PASCOS 2021 26th International Symposium on Particles, Strings & Cosmology

Chiral gravitational effect in primordial thermal plasma

June 14th, 2021

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Based on:

K. Kamada, **JK** and Y. Yamada, JHEP05(2021)292

Contents

- Cosmology and quantum anomaly
- Chiral gravitational anomaly
- Chiral Gravitational Effect
- Discussion & Summary

Cosmology and quantum anomaly

- Quantum anomaly

Break down of the classical symmetry due to the quantum effects

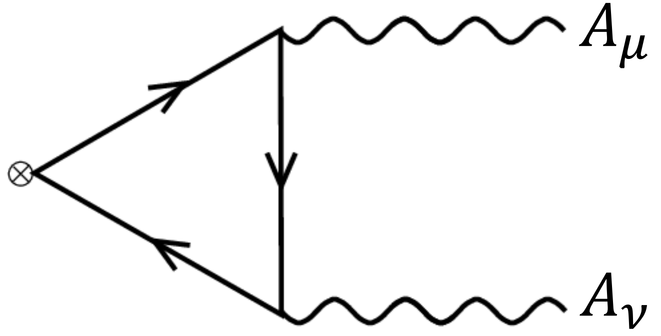
ex.) global chiral U(1) symmetry in massless QED (S. Adler 1969, J Bell & R. Jackiw 1969)

$$\partial_\mu \langle \bar{\psi}\gamma^\mu \gamma_5 \psi \rangle = -\frac{e^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} \equiv -\frac{e^2}{8\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

→**Chiral anomaly**

This anomaly explains experimentally observed non-zero π^0 decay constant: $\Gamma_{\pi^0 \to \gamma\gamma}$

— Any prediction **in cosmology**?



Cosmology and quantum anomaly

Chiral Magnetic Effect (CME) (A. Vilenkin 1980, K. Fukushima et al. 2008)

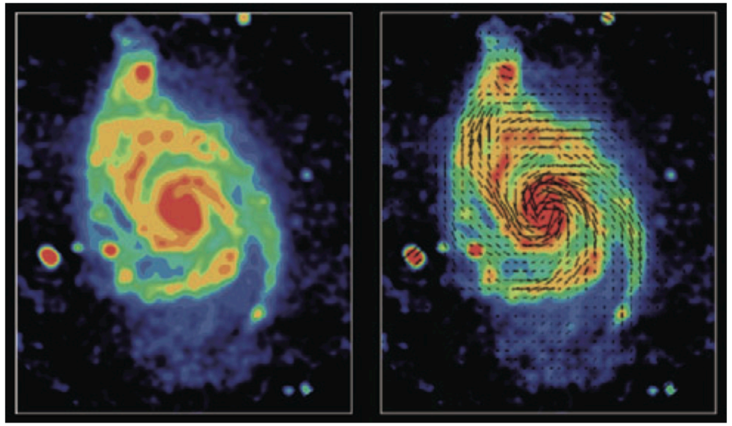
In thermal plasma consists of chiral fermion ($n_5 \sim \mu_5 T^2$, $\mu_5 \ll T$),
magnetic field induces electric current:

$$\vec{J}_{\rm EM} = \frac{e^2}{2\pi^2}\mu_5 \vec{B}$$

Chiral anomaly fixes the coefficient

Anomalous current causes
instability of the large-scale magnetic field.
(Chiral Plasma Instability)
→**Origin of the primordial magnetic fields**
(M. Joyce & M. Shaposhinikov 1997, Y. Akamatsu & N. Yamamoto 2013)

Magnetic fields in galaxy M51



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Cosmology and quantum anomaly

Chiral Magnetic Effect (CME) (A. Vilenkin 1980, K. Fukushima et al. 2008)

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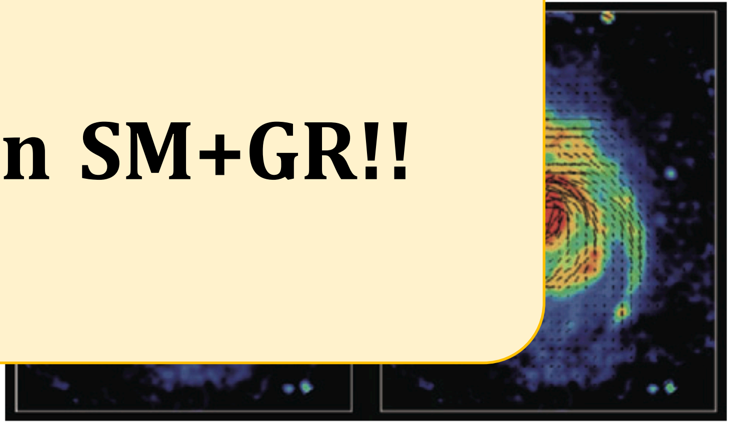
mag

$\vec{J}_{\mathrm{EM}}$

Anomalous effects seem to play some roles in cosmology.

–We have "gravitational" one in SM+GR!!

M51



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→O of the primordial magnetic fields

(M. Joy & M. Shaposhinikov 1997, Y. Akamatsu & N. Yamamoto 2013)

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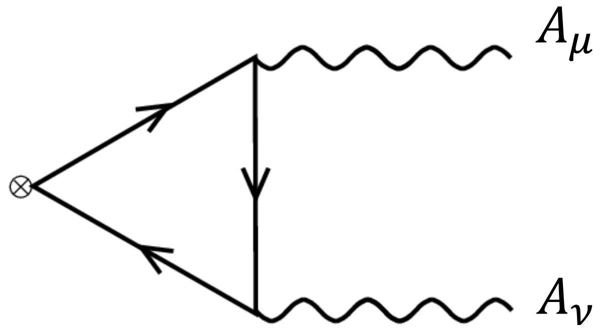
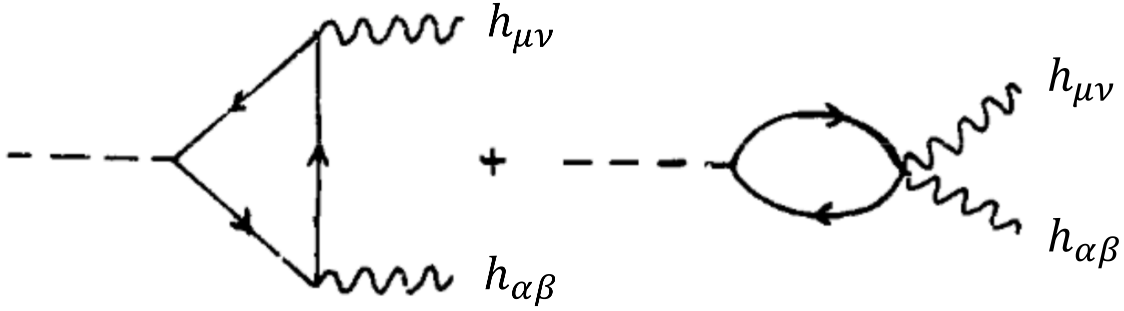
Chiral gravitational anomaly

- **Chiral gravitational anomaly** (R. Delbourgo & A. Salam 1972, L. Alvarez-Gaume & E. Witten 1984)

Fermion loop calculation with **external graviton**:



(R. Delbourgo & A. Salam 1972)



One-fermion loop contributions to the two-graviton mode.

Axial current relates to the gravitational Chern-Pontryagin density:

$$\nabla_\mu j_5^\mu = \frac{1}{384\pi^2}\frac{\epsilon^{\alpha\beta\lambda\rho}}{\sqrt{-g}}R^{\mu\nu}{}_{\alpha\beta}R_{\mu\nu\lambda\rho} \equiv -\frac{1}{12(16\pi^2)}R\tilde{R}$$

Chiral gravitational anomaly

Why "**gravitational**"??

- **Chiral gravitational anomaly** (R. Delbourgo & A. Salam 1972, L. Alvarez-Gaume & E. Witten 1984)

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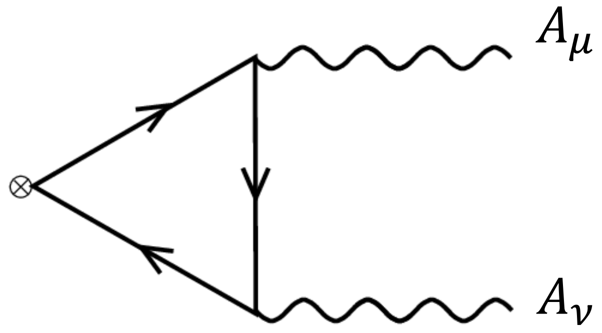
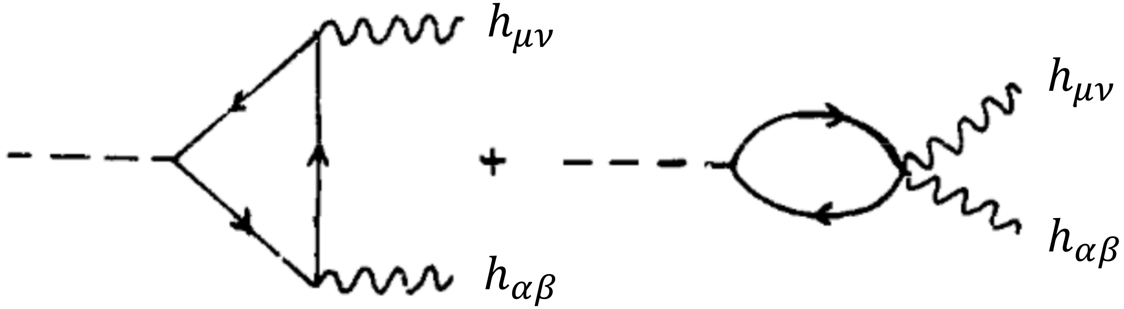


(R. Delbourgo & A. Salam 1972)



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Chiral gravitational anomaly

- **Chiral gravitational anomaly** (R. Delbourgo & A. Salam 1972, L. Alvarez-Gaume & E. Witten 1984)

n_R/n_L **asymmetry**

$$\nabla_\mu j_5^\mu = -\frac{1}{12(16\pi^2)} R\tilde{R}$$

h_k^R/h_k^L **asymmetry**

Chiral gravitational anomaly

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h_k^R/h_k^L **asymmetry**

Chiral fermions

Non-trivial interplay

Cosmological GWs

Primordial plasma may carry the **chirality**!!

- **Inflation**
- **Preheating**
- **Phase transition**
- **Cosmic string** …etc

Chiral gravitational anomaly

Chiral anomaly (in U(1) theory)

$$\partial_\mu \langle \bar{\psi} \gamma^\mu \gamma_5 \psi \rangle = -\frac{e^2}{16\pi^2} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

Chiral gravitational anomaly

$$\nabla_\mu j_5^\mu = \frac{1}{384\pi^2} \frac{\epsilon^{\alpha\beta\lambda\rho}}{\sqrt{-g}} R^{\mu\nu}{}_{\alpha\beta} R_{\mu\nu\lambda\rho}$$

Chiral Magnetic Effect

$$\vec{J}_{\rm EM} = \frac{e^2}{2\pi^2} \mu_5 \vec{B}$$

Chiral Gravitational Effect

？？？

Chiral gravitational anomaly

- **Chiral "Gravitational" Effect** !? (A. Sadofyev & S. Sen 2017)

In static thermal fermion background with chiral asymmetry,

gravitational anomaly leads to the response: (J. Mans & M. Valle 2012, K. Jensen et al. 2012)

$$\delta\langle T^{ij}\rangle = -\frac{\mu_5}{192\pi^2}\epsilon^{ilm}\delta^{jk}\nabla^2\partial_l\tilde{h}_{km} + (i \leftrightarrow j) \quad \Longleftrightarrow \quad \vec{J}_{\mathrm{EM}} = \frac{e^2}{2\pi^2}\mu_5\vec{B}$$

Application: **GWs propagating in primordial chiral plasma**?

But chirality is dynamically violated…

Chiral gravitational anomaly

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Application: **GWs propagating in primordial chiral plasma**?

But chirality is dynamically violated...

Incorporate non-trivial time dependence ? 🤔

–**Effective theoretic viewpoint** provides such an extension.

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Chiral Gravitational Effect -gravitational counterpart of CME-

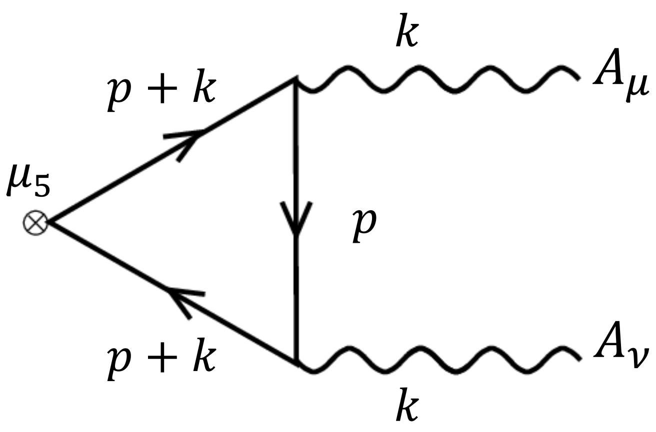
- CME from effective action (A. N. Redlich & L. C. R. Wijewardhana 1985)

$$S_\mu^{\rm eff} = \int d^4x \mu_5 \bar{\psi}\gamma^0\gamma^5\psi$$ μ_5: chiral chemical potential

Integrating out fermions...

$$S_A^{\rm eff} = \mu_5 \frac{e^2}{8\pi^2}\epsilon^{0\nu\rho\sigma}A_\nu F_{\rho\sigma} = -\frac{e^2}{8\pi^2}\theta F\tilde{F}$$

$\partial_\mu\theta(x) = (\mu_5, 0, 0, 0)$ (See e.g. A. Boyarsky et al. 2015)



Induced CS term contributes to the Maxwell eq. as

$$\vec{\nabla}\times\vec{B} - \frac{\partial\vec{E}}{\partial t} = \vec{J} + \frac{e^2}{2\pi^2}\dot{\theta}\vec{B} \simeq \sigma\vec{E} + \underline{\underline{\frac{e^2}{2\pi^2}\mu_5\vec{B}}}$$ → **CME is reproduced!!**

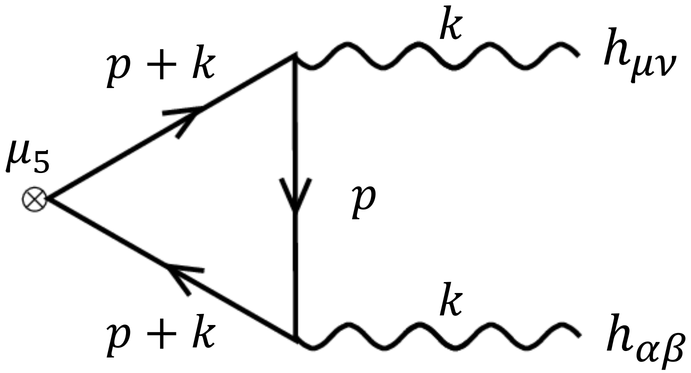
Chiral Gravitational Effect -gravitational counterpart of CME-

- Graviton in the chiral medium (N. D. Barrie & A. Kobakhidze 2017)

Effective action of **graviton** around the Minkowski spacetime:

$$S^{\text{eff}} = \int d^4x (\partial_\mu \theta) 2K^\mu = -\int d^4x \sqrt{-g}\,\theta R\tilde{R}$$

$$\partial_\mu \theta(x) = \left(\frac{\mu_5}{192\pi^2}, 0, 0, 0\right)$$



$$\ddot{h}^j{}_i - \nabla^2 h^j{}_i = -\frac{8}{M_{\text{Pl}}^2}\epsilon^{jmn}\partial_m\left(\dot{\theta}\ddot{h}_{in} - \dot{\theta}\nabla^2 h_{in} + \ddot{\theta}\dot{h}_{in}\right)$$

Induced CS term = **anomalously induced energy momentum tensor**

$$T^j{}_i = \frac{\epsilon^{jmn}}{48\pi^2}\left\{\mu_5 \partial_m(\partial_t^2 - \nabla^2) + \dot{\mu}_5 \partial_m \partial_t\right\} h_{in}$$

$\dot{\mu}_5$ **dependence appears!!**

(K. Kamada, **JK** & Y. Yamada 2021)

Chiral Gravitational Effect -gravitational counterpart of CME-

	Chiral Magnetic Effect	Chiral Gravitational Effect
Anomalous induction	Electric current $\vec{J}_{\mathrm{EM}} = \frac{e^2}{2\pi^2}\mu_5 \vec{B}$	Energy momentum tensor $T^j{}_i \simeq \frac{\epsilon^{jmn}}{48\pi^2}\left\{\mu_5 \partial_m \left(\partial_t^2 - \nabla^2\right) + \underline{\dot{\mu}_5 \partial_m \partial_t}\right\} h_{in}$
Modified dynamics of the field	Instability in large scale B field $\dot{b}_{\mathrm{L/R}}(t) = -\frac{1}{\sigma}\left(k^2 \pm \underline{\frac{e^2}{2\pi^2}\mu_5 k}\right) b_{\mathrm{L/R}}(t)$	?
Backreaction to chiral plasma	Decay of μ_5 →weakening the instability $\frac{d\mu_5}{dt} \simeq -\frac{1}{T^2}\frac{e^2}{4\pi^2}\dot{h}_B$	?

Chiral Gravitational Effect -gravitational counterpart of CME-

- Gravitational birefringence and the memory effect (K. Kamada , **JK** & Y. Yamada 2021)

GWs propagating in z-direction:

$$h_{ij} = \sum_{A=R,L} h_A(t,z) p_{ij}^A \quad \text{with} \quad h_A(t,z) = h_A(t) e^{ikz} + h_{\bar{A}}^*(t) e^{-ikz}$$

$$\Rightarrow \left(\frac{d^2}{dt^2} + k^2\right) h_A(t) = -\frac{\lambda_A k}{24\pi^2 M_{\rm Pl}^2} \left\{ \underline{\underline{\mu_5 \left(\frac{d^2}{dt^2} + k^2\right)}} + \underline{\underline{\dot{\mu}_5 \frac{d}{dt}}} \right\} h_A(t) \qquad \lambda^R = +1,\ \lambda^L = -1$$

Induced EMT causes **Gravitational birefringence**.

But..., **Strong coupling & Ghost issue**!!

We assume $\mu_5 k \ll M_{Pl}^2$ (also $\dot{\mu}_5 \ll M_{Pl}^2$) where CS term is perturbative.

$\rightarrow$ $\dot{\mu}_5 \partial_t$ gives non-trivial correction to GWs!!

Chiral Gravitational Effect -gravitational counterpart of CME-

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Induced EMT causes **Gravitational birefringe**

$$\left(1 - \lambda_A \frac{k\mu_5}{24\pi^2 M_{\rm Pl}^2}\right)\frac{d^2}{dt^2}$$

But..., **Strong coupling & Ghost issue**!!

We assume $\mu_5 k \ll M_{Pl}^2$ (also $\dot{\mu}_5 \ll M_{Pl}^2$) where CS term is perturbative.

→ $\dot{\mu}_5 \partial_t$ gives non-trivial correction to GWs!!

Chiral Gravitational Effect -gravitational counterpart of CME-

- Gravitational birefringence and the memory effect (K. Kamada , **JK** & Y. Yamada 2021)

0th order solution : $h_{\mathrm{L}}^{(0)}(t) = Ae^{-ikt}$

Corrected solution can be evaluated as

$$h_{\mathrm{L}}^{(1)}(t) = A\left(1 + \underline{\underline{\frac{k}{48\pi^2 M_{\mathrm{Pl}}^2}\Delta\mu_5(t)}}\right)e^{-ikt} - \frac{Ak}{48\pi^2 M_{\mathrm{Pl}}^2}\left(\underline{\underline{\int_{-\infty}^{t} dt' \dot{\mu}_5(t') e^{-2ikt'}}}\right)e^{ikt}$$

※For R mode $h_{\mathrm{R}}^{(0)}(t) = Ae^{-ikt}, \quad \mu_5 \to -\mu_5$

$\dot{\mu}_5 \neq 0$ changes the dynamics ← External chirality-changing process.

Chiral Gravitational Effect -gravitational counterpart of CME-

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0th order solution : $h_{\mathrm{L}}^{(0)}(t) = Ae^{-ikt}$

Corrected solution can be evaluated as

non-trivial phase $e^{-2ikt'}$
→ **Memory Effect!!**

$$h_{\mathrm{L}}^{(1)}(t) = A\left(1 + \frac{k}{48\pi^2 M_{\mathrm{Pl}}^2}\Delta\mu_5(t)\right)e^{-ikt} - \frac{Ak}{48\pi^2 M_{\mathrm{Pl}}^2}\left(\int_{-\infty}^{t} dt' \dot{\mu}_5(t') e^{-2ikt'}\right)e^{ikt}$$

※For R mode $h_{\mathrm{R}}^{(0)}(t) = Ae^{-ikt}$, $\mu_5 \to -\mu_5$

$\dot{\mu}_5 \neq 0$ changes the dynamics ← External chirality-changing process.

The **information of chirality violation** is **accumulated on the helicity**

even if $\Delta\mu_5(t) = \mu_5(t) - \mu_5(-\infty) = 0$.

Chiral Gravitational Effect -gravitational counterpart of CME-

- Backreaction from GWs (K. Kamada, **JK** & Y. Yamada 2021)

External chirality-changing process dominates evolution of μ_5.

→Small modulation (or backreaction) is evaluated from the anomaly eq.

$$\mu_5(t) = \mu_5^{(0)}(t) + \mu_5^{(1)}(t)$$

$$\dot{\mu}_5^{(1)}(t) = \delta \left\{ \dot{\mu}_5^{(0)}(t) - 2k \int_0^t dt' \dot{\mu}_5^{(0)}(t') \sin 2k(t-t') \right\} + \mathcal{O}(\delta^2) \qquad \delta \equiv \frac{2C}{\pi^2} A^2 \frac{k^4}{T^2 M_{\mathrm{Pl}}^2}$$

For example, $\dot{\mu}_5^{(0)} = \Gamma \exp(-t/\tau) \quad \text{for} \quad t \geq 0$ (←mimic heavy particle decay)

Chiral Gravitational Effect -gravitational counterpart of CME-

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For example, $\dot{\mu}_5^{(0)} = \Gamma \exp(-t/\tau) \quad \text{for} \quad t \geq 0$

$$\Delta\mu_5(t) = \int_{-\infty}^t dt' (\dot{\mu}_5^{(0)}(t') + \dot{\mu}_5^{(1)}(t'))$$

$$\simeq \Gamma\tau \left(1 - e^{-\frac{t}{\tau}}\right) - \frac{\Gamma\tau\delta}{1+4k^2\tau^2} \left(\underline{\underline{\cos(2kt) + 2k\tau \sin(2kt)}} - e^{-\frac{t}{\tau}}\right) + \mathcal{O}(\delta^2)$$

Small but **non-decaying oscillation**!!
←sourced by **early time memory**.

Chiral Gravitational Effect -gravitational counterpart of CME-

- Backreaction from GWs (K. Kamada, **JK** & Y. Yamada 2021)

For the flipping process:

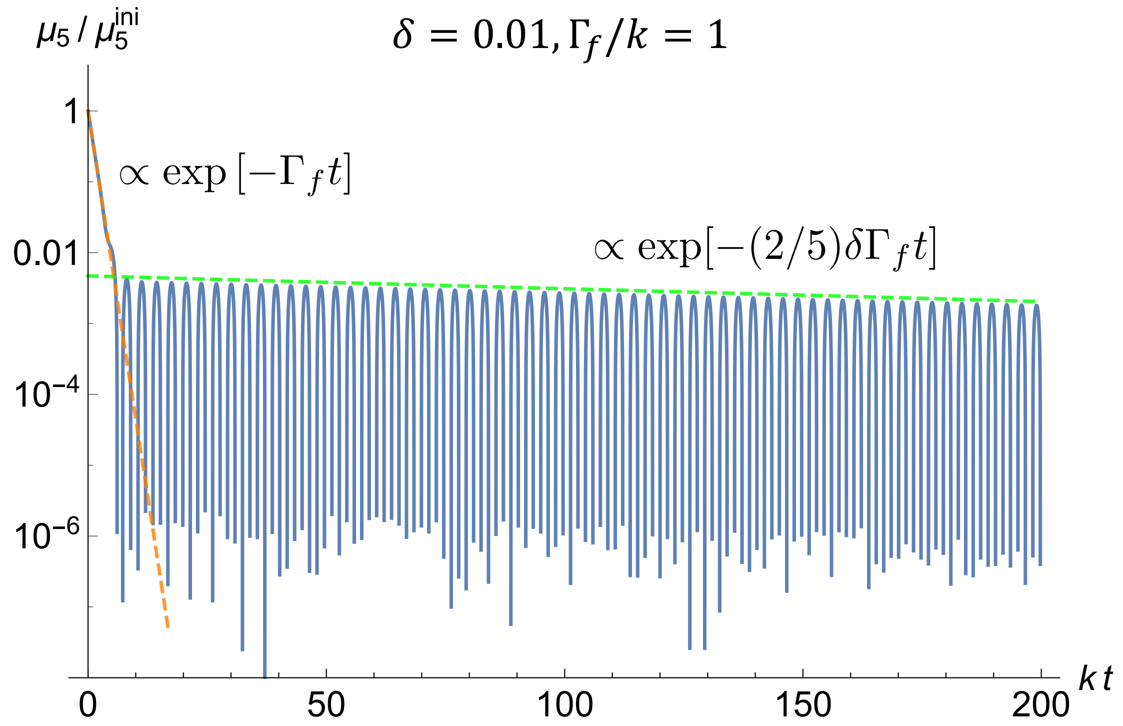
$\dot{\mu}_5 \ni -\Gamma_f \mu_5$

with initial asymmetry μ_5^{ini}

Dominant process $\mu_5 \propto \exp[-\Gamma_f t]$

and slightly damped oscillation with

decay rate $\sim \delta\Gamma_f$.



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Modified dynamics of the field	Instability in large scale B field $$\ddot{b}_{\mathrm{L/R}}(t) = -\frac{1}{\sigma}\left(k^2 \pm \underline{\frac{e^2}{2\pi^2}\mu_5 k}\right) b_{\mathrm{L/R}}(t)$$	Birefringence and Memory effect of GW $$\delta h_{\mathrm{L/R}}(t) = \pm\frac{Ak}{48\pi^2 M_{\mathrm{Pl}}^2}\left\{\Delta\mu_5(t)e^{-ikt} \mp \left(\int_{-\infty}^{t} \underline{dt'\dot{\mu}_5(t')e^{-2ikt'}}\right) e^{ikt}\right\}$$
Backreaction to chiral plasma	Decay of μ_5 →weakening the instability $$\frac{d\mu_5}{dt} \simeq -\frac{1}{T^2}\frac{e^2}{4\pi^2}\dot{h}_B$$	μ_5 oscillates due to the early time memory. $$\dot{\mu}_5^{(1)}(t) \simeq \delta\left\{\dot{\mu}_5^{(0)}(t) - 2k\int_0^t dt'\dot{\mu}_5^{(0)}(t')\sin 2k(t-t')\right\}$$

Chiral Gravitational Effect -gravitational counterpart of CME-

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M		Birefringence and Memory effect of GW $\delta h_{\rm L/R}(t) = \pm \frac{Ak}{48\pi^2 M_{\rm Pl}^2}\left\{\Delta\mu_5(t) e^{-ikt} \mp \left(\int_{-\infty}^{t} \underline{dt' \dot{\mu}_5(t') e^{-2ikt'}}\right) e^{ikt}\right\}$
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This is not the end of story!!

How about **expanding background**!?

Chiral Gravitational Effect -curvature dependence-

- CGE in expanding background (K. Kamada, **JK** & Y. Yamada 2021)

gravitons are **not conformal…!!** → **background curvature dependence**

ex.) diluting chemical potential in RD era: $\mu_5(\eta) \simeq \frac{a_0}{a(\eta)}\left(\mu_5^{(0)} + \mu_5^{(1)}(\eta)\right)$ $\quad a(\eta) = a_0(\eta/\eta_0)$

$$(h_{\mathbf{k}}^A)'' + \frac{2}{\eta}(h_{\mathbf{k}}^A)' + k^2 h_{\mathbf{k}}^A = -\frac{\lambda_{\mathbf{k}}^A k}{24\pi^2 M_{\mathrm{Pl}}^2}\left\{\frac{\mu_5}{a(\eta)}\left(\frac{d^2}{d\eta^2} + \frac{2}{\eta}\frac{d}{d\eta} + k^2\right) + \underline{\underline{\left(\frac{\mu_5}{a(\eta)}\right)'}}\frac{d}{d\eta}\right\} h_{\mathbf{k}}^A$$

$$\Rightarrow \quad h^{L,R}(\eta) \sim \frac{A}{k_0\eta}\left(\sin k_0\eta \pm \frac{k_0^3\eta_0^2\mu_5^{(0)}}{216\pi^2 M_{\mathrm{Pl}}^2 a_0^2}\{3\pi\cos(k_0\eta) + (6\gamma - 5 + 6\log(2k_0\eta_0))\sin(k_0\eta)\}\right)$$

→helicity accumulates the non-zero initial asymmetry.

※backreaction drives oscillation but rapidly decays.

(For MD era, see N. D. Barrie & A. Kobakhidze 2017)

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Discussion

- Non-local contribution (J. Mans & M. Valle 2012, A. Sadofyev &S. Sen 2017)

While we consider local contribution with covariant Lagrangian, non-local ones (e.g. thermal correction) are also found:

dominant for $k \ll T$
→relevant to LIGO etc.

$$T_{11} = -T_{22} = i\xi_T(\omega, q)q_3 \; h_{12}$$
$$T_{12} = -i\xi_T(\omega, q)q_3 \; h_{11} \, .$$

$$\xi_T(\omega, q) = -\frac{1}{96\pi^2}\underline{\underline{\mu_5(\mu_5^2 + \pi^2 T^2)}}\left(2 + \frac{Q^2}{q^2} + \frac{3Q^4}{q^4}L(\omega, q)\right)$$

For such contributions, we may need non-covariant Lagrangian.

As an example, effective action reproducing vortical conductivity:

$$S_{\rm CS}[h] = \int d^4x \, h_{\mu\nu}\left[-\frac{1}{192\pi^2}\epsilon^{\mu\rho\kappa\lambda}b_\kappa\partial_\lambda\left(\Box h_\rho{}^\nu - \partial^\nu\partial^\sigma h_{\rho\sigma}\right) - \frac{T^2}{12}b_0\epsilon^{\mu\rho\kappa\lambda}u_\kappa\partial_\lambda \underline{\underline{\left(\frac{\partial_0\partial^\nu}{\Box} - u^\nu\right)}}\,\underline{\underline{\left(\frac{\partial_0\partial^\sigma}{\Box} - u^\sigma\right)}} h_{\rho\sigma}\right]$$

(J. F. Assuncao et al. 2018)

Summary

- CGE = **Energy momentum tensor induction** in the chiral media

$\dot{\mu}_5$ and background curvature dependence are specific to CGE.

※For $k \ll T$, non-local contribution ($\propto \mu_5(T^2 + \mu_5^2)$) needs to be investigated.

- **Memory of the chiral imbalance** is imprinted on high freq. GWs

Birefringence takes place according to dominant chirality changing process.

Gravity couples to everything → Potential probe for the "**chiral dark sector**".

- Backreaction does not spoil memory while causing small $\boldsymbol{\mu_5}$ **oscillation**

Smallness of backreaction originates from perturbativity of CS term (or consistency).

The late time oscillation caused by the early time memory.