        | [55.724, 60.383]                    |
|           | $n_s$              | [0.9607, 0.9643]                            | 0.9643                       | [0.96435, 0.967]                    |
|           | $r$                | [0.003867, 0.004683]                        | 0.003899                     | [0.003326, 0.003896]                |
|           | $N_{\text{re}}$    | [0, 20.9624]                                | 0                            | [0, 37.5836]                        |
|           | $T_{\text{re}}$    | $[3.9 \times 10^8, 2.4 \times 10^{15}]$ GeV | $2.4 \times 10^{15}$ GeV     | $[10^{-3}, 2.4 \times 10^{15}]$ GeV |
| 0.8       | $N_k^{\text{inf}}$ | [50.6026, 55.17]                            | 55.1749                      | [55.1761, 59.8198]                  |
|           | $n_s$              | [0.9607, 0.9639]                            | 0.9639                       | [0.9639, 0.966706]                  |
|           | $r$                | [0.001009, 0.001198]                        | 0.00101                      | [0.0008604, 0.00101]                |
|           | $N_{\text{re}}$    | [0, 18.4401]                                | 0                            | [0, 37.4696]                        |
|           | $T_{\text{re}}$    | $[2.2 \times 10^9, 2.1 \times 10^{15}]$ GeV | $2.1 \times 10^{15}$ GeV     | $[10^{-3}, 2.1 \times 10^{15}]$ GeV |

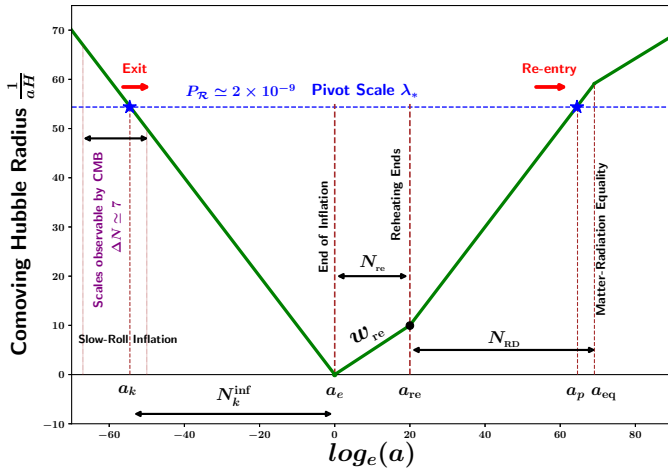
$\Rightarrow$  Degeneracies Broken

# Primordial Gravitational Waves

## The Gravitational Wave Spectrum



## Tensor fluctuations $\rightarrow$ GWs (Physically)



# Present-day Frequency of GWs

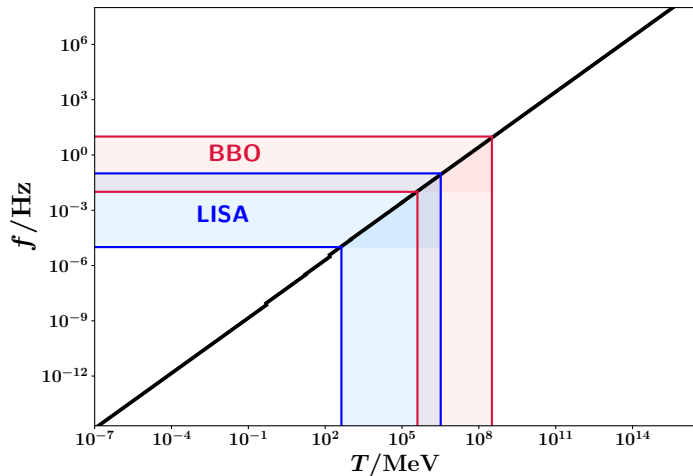
$$f = \frac{1}{2\pi} \left( \frac{k}{a_0} \right) = \frac{1}{2\pi} \left( \frac{a}{a_0} \right) H$$

$$f = 7.36 \times 10^{-8} \text{ Hz} \left( \frac{g_0^s}{g_T^s} \right)^{\frac{1}{3}} \left( \frac{g_T}{90} \right)^{\frac{1}{2}} \left( \frac{T}{\text{GeV}} \right)$$

| Epoch                          | Temperature $T$        | GW $f$ (in Hz)        |
|--------------------------------|------------------------|-----------------------|
| Matter-radiation equality      | $\sim 1 \text{ eV}$    | $1.7 \times 10^{-17}$ |
| CMB pivot scale re-entry       | $\sim 5 \text{ eV}$    | $8.5 \times 10^{-17}$ |
| Big Bang Nucleosynthesis       | $\sim 1 \text{ MeV}$   | $1.8 \times 10^{-11}$ |
| Electro-weak symmetry breaking | $\sim 100 \text{ GeV}$ | $2.7 \times 10^{-6}$  |

Table:

# Present-day Frequency of GWs



# Gravitational Wave Spectrum

Definition:

$$\Omega_{\text{GW}}(f) \equiv \frac{1}{\rho_{0c}} \frac{d\rho_{\text{GW}}^0(f)}{d \log f}$$

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$$n_{\text{GW}}^{\text{PI}} = \frac{d \log \Omega_{\text{GW}}(k)}{d \log k} = \frac{d \log \Omega_{\text{GW}}(f)}{d \log f}$$

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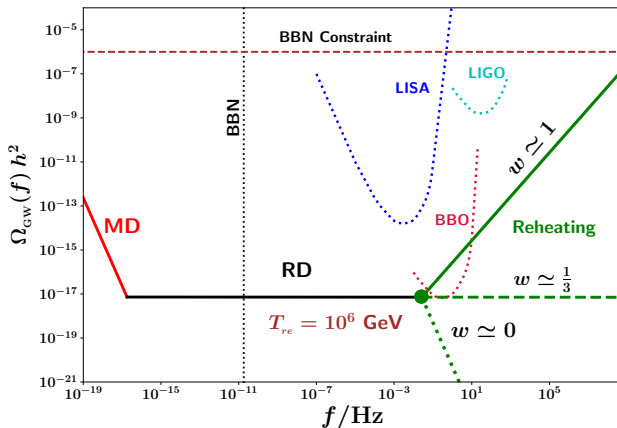
$$n_{\text{GW}}^{\text{PI}} \leq 0 \quad \text{for } w < 1/3 \Rightarrow \textbf{Red Tilt}$$

$$n_{\text{GW}}^{\text{PI}} \simeq 0 \quad \text{for } w = 1/3$$

$$n_{\text{GW}}^{\text{PI}} \geq 0 \quad \text{for } w > 1/3 \Rightarrow \textbf{Blue Tilt}$$

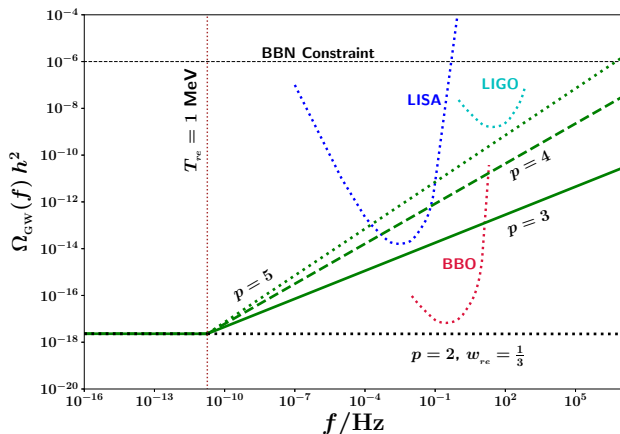
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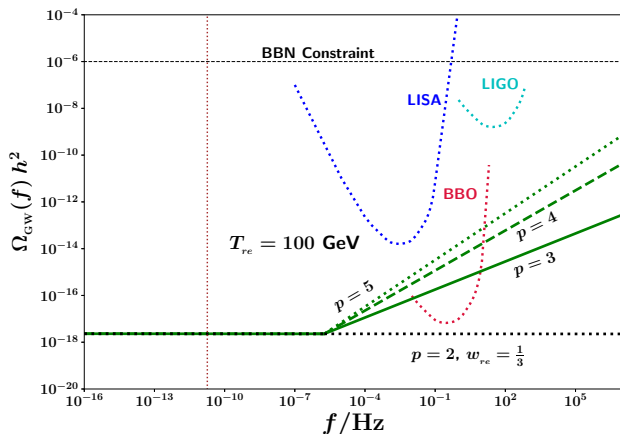
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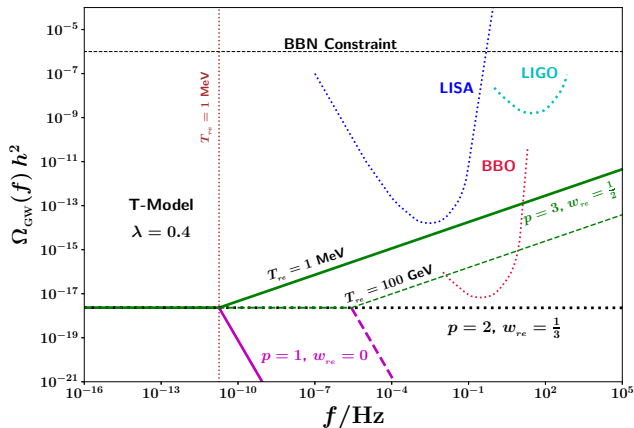


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# GW Spectrum – Breaking Degeneracies



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- Relic Gravitational Waves as the cleanest probe of the Early Universe including the epoch of reheating

# Extra Slides