Quantum Gravity Conjectures in String Theory

- 2005.10837 and 2012.00766
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- 2011.00024
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The Swampland Program

Which EFT can be coupled to a fundamental theory of QG?

Swampland		Landscape
EFT consistent as QFT but not as QG	$\longleftrightarrow$	EFT fully consistent as QG

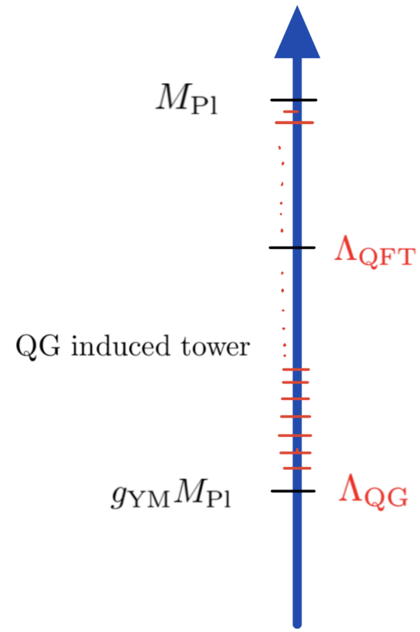
Swampland Conjectures: [initiated by Vafa '05]

Proposals for criteria to distinguish both sides

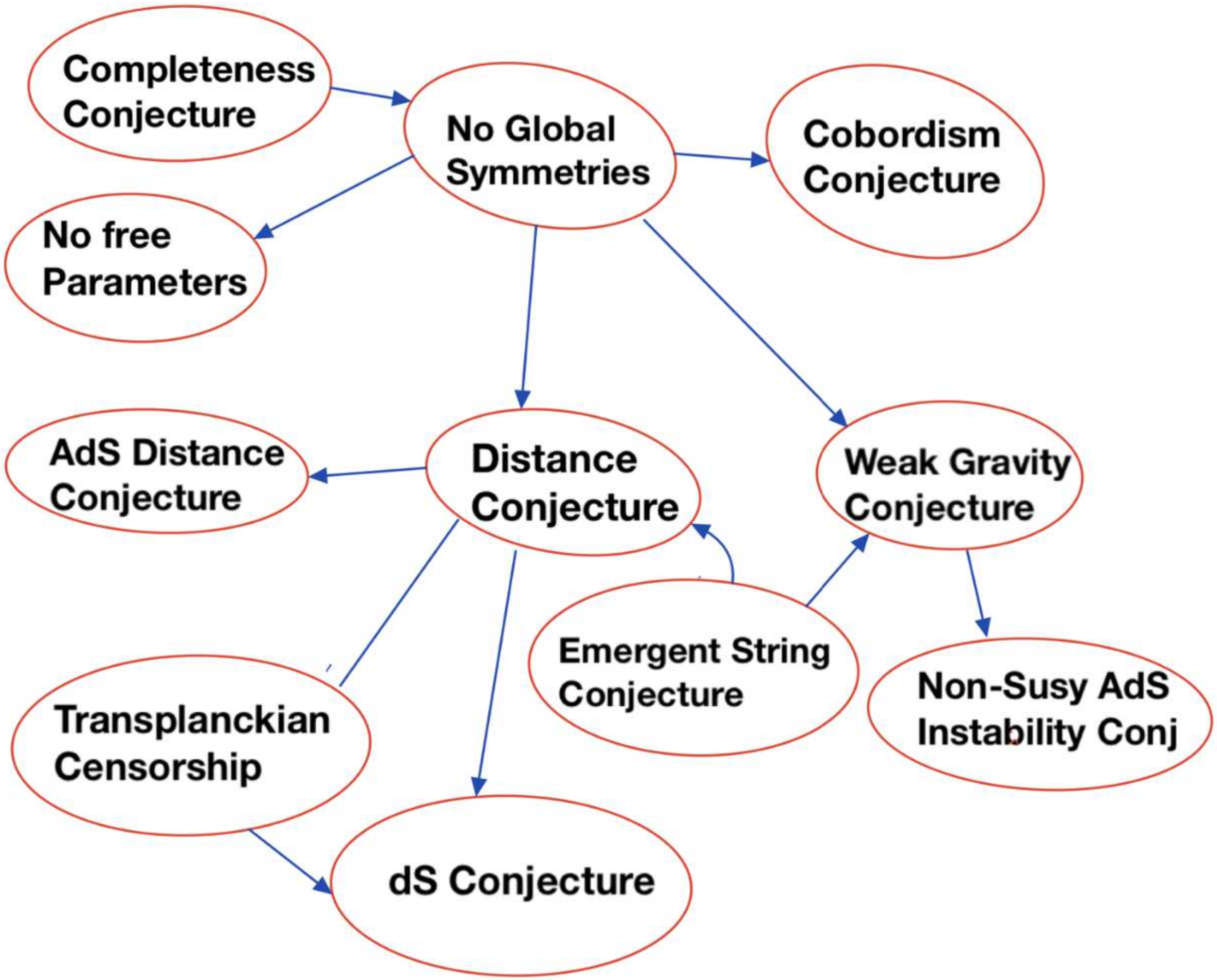
Typical arguments:

- Consistency of additional probe objects enforced by QG constrains the original theory
- QG imposes cutoff for EFT parametrically below naive cutoff $M_{\rm Pl}$

$$\Lambda = g_{\rm YM} M_{\rm Pl} \ll M_{\rm Pl} \quad \text{if} \quad g_{\rm YM} \to 0$$



A Web of Conjectures



Reviews: [Brennon,Carta,Vafa'17] [Palti,'19] [Beest,Calderon,Mirfendereski,Valenzuela'21]

Swampland Distance Conjecture

Swampland Distance Conjecture: [Ooguri,Vafa'06]

Infinite tower of states becomes massless at infinite distance in moduli space:

$$m(\phi) = m_0 \, e^{-c \frac{\Delta\phi}{M_{\rm Pl}}}$$

Refined Swampland Conjecture:

[Klaewer,Palti'16]: $c = \mathcal{O}(1)$

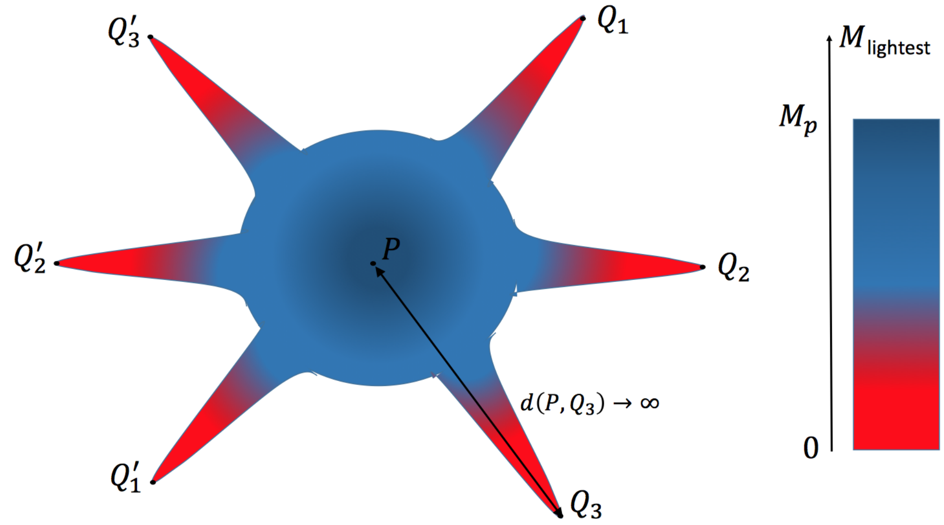


Image: Palti,1903.06239

Numerous proofs in various corners of string landscape, beginning with complex structure moduli of Type II compactifications (4d N=2): [Grimm,Palti,Valenzuela'18] [Grimm,Li,Palti'18] [Klemm,Joshi'19] [Font,Herraez,Ibanez'19] [Blumenhagen,Klaewer,Schlechter,Wolf'18], [Klaewer - to appear]

Swampland Distance Conjecture

What is the origin of the towers of states at infinite distance?

What is physics at the asymptotic boundary?

Compare:

- In QFT, boundaries of moduli space at finite distance lead to new phenomena in physics - e.g. strongly coupled CFT points (Argyres-Douglas type or 6d N=(1,0) SCFTs).
- Are there new phases of quantum gravity at the boundaries at infinite distance?
  Can the infinite distance regime still described by an effective field theory?

$\Longrightarrow$ **Emergent String Conjecture** [Lee,Lerche,TW'19]

String Emergence

Proposal: [Lee,Lerche,TW'19]

String emergence at infinite distance

If a quantum gravity theory admits an infinite distance limit, then

- *either it reduces to a weakly coupled string theory*
  $\Rightarrow$ *infinite tower of string excitations*
- *or it decompactifies*
  $\Rightarrow$ *infinite tower of Kaluza-Klein excitations*

Confirmed in highly-non-trivial (non-perturbative) setups:

Existence and uniqueness of emergent critical string $\Longleftrightarrow$ (Quantum) geometry of string compactification

[Lee,Lerche,TW'18/19/20]: F/M-theory in 6d/5d/4d

[Baume,Marchesano,Wiesner'19]: 4d N=2 hypermultiplets

[Xu'20]: M-theory on G_2

[Lee,Kläwer,TW,Wiesner'20]: 4d N=1+ corrections

Distant Axionic String Conjecture

Independent proposal: [Lanza,Marchesano,Martucci,Valenzuela'20/21]

Distant Axionic String Conjecture

In a 4d N=1 QG theory, any infinite distance limit an be understood as a renormalisation group flow endpoint towards an asymptotically tensionless string.

- Both conjectures are compatible
- Main difference:

  Emergent String Conjecture focuses on the leading tower due to unique, critical string (or KK tower)

  Axionic String Conjecture also refers to strings above the leading tower, which need not be critical strings

This Talk

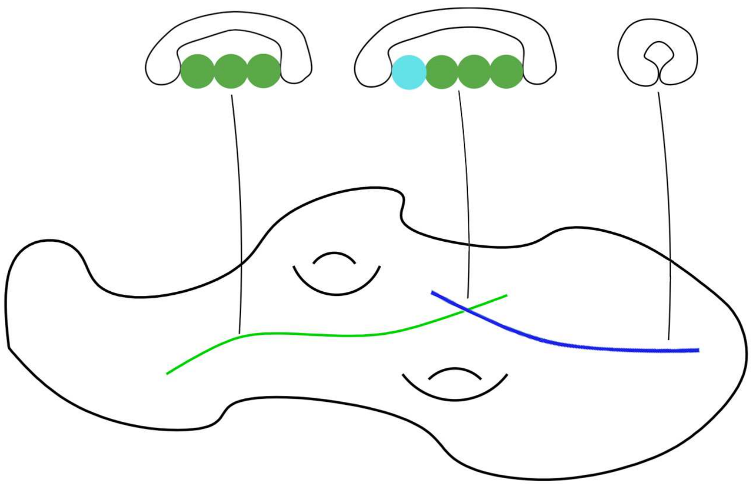
I) Introduction

II) Brief Review:
Emergent String Conjecture in Kähler moduli space of
4d N=1 F-theory

III) Infinite Distance Limits in Brane Moduli Space in F-theory

F-theory in 4d

F-theory in 4d N=1 $\Longleftrightarrow$ Type IIB on $\mathbb{R}^{1,3} \times B_3$ with 7-branes



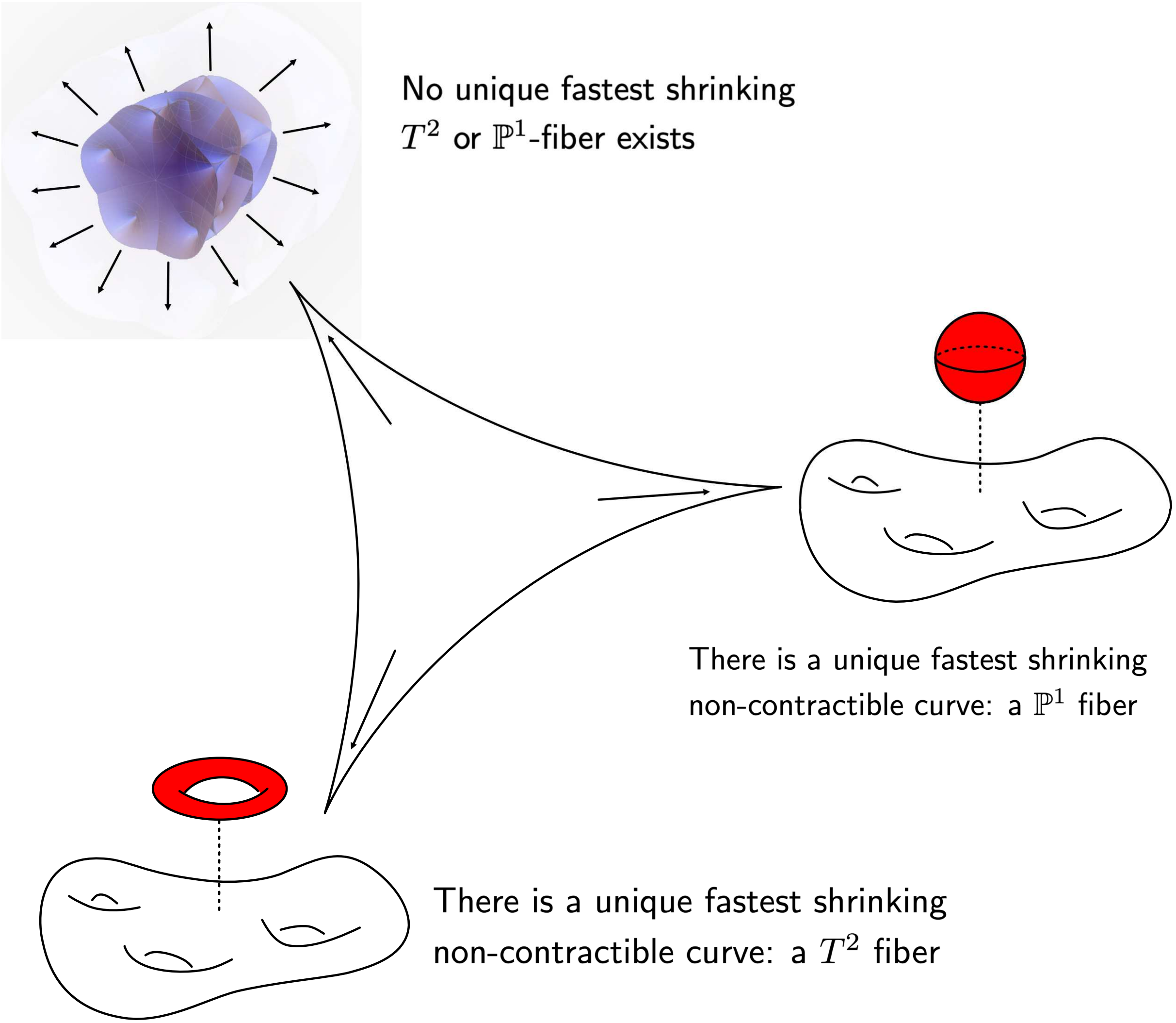
- B_3 = compact Kähler 3-fold
  $\Longrightarrow$ dynamical gravity
- 7-branes on complex surface $\mathbf{S} \subset B_3$
  $\Longrightarrow$ gauge symmetry
- Gauge fluxes on $\mathbf{S}$ required for chiral spectrum

Couplings: (IIB Einstein frame)

$$\frac{\mathbf{M}_{\mathrm{Pl}}^{\mathbf{2}}}{\mathbf{M}_{\mathrm{IIB}}^{\mathbf{2}}} = \mathbf{4}\pi\mathcal{V}_{\mathbf{B_3}} \qquad \frac{\mathbf{1}}{\mathbf{g}_{\mathrm{YM}}^{\mathbf{2}}} = \frac{\mathbf{1}}{\mathbf{2}\pi}\mathcal{V}_{\mathbf{S}}$$

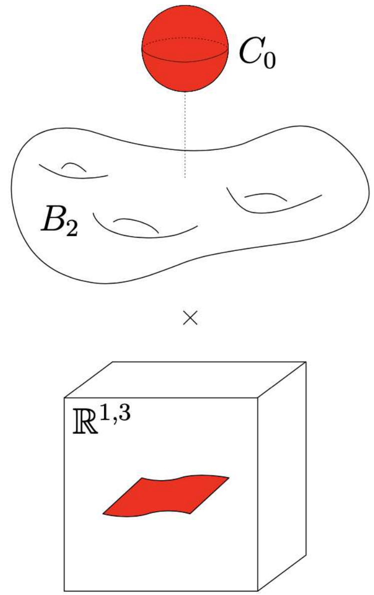
Infinite distance limits - Classical

Detailed classification of limits [Lee,Lerche,TW '19; Klaewer,Lee,TW,Wiesner'20]:



Infinite distance limits

- D3-brane on shrinking $\mathbb{P}^1/T^2$ gives unique asymptotically tensionless heterotic/Type II string
- All other Kähler limits are decompactification limits
- Classical picture survives quantum corrections in 4d N=1



String emergence $\Longrightarrow$ Weak Gravity Conjecture

In gauge theories on 7-brane with weak coupling limit $g_{\rm YM} \to 0$, excitations of asymptotically tensionless heterotic string contain a sublattice of super-extremal states [Lee,Lerche,TW'19/20]

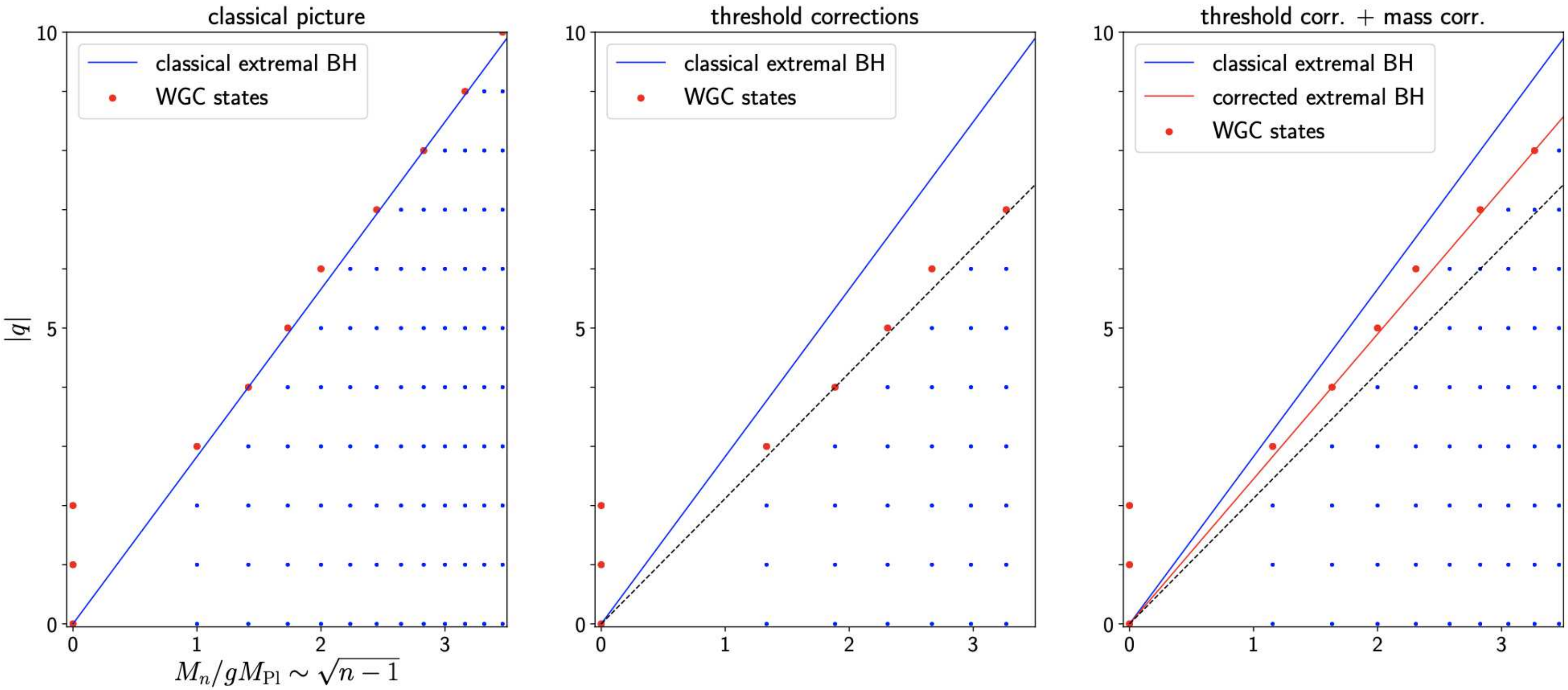
$$\frac{q_k^2 g_{\rm YM}^2}{M_k^2}\Big|_{\rm state} \overset{!}{\geq} \frac{Q^2 g_{\rm YM}^2}{M^2}\Big|_{\rm B.H.}$$

Weak Gravity Conjecture

WGC relation at 1-loop : $$\frac{g_{\mathrm{YM}}^2 q_k^2}{M_k^2} \overset{!}{\geq} \frac{1}{M_{\mathrm{Pl}}^2}\left(1 - \frac{1}{2}\Delta\right)$$

$-\frac{1}{2}\Delta$ is a quantum correction:

- 1-loop corrections to g_{YM}
- mass renormalisation effects on string states [Klaewer,Lee,TW,Wiesner'20]



Infinite distance for brane moduli

Goal: Understand possible infinite distance limits in open string/brane moduli space

Naive expectation: Open string moduli space should be compact

- position of branes on compact internal space
- Coulomb/Higgs branches of gauge theories

Framework for definiteness:

Brane moduli for 7-branes in F-theory compactified to 8d

Main Message:

- Infinite distance directions describe enhancements

$$\text{Lie algebra } G \longrightarrow \text{loop algebra } \hat{G}^{a}$$

- This leads to an effective, dual decompactification limit.

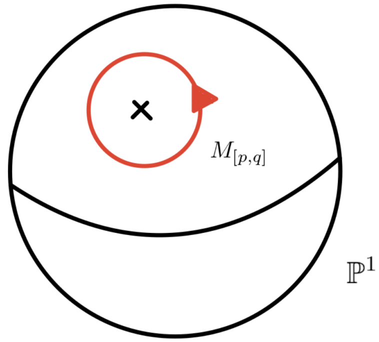
[a] This includes the possibility $G = \emptyset$ and $\hat{G} = \hat{\emptyset}$.

Review: $[p,q]$-7-branes

$\begin{pmatrix} p \\ q \end{pmatrix}$ strings end on $[p,q]$-7-branes with $SL(2,\mathbb{Z})$ monodromy

$$\begin{pmatrix} r \\ s \end{pmatrix} \to M_{[p,q]} \begin{pmatrix} r \\ s \end{pmatrix}$$

$$M_{[p,q]} = \begin{pmatrix} 1+pq & -p^2 \\ q^2 & 1-pq \end{pmatrix}$$



ADE Lie algebras by collision of $[p,q]$-branes of type [Gaberdiel,Zwiebach'97]

$A = X_{(1,0)}, \qquad B = X_{(1,-1)}, \qquad C = X_{(1,1)}$ [DeWolfe,Zwiebach'98]

G	branes	Monodromy M_G
A_{N-1}	A^N	$\begin{pmatrix} 1 & -N-1 \\ 0 & 1 \end{pmatrix}$
D_N	$A^N BC$	$\begin{pmatrix} -1 & -N-4 \\ 0 & -1 \end{pmatrix}$
E_N	$A^{N-1}BCC$	$\begin{pmatrix} -2 & 2N-9 \\ -1 & N-5 \end{pmatrix}$

Affine Enhancements

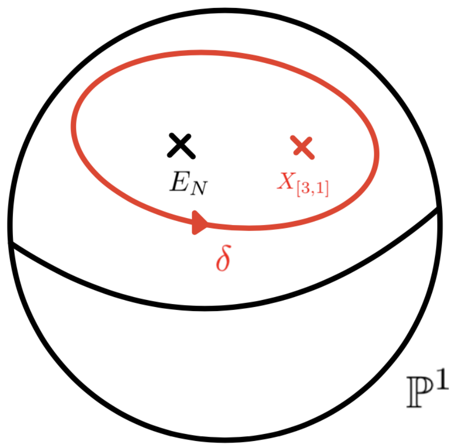
Analysis of monodromies shows: [DeWolfe,Hauer,Iqbal,Zwiebach'98]

$$\text{Affine enhancement} \qquad E_N \xrightarrow{+X_{[3,1]}} \hat{E}_N$$

$$\hat{E}_N = E_N X_{[3,1]} = A^{N-1}BCCX_{[3,1]} = A^{N-1}BCBC\,, \qquad M_{\hat{E}_N} = \begin{pmatrix} 1 & 9-N \\ 0 & 1 \end{pmatrix}$$

- $M_{\hat{E}_N}\delta = \delta\,, \qquad \delta = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

  $\Longrightarrow$ $\delta =$ string encircling $\hat{E}_N$ gives BPS state, massless for coincident E_N and $X_{[3,1]}$



- $\hat{E}_N$ [a] is affine extension of finite Lie algebra E_N
  simple roots: $\{\alpha_i\}_{E_N}$, δ: imaginary root $\delta \cdot \delta = 0$, $\delta \cdot \alpha_i = 0$

[a] E_8 has two equivalent enhancements: 1) $E_8 \to \hat{E}_8 = E_8 X_{[3,1]}$ 2) $E_8 \to E_9 = AE_8$

Loop Enhancements

Rank-one enhancements: [DeWolfe,Hauer,Iqbal,Zwiebach'98]

$\hat{E}_{6,7,8}$,

$\hat{E}_5 = \hat{D}_5,\ \hat{E}_4 = \hat{A}_4,\ \hat{E}_3 = \widehat{A_2 \oplus A_1},\ \hat{E}_2 = \widehat{A_1 \oplus u(1)},\ \hat{E}_1 = \hat{A}_1,\ \hat{E}_0 = \hat{\emptyset}$

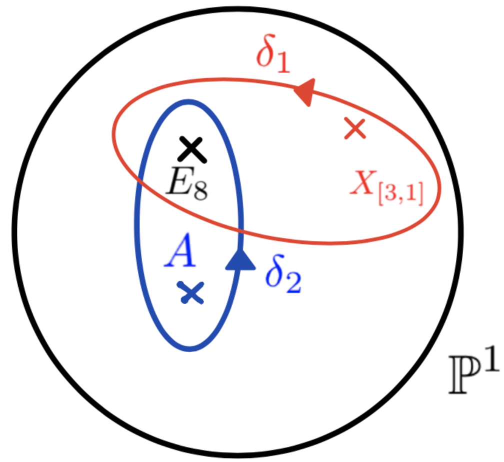
Rank-two enhancements: Monodromy analysis alone admits several options

Relevant for us:

$\hat{E}_8 \longrightarrow \hat{E}_9 = AE_8X_{[3,1]}$:

- loop extension of affine $\hat{E}_8$
- $M_{\hat{E}_9} = 1 \Longrightarrow$ imaginary roots δ_1, δ_2:

$$\delta_i \cdot \delta_j = 0,\ \delta_i \cdot \alpha_{E_8} = 0$$



Monodromy analysis does not address:

1) What's the physics interpretation?

2) Which loop enhancements can occur?

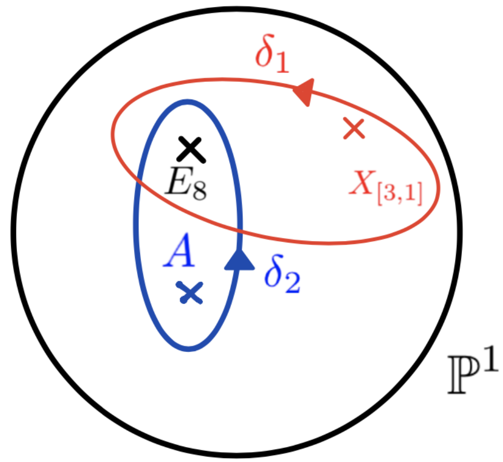
3) How are loop algebras realised on elliptic K3?

Affinisation - Physics

imaginary root $\delta \leftrightarrow$ BPS state from encircling (p, q)-string

winding number $n \in \mathbb{Z}$

$\Longrightarrow$ 1 BPS tower per imaginary root



Interpretation: [Lee,Lerche,TW- to appear]

Affine and loop enhancement signal an infinite distance degeneration in brane moduli space of 8d F-theory

BPS towers $=$ KK modes associated to dual decompactification along S^1s

- affine series $\hat{E}_N$, $N \leq 8$: decompactification 8d $\to$ 9d
- double loop algebra $\hat{E}_9$: full decompactification 8d $\to$ 10d

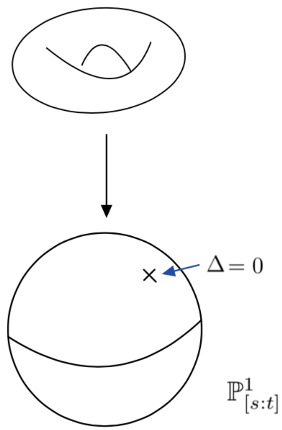
  $\delta_1 \cdot \delta_2 = 0$ signals decompactification on $S^1_1 \times S^1_2$
- interpretation backed up by heterotic dual (see later)

Weierstrass realisations

K3: Elliptic fibration over $\mathbb{P}^1_{[s:t]}$

$$y^2 = x^3 + f_8(s,t)xz^4 + g_{12}(s,t)z^6$$
$$\Delta = 4f^3 + 27g^2$$

7-branes $\leftrightarrow$ $\Delta = 0$



finite enhancements $\longleftrightarrow$ Kodaira-Néron classification

Branes	Algebra	$\mathrm{ord}(f)$	$\mathrm{ord}(g)$	$\mathrm{ord}(\Delta)$
A^{n+1}	A_n	0	0	n
A^nBC	D_n	2	3	n+2
A^5BC^2	E_6	≥ 3	4	8
A^6BC^2	E_7	3	≥ 5	9
A^7BC^2	E_8	≥ 4	5	10
	non-min	≥ 4	≥ 6	≥ 12

Weierstrass realisations

Claim:

[Lee,Lerche,TW- to appear]

- Loop enhancements

  $\leftrightarrow$ non-minimal fibers

Branes	Algebra	ord(f)	ord(g)	ord(Δ)
A^{n+1}	A_n	0	0	n
$A^n BC$	D_n	2	3	n+2
$A^5 BC^2$	E_6	≥ 3	4	8
$A^6 BC^2$	E_7	3	≥ 5	9
$A^7 BC^2$	E_8	≥ 4	5	10
$A^{n-1} BC^2 X_{[3,1]}$	$\hat{E}_n$	≥ 4	≥ 6	≥ 12

- Such enhancements known to require sequence of blowups on the base

  [Aspinwall,Morrison'97] [Aspinwall'98],...

  $\longrightarrow$ Degeneration of K3 into a chain of intersecting components (Kulikov degenerations)

- After blowup: [Lee,Lerche,TW- to appear]

$$\hat{E}_{9-N} \quad \longleftrightarrow \quad (\mathrm{ord}(f), \mathrm{ord}(g), \mathrm{ord}(\Delta)) = (4, 6, 12-N)$$

on dP_9 components away from intersections

$E_7 \times E_8$ Weierstrass model

$$f_8 = t^3 s^4 (a\,t + c\,s)\,, \qquad g_{12} = t^5 s^5 (d\,s^2 + b\,s\,t + e\,t^2)$$

- $[t : s]$ homogeneous coordinates on $\mathbb{P}^1$
- a, b, c, d, e: 5 parameters modulo 2 rescaling symmetries
  $\longrightarrow$ 3 complex dimensional moduli space

Affine enhancements:

1. (4,6,12) at $t = 0$: $c \to 0$ and $d \to 0$
2. (4,6,12) at $s = 0$: $e \to 0$

By rescaling: w.l.o.g. consider 1) in patch $e = 1$
Realise as 1-parameter family $c(u)$, $d(u)$, ...

$$c \sim u^n\,, \quad d \sim u^m \qquad n \geq 1, m \geq 1$$

$l = \min(n, m)$ blowups resolve the non-Kodaira singularity at $t = 0$

$E_7 \times E_8$ Weierstrass model

$$f = t^3 s^4 (a\,t + c\,s)\,, \qquad g = t^5 s^5 (d\,s^2 + b\,s\,t + e\,t^2)$$

$$4a^3 + 27b^2 \sim u^k\,, \quad c \sim u^n\,, \quad d \sim u^m \qquad n \geq 1, m \geq 1$$

- $k = 0$: Type II limit: $\hat{E}_9 \times \hat{E}_9$
  Full decompactification to 10d with non-ab. gauge group $E_8 \times E_8$
- $k \geq 1$: Type III limit:
  decompactification to 9d, with variety of further enhancements

towers from $\hat{G}_1$ and $\hat{G}_2$ are equivalent and identified

Loop algebra	non-ab part in 9d
$\hat{E}_7 \times \hat{E}_8$	$E_7 \times E_8$
$\hat{E}_7 \times \hat{E}_8 \times SU(2)$	$E_7 \times E_8 \times SU(2)$
$\hat{E}_7 \times \hat{E}_8 \times SU(3)$	$E_7 \times E_8 \times SU(3)$
$\hat{E}_8 \times \hat{E}_8$	$E_8 \times E_8$
$\hat{E}_8 \times \hat{E}_8 \times SU(2)$	$E_8 \times E_8 \times SU(2)$

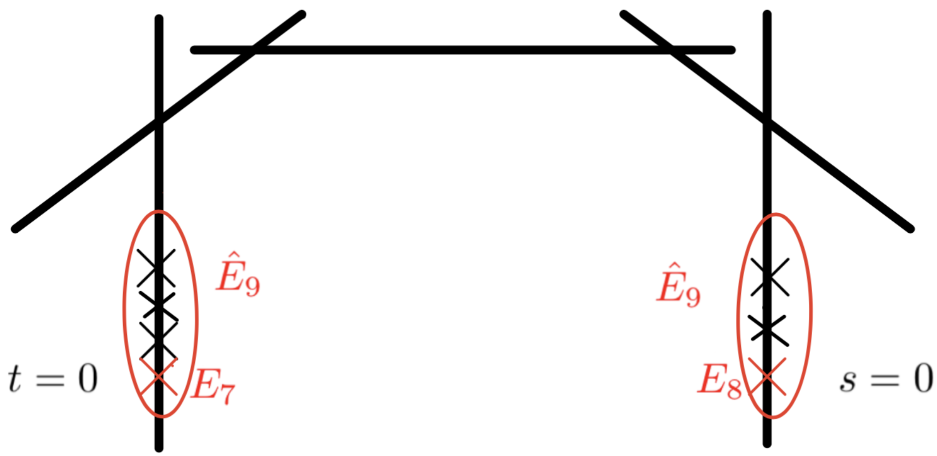
$E_7 \times E_8$ Weierstrass model

$$f_8 = t^3 s^4 (a\,t + c\,s)\,, \qquad g_{12} = t^5 s^5 (d\,s^2 + b\,s\,t + e\,t^2)$$

Example : $\quad 4a^3 + 27b^2 \sim u^k\,, \qquad c \sim u^4\,, \quad d \sim u^4 \qquad u \to 0$

$k = 0$: Kulikov Type II

$K3 \to V_0 \cup V_1 \cup V_2 \cup V_3 \cup V_4$



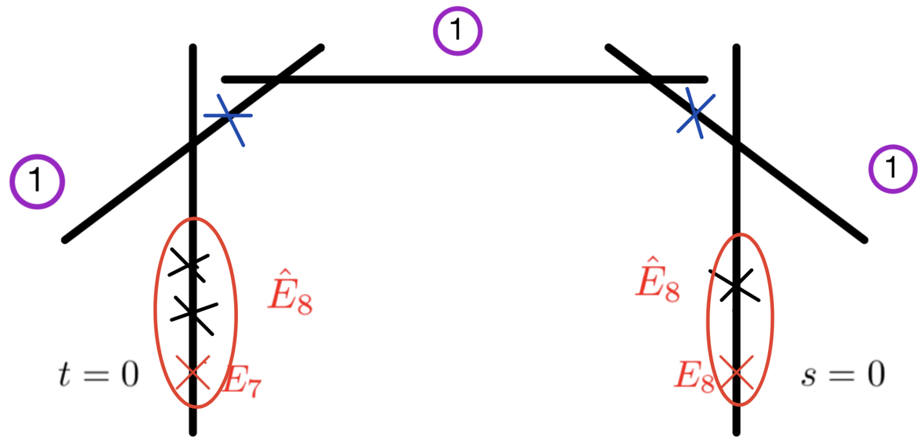
- V_0 and V_4 are dP$_9$, $\qquad V_1,\, V_2,\, V_3 \simeq T^2 \times \mathbb{P}^1, \qquad V_i \cap V_j = T^2$
- V_0 and V_4 contain those branes which coalesce in inf. distance limit $\Longrightarrow \hat{E}_9 \times \hat{E}_9$
- Each $\hat{E}_9$ gives two towers, and towers from V_0 and V_4 are isomorphic $\Longrightarrow$ decompactification 8d $\to$ 10d

$E_7 \times E_8$ Weierstrass model

Example : $\quad 4a^3 + 27b^2 \sim u^k \,, \qquad c \sim u^4 \,, \quad d \sim u^4$

$k = 1$, otherwise generic:

Kulikov Type III



- V_0, V_4: dP$_9$, $\quad V_1, V_2, V_3$: rational fibration over $\mathbb{P}^1$, $\quad V_i \cap V_j = \mathbb{P}^1$
- From V_0, V_4: $\hat{E}_8 \times \hat{E}_8$ in infinite distance limit
- Interpretation:

  1 BPS tower from each $\hat{E}_8$, both identified

  $\Longrightarrow$ decompactification $8d \to 9d$ with $G^{\rm 9d}_{\rm non-ab} = E_8 \times E_8$

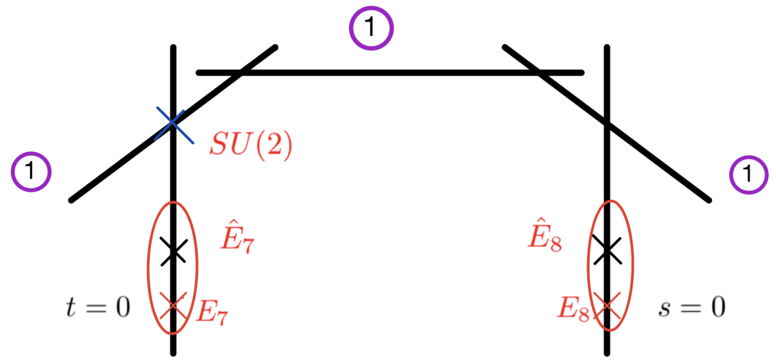
$E_7 \times E_8$ Weierstrass model

Type III degeneration

$4a^3 + 27b^2 \sim u$

Specialisation $c = -i\frac{\sqrt{3}}{\sqrt{a}}d$

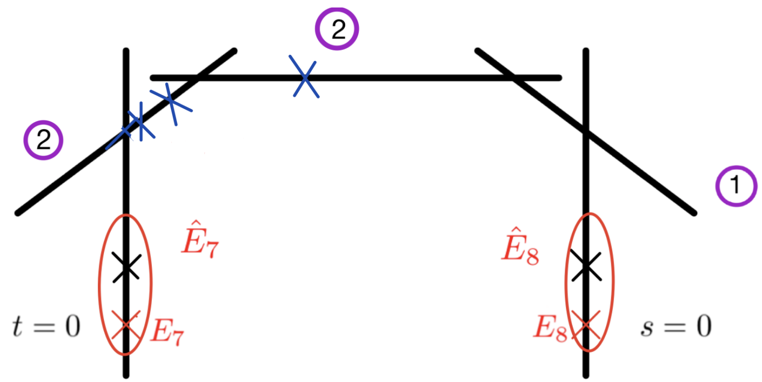
$E_7 \times E_8 \times SU(2)$ in 9d



$4a^3 + 27b^2 \sim u^2$

Specialisation $c = -i\frac{\sqrt{3}}{\sqrt{a}}d$

$E_7 \times E_8$ in 9d



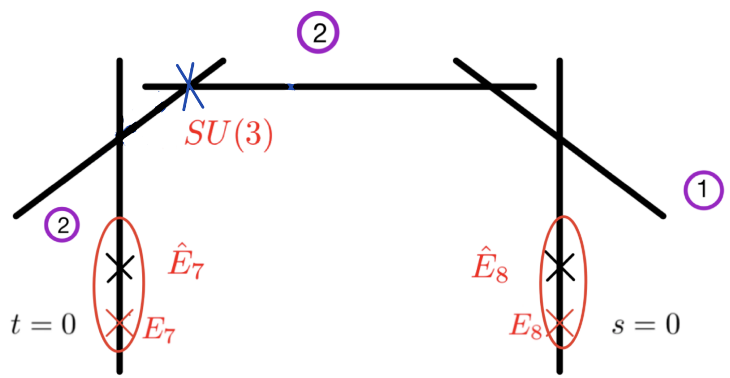
$4a^3 + 27b^2 \sim u^3$

Specialisation $c = -i\frac{\sqrt{3}}{\sqrt{a}}d$

$+$ 1 more tuning

Type III degeneration

$E_7 \times E_8 \times SU(3)$ in 9d



Heterotic dual

Match with dual heterotic on T^2 cf. [Malmendier,Morrison'14] [Jockers,Gu'15]

[Klemm, Poretschkin,Schimannek,Raum'15]

Map to Siegel modular forms [Font,Garcia-E.,Lust,Massai,Mayrhofer'16]

$$a = -\frac{\psi_4(\underline{\tau})}{48}\,, \quad b = -\frac{\psi_6(\underline{\tau})}{864}\,, \quad c = -4\chi_{10}(\underline{\tau})\,, \quad d = \chi_{12}(\underline{\tau})\,, \quad e = 1\,.$$

$\underline{\tau} = \begin{pmatrix} \tau & z \\ z & \rho \end{pmatrix}$ τ: compl. struct. ρ: Kähler mod. , z: Wilson line

$$c \sim \chi_{10} \sim q_\tau q_\rho(-2 + \xi + \frac{1}{\xi}) + \ldots \sim u^4 \quad \to 0$$

$$d \sim \chi_{12} \sim q_\tau q_\rho(10 + \xi + \frac{1}{\xi}) + \ldots \sim u^4 \quad \to 0$$

$$4a^3 + 27b^2 \sim (\psi_4^3 - \psi_6^2) \sim q_\tau + q_\rho + \ldots \sim u^k$$

$$q_\tau = e^{2\pi i\tau}\,, \qquad q_\rho = e^{2\pi i\rho}\,, \qquad \xi = e^{2\pi i z}$$

$k = 0$ (Type II): $\rho \to i\infty$, τ finite: $\longrightarrow$ 10d limit ✓

$k > 0$ (Type III): $\rho \to i\infty$, $\tau \to i\infty$, $\tau/\rho = \mathcal{O}(1)$: $\longrightarrow$ 9d limit ✓

Summary

Found further evidence for Emergent String Conjecture:

Every infinite distance limit in quantum gravity either leads to a tensionless fundamental string or to decompactification.

✓ Previously confirmed in various setups involving Kähler moduli

Swampland Distance Conjecture in 'open string/brane moduli space' in F-theory [Lee,Lerche,TW - to appear]

- Infinite distance directions lead to the affine or loop enhancements $\hat{E}_N$, $N = 0, 1, \ldots, 9$ introduced by [DeWolfe,Hauer,Iqbal,Zwiebach'98]
- Imaginary root $\Longrightarrow$ BPS tower $\Longrightarrow$ dual decompactific'n to 10d or 9d
- Interpretation in agreement with heterotic dual description
- Systematic treatment via blowups of non-Kodaira singularities in codimension one on base - in agreement with 9d enhancements in [Font,Grana,Fraiman,Nunez,Freitas'20]

A BIG THANK YOU TO THE ORGANISERS FOR THEIR HARD AND SUCCESSFUL WORK!

Appendix 1: Kulikov models

Kulikov Type II: $K3 \to V_0 \cup V_1 \cup \ldots \cup V_n$

- V_0, V_n: smooth rational surfaces
- V_1, ..., V_{n-1}: birational to ruled elliptic surface
- $V_i \cap V_{i+1}$: elliptic curve, all of same complex structure

Kulikov Type III: $K3 \to$ multiple components V_i

- Each is V_i a rational surface
- $V_i \cap V_j$ is a rational curve or empty

Claim: [Lee,Lerche,TW- to appear]

1. Enhancement to $\hat{E}_9 \leftrightarrow$ Type II model after blowup
   Full dual decompactification on $S^1 \times S^1$
2. Enhancements to $\hat{E}_n$, $n = 0, 1, \ldots, 8 \leftrightarrow$ Type III models after blowup
   Dual decompactification on S^1
   $\longrightarrow$ rich enhancement structure governing gauge algebras in 9d

Appendix 2: Complex structure

Open string moduli of F-theory contained within complex structure deformations of elliptic fibration

For Type II on general CY_3 and CY_4 studied in detail in

[Grimm,Palti,Valenzuela'18] [Grimm,Li,Palti'18] [Klemm,Joshi'19] [Grimm,Li,Valenzuela'19]

For F-theory on K3, similar reasoning predicts

- Kulikov Type II: 2 towers of BPS states
- Kulikov Type III: 1 tower of BPS states

- in agreement with our findings

For those Type II/III Kulikov degenerations corresponding to open string infinite distance limits in our sense have identified the towers as dual KK towers.